Inflation Determination with Taylor Rules: A Critical Review

John H. Cochrane*

First draft: April 2005. Current draft: September 12, 2007

Abstract

The new-Keynesian, Taylor-rule theory of inflation determination relies on explosive dynamics. By raising interest rates in response to inflation, the Fed does not directly stabilize future inflation. Rather, the Fed threatens hyperinflation or deflation, unless inflation jumps to one particular value on each date. However, there is nothing in economics to rule out hyperinflationary or deflationary solutions. Therefore, inflation is just as indeterminate under "active" interest rate targets as it is under standard fixed interest rate targets. Inflation determination requires ingredients beyond an interest-rate policy that follows the Taylor principle.

*Graduate School of Business, University of Chicago and NBER. 5807 S. Woodlawn Ave. Chicago IL 60637. 773 702 3059. john.cochrane@chicagogsb.edu. I gratefully acknowledge research support from the CRSP and the NSF via a grant administered by the NBER. I am grateful for many comments and suggestions to Fernando Alvarez, Marco Bassetto, David Backus, Mark Gertler, Eric Leeper, Lars Hansen, Edward Nelson, Christopher Sims, Michael Woodford, and participants at the Spring 2005 Inidiana University Monetary Economics Conference and participants at the Fall 2006 NBER EFG conference. Updates and appendices are available on my website
http://faculty.chicagogsb.edu/john.cochrane/research/Papers/

1 Introduction

How is the price level determined, in modern fiat-money economies in which the central bank follows an interest rate target, ignoring monetary aggregates? The new-Keynesian, Taylor-Rule approach to monetary economics provides the current "standard answer" to this, perhaps the most fundamental question of macroeconomics. In this theory, inflation is determined because the Fed systematically raises nominal interest rates more than one-for-one with inflation. This "active" interest rate target is thought to eliminate the indeterminacy that results from fixed nominal interest rate targets.

New-Keynesian models *do not* say that higher inflation causes the Fed to raise real interest rates, which in turn lowers "demand" and reduces future inflation. That's "old-Keynesian" logic. That logic produces stable dynamics, in which we solve backward for endogenous variables as a function of current and past shocks. New-Keynesian models specify exactly the opposite dynamics: the Fed commits to raise future inflation explosively in response to higher inflation today. For only one value of inflation today will we fail to see such an explosion. Ruling out explosive paths, or, more generally, "non-local" paths, new-Keynesian modelers conclude that inflation today will jump to that unique value. One theme of this paper is the striking difference between new-Keynesian models and the old-Keynesian intuition. You simply cannot use the old intuition to discuss how the new models work, nor can the new models be used to buttress old-Keynesian discussions.

I argue that the Taylor principle, in the context of new-Keynesian models, does not, in fact, determine inflation or the price level. Nothing in economics rules out explosive or "non-local" nominal paths. Transversality conditions can rule out *real* explosions, but not nominal ones. Therefore, nothing in economics generically allows us to insist on the unique "locally-bounded" equilibrium of a new-Keynesian model. The inflation indeterminacy of nominal interest rate targets is not cured by the Taylor principle.

"Review"

Part of this paper's title is "review," and so is much of its body. Surely, the great economists of the last 20 years who assembled the new-Keynesian structure are aware of such a basic point? They are, so to be convincing I must document what they say about it, and that they offer no obvious rejoinder. Now, most new-Keynesian theorizing simply rules out unpleasant equilibria, typically just writing something like "we restrict attention to the unique locally-bounded equilibrium." Pruning equilibria is, sensibly, a boring technical detail to be gotten over quickly on the way to more interesting issues, such as characterizing the joint dynamics of inflation and macroeconomic variables, or characterizing optimal policy to produce better dynamics. However, as I will show, the authors that do grapple with this question agree, if you read carefully enough: the explosive or non-local equilibria *are* perfectly valid equilibria, there really is no good reason to throw them out.

This paper is a review in another sense. None of the modeling is novel. Almost all of the equations have appeared elsewhere. The larger issue of how to rule out multiple equilibria in monetary models goes back decades, and my conclusion that "monetary policy must be considered jointly with fiscal policy" has been emphasized for at least half a century.[1] My contribution is to apply these standard ideas to the standard new-Keynesian Taylor rule setup, and to advocate different conclusions from the familiar equations of that setup.

