Comments on "Volatility, the Macroeconomy and Asset Prices," by Ravi Bansal, Dana Kiku, Ivan Shaliastovich, and Amir Yaron, and "An Intertemporal CAPM with Stochastic Volatility" John Y. Campbell, Stefano Giglio, Christopher Polk, and Robert Turley

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April 13 2012
A skeptic’s questions

- Do long run risks/recursive utility matter?

\[(E_{t+1} - E_t) \ln m_{t+1} = -\gamma \Delta c_{t+1} + (1 - \gamma) \sum_{j=1}^{\infty} \beta^j \Delta c_{t+j}\]

\[E(R^e_{t+1}) = \text{cov}(R^e_{t+1}, \Delta c_{t+1}) \gamma + \text{cov}(R^e_{t+1}, \sum_{j=1}^{\infty} \beta^j \Delta c_{t+j}) (\gamma - 1)\]

- Are people really afraid of \(\sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j}\) holding constant \(c_t\)?
- Is \(\sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j}\) the “second factor”?
- Is there really much variation in \(E_t \sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j}\) not reflected in current state variables (\(\Delta c_{t+1}\)?)?
- “Long run risks” does not necessarily mean recursive utility, sensitivity to news
More skeptic’s questions

\[(E_{t+1} - E_t) \ln m_{t+1} = -\gamma \Delta c_{t+1} + (1 - \gamma) \sum_{j=1}^{\infty} \beta^j \Delta c_{t+j}\]

\[E(R^e_{t+1}) = \text{cov}(R^e_{t+1}, \Delta c_{t+1}) \gamma + \text{cov}(R^e_{t+1}, \sum_{j=1}^{\infty} \beta^j \Delta c_{t+j}) (\gamma - 1)\]

- Does time-varying consumption volatility \(\sigma^2_t(\Delta c_{t+1})\) generate time-varying expected returns \(\sigma_t \ln m_{t+1}\)? (Vs. time-varying risk aversion; habits or leverage, etc.)
- Is there really enough variation in \(\sigma_t(\Delta c_{t+1})\)? (factor of 2)
- Is there really much variation in \(\sum_{j=1}^{\infty} \beta^j \sigma^2_t(\Delta c_{t+1+j})\)?
Paper 1 answers: persistent volatility?

- Is there really much variation in $E_t \sum_{j=1}^{\infty} \beta^j \Delta c_{t+1+j}$, $\sigma_t(\Delta c_{t+1})$?
  $$\sum_{j=1}^{\infty} \beta^j \sigma_t^2(\Delta c_{t+1+j})$$

1. $RV_t = \frac{1}{12} \sum_{j=0}^{11} \Delta ip_{t-j/12}^2$ = realized industrial production volatility.

2. Forecast $RV_{t+1}$ from VAR using annual data from 1930

   \[
   \begin{array}{cccccc}
   \Delta c_t & \Delta y_t & r_t & pd_t & RV_t & R^2 \\
   RV_{t+1} & -0.007 & -0.005 & 0.001 & -0.001 & 0.291 & 0.33 \\
   & (0.001) & (0.001) & (0.001) & (0.001) & (0.004) & \\
   \end{array}
   \]

