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Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations

JOHN H. COCHRANE*

ABSTRACT

This paper describes a production-based asset pricing model. It is analogous to the standard consumption-based model, but it uses producers and production functions in the place of consumers and utility functions. The model ties stock returns to investment returns (marginal rates of transformation) which are inferred from investment data via a production function. The production-based model is used to examine forecasts of stock returns by business-cycle related variables and the association of stock returns with subsequent economic activity.

THIS PAPER DESCRIBES A production-based asset pricing model. It is analogous to the standard consumption-based model, but it uses producers and production functions in the place of consumers and utility functions. The production-based model is used to explain two links between stock returns and economic fluctuations that have been the focus of much recent empirical research in finance. These are: 1) a number of variables forecast stock returns, including the term premium, the default premium, lagged returns, dividend-price ratios, and investment; and 2) many of the same variables, and stock returns in particular, forecast measures of economic activity such as investment and GNP growth.¹

Since the production-based model is explicitly analogous to the consumption-based model, I start with a review of that model's logic. The consumption-based model ties asset returns to marginal rates of substitution which are inferred from consumption data (or state variables presumed to drive consumption) through a utility function. It is derived from the consumer's first order conditions for optimal intertemporal consumption demand. Its

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¹References include the following. Forecasts of stock returns based on lagged returns: Fama and French (1988a), Lo and MacKinlay (1988), Poterba and Summers (1988); based on other variables: Cochrane and Sbordone (1988), Fama (1990a), Fama and French (1988b); quantity variable forecasts based on term premia: Estrella and Hardouvelis (1991), Harvey (1989), Stock and Watson (1989); based on stock returns: Barro (1990), Fama (1981, 1990b), Fama and Gibbons (1982).

testable content is a restriction on the joint stochastic process of consumption and returns.² This restriction can be interpreted in two ways. If we fix or model the return process and make predictions about consumption behavior, it is a theory of consumption, as in the permanent income hypotheses. If we fix or model the *consumption* process and make predictions about *returns*, it is the consumption-based asset pricing model. For example, the consumption-based asset pricing model might say “expected returns are high because consumption growth is high”.³

The logic of the production-based model is exactly analogous. It ties asset returns to marginal rates of *transformation*, which are inferred from data on investment (and, potentially, output and other production variables) through a *production* function. It is derived from the *producer's* first order conditions for optimal intertemporal *investment* demand. Its testable content is a restriction on the joint stochastic process of *investment* (and/or other production variables) and asset returns. This restriction can also be interpreted in two ways. If we fix the return process, it is a version of the *q* theory of investment. If we fix the investment process, it is a production-based asset pricing model. For example, the production-based asset pricing model can make statements like “expected returns are high because (a function of) *investment* growth is high”.

The model is thus just a statement of the producer's first order conditions.⁴ The central concept is the *investment* return, or the stochastic intertemporal marginal rate of *transformation*. Consider a firm that employs labor and capital to produce a consumption good. Suppose the firm reduces sales of the consumption good at date *t* by one unit, and increases investment. It can then sell extra units of the consumption good at date *t* + 1 while leaving its capital stock and sales plan unchanged for dates *t* + 2, *t* + 3, etc. The *investment return* is the rate at which the firm can transform date *t* consumption goods to date *t* + 1 consumption goods with this operation. Note that the investment return is not risk free: the additional sales at *t* + 1 depend on events at *t* + 1 that are not known at time *t*, including changes in productivity, and investment and labor demand decisions made in response to events at *t* + 1.

²For example, a simple version of the consumption-based model is

$$1 = E_t \left(\rho \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1} \right) = E_t \left(\rho \left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} R_{t+1} \right),$$

where ρ is the subjective discount factor, c_t = consumption, and R_{t+1} is the gross real return on any asset. The second equality imposes the utility function $u'(c) = c^{-\alpha}$.

³Actually, the variables that forecast returns forecast *excess* returns and thus risk premia, so one must find changes in the *conditional covariance* of marginal utility growth with returns, not just the conditional mean of marginal utility growth.

⁴Roughly similar statements of firm's first order conditions are derived in the investment literature, including Abel and Blanchard (1986), Chirinko (1988), Craine (1975), Lucas and Prescott (1981), Sargent (1980), and Shapiro (1986). Several important differences are discussed below.

The producer's first order conditions relate the investment return to asset returns. To derive this relation, assume that firm managers have access to complete financial markets. Then they can trade a portfolio of assets whose payoffs across states of nature at date $t + 1$ mimic exactly those of the investment return. If the price of this mimicking portfolio is greater than 1, the managers should short the portfolio, invest one dollar of the proceeds, pay off the mimicking portfolio with the investment return, and make a sure profit (and vice versa). Firms will continue to adjust their investment and production plans until the investment return equals the mimicking portfolio return. Equivalently, firms remove arbitrage opportunities between asset returns and investment returns.

The producer's first order conditions imply that the investment return and the mimicking portfolio return should be equal ex post, in every state of nature. This is initially a surprising result, since most of the investment literature only studies the weaker relation that the *expected* investment return should equal the *expected* return on some asset. It is a feature of complete markets: if markets are complete, then portfolios exist whose payoffs are proportional ex post to *any* function of state variables.⁵

In the empirical section of this paper, I construct investment returns from investment data and a production function that features adjustment costs of investment. The production function has the useful property that the mimicking portfolio return is the return on the firm's own stock, so the model predicts that the investment return should equal the stock return. Then I run regressions to test whether forecasts of the stock return are equal to forecasts of the investment return and whether stock return forecasts of future activity are equal to investment return forecasts of future activity. To the extent that they are, the production-based model gives a partial equilibrium explanation of the relations between stock returns and economic fluctuations listed in the first paragraph.

One might suspect that the results are sensitive to the assumed production function and parameter choice. However, the investment return calculated with an adjustment cost production function is approximately a monotone function of investment growth. As a result, relations between asset returns and investment *growth* drive the relations between asset returns and investment *returns*, and the results are not sensitive to the particular form of the adjustment cost technology or, as it turns out, to the production function parameters.

The consumption-based asset pricing model is the conventional approach to understanding a link between real activity and expected stock returns. There are several reasons to hope that a production-based model may prove more useful for this purpose. The production-based model ties asset returns di-

⁵The consumption-based model with complete markets also predicts the existence of a return to which marginal utility growth is equal ex post. The return is $q_{t+1} / \rho E(q_{t+1}^2)$ in the notation of equation (4) below, and r^* in the notation of Hansen and Richard (1987). Also, inverse marginal utility growth equals the market return in the log utility CAPM.

rectly to production variables such as output and investment, whose relatively large movements characterize economic fluctuations, rather than to the relatively smooth nondurable and services consumption series. Also, firms are larger than consumers, so the transactions and information costs, lumpiness of goods, etc., that may be responsible for some of the difficulties of consumption-based models may not apply to a production-based model (see Cochrane (1989a) for a description of this view). However, consumption-based and production-based models are not competitors for anything but research and reading time. One does not have to be “wrong” for the other to be “right”. The production-based model will be an interesting complement to the consumption-based model, even if a specification of the latter is found that works perfectly.

Both consumption-based and production-based models are partial equilibrium models. General equilibrium models with nontrivial production sectors have also been used to investigate the link between stock returns and economic fluctuations.⁶ General equilibrium models make more powerful predictions, but those predictions are more sensitive to misspecifications. First, partial equilibrium models don’t explain *why* certain variables forecast consumption or investment growth and hence returns. They just state that *if* a variable forecasts consumption or investment growth then it should also forecast returns in a specific way. But they also don’t *have to* explain why certain variables forecast consumption and investment growth and asset returns. They don’t have to present a structural explanation of the source (production shocks? monetary shocks?, etc.) and nature of economic fluctuations, and a misspecification here need not influence their results. Second, general equilibrium models include the consumption-based model, so they *have to* resolve the specification issues and empirical shortcomings of the consumption-based model, or at least explain how adding a rich production sector to a model whose partial equilibrium implications are easily rejected will result in a greater empirical success.⁷ The restrictions between asset returns and production variables predicted by a production-based model should hold no matter what consumers do, just as the restrictions between asset returns and consumption predicted by a consumption-based model should hold for any technology. Hence a production-based model can simply *ignore* the specification and empirical difficulties of the consumption-based model.

⁶Examples are Balvers, Cosimano, and McDonald (1990), Sharathchandra (1989), and Rouwenhorst (1990). See also Brock (1982). “Nontrivial production sectors” is an important qualification. Partial equilibrium consumption-based models are often called “general equilibrium” following Lucas (1978) by treating the consumption stream as an endowment. Since actual economies have storage and production, empirical applications of these models in fact only exploit a partial equilibrium relation between consumption and asset returns.

⁷One common resolution is to claim that consumption data are poorly measured, so the consumption Euler equation can be ignored. However, many general equilibrium models imply consumption processes so drastically different from observed consumption that this may not be a successful argument.