This paper is about theory. Theories ultimately rise and fall on their ability to organize facts. The central *empirical* argument for the new-Keynesian Taylor rule view of inflation determination is the claim that inflation got out of control in the 1970s because the Fed reacted insufficiently to inflation, and was controlled in the 1980s when the Fed began to react more than one-for-one to inflation, as captured most famously by Clarida, Galí and Gertler's (2000) regressions. A companion paper, Cochrane (2007), argues that these facts are misinterpreted, since the Taylor coefficient required to stabilize inflation in the new-Keynesian model is not identified in the data.

These papers are about inflation determination. They are not a criticism of new-Keynesian economics in general. In particular they do not argue that anything is wrong with its basic ingredients, an intertemporal, forward-looking model of the real economy, as boiled down to the intertemporal "IS" curve of the simple model, or an intertemporally-optimizing, forward-looking model of price-setting subject to frictions, as captured in the "new-Keynesian Phillips curve." If anything, in fact, these paper praises those ingredients, by insisting that we take their dynamics seriously and not as confusing cover for muddy old-Keynesian stories.

Overview

Section 2 introduces the simplest model. I combine a Fisher equation for interest rates and inflation together with a Taylor rule. I show the nature of indeterminacy, and how an "active" Taylor rule can make all solutions but one explode.

To judge whether explosive solutions really are valid, we need a model. Section 3 reviews the explicit economic underpinnings to show that in fact, the nominal explosions

[1] In the modern era, nominal ideterminacy goes back to Patinkin (1949), (1965). Friedman (1968) pointed out the danger of accelerating inflation with a fixed interest rate target. Sargent and Wallace (1975) went further, establishing the modern sense of indeterminacy, i.e. multiple equilibria. McCallum (1981) first advocated the idea that an interst rate target that varied with endogenous variables in the economy could overcome Sargent and Wallace's indeterminacy. Taylor's (1993) famous "rule" was really an empirical observation about the behavior of central banks, and only later, when paired with new-Keynesian models became a theory of inflation determinacy.

A vast parallel literature, that I will not review, discusses determinacy under fixed money supply (rather than interest rate) regimes. Cagan (1956) pointed out that when money-demand is interest elastic, there can be multiple price level paths even with fixed money supply. The equations

$$\begin{aligned} m_t + v(i_t) &= p_t + y_t \\ i_t &= r + E_t p_{t+1} - p_t \end{aligned}$$

have multiple solutions. Again, there is usually only one non-explosive or forward-looking solution, but the question remains why one should pick only that solution since it features only a nominal, not a real, explosion. As one can see in the discussions by Sims (1994, 2003) and Woodford (1995, 2003), the same problems arise.

Eagle (2006, 2007) also criticizes the selection of only locally-bounded nominal solutions, with a larger literature review. He emphasizes the counterintuitive signs of off-equilibrium responses.

are perfectly valid equilibria. I also review how a non-Ricardian regime works, and how it can lead to a determinate price level and inflation rate when combined with interest rate targets.

Section 4 reviews Woodford (2003) and Benhabib, Schmitt-Grohé and Uribe (2002), which are two of the most thoughtful and authoritative new-Keynesian analyses of determinacy. I verify that they agree that non-local equilibria are valid, and I review how they struggle to rule them out. I show how, ultimately, most of their proposals add the ingredient of a non-Ricardian fiscal regime. However, I show that their proposals to trim multiple equilibria have the government commit not just to non-Ricardian regimes, but to inconsistent or impossible regimes. Essentially, the government threatens to blow up the world should any but one equilibrium occur. I opine that this not a credible threat, or, more to the point, it is not likely to be a good characterization of people's expectations.

If a threat to blow up the world is not credible, perhaps the Fed can threaten to take us to a region in which the world will blow up on its own, thus ruling out equilibria. I review the related attempts to rule out inflationary or deflationary paths on this basis. I argue that, first, these models are fragile economically, and second, the Fed's threat to take the world to a region in which the world blows up is just as incredible as the Fed's threat to blow up the world explicitly.

In all these attempts to prune equilibria, we need to keep in mind they need to be convincing descriptions of people's *current* expectations. If they are only policy proposals of commitments the government might make in the future, the resulting model cannot apply to today's data.