3. Assume $\sigma_t^2(\Delta c_{t+1}) = E_t RV_{t+1}$! (“permanent income?” GE?)

4. Even so, little persistence, little long-run volatility
  $$\sum_{j=1}^{\infty} \beta^j \sigma_t^2(\Delta c_{t+1+j})$$
Table 8: VAR Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_t$</th>
<th>$\Delta y_t$</th>
<th>$r_t$</th>
<th>$pd_t$</th>
<th>$RV_t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t+1}$</td>
<td>0.447</td>
<td>0.014</td>
<td>0.057</td>
<td>-0.011</td>
<td>-2.681</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.036)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.193)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t+1}$</td>
<td>0.283</td>
<td>0.350</td>
<td>0.030</td>
<td>-0.001</td>
<td>-3.295</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.078)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.452)</td>
<td></td>
</tr>
<tr>
<td>$r_{t+1}$</td>
<td>-2.883</td>
<td>1.164</td>
<td>-0.009</td>
<td>-0.075</td>
<td>-9.629</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(4.335)</td>
<td>(5.448)</td>
<td>(0.110)</td>
<td>(0.075)</td>
<td>(30.751)</td>
<td></td>
</tr>
<tr>
<td>$pd_{t+1}$</td>
<td>-3.553</td>
<td>1.113</td>
<td>-0.338</td>
<td>0.902</td>
<td>-7.939</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(4.060)</td>
<td>(5.026)</td>
<td>(0.104)</td>
<td>(0.085)</td>
<td>(33.888)</td>
<td></td>
</tr>
<tr>
<td>$RV_{t+1}$</td>
<td>-0.007</td>
<td>-0.005</td>
<td>0.001</td>
<td>-0.001</td>
<td>0.291</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.004)</td>
<td></td>
</tr>
</tbody>
</table>
Is volatility an important Merton state variable?

- Does volatility matter to long-run investors?
- Is volatility the missing second factor for which hml is proxy?

\[
\begin{align*}
  w_t &= \frac{1}{\gamma_t} \Sigma_t^{-1} E_t(R^e_{t+1}) + \beta R,z \frac{\eta_t}{\gamma_t} = \text{market-time + hedge} \\
  \gamma_t &= -\frac{W V_{W W}(W, z)}{V_W(W, z)}; \eta_t = \frac{V_{W z}(W, z)}{V_W(W, z)} \\

  E_t(R^e_{t+1}) = cov_t(R^e_{t+1}, R^{em}_{t+1}) \gamma^m_t - cov_t(R^e_{t+1}, z_{t+1}) \eta^m_t
\end{align*}
\]

- Is \( \eta \) large for \( z_t = \sigma^2_t \)? (Theory)
- Is \( cov_t(hml_t, \sigma^2_t) \) large? (CGPR is about the hedging component / unconditional means only)
Is $\eta = \frac{V_{Wz}}{V_W}$ large for $z_t = \sigma_t^2$? Example 1.

1. Campbell/Wachter: Long-run bond is the riskless asset for a long-run investor

2. The long run bond investor does not care about volatility. $V(W, \text{yield})$ not $V(W, \sigma^2(\text{yield}))$

3. (Long run: Thinking about long-run bond investing as one period mean/variance, + state variable hedging, is nuts.)
Do long-run investors care about volatility?

1. Ex. 2: Max $EU(W_T)$ investor correctly ignores “short term” changes in volatility because it does not much affect $\sigma^2(R_{0\rightarrow T}) = \sigma^2(\sum r_{t+j})$

2. $\Leftrightarrow$ Is there variation in long-run volatility?

- Paper: Do it right with theory: Recursive utility machinery to derive $\eta, V_{Wz}$. 
Do long-run investors care about volatility?

- Doing it right with Theory

1. Recursive utility machinery to derive $\eta, V_{Wz}$.

2. Yes, this investor cares ($\omega$), about *long-run* volatility

\[
E_t r_{i,t+1} - r_{ft} = \gamma \text{cov}_t (r_{i,t+1}, N_{CF,t+1}) + \text{cov}_t (r_{i,t+1}, N_{DR,t+1}) + \frac{1}{2} \omega(\gamma, ..) \text{cov}_t [r_{i,t+1}, N_{V,t+1}] 
\]

3. Is there variation in *long-run* volatility? (Again)

\[
N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \sigma^2_{t+j}(r_{t+j})
\]
**Paper 2: Is there variation in long run volatility?**

Step 1: Create EVAR forecast of realized volatility.