The production-based model predicts a contemporaneous relationship between asset returns and investment returns. At first glance, this seems contradictory to the results of Fama (1981, 1990b), Fama and Gibbons (1982), and Barro (1990). These papers document that *ex post* stock returns are associated with *subsequent* changes in GNP or cash flows. However, these results do not necessarily contradict the production-based model, since investment is a leading indicator. For example, suppose investors find out that earnings and output will be higher in the future. Then, the stock price rises, leading to a high *ex post* return from last period to this period, which forecasts the rise in earnings and output. But if the stock price rises this period, firms should raise investment *immediately* since the price has risen relative to the cost of capital. Investment growth and hence the investment return from last period to this period will then rise, *at the same time* as the increased stock return. (If there are lags in the investment process, then investment will not rise for a few periods, but orders or investment plans rise immediately. If the production function recognizes such lags, the investment return should still move at the same time as the stock return.)

Section I shows formally that the investment return should equal the mimicking portfolio return, introduces a functional form for technology, and shows that with that technology the mimicking portfolio return is equal to the stock return. Section II presents regressions of investment returns and stock returns on a variety of variables, designed to address the links between stock returns and economic fluctuations outlined in the first paragraph. Section III contains a summary and concluding remarks.

I. Producers' First Order Conditions and Investment Returns

This section presents a derivation of producers' first order conditions in complete markets. It introduces a parametric form for the production technology and shows that with that technology the investment return is equal to the return on the firm's own stock. It then compares the production-based model to the q theory of investment. To simplify the mathematics, the model is presented in a discrete time, one consumption good, nonmonetary economy with a finite number of states, but these features are not essential.

A. Asset Prices and Contingent Claim Prices

The first step is to relate asset prices and implied contingent claim prices. Ingersoll (1988) and Hansen and Richard (1987) derive similar equations, the latter with an infinite-dimensional state space and incomplete markets. They are rederived here to keep the presentation and notation self-contained.

Uncertainty comes from a variable s_t that generates a state tree. The state or cumulative history of shocks at date t is $s^t = \{s_0, s_1, s_2, \dots, s_t\}$. $P(s^t)$ is the time 0 price to a claim to a unit of a single consumption good $c(s^t)$ delivered at date t in state s^t . An asset is a claim to a contingent stream of "dividends" $\{d(s^1), d(s^2), \dots\}$, where the list extends over all dates and

states. The asset's price at time t in state s^t (i.e., with $c(s^t)$ as numeraire) is

$$P^A(s^t) = \sum_{\{s^\tau \text{ that follow } s^t\}} \frac{P(s^\tau)}{P(s^t)} d(s^\tau). \quad (1)$$

Let $p(s^{t+1}) = P(s^{t+1})/P(s^t)$ denote the one period ahead contingent claims price, i.e., the price at time t in state s^t of a unit delivered in a state s^{t+1} that follows s^t , and let

$$R^A(s^{t+1}) = \frac{P^A(s^{t+1}) + d(s^{t+1})}{P^A(s^t)}$$

denote a one period asset return from s^t to a state s^{t+1} that follows s^t . Then, equation (1) implies that

$$1 = \sum_{s_{t+1}} p(s^{t+1}) R^A(s^{t+1}). \quad (2)$$

Equations (1) and (2) are conventionally written in terms of scaled prices Q and q , defined by

$$Q(s^t) = P(s^t)/\rho^t \pi(s^t); \quad q(s^{t+1}) = p(s^{t+1})/\rho \pi(s_{t+1} | s^t).$$

$\pi(s^t)$ is an unconditional probability of state s^t , $\pi(s_{t+1} | s^t)$ is a conditional probability of s_{t+1} given s^t and ρ is a number $\rho < 1$. (Variables π and ρ can be the representative consumer's subjective probabilities and discount factor, but they need not be.) It is also conventional to delete the reference to state in writing random variables, so $Q(s^t)$ (or $Q_t(\omega)$) is commonly written Q_t , etc. With this notation, equation (2) is equivalent to

$$1 = \sum_{s_{t+1}} \rho \pi(s_{t+1} | s^t) q(s^{t+1}) R^A(s^{t+1}) \quad (3)$$

or

$$1 = \rho E_t(q_{t+1} R_{t+1}^A), \quad (4)$$

and equation (1) is equivalent to

$$P_t^A = E_t \left(\sum_{\tau=1}^{\infty} \rho^\tau \frac{Q_{t+\tau}}{Q_t} d_{t+\tau} \right). \quad (5)$$

I use (1) and (2) rather than (4) and (5) to emphasize that probability assessments do not enter the firm's optimization problem.

B. Producers' First Order Conditions

The firm chooses a production plan for sales, investment, output, capital stocks, and labor inputs $\{c(s^t), I(s^t), y(s^t), k(s^t), l(s^t)\}$ (the list extends across

all dates and states) to maximize its time zero contingent claim value,⁸ given an initial stock of capital k_0 and a sequence of contingent claims prices $\{P(s^t)\}$ and wages $\{w(s^t)\}$,

$$\max_{\{\text{all states}\}} \sum P(s^t)(c(s^t) - w(s^t)l(s^t)). \quad (6)$$

The constraints are

$$\text{Production:} \quad y_t = f(k_t, l_t, s_t) \quad (7)$$

$$\text{Resources:} \quad y_t = c_t + I_t \quad (8)$$

$$\text{Capital accumulation:} \quad k_{t+1} = g(k_t, I_t) \quad (9)$$

The capital accumulation function $g(\cdot)$ allows for adjustment costs to investment. Subtracting an adjustment cost from output yields very similar results.

The first order conditions include

$$1 = \sum_{s^{t+1}} p(s^{t+1}) R^I(s^{t+1}) \quad (10)$$

or, equivalently,

$$1 = \rho E_t(q_{t+1} R^I_{t+1}). \quad (11)$$

where R^I is the investment return from state s^t to state s^{t+1} ,

$$R^I(s^{t+1}) = \left(f_k(t+1) + \frac{g_k(t+1)}{g_I(t+1)} \right) g_I(t). \quad (12)$$

(The notation (t) means “evaluated with respect to the appropriate arguments at time t in state s^t ,” and the subscripts denote partial derivatives, e.g., $g_I(t) = \partial g(k_t, I_t) / \partial I_t$.) When there are several technologies (firms), producer’s first order conditions specify equation (10) or (11) for each investment return separately.

To derive (10) or (11), consider marginal changes in investment at time t and at time $t + 1$, arranged so the production plan is unchanged for $t + 2$ and beyond. The marginal cost of increasing I_t by dI_t is a lost unit of sales dI_t . The increased investment gives rise to increased capital $dk_{t+1} = g_I(t) dI_t$. This increased capital gives rise to increased output:

$$dy_{t+1} = f_k(t+1) dk_{t+1} = f_k(t+1) g_I(t) dI_t.$$

Also, I_{t+1} must be simultaneously decreased to hold k_{t+2} unchanged:

$$dk_{t+2} = g_k(t+1) dk_{t+1} + g_I(t+1) dI_{t+1} = 0,$$

so

$$dI_{t+1} = - \frac{g_k(t+1)}{g_I(t+1)} dk_{t+1} = - \frac{g_k(t+1)}{g_I(t+1)} g_I(t) dI_t.$$

⁸As equation (1) is equivalent to (5), (6) is equivalent to an expected discounted present value

$$\max E_0 \sum_{\tau=0}^{\infty} \rho^\tau Q_\tau (c_\tau - w_\tau l_\tau).$$

Both the increased output and decreased investment at $t + 1$ can be sold. These benefits occur in every state s^{t+1} that follows s^t , so marginal cost = marginal benefit is:

$$P_t dI_t = \sum_{s_{t+1}} P_{t+1} \left(f_k(t+1) + \frac{g_k(t+1)}{g_I(t+1)} \right) g_I(t) dI_t.$$

Dividing by $P_t dI_t$ and using the definitions of $p(s^{t+1})$ or $q(s^{t+1})$ yields equations (10) and (11).

There are (at least) three ways to interpret these first order conditions. First, note that equation (10) or (11) has the same form as (2) or (4). Equation (2) or (4) describes a linear space in which all asset returns must lie to prevent arbitrage from portfolio formation. Thus, (10) or (11) states that the firm should adjust investment until no arbitrage possibilities remain by forming portfolios of asset returns and the investment return. Second, since markets are complete, there is always a unique asset return (satisfying (2) or (4)) that is *proportional* to the investment return. Therefore, equation (10) or (11) indicates that the firm should adjust investment until the investment return *equals* this mimicking portfolio return. In both of these interpretations, the idea is that the firm exhausts its possibilities of shorting a portfolio of assets, investing the proceeds, and making risk-free profits. Third, most asset pricing theories can be summarized by their implications for the contingent claims price or benchmark q_{t+1} . Then, equation (2) or (4) gives the model's pricing predictions for all assets, and (10) or (11) says that the firm should adjust investment until the benchmark that prices all other asset payoffs also correctly prices the investment return.