Perhaps the focus on frictionless models with no real interest rate variation is at fault. In such models, the only way the Fed can raise interest rates following inflation is by raising future inflation. "Of course" one might say, despite the explicit claim to the contrary, "a Taylor rule in *that* world would lead to hyperinflation. What about a world in which the Fed can control *real* interest rates?" Section 5 studies determinacy in the standard new-Keynesian model. I verify that the issues are the same, and the Fed does in fact determine inflation by threatening hyperinflation, not by stabilizing past inflation. The key is that the new-Keynesian model is forward-looking so all the standard dynamics are reversed. Higher real interest rates mean output grows *faster*, larger output is associated with *declining* inflation, and expected future values are on the right hand side of structural equations.

Section 6 answers the natural question, "How does Taylor think the Taylor rule works?", and through him exposits a standard old-Keynesian model in which the Fed does stabilize past inflation. Far from being a "reduced form" or an "approximation," I show how fundamentally different the new and old Keynesian model dynamics are, and emphasize that we cannot use new-Keynesian models for the old-Keynesian intuition embodied in this set of equations.

Section 7 investigates how sensitive the analysis is to Taylor rules that react to lagged, current, and expected future inflation, as well as to rules that allow output responses. The usual claim is that there is not much difference, that Taylor rule determinacy is "robust." That claim is dramatically false in new-Keynesian models: which inflation the Fed responds to matters a good deal, and local determinacy rests on all parameters of the model, not just the policy rule. I also review the recent proposals by Loisel (2007) and Adão, Correia and Teles (2007). These rules seem to eliminate all the multiple equilibrium problems, giving a single determinate price level or inflation rate. I show that they are limits of the standard logic, in which the threatened explosions happen infinitely fast – the eigenvalues greater than one are infinite.

Section 8 concludes, and offers some interpretation, in particular some answers to the obvious question, if not this theory, what theory *can* account for price-level determination in a modern fiat-money economy in which the central bank follows an interest rate target? I conclude that adding a non-Ricardian fiscal regime is the most plausible route to such a theory.

2 Determinacy in a simple model

Start with a very simple model consisting only of a Fisher equation and a Taylor rule describing Fed policy:

$$i_t = r + E_t \pi_{t+1} \tag{1}$$

$$i_t = r + \phi \pi_t + x_t \tag{2}$$

where i_t = nominal interest rate, π_t = inflation, r = constant real rate, and x_t = random component to monetary policy. The coefficient ϕ measures how sensitive the central bank's interest rate target is to inflation.

The basic points do not require the Phillips - IS curve features of new-Keynesian models, and thus they do not need any frictions. This claim needs to be shown, and I do so by expanding the analysis to include fully-specified new-Keynesian models below. While initially surprising to those of us brought up to think that monetary economics requires some frictions, it is routine in the new-Keynesian literature to study inflation determinacy in such a stripped down and frictionless model (King 2000 p. 76, Woodford 2003 for example). In fact, it is a major accomplishment of this literature to be able to discuss inflation determination with no mention of money, following central bank practice.

We can solve this model by substituting out the nominal interest rate, leaving only inflation,

$$E_t \pi_{t+1} = \phi \pi_t + x_t. \tag{3}$$

Equation (3) has many solutions, and this observation forms the classic doctrine that inflation (to say nothing of the price level) is indeterminate with an interest rate target. We can write the equilibria of this model as

$$\pi_{t+1} = \phi \pi_t + x_t + \delta_{t+1};\ E_t \left(\delta_{t+1}\right) = 0 \tag{4}$$

and hence

$$\pi_t = \pi_0 \phi^t + \sum_{j=1}^{t} \phi^{j-1} x_{t-j} + \sum_{j=0}^{t-1} \phi^j \delta_{t-j}$$

where δ_t is any mean-zero random variable. Here, inflation indeterminacy is indexed not only by initial inflation π_0, but also by the arbitrary random variables or "sunspots" δ_t.

However, if $\phi > 1$, all of these solutions except one eventually explode. If we disallow such explosive solutions – if we require that $E_t(\pi_{t+j})$ be bounded – then a unique solution remains. We find this solution by solving the difference equation (3) forward,

$$\pi_t = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t(x_{t+j}). \tag{5}$$

Equivalently, by this criterion we choose the unique initial value π_0 and forecast error $\{\delta_t\}$ at each date (these variables index solutions) that lead to a nonexplosive path for $E_t(\pi_{t+j})$.