<table>
<thead>
<tr>
<th>( \text{RVAR}_{t+1} )</th>
<th>( r_t )</th>
<th>( \text{RVAR}_t )</th>
<th>( \text{PE}_t )</th>
<th>( \text{TY}_t )</th>
<th>( \text{DEF}_t )</th>
<th>( \text{VS}_t )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.017</td>
<td>0.30</td>
<td>0.013</td>
<td>-0.002</td>
<td>0.024</td>
<td>0.001</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.061)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.008)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{EVAR}_t = E_t(\text{RVAR}_{t+1}) \], keeping all the coefficients.
This graph is a dramatic *failure*.
Step 2: VAR for EVAR

<table>
<thead>
<tr>
<th>Variable</th>
<th>$r_t$</th>
<th>EVAR$_t$</th>
<th>PE$_t$</th>
<th>TY$_t$</th>
<th>DEF$_t$</th>
<th>VS$_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{t+1}$</td>
<td>0.12</td>
<td>0.66</td>
<td>-0.054</td>
<td>0.007</td>
<td>-0.029</td>
<td>-0.017</td>
</tr>
<tr>
<td>(se)</td>
<td>(0.082)</td>
<td>(0.93)</td>
<td>(0.039)</td>
<td>(0.009)</td>
<td>(0.028)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>EVAR$_{t+1}$</td>
<td>-0.004</td>
<td>0.34</td>
<td>0.012</td>
<td>-0.001</td>
<td>0.018</td>
<td>0.005</td>
</tr>
<tr>
<td>(se)</td>
<td>(0.005)</td>
<td>(0.085)</td>
<td>(0.007)</td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>PE$_{t+1}$</td>
<td>0.19</td>
<td>0.57</td>
<td>0.96</td>
<td>0.007</td>
<td>-0.024</td>
<td>-0.004</td>
</tr>
<tr>
<td>(se)</td>
<td>(0.079)</td>
<td>(0.88)</td>
<td>(0.037)</td>
<td>(0.008)</td>
<td>(0.027)</td>
<td>(0.044)</td>
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<tr>
<td>TY$_{t+1}$</td>
<td>-0.16</td>
<td>2.91</td>
<td>-0.002</td>
<td>0.85</td>
<td>0.099</td>
<td>0.044</td>
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<tr>
<td>(se)</td>
<td>(0.37)</td>
<td>(4.01)</td>
<td>(0.160)</td>
<td>(0.039)</td>
<td>(0.13)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>DEF$_{t+1}$</td>
<td>-0.45</td>
<td>2.23</td>
<td>-0.033</td>
<td>-0.003</td>
<td>0.87</td>
<td>0.035</td>
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<tr>
<td>(se)</td>
<td>(0.20)</td>
<td>(1.82)</td>
<td>(0.080)</td>
<td>(0.020)</td>
<td>(0.064)</td>
<td>(0.10)</td>
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<tr>
<td>VS$_{t+1}$</td>
<td>0.066</td>
<td>0.97</td>
<td>-0.010</td>
<td>-0.005</td>
<td>-0.001</td>
<td>0.93</td>
</tr>
<tr>
<td>(se)</td>
<td>(0.073)</td>
<td>(0.74)</td>
<td>(0.033)</td>
<td>(0.008)</td>
<td>(0.025)</td>
<td>(0.041)</td>
</tr>
</tbody>
</table>

Step 3: Calculate $N_V = \left( E_t - E_{t-1} \right) \sum \rho^{j-1} EVAR_{t+j}$ Using these point estimates

Danger: spurious forecasts from slow moving variables dominate long-run
The big recent data point

Issue 1: Why do you hold the market as $\sigma$ rises from $0.18^2 = 0.0324$ to $0.80^2 = 0.64$? $w_t = \frac{1}{\gamma_t} \frac{E_t(R_t^{e})}{\sigma_t^2(R_t^{e})} + \beta_R z_t \frac{\eta_t}{\gamma_t}$
The big recent data point

Issue 2: is hml really a great volatility hedge? Is volatility really the explanation for hml?
- Is volatility the extra state variable that explains the value effect?
- We should just be pricing hml, not the 25!