C. A Functional Form For Technology and Investment Returns

The empirical section of this paper uses the following parametric form of the technology.

$$\text{Production:} \quad y_t = mp_t k_t + mpl_t l_t \quad (13)$$

$$\text{Resources:} \quad y_t = c_t + I_t$$

$$\text{Capital accumulation:} \quad k_{t+1} = (1 - \delta) \left[k_t + \left(1 - \frac{\alpha}{2} \left(\frac{I_t}{k_t} \right)^2 \right) I_t \right] \quad (14)$$

where y_t denotes output, k_t capital, l_t labor, and I_t investment. Values mp_t and mpl_t are the marginal products of capital and labor, δ is the depreciation rate, and α is the adjustment cost parameter. As the investment/capital ratio increases, larger fractions of investment are lost, which is the adjustment cost.

The one-period investment return is, from its definition, equation (12):

$$R^I(t \rightarrow t + 1) = (1 - \delta) \left(mp_{t+1} + \frac{1 + \alpha(I_{t+1}/k_{t+1})^3}{1 - (3/2)\alpha(I_{t+1}/k_{t+1})^2} \right) \cdot \left(1 - \frac{3}{2}\alpha \left(\frac{I_t}{k_t} \right)^2 \right). \quad (15)$$

The notation $R(t \rightarrow t + 1)$ distinguishes a quarterly return from an annual return, $R(t \rightarrow t + 4)$.

If the investment/capital ratio at time t is high, the investment return is low, because investment runs into a stiff adjustment cost. (If one thinks of this model as demand for investment, investment is high when returns are low, as expected.) When the investment/capital ratio at time $t + 1$ is high, however, the investment return from t to $t + 1$ is *high* (the squared term in the denominator outweighs the cubed term in the numerator for small investment/capital ratios). In creating a return from t to $t + 1$, the firm *disinvests* at time $t + 1$ to restore its original capital plan. A time when adjustment costs are *high* is a good time to *lower* investment, because the firm can sell a larger quantity of the consumption good for every unit by which it lowers the capital stock.

The investment return has roughly the same sensitivity (partial derivative) to investment/capital ratios at t and at $t + 1$, though with opposite sign. Hence, the investment return is roughly proportional to the change in the investment/capital ratio, or, since capital changes less than investment, to investment growth.

D. The Mimicking Portfolio Return and the Stock Return

With this technology, in equilibrium, the mimicking portfolio return is the return to owning a unit of capital, which we can identify with the stock return.

The firm can transform a marginal unit of the consumption good at t into $g_I(t)$ units of installed capital at $t + 1$ via the investment equation $k_{t+1} = g(I_t, k_t)$. Thus the price at time t of a claim to a unit of time $t + 1$ installed capital must be

$$P_t^{k_{t+1}} = \frac{1}{g_I(I_t, k_t)} = \frac{1}{(1 - \delta)(1 - (3/2)\alpha(I_t/k_t)^2)}. \quad (16)$$

Now, what is the (market) return available from buying some capital and holding it for a period? Buying one unit of capital at time t costs $P_t^{k_{t+1}}$. In return, the buyer gets the marginal product of that capital at period $t + 1$, $f_k(t + 1)$. An extra unit of capital at $t + 1$ depreciates and becomes $g_k(t + 1)$ units of capital at $t + 2$. This may be sold at time $t + 1$ for $P_{t+1}^{k_{t+2}}$. Thus the

return from buying capital and holding it for a period is

$$\text{Return} = \frac{f_k(t+1) + g_k(t+1)P_{t+1}^{k_{t+1}^2}}{P_t^{k_{t+1}}}$$

Substituting $P_t^{k_{t+1}} = 1/g_I(t)$ and $P_{t+1}^{k_{t+1}^2} = 1/g_I(t+1)$ from equation (16), we obtain the investment return (12) again. Thus, in equilibrium, *the investment return and mimicking portfolio return are equal to the return to owning capital for a period*. If we model a firm as a claim to the capital of a single technology or a claim to a constant linear combination of technologies, the investment return is the same as the return on a share of the firm.

E. Comparison with q Theory

The technology and economics are similar to those of the theory of investment demand. For example, equation (16) can be inverted to express investment as a function of the price of capital relative to replacement cost, which is $1/(1 - \delta)$:

$$I_t = k_t \left(\frac{2}{3} \alpha \left(1 - \frac{1}{(1 - \delta) P_t^{k_{t+1}}} \right) \right)^{1/2} \quad (17)$$

The q theory of investment fits poorly in many empirical applications, and one might wonder why reinterpreting it as a production-based asset pricing model will be any more successful. One answer is that the production-based model studied here is expressed as a relation between returns, while most of the investment literature studies present value versions of the model. In theory, the two specifications are the same. One can iterate equations (11) and (12) to obtain an equivalent statement that marginal cost of investment today equals an infinite stream of marginal products of depreciating capital at all future dates (see Abel and Blanchard (1986)). However, one might expect the return formulation to work better in practice for two reasons. First, returns emphasize high frequency aspects of the data that the models may be better able to capture in the presence of slow moving and unobserved changes in technology. The stock *price* may drift from the relation to investment/capital ratios predicted by equation (17), but *returns*, which are dominated by price *changes*, may still be fairly well modeled by investment returns (see Craine (1990)). Second, most present value models (both consumption-based and production-based) exclude time varying risk premia for tractability. They typically relate changes in investment to changes in interest rates, not changes in stock returns. In the context of equation (11), proxies for q_t are constructed that vary only in response to changes in interest rates. Yet the data display evidence of time varying risk premia (forecastability of excess returns). Models expressed as relations between returns can capture firms' responses to time-varying risk premia by relating investment to stock returns rather than interest rates.

II. Empirical Relation between Stock Returns and Investment Returns

To examine the relation between stock returns and economic activity with a very simple specification of the production-based model, I assume that the CRSP value weighted NYSE portfolio is a claim to the capital stock corresponding to gross fixed private domestic investment.⁹ Lacking explicit data on the productivity shock, I assume it is constant.¹⁰ The production-based model then predicts that the real value-weighted return and the return calculated from this investment series should be the same:

$$R^{VW}(t-1 \rightarrow t) = R^I(I_{t-1}/k_{t-1}, I_t/k_t). \quad (18)$$

The empirical work exploits the following implication of (18): if ex post stock returns and investment returns are equal, then the coefficients in regressions of stock returns and investment returns on *any* set of variables should also be equal. Equivalently, the coefficients in regressions of the difference between stock and investment returns on any set of variables should be zero.

I use three sets of variables. First, I regress investment and stock returns from $t-1$ to t on variables dated $t-2$ or earlier that forecast stock returns, to see if forecasts of stock returns are equal to forecasts of investment returns. Second, I regress the returns on variables dated $t+1$ and later, to see if stock and investment returns have the same association with subsequent activity. I also run these regressions backwards, so they can be interpreted as return forecasts of subsequent activity. Third, I regress the returns on investment/capital ratios from $t-8$ to $t+8$, to see if the strong relation between investment returns and investment/capital ratios is also found in stock returns.

The structure of the model suggests that regressions on investment/capital ratios are an important diagnostic. (There are not quite a test, as explained below.) By construction, the investment return from $t-1$ to t is strongly

⁹For simplicity and to keep the results comparable to the return forecasting literature, I ignored the bond portion of claims to firms in the NYSE. This should not have much effect on the results. First, bond returns are much less volatile than stock returns, so adding the bond portion of the claims should just change the mean and/or standard deviation of the investment return while not changing its correlation with other variables greatly. The mean and variance of the investment return are essentially free parameters picked by the production function parameters. Second, much investment is financed from retained earnings, so marginal investment may be fully reflected in stock returns alone.

¹⁰It is possible in principle to infer productivity shocks, since output is observed. (Preference shocks are not similarly measurable since utility is not observed.) For example, in the model $mp_t = (y_t - mpl_t l_t)/k_t$, y_t may be measured as output, and $mpl_t l_t$ may be measured as the wage bill. However, note from equation (16) that the terms in the investment/capital ratio measure changes in prices. To the extent that price changes are more important than dividend changes, leaving out changes in marginal products has a small effect on the results that may not warrant the additional measurement difficulties. Also, a constant productivity shock isolates firms' responses to other events (movements along a curve), rather than changes in investment returns due merely to good or bad luck. It is interesting that the variables that forecast returns forecast changing investment decisions and not just changes in productivity.

related to investment/capital ratios at $t - 1$ and t , and only to these variables. For example, another variable can only forecast the investment return if it forecasts investment/capital ratios at $t - 1$ and t . Hence, it is natural to check whether stock returns have the same strong relation to investment/capital ratios. For example, if the basic model is wrong, we might see no relation of stock returns to investment/capital ratios or a relation that is not at all similar to that of the investment return. If the basic model is right but the assumed production function is wrong, the regression of the stock return on investment/capital ratios will measure the gradient of the true investment return function with respect to investment/capital ratios and thus suggest modifications to the production function that might improve the model.

The empirical work does not include formal statistical tests of the model. Equation (18), taken literally, predicts that the investment and stock return should be equal *at every data point*. Since there is no choice of parameters for which the stock and investment return are exactly equal, data point by data point, the model, taken literally, is formally rejected at any level of significance.

Of course, one would not want to take equation (18) literally, since it contains obvious specification and measurement errors. The value-weighted portfolio is not exactly a claim to the capital stock corresponding to gross fixed private investment. This investment series includes investment by firms not listed on the NYSE and residential investment which may be inadequately represented in the value-weighted portfolio, and the bond portion of claims to the firms on the NYSE is not included. Also, I make no adjustment for taxes, no correction for measurement error in the investment series, I don't try to measure the productivity shock, let alone account for error in its measurement, and I make a crude adjustment for time aggregation (described below). While one can make some progress on all these issues, I doubt that a specification can be achieved in which one can reasonably expect *zero* measurement and specification error.

With *no* knowledge about the errors, the model can *never* be rejected, at any level of significance. This is just as valid a statement as the statement that the model taken literally is *always* rejected. To conduct a formal test of equation (18), one must add statistical assumptions about the errors. For example, the regressions described above could be defended as formal tests of the model by assuming that the error is uncorrelated with the right hand variables. But measurement, and especially specification, errors, unlike forecast errors, do not obey any useful statistical properties, so I doubt that this is a productive strategy.¹¹

¹¹The consumption-based model suffers from the same problems: unobserved preference shocks, components of consumption that enter nonseparably in the utility function (for example, the service flow from durables), and measurement error all contribute to the error term, and there is no reason to expect these errors to obey the orthogonality restrictions that the forecast error obeys. Empirical work on consumption-based models focuses on the forecast error since it has so many useful properties, but the importance in practice of these other sources of error may be part of the reason for its empirical difficulties.

In summary, the model predicts that the investment and stock returns are exactly equal. Taken literally, this is a silly proposition, but there is no formal way to test propositions like “stock returns are *approximately* equal to investment returns”. For this reason, it seems more appropriate at this stage of the research to see if a very simple specification of the model based on easily available broad aggregates can provide a useful description of the links between returns and fluctuations through the regressions described above, rather than to concentrate on formal testing. (In a similar spirit, Sargent (1989) and Watson (1990) present useful models that contain singularities, and hence are formally rejected, along with several suggestions for evaluating them.)

A. Construction of Investment Returns

For each choice of parameters, I constructed a capital stock series by accumulating past investment, using the capital accumulation rule, equation (14). (The procedure and choice of initial value are detailed in the Appendix.) Then, I calculated quarterly returns (from $t - 1$ to t) from investment/capital ratios at $t - 1$ and t according to equation (15) and overlapping quarterly observations of annual investment returns from $t - 4$ to t by accumulating the quarterly returns.

Investment is a quarterly aggregate, but stock returns are point to point. As a crude adjustment for this difference, I shifted the stock returns so that they go from approximately the center of the initial quarter to the center of the final quarter. This dating convention is illustrated in Figure 1. Other variables have conventional dating: returns dated t used as forecasting variables are from the beginning to the end of quarter t , real variables dated t are aggregates for quarter t .

Three parameters govern the relation between investment returns and investment/capital ratios: the adjustment cost parameter α , depreciation δ , and the productivity of capital mp . From equation (15) one can see that the depreciation δ and marginal product mp together just raise or lower the investment return by a constant and thus determine the mean investment return. The adjustment cost parameter α controls the sensitivity of the investment return to investment/capital ratios at t and $t + 1$, and thus it controls the standard deviation of the investment return. But α has very

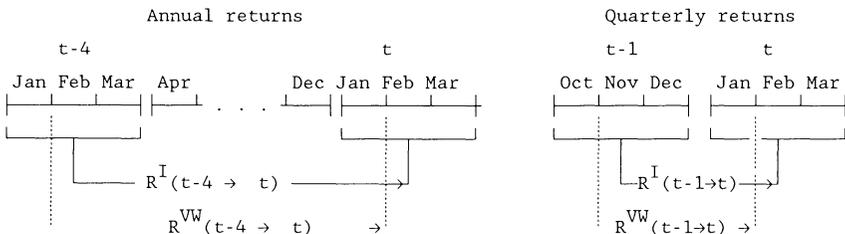


Figure 1. Dating convention for real value weighted returns (R^{VW}) and investment returns (R^I).

little effect on the *relative* sensitivity of the investment return to the investment/capital ratios at different dates or on the *ratio* of the partial derivatives of the investment return from t to $t + 1$ with respect to $I/k(t)$ and $I/k(t + 1)$. Therefore, the parameters have almost no effect on the *correlation* of the investment return with investment/capital ratios and with other variables.

Given that the parameters $\{\alpha, \delta, \text{ and } mp\}$ control the mean and standard deviation of investment returns but have little impact on its timing or correlation with other variables, I chose the parameters of the investment return as follows: 1) I set depreciation δ arbitrarily to 0.1; 2) I chose the marginal product mp and the adjustment cost parameter α to equate mean investment and stock returns and to equate the standard deviation of the fitted values of a regression of the stock return on eight leads and lags of the investment/capital ratio to the standard deviation of the fitted value of the same regression for the investment return. (Since the coefficients on long lags of the investment/capital ratio are small, the results are similar using different numbers of leads and lags.) The resulting parameters are:

$$\begin{array}{lll} \text{Quarterly returns:} & \delta = 0.10 & \alpha = 13.04 & mp = 0.15 \\ \text{Annual returns:} & \delta = 0.10 & \alpha = 13.22 & mp = 0.16. \end{array}$$

The reason for this choice of standard deviation is that the regression of the stock return on investment/capital ratios leaves a larger residual (lower R^2) than the corresponding investment return regression. This choice of the investment return standard deviation is designed to produce a series of about the same standard deviation as the investment return component of stock returns. Since most of the results are driven by the *correlation* of investment and stock returns, this scaling is not crucial to the results. The constructed investment return is also quite insensitive to the arbitrary choice of δ , so long as mp is simultaneously adjusted to match the mean investment and stock return. Also, the correlation of the investment return with other variables is quite insensitive to any of the parameters, α in particular.

A puzzle of the q theory is that adjustment cost estimates often seem implausibly high. They imply that very large fractions of GNP (often greater than 1) are lost to adjustment costs. This is analogous to the puzzle that large coefficients of risk aversion seem to be required in the consumption-based model. With the technology of equations (13) and (14), the fraction of investment lost to adjustment costs is $(\alpha/2)(I/k)^2$. α is around 13, I/k is about the same as depreciation (0.1), so the fraction of investment lost to adjustment costs is about 7%. The fraction of output lost is $I/y \times 7\%$, or around 1%. Thus the puzzle of implausibly high adjustment costs is not present in these parameters.

Table I presents means, standard deviations and autocorrelations of investment/capital ratios, investment returns, and value-weighted returns. The important thing to notice in this table is that the investment/capital ratio is highly autocorrelated. This feature drives some of the regression results that follow.

Table I
Means, Standard Deviations, and Autocorrelations of
Investment/Capital Ratios, Investment Returns, and Stock
Returns

Data are quarterly, 1947:1–1987:4. Annual returns are overlapping quarterly observations. All returns are expressed as percentages. Autocorrelations are calculated from single regression slope coefficients. Stock returns are the CRSP value weighted portfolio deflated by the CPI; the investment return is constructed from gross fixed investment data.

	Investment/ Capital Ratio	Quarterly		Annual	
		Investment Return	Stock Return	Investment Return	Stock Return
Mean	0.137	1.70	1.70	7.33	7.33
Standard deviation	0.009	3.42	7.24	9.37	15.53
Autocorrelations	1	0.90	0.45	0.11	
(by lag, in quarters)	2	0.71	0.10	0.04	
	3	0.49	–0.06	–0.04	
	4	0.28	–0.19	–0.03	–0.18
	5	0.12	–0.23	–0.10	
	6	0.00	–0.13	–0.07	
	8	–0.17	–0.18	0.06	–0.20
	12	–0.36	–0.19	0.04	–0.23
					0.07

B. Correlation Between Investment and Value-Weighted Returns

Figure 2 presents a plot of quarterly observations of annual stock returns and corresponding annual investment returns and shows that they are positively correlated.

Table II presents some regressions and correlations designed to assess the statistical significance of the correlation between the investment return and real stock return apparent in Figure 2.

The message of Table II is that the correlation visible to the eye in Figure 2 is statistically significant at conventional levels. The correlation coefficient between stock and investment returns ranges from 0.241 for quarterly returns to 0.385 for annual returns and is as high as 0.449 for first quarter annual returns.