\[
\begin{align*}
FF & : \ E(R_{ei}) = b_i E(rmrf) + h_i E(hml) + s_i E(smb) \\
R_{ei} & = b_i rmrf + h_i hml + s_i smb; \ R^2 = 0.95
\end{align*}
\]
Panel A: Mean returns versus consumption covariances

Stockholders

Nonstockholders

Top stockholders

Aggregate

$R^2 = 0.63$

$R^2 = 0.66$

$R^2 = 0.32$

$R^2 = 0.40$
Why does beta spread disappear in the earlier period?
Why to the volatility betas change sign in the earlier period?
Where do the betas come from? (Cash flow, discount rate, volatility)

<table>
<thead>
<tr>
<th>( \hat{\beta}_V )</th>
<th>Growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Value</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.61</td>
<td>0.37</td>
<td>0.24</td>
<td>0.19</td>
<td>0.02</td>
<td>-0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>0.42</td>
<td>0.24</td>
<td>0.19</td>
<td>0.08</td>
<td>-0.60</td>
</tr>
<tr>
<td>3</td>
<td>0.67</td>
<td>0.36</td>
<td>0.27</td>
<td>0.13</td>
<td>0.15</td>
<td>-0.52</td>
</tr>
<tr>
<td>4</td>
<td>0.63</td>
<td>0.36</td>
<td>0.19</td>
<td>0.17</td>
<td>0.11</td>
<td>-0.53</td>
</tr>
<tr>
<td>Large</td>
<td>0.47</td>
<td>0.36</td>
<td>0.18</td>
<td>0.12</td>
<td>0.14</td>
<td>-0.33</td>
</tr>
<tr>
<td>Diff</td>
<td>-0.14</td>
<td>-0.01</td>
<td>-0.06</td>
<td>-0.07</td>
<td>0.12</td>
<td>0.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \hat{\beta}_V )</th>
<th>Growth</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Value</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>-0.72</td>
<td>-0.79</td>
<td>-0.85</td>
<td>-0.82</td>
<td>-0.88</td>
<td>-0.16</td>
</tr>
<tr>
<td>2</td>
<td>-0.50</td>
<td>-0.52</td>
<td>-0.60</td>
<td>-0.62</td>
<td>-0.82</td>
<td>-0.32</td>
</tr>
<tr>
<td>3</td>
<td>-0.48</td>
<td>-0.38</td>
<td>-0.53</td>
<td>-0.55</td>
<td>-0.85</td>
<td>-0.37</td>
</tr>
<tr>
<td>4</td>
<td>-0.21</td>
<td>-0.35</td>
<td>-0.43</td>
<td>-0.58</td>
<td>-0.87</td>
<td>-0.65</td>
</tr>
<tr>
<td>Large</td>
<td>-0.22</td>
<td>-0.23</td>
<td>-0.44</td>
<td>-0.67</td>
<td>-0.68</td>
<td>-0.46</td>
</tr>
<tr>
<td>Diff</td>
<td>0.50</td>
<td>0.56</td>
<td>0.41</td>
<td>0.16</td>
<td>0.19</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Betas

- Why does beta spread disappear in the earlier period?
- Why do the volatility betas change sign in the earlier period?
- Where do the betas come from? (Cash flow, discount rate, volatility)
- What about the FF factor structure? How much $R^2$ is absorbed by variance in the time-series regression

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \epsilon_t^i; R^2 = 0.95$$

$$R_t^{ei} = a_i + d_i N_{DR}t + c_i N_{CF}t + v_i N_{V}t \; \epsilon_t^i; \; R^2 = ?$$

- Again, Fama and French tell us that you price the 25 if you price hml. Does this price hml? What’s the correlation of hml and $N_V$?
Bottom line:

1. Hooray for the long run!

\[ p_t - d_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \]

\[ \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - (p_t - d_t) \]

1.1 Prices, long-run returns not one-period returns
1.2 Long run betas are all cashflow betas
1.3 State variables disappear from long run portfolio / equilibrium problems.

2. Not convinced yet on recursive utility, long run news shocks, that volatility is the crucial state variable (not, say nontraded income) explaining value, or very persistent volatility.