Table II also includes the correlation of stock returns with investment growth and GNP growth. Both have about the same correlation with stock returns as does the investment return, and graphs of scaled investment and GNP growth against stock returns look very much like Figure 2. Thus, the correlation between stock returns and investment returns visible in Figure 2 is not a sensitive result of the specific model relating investment returns to investment data.

C. Forecasts of Investment Returns and Stock Returns

Table III compares forecasts of stock returns and forecasts of investment returns. The forecasting variables are the term premium, the corporate

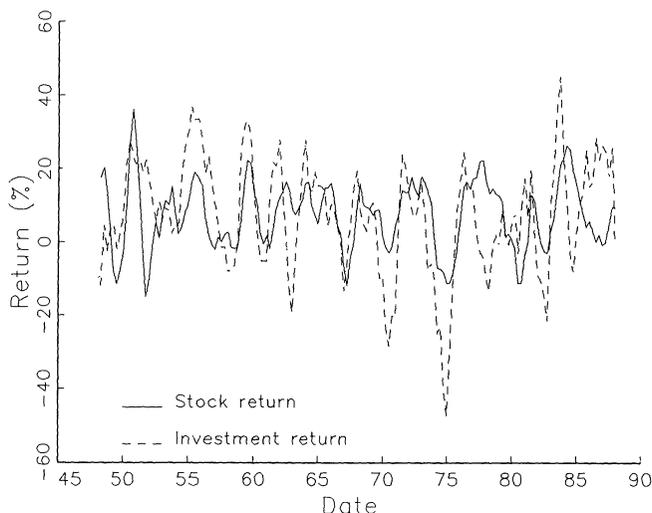


Figure 2. Quarterly observations of annual (from $t - 4$ to t) real returns on the value weighted NYSE portfolio, and annual investment returns.

premium, the lagged real stock return, the dividend-price ratio, and the investment/capital ratio. I added the investment/capital ratio to the familiar list because the investment return model strongly links returns to contemporaneous investment/capital ratios, and since investment/capital ratios are serially correlated, it suggests that the investment/capital ratio should also forecast returns. These are by no means all the variables that are known to forecast stock returns. These are just a few well known representative variables, picked in particular for their association with economic activity.

Panel A of Table III presents single regressions of quarterly and annual returns on the forecasting variables. The coefficients of stock returns on each of the forecasting variables are significant at conventional levels, except lagged returns for annual returns. The coefficients in the investment return regressions are of the same sign and roughly of the same magnitude as the coefficients in the stock return regressions, with the exception of the dividend-price ratio. To test whether the coefficients are in fact equal, I regressed the difference between the stock return and the investment return on the forecasting variables, in the column marked "Stock-Inv." As the table shows, we cannot reject that the single regression coefficients are equal for all the forecasting variables except the dividend-price ratio.

Panel B of Table III presents multiple regression forecasts of returns using all the forecasting variables together. It also reports the probability values for tests of joint significance from multiple regressions on subsets of the forecasting variables. I omitted the individual coefficients of the latter regressions to save space, since they are similar to those reported.

The forecasting variables are jointly significant predictors of stock returns: the χ^2 test for the joint significance has a probability value of 0.03% for

Table II
**Regression of Real Stock Returns on Investment Returns,
Investment Growth, and GNP Growth**

This table documents that the correlation between stock returns and investment returns visible in Figure 2 is statistically significant. The data sample is 1947:1-1987:4.

Panel A. Quarterly Returns				
Stock Return ($t - 1 \rightarrow t$) = $\alpha + \beta$ Right Hand Variable ($t - 1 \rightarrow t$) + $\varepsilon(t)$				
Right Hand Variable	t - stat.	% p value ^a	Correlation of stock, R.H.V.	Std. error of correlation
Investment returns	3.163	0.186	0.241	0.069
Investment growth	3.103	0.226	0.237	0.068
GNP growth	3.914	0.013	0.294	0.074
Panel B. Overlapping Annual Returns, with Corrected Standard Errors ^b				
Stock Return ($t - 4 \rightarrow t$) = $\alpha + \beta$ Right Hand Variable ($t - 4 \rightarrow t$) + $\varepsilon(t)$				
Right Hand Variable	t - stat.	% p value ^a	Correlation of stock, R.H.V	Std. error of correlation
Investment returns	2.820	0.541	0.385	0.113
Investment growth	3.060	0.259	0.360	0.103
GNP growth	3.921	0.012	0.404	0.097
Panel C. Annual Returns with No Overlap (First Quarter to First Quarter, etc.)				
Stock Return ($t - 4 \rightarrow t$) = $\alpha + \beta$ Investment Return ($t - 4 \rightarrow t$) + $\varepsilon(t)$				
Data sample	t - stat.	% p value ^a	Correlation of of stock, inv. return	Std. error of correlation
First quarter	2.885	0.634	0.449	0.128
Second quarter	2.578	1.384	0.407	0.139
Third quarter	1.851	7.173	0.306	0.141
Fourth quarter	2.569	1.412	0.404	0.137

^aPercent probability value of a two sided test for $\beta = 0$.

^bThese standard errors are constructed as in Hansen (1982) and Newey and West (1987) to correct for serial correlation due to overlap, using 8 positive and negative covariances (twice the overlap). (An undamped sum of 4 covariances is appropriate if the overlap is the only source of serial correlation, but this is not always positive. A damped sum of 4 covariances is positive, but does not account for all the serial correlation due to overlap. I used a damped sum of 8 covariances so that the first four are adequately weighted, but the standard error is positive.)

quarterly stock returns and 0.01% for annual stock returns, with R^2 values of 0.12 and 0.22. The dividend-price ratio is an individually significant predictor of stock returns. The other variables are individually insignificant but jointly significant, both in multiple regressions that include the dividend-price ratio and in those that exclude it.

Contrast these results to the forecasts of the difference between stock and investment returns, in the column "Stock-Inv." All the individual variables except the dividend-price ratio and the investment/capital ratio with annual returns (p value 3.94%) are still individually insignificant. More importantly, all variables except the dividend-price ratio are now jointly insignifi-

cant predictors of the return difference. Thus, we cannot reject that the investment return and stock return forecasts based on all variables except the dividend-price ratio are the same.

Panel B of Table III also documents the similarity of the multiple regression forecasts by their correlation. Without the dividend-price ratio, the correlation of the two forecasts is 0.875 quarterly and 0.938 annually, and statistically significant. Figure 3 plots these forecasts of quarterly real stock and investment returns and demonstrates their correlation to the eye. (Fitted values of single regression forecasts are perfectly correlated by construction.

Table III

Forecasts of Stock Returns and Investment Returns

The forecasting variables are as follows: Term is the 10-year government bond return less treasury bill return. Corp is the corporate bond return less the treasury bill return. Ret is the real value weighted return. d/p is the dividend-price ratio. I/k is the investment/capital ratio. Term and d/p are based on returns for the year ending in the indicated quarter ($t - 5$ or $t - 2$), Ret and Corp are returns for the quarter $t - 5$ or $t - 2$. The data sample is 1947:1-1987:4.

" β " gives OLS slope coefficients. "% p value" gives the percent probability values of two sided t -tests of the corresponding slope coefficients. "Joint χ^2 " gives the percent probability values for a χ^2 test of the joint significance of the coefficients. "Joint χ^2 all but d/p " gives the percent probability value of a χ^2 test for the joint significance of all variables except the dividend-price ratio. "Regressions without d/p " give partial results for corresponding multiple regressions that exclude the dividend-price ratio.

Annual return standard errors are adjusted using a Hansen (1982)-Newey-West (1987) correction, using 8 covariances, or twice the overlap. All correlation standard errors include this correction.

Panel A. Single Regression

1. Quarterly Returns

$$\text{Return}(t-1 \rightarrow t) = \alpha + \beta X(t-2) + \varepsilon(t)$$

Forecasting Variable	Stock Return		Investment Return		Stock-Inv.
	β	% p value	β	% p value	% p value
Term	0.16	0.53	0.10	0.05	24.10
Corp	0.35	0.94	0.16	0.23	12.44
Ret	0.16	2.51	0.15	0.00	88.56
d/p	1.32	0.26	0.11	70.70	1.22
I/k	-1.53	2.12	-1.71	0.00	79.96

2. Annual Returns

$$\text{Return}(t-4 \text{ to } t) = \alpha + \beta X(t-5) + \varepsilon(t)$$

Forecasting Variable	Stock Return		Investment Return		Stock-Inv.
	β	% p value	β	% p value	% p value
Term	0.35	1.12	0.35	2.51	99.57
Corp	0.68	1.23	0.59	0.32	70.99
Ret	0.12	50.97	0.24	0.66	48.86
d/p	5.02	0.28	0.80	48.47	0.02
I/k	-4.74	4.34	-7.40	0.00	25.35

Table III—Continued

Panel B. Multiple Regressions					
1. Quarterly Returns					
Return (t - 1 to t) = $\alpha + \beta_1 \text{Term}(t - 2) + \dots + \beta_5 I/k(t - 2) + \varepsilon(t)$					
Forecasting Variable	Stock Return		Investment Return		Stock—Inv.
	β	% p value	β	% p value	% p value
Term	0.09	11.02	0.06	3.47	55.26
Corp	0.17	20.71	-0.04	52.47	11.86
Ret	0.03	69.18	0.10	0.03	33.79
d/p	1.08	1.06	-0.28	25.15	0.54
I/k	-0.77	26.33	-1.53	0.00	28.04
R ²		0.12		0.29	0.07
Joint χ^2 all variables		0.03		0.00	2.32
Joint χ^2 all but d/p		2.32		0.00	24.79
Correlation of stock, investment return forecasts: 0.664, s.e.: 0.088					
Regression without d/p:					
R ²		0.09		0.28	0.02
Joint χ^2 all variables		0.61		0.00	49.24
Correlation of stock, investment return forecasts: 0.875, s.e.: 0.035					
2. Annual Returns					
Return (t - 4 to t) = $\alpha + \beta_1 \text{Term}(t - 5) + \dots + \beta_5 I/k(t - 5) + \varepsilon(t)$					
Forecasting Variable	Stock Return		Investment Return		Stock—Inv.
	β	% p value	β	% p value	% p value
Term	0.26	18.17	0.23	2.36	92.19
Corp	0.41	12.91	0.06	51.05	18.64
Ret	-0.30	8.02	-0.05	46.05	13.44
d/p	4.60	0.05	-0.57	39.89	0.00
I/k	-2.83	14.81	-7.10	0.00	3.94
R ²		0.22		0.52	0.18
Joint χ^2 all variables		0.01		0.00	0.00
Joint χ^2 all but d/p		1.29		0.00	7.09
Joint χ^2 all but d/p, I/k		1.01		5.42	30.68
Correlation of stock, investment return forecasts: 0.610, s.e.: 0.112					
Regression without d/p:					
R ²		0.11		0.51	0.03
Joint χ^2 all variables		4.03		0.00	58.61
Correlation of stock, investment return forecasts: 0.938, s.e.: 0.179					

But fitted value of multiple regression forecasts are not, and the correlation has mean 0 if the forecasted variables are independent.)

However, the dividend-price ratio significantly forecasts the difference between stock and investment returns and lowers the correlation between the two forecasts.

The pattern of these results suggest that all variables except the dividend-price ratio have a common "business cycle" component that forecasts stock and investment returns equally. The dividend-price ratio contains another,

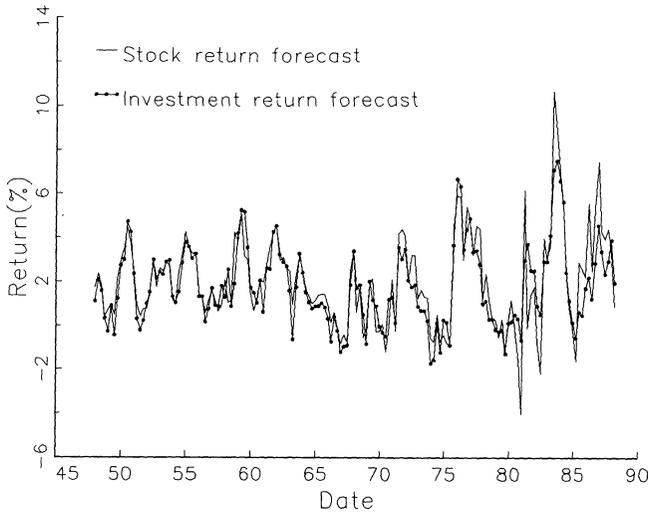


Figure 3. Forecasts of quarterly stock returns and investment returns. Forecasts are from linear regressions of returns on the term premium, corporate premium, lagged return and investment to capital ratio.

longer-term component that forecasts a long-term component in the stock return not found in the investment return. The fact that each of the variables except the dividend-price ratio is significant in single regressions, and individually insignificant but jointly significant in multiple regressions, suggests that these variables are all forecasting the same component of stock returns. Since these variables do not forecast the difference between stock and investment returns, singly or jointly, the forecastable component is the same in stock and investment returns. The fact that the dividend-price ratio is individually significant in multiple regression stock return forecasts suggests that it forecasts a different component of stock returns than the other variables. Since it forecasts the return difference, that component is not found in the investment return. The strong serial correlation of the dividend price ratio, and hence its forecast of returns, suggests the “long horizon” label.

D. Regressions of Returns on Investment/Capital Ratios

Figure 4 and Table IV present regressions of stock returns, investment returns, and their difference on investment/capital ratios. The regressions include investment/capital ratios before, contemporaneous, and subsequent to the return dates, so these regressions address all three issues—whether forecasts of the two returns from investment/capital ratios are the same, whether the association of the two returns with subsequent investment/capital ratios is the same, and whether the projections of returns on investment/capital ratios at many dates are the same. I start with the last issue and then consider the first two.

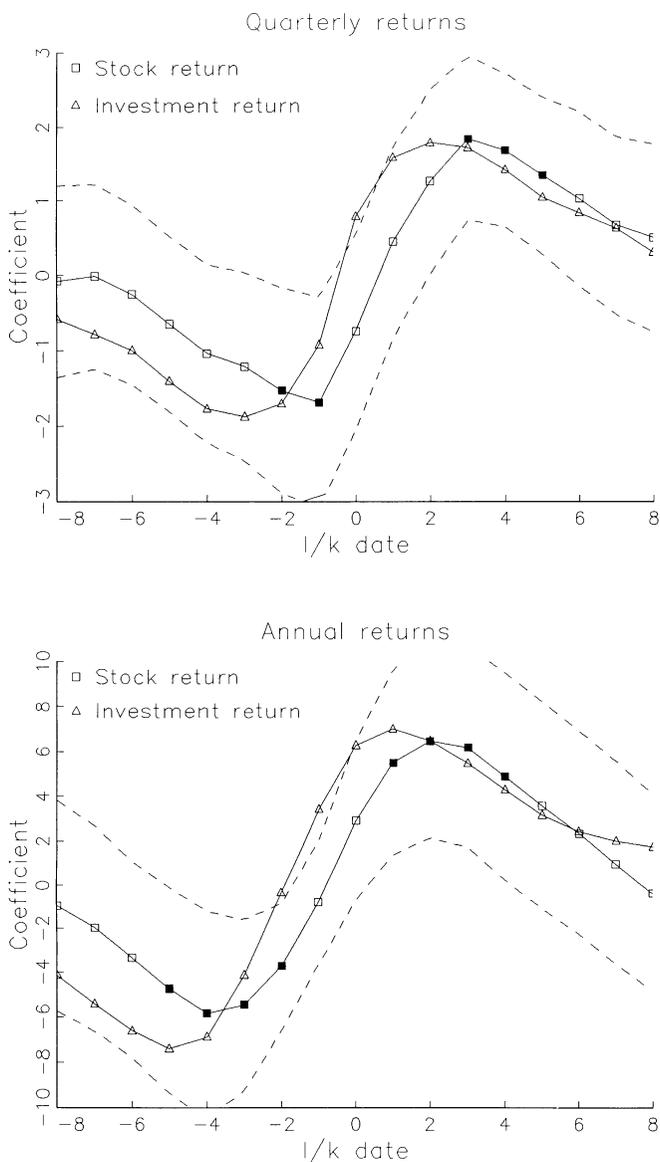


Figure 4. Single regression slope coefficients of quarterly (from $t - 1$ to t) and annual (from $t - 4$ to t) investment returns and stock returns on investment/capital ratios. The two standard error bands on the stock return coefficients (dashed lines) are standard errors bands for the coefficient of stock return-investment return on investment/capital ratios. If the investment return coefficient lies outside the band indicated by dashed lines, the stock-investment coefficient is significant at 5%. Solid symbols for stock return coefficients indicate significance at 5% level. All investment return coefficients are significant. Annual standard errors are calculated using the Hansen (1982)-Newey-West (1987) procedure, using eight covariances. Then data sample is 1947:1-1987:4.

Table IV

Multiple Regressions of Returns on Investments/Capital Ratios

“β” gives OLS slope coefficients. “% p value” gives percent probability values for two sided *t*-tests of the slope coefficients. “Grad” gives the partial derivative of the investment return with respect to investment/capital ratios, evaluated at the “steady state” investment/capital ratio *i** (see Appendix). “Joint χ² p val” gives the percent probability value of a χ² test for joint significance of the coefficients listed in “variables”. Annual return probability values include a Hansen (1982)–Newey–West (1987) correction for serial correlation due to overlap, using eight covariances. Data sample is 1947:1–1987:4.

Panel A. Quarterly Returns										
Return(<i>t</i> - 1 → <i>t</i>) = α + β ₄ <i>I</i> / <i>k</i> (<i>t</i> - 4) + ⋯ + β ₋₂ <i>I</i> / <i>k</i> (<i>t</i> + 2) + ε(<i>t</i>)										
Column:	Stock Return				Investment Return			Stock–Inv.		
	(1)		(2)		(3)		(4)	(5)		
	β	% p value	β	% p value	β	% p value	Grad	β	% p value	
<i>I</i> / <i>k</i> (<i>t</i> - 4)			-0.75	59.05	-0.01	87.97		-0.74	60.07	
<i>I</i> / <i>k</i> (<i>t</i> - 3)			1.39	58.46	0.08	63.17		1.31	60.85	
<i>I</i> / <i>k</i> (<i>t</i> - 2)			1.78	57.66	-0.10	61.74		1.88	55.97	
<i>I</i> / <i>k</i> (<i>t</i> - 1)	-5.33	0.01	-6.14	7.19	-8.47	0.00	-8.70	2.34	50.02	
<i>I</i> / <i>k</i> (<i>t</i>)	4.05	0.19	0.17	96.10	8.46	0.00	8.63	-8.29	1.93	
<i>I</i> / <i>k</i> (<i>t</i> + 1)			1.29	68.72	0.00	98.41		1.28	69.26	
<i>I</i> / <i>k</i> (<i>t</i> + 2)			2.49	9.45	-0.03	77.46		2.53	9.48	
<i>R</i> ²		0.09		0.17		0.98			0.14	
Joint χ ² p val		0.05		0.06		0.00		0.01	2.06	
Variables		All		All		All		All	No <i>t</i> - 1, <i>t</i>	
Panel B. Annual Returns										
Return(<i>t</i> - 4 → <i>t</i>) = α + β ₇ <i>I</i> / <i>k</i> (<i>t</i> - 7) + ⋯ + β ₋₂ <i>I</i> / <i>k</i> (<i>t</i> + 2) + ε(<i>t</i>)										
Column:	Stock Return				Investment Return			Stock–Inv.		
	(6)		(7)		(8)		(9)	(10)	(11)	
	β	% p value	β	% p value	β	% p value	Grad	β	% p value	
<i>I</i> / <i>k</i> (<i>t</i> - 7)					2.73	34.49	0.01	96.84	2.72	37.97
<i>I</i> / <i>k</i> (<i>t</i> - 6)					-1.77	49.03	0.21	46.38	-1.98	44.86
<i>I</i> / <i>k</i> (<i>t</i> - 5)					2.21	42.24	-0.20	30.18	2.41	38.48
<i>I</i> / <i>k</i> (<i>t</i> - 4)	-7.24	0.33	-1.11	74.06	-3.99	26.50	-9.31	0.00	-9.45	5.33
<i>I</i> / <i>k</i> (<i>t</i> - 3)			-2.98	39.94	-2.67	39.07	-0.00	99.00	-0.09	-2.67
<i>I</i> / <i>k</i> (<i>t</i> - 2)			-2.07	43.39	-2.07	44.07	-0.15	54.05	-0.09	-1.91
<i>I</i> / <i>k</i> (<i>t</i> - 1)			-9.96	1.09	-4.52	11.07	-0.00	99.87	-0.09	-4.52
<i>I</i> / <i>k</i> (<i>t</i>)	4.98	1.43	15.20	0.03	5.67	6.59	9.08	0.00	9.36	-3.41
<i>I</i> / <i>k</i> (<i>t</i> + 1)					4.57	10.21	-0.07	78.30		4.64
<i>I</i> / <i>k</i> (<i>t</i> + 2)					2.11	42.18	0.04	90.10		2.07
<i>R</i> ²		0.18		0.26		0.30		0.99		0.16
Joint χ ² p val		0.90		0.85		0.76		0.00		0.00
Variables		All		All		All		All		All but <i>t</i> - 4, <i>t</i>

Figure 4 presents single regression coefficients of returns on investment/capital ratios. The pattern of the coefficients is quite similar, but the stock return coefficients appear slightly shifted in time: the two sets of coefficients would line up almost perfectly if the stock return coefficients were shifted to the left one or two quarters. Though this shift is eye-catching, only the $t - 1$ and t quarterly and $t - 2$ to t annual coefficients are significantly different.

Table IV presents multiple regressions of returns on contemporaneous, leading, and lagging investment/capital ratios. The investment return columns (3, 4, 9, and 10) give the predictions of the model. Since the investment/capital ratio is strongly serially correlated, multiple regressions of stock returns on investment/capital ratios depend on exactly which investment/capital ratios one includes. Table IV includes several choices.

Columns 1 and 6 use only the investment/capital ratios at $t - 1$ and t quarterly, and $t - 4$ and t annual, which should have the largest coefficients. The stock return coefficients in columns 1 and 6 have the right signs and approximately the right relative magnitudes but are slightly lower in absolute value than the corresponding investment return coefficients. The difference in magnitude results from the choice of the investment return standard deviation. Column 7 presents a regressions of annual stock returns on all the contemporaneous investment/capital ratios from $t - 4$ to t . The $t - 4$ to $t - 1$ investment/capital ratios enter negatively as they should but not with the relative magnitudes predicted by the model in columns 9 and 10. The model predicts a much larger coefficient for $t - 4$ and t than for $t - 3$, $t - 2$, and $t - 1$, but the stock return regression coefficients smoothly decline from $t - 1$ to $t - 4$.

Columns 2 and 8 add leads and lags of the investment/capital ratio, both to see how the results are affected by adding more investment/capital ratios and to check that noncontemporaneous investment/capital ratios do not enter the stock return regressions (as they do not enter the investment return regressions). With this set of investment/capital ratios, the positive coefficient at t seems spread forward to $t + 1$ or $t + 2$ as the single regression coefficients were slightly shifted forward. Otherwise the pattern doesn't change much, and the noncontemporaneous investment/capital ratios do not enter the regression significantly.

Columns 5 and 11 present multiple regressions of the difference between stock and investment returns in investment/capital ratios to test whether the investment and stock return coefficients are equal. The χ^2 statistics reject the hypothesis that all the multiple regression coefficients are equal. However, most of this rejection is due to the coefficients contemporaneous to returns, as seen in the joint χ^2 statistics for only the other coefficients.

In summary, the shape of relation between the returns and investment/capital ratios matches in many respects but not perfectly. The basic pattern of negative relation to early investment/capital ratios and positive relation to later investment/capital ratios is found in both single and multiple stock return regressions. However, the stock return regression coefficients are slightly shifted forward in time, and the annual stock return

multiple regression coefficients display a different pattern than the investment return coefficients. These two features seem to account for the statistical rejection of the hypothesis that all coefficients are equal rather than a difference in the basic shape of the coefficients, the absence of a relation between stock returns and investment/capital ratios, or a strong influence of noncontemporaneous investment/capital ratios on stock returns. The conclusion contains some speculations about modifications to the production function that might account for these differences.

As in Table III, the regressions of Figure 4 and Table IV do not reject that stock return and investment return forecasts based on investment/capital ratios are the same.

The single regressions in Figure 4 and multiple regressions in Table IV also contain evidence on the association of returns with subsequent investment/capital ratios. In the single regressions of Figure 4, investment and stock returns are both strongly associated with future investment/capital ratios. But, in both the single regressions of Figure 4 and the multiple regressions of Table IV, we cannot reject that the investment and stock return coefficients on future investment/capital ratios are the same.

The fact that stock returns are associated with future investment/capital ratios is perhaps not surprising, given that stock returns are known to forecast other measures of activity. The investment return is a little more surprising. Notice from Table IV, columns 3, 4, 9, and 10, or the definition equation (15) that the investment return is only associated with contemporaneous ($t - 1$ and t quarterly, $t - 4$ to t annual) investment/capital ratios in a functional or multiple regression sense. Yet the investment return has a large and significant association with future investment/capital ratios (Figure 4) in single regressions. In fact, these single regression coefficients are larger than the single regression coefficients on the contemporaneous investment/capital ratios. The explanation is that the investment/capital ratio is serially correlated. In this sense, the association of investment returns with future investment/capital ratios is spurious. It disappears in multiple regressions that include contemporaneous investment/capital ratios (Table IV, columns 3 and 9). The model predicts that stock returns should display the same behavior. They *should* be associated with future investment/capital ratios in single regressions, but *not* in multiple regressions that include either the investment return or contemporaneous investment/capital ratios. They should have no *marginal* forecast power. And, in fact, we do not reject that the single regression coefficients in Figure 4 and the multiple regression coefficients in Table IV on future investment/capital ratios are the same, verifying both points.

E. Forecasts of GNP Growth from Investment Returns and Value-Weighted Returns

Table V takes up the last issue in more detail by presenting forecasts of GNP growth from lagged returns. In Table V GNP growth is regressed on

returns in the style of forecasting regressions, not the other way around as in Tables III and IV and Figure 4.

Panel A of Table V presents single regressions. Stock returns from $t - 3$ to t are individually significant for quarterly GNP and stock returns from $t - 4$ to t are individually significant for annual GNP, confirming Fama's (1981, 1990b) and Barro's (1990) results. The pattern of the point estimates of investment return coefficients is roughly the same, though their overall magnitude is larger, and when graphed they again seem slightly shifted in time, as in Figure 4. However, the return difference regressions do not find very significant differences in the investment and stock return coefficients.

Table V
Return Forecasts of GNP Growth

" β " gives OLS slope coefficients. "% p value" gives the percent probability value of two sided test for $\beta = 0$ using the t -statistic. "Joint χ^2 p value" gives the percent probability value of the χ^2 test for joint significance of all returns used to forecast GNP growth. Annual return standard errors include a Hansen (1982)-Newey West (1987) correction for serial correlation due to overlap, using eight covariances (twice the overlap). Data are quarterly, 1947:1-1987:4.

Panel A. Single Regressions						
Return Used to Forecast GNP						
1. Quarterly GNP Growth on Quarterly Returns						
$GNP(t)/GNP(t - 1) = \alpha + \beta \text{ Return}(t - x - 1 \rightarrow t - x) + \varepsilon(t)$						
Return Date	Stock Return		Investment Return		Stock-Investment	
	β	% p value	β	% p value	β	% p value
$t - 4$	1.33	29.19	2.40	39.23	0.75	52.78
$t - 3$	2.80	1.74	3.54	17.02	1.93	7.30
$t - 2$	3.85	0.06	7.63	0.32	1.99	8.02
$t - 1$	4.87	0.00	13.11	0.00	1.69	19.66
t	4.45	0.04	17.67	0.00	0.44	71.63
$t + 1$	-0.87	47.59	7.82	0.98	-2.64	4.06
$t + 2$	-1.12	31.83	-1.19	69.74	-0.84	51.20
2. Annual GNP Growth on Annual Returns						
$GNP(t)/GNP(t - 4) = \alpha + \beta \text{ Return}(t - x - 4 \rightarrow t - x) + \varepsilon(t)$						
Return Date	Stock Return		Investment Return		Stock-Investment	
	β	% p value	β	% p value	β	% p value
$t - 6$	-0.76	64.50	-3.17	40.52	0.57	81.53
$t - 5$	1.95	24.74	-0.18	95.76	2.29	29.52
$t - 4$	5.05	0.60	5.03	7.93	3.45	7.61
$t - 3$	8.30	0.00	11.06	0.00	4.42	3.27
$t - 2$	10.28	0.00	16.97	0.00	4.06	6.97
$t - 1$	9.98	0.00	20.48	0.00	2.33	32.10
t	7.60	0.08	19.24	0.00	0.24	92.10
$t + 1$	3.01	18.00	12.47	0.13	-2.00	47.86
$t + 2$	-1.32	53.80	2.87	49.89	-2.72	39.10

Table V—Continued

Panel B. Multiple Regressions						
Return Used to Forecast GNP						
1. Quarterly GNP Growth on Quarterly Returns						
$GNP(t)/GNP(t-1) = \alpha + \beta_1 R(t-2 \rightarrow t-1) + \dots + \beta_4 R(t-5 \rightarrow t-4) + \varepsilon(t)$						
Return Date	Stock Return		Investment Return		Stock—Investment	
	β	% <i>p</i> value	β	% <i>p</i> value	β	% <i>p</i> value
<i>t</i> - 4	1.03	38.29	3.14	31.26	0.74	52.68
<i>t</i> - 3	2.10	4.19	-0.23	93.91	1.71	9.83
<i>t</i> - 2	3.02	0.41	1.57	57.92	1.72	13.00
<i>t</i> - 1	4.39	0.01	13.27	0.00	1.58	23.87
R^2		0.16		0.19		0.04
Joint χ^2 % <i>p</i> value		0.00		0.00		11.21
2. Annual GNP Growth on Annual Returns						
$GNP(t)/GNP(t-4) = \alpha + \beta_1 R(t-8 \rightarrow t-4) + \dots + \beta_4 R(t-5 \rightarrow t-1) + \varepsilon(t)$						
Return Date	Stock Return		Investment Return		Stock—Investment	
	β	% <i>p</i> value	β	% <i>p</i> value	β	% <i>p</i> value
<i>t</i> - 4	-0.27	89.79	6.69	11.46	0.09	97.22
<i>t</i> - 3	2.33	12.58	-2.49	58.90	2.66	21.19
<i>t</i> - 2	4.05	0.61	-5.24	18.17	3.13	8.10
<i>t</i> - 1	5.79	1.53	25.53	0.00	-1.13	69.02
R^2		0.33		0.47		0.06
Joint χ^2 % <i>p</i> value		0.00		0.00		18.87

Only one of the coefficients on lagged returns is significant at the 5% level for the return difference, though four are significant at 10%. The 10% rejections are also concentrated around *t* - 3 and *t* - 2 rather than near *t* - 1 or *t* where the magnitudes of the coefficients and the magnitude of their difference are largest. In particular, the large difference between the coefficients near *t* is not statistically significant. In quarterly returns, the *t* + 1 investment return is still associated with time *t* GNP growth, but the *t* + 1 stock return is not, so the “shift” is statistically significant here.

Panel B of Table V presents multiple regressions of GNP growth on lagged investment returns and stock returns. (I ran multiple regressions using up to eight lags, but the additional lags were small and insignificant.) In both cases the nearest returns are the most individually significant predictors of GNP. Though lagged stock returns are individually and jointly significant predictors of GNP, lagged return differences are not, so we do not reject that the stock and investment return forecasts of GNP growth are the same.

III. Concluding Remarks

The simple implementation of a production-based asset pricing model in this paper predicts that stock returns and investment returns should be equal.

This idea is used to give a partial equilibrium explanation of the forecastability of stock returns and the fact that stock returns forecast real variables including investment and GNP. Regressions of returns on contemporaneous investment/capital ratios are also included as a diagnostic.

Forecasts of investment returns and stock returns appear to be the same for most of the forecasting variables. Conversely, forecasts of future investment/capital ratios and GNP growth from investment returns and stock returns also appear to be the same. Other successes included findings that ex post investment returns and stock returns are highly correlated and that the projection of investment and stock returns on investment/capital ratios matches in many respects.

However, investment returns do not explain the component of stock returns forecastable by dividend-price ratios. Dividend-price ratios seem to forecast a long horizon component in stock returns not present in investment returns. This component of stock returns might reflect a long-term movement in productivity, which is assumed to be constant here.

Also, the shape of the function relating stock returns to investment/capital ratios is significantly different from that of the investment returns. The single regression coefficients are significantly different near time t , and the pattern of multiple regression coefficients, though qualitatively similar, is quantitatively different, and the difference is statistically significant. Uncertainties in the timing of investment, gestation lags, and adjustment costs to the level of investment have been found important in other studies (e.g., Rosen and Topel (1988)) and may account for some of these differences. For example, if investment purchased this quarter does not give rise to productive capital until next quarter, this could account for a one quarter shift in the investment return as measured here relative to the true investment return.

There are several promising directions in which this model can be extended. Alternate forms for technology may improve the fit, and variations in marginal products can be estimated. By not attempting to construct a mimicking portfolio, producers' first order conditions can be estimated and tested by generalized method of moments. Most importantly, one can check the production-based model's implications of *cross sectional* as well as time-series variation in return. These implications are lost here by aggregation to a single technology. They can be explored using components of investment or industry or firm investment data to generate multiple investment returns.

APPENDIX

Data Sources and Transformations

The investment series is gross private domestic investment, seasonally adjusted, from CITIBASE (series GIF82). The stock return and dividend price ratio series are derived from the CRSP value-weighted NYSE portfolio (VWRET and VWRETX). The treasury bill, government bond, corporate bond, and CPI are from the Ibbotson-Sinquefeld data set (USTR, GBTR, CBTR, and CPI). The sample is 1947:1-1987:4.

I constructed the investment/capital ratio (I_t/k_t) as follows. The capital accumulation rule equation (14) implies that $i_t \equiv I_t/k_t$ follows:

$$i_{t+1} = \frac{I_{t+1}}{I_t} \frac{i_t}{(1 - \delta)(1 + i_t - (\alpha/2)i_t^3)}. \quad (\text{A.1})$$

I set the investment/capital ratio to its "steady state" value i^* in 1947:1, where i^* is defined by the fixed point of equation (A.1) with investment growth set to its mean value. (i^* is a close approximation to the mean investment/capital ratio.) I then used (A.1) to find investment/capital ratios at all other dates.

I formed the forecasting variables as follows. Term is GBTR – USTR, Corp is CBTR – USTR. VWRET and VWRETX were both accumulated for a year. Then $d/p = (\text{annual VWRET} - \text{annual VWRETX}) / (1 + \text{annual VWRETX})$ forms dividends brought forward at the market return (VWRET), divided by end of period price. (See the Appendix to Cochrane (1989b).)

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