Thus we have it: *if* the central bank's interest rate target reacts sufficiently to inflation – if $\phi > 1$ – then it seems that a pure interest rate target, with no control of monetary aggregates, no commodity standard or peg, and no "backing" beyond pure fiat, can determine at least the inflation rate, if not quite the price level. It seems that making the peg react to economic conditions overturns the classic doctrine that inflation is indeterminate under an interest-rate peg.

This example makes it crystal-clear that inflation determination comes from a threat to increase future inflation if current inflation gets too high, not from the stabilization of inflation common in verbal and old-Keynesian stories. Also, as King (2000) emphasizes, $\phi < -1$ works just as well as $\phi > 1$ to ensure determinacy. If the Fed threatens oscillating hyperinflation and deflation, that threat is just as effective in ruling out equilibria other than (5).

3 Ruling out non-local equilibria

This is a beautiful result, and writers who are in a hurry to get to price-quantity dynamics, optimal monetary policy and so forth may be forgiven for choosing the unique nonexplosive equilibrium and moving on to those issues. But for my purpose, understanding inflation determination, we need to take a closer look. How, exactly, did we rule out the explosive equilibria?

In fact, nothing in *economics* rules out nominal explosions, and hyperinflations are historic realities. Economics (transversality conditions) can rule out *real* explosions. That's why we choose the unique forward-looking solution for price = present value of dividends. Furthermore, in the full nonlinear model, the "non-local" equilibria are not always explosive, making it harder still to rule them out. In sum, once we think about it for a moment, we cannot in fact say that an interest rate target that follows the Taylor rule, by itself, determines the inflation rate in this class of models.

To make these claims, I really must write down a model in a more explicit fashion. To keep the discussion compact, I just simplify the standard source, Woodford (2003).

3.1 The model

Consumers maximize a standard utility function

$$\max E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j})$$

Consumers receive a constant nonstorable endowment $Y_t = Y$. They trade in complete financial markets described by real contingent claims prices m_{t+1} and hence nominal contingent claims prices

$$Q_{t,t+1} = \frac{P_t}{P_{t+1}} m_{t+1}.$$

The interest rate is related to contingent claim prices by

$$\frac{1}{1+i_t} = E_t \left[Q_{t,t+1} \right].$$

The government issues one-period nominal debt; the face value issued at time $t-1$ and coming due at date t is $B_{t-1}(t)$. The government also levies lump-sum taxes T_t, net of transfers, which it uses to fund any difference between debt redemptions and new debt sales.

I follow Woodford (2003) and many others in describing a frictionless economy. One may be a bit disturbed by the presence of prices and no money, but this specification does make sense. At the simplest level, we can specify that the government pays interest on money, equal to the interest it pays on one-period nominal debt. At this point, money is exactly equivalent to nominal debt, so there is no point in carrying around two letters for the same thing. (Woodford 2003 trades a few more symbols for a bit of comfort and presents the model with interest-paying money in this way.) Alternatively, remember that money M in monetary models represents money held overnight, usually subject to an interest cost. Thus, a "cashless economy" can operate well if agents exchange maturing government bonds for cash in the morning, use the cash for transactions during the day, and then pay taxes and buy new government debt with cash (execute repurchase agreements) at the end of the day, holding no money overnight when money is counted and interest is charged. The "price level" in this "cashless economy" still refers to the tradeoff between cash and goods. (Cochrane 2005 describes this situation in a cash-in-advance model.) Why the price level is *determinate* in such an economy is the question we are after, but if it is, there is no harm in talking about nominal prices even though no money is held overnight, $M = 0$. The economy can also be truly cashless, using electronic claims to maturing government debt as medium of exchange. A "dollar" is then defined by the right to exchange a "dollar" of maturing debt to extinguish a "dollar" of tax liability. This economy truly is cashless, but the price level, if determinate, is still a well-defined object.

The consumer faces a present-value budget constraint

$$E_t \sum_{j=0}^{\infty} Q_{t,t+j} P_{t+j} C_{t+j} = B_{t-1}(t) + E_t \sum_{j=0}^{\infty} Q_{t,t+j} P_{t+j} \left(Y_{t+j} - T_{t+j} \right). \tag{6}$$

or, in real terms,

$$E_t \sum_{j=0}^{\infty} m_{t,t+j} C_{t+j} = \frac{B_{t-1}(t)}{P_t} + E_t \sum_{j=0}^{\infty} m_{t,t+j} \left(Y_{t+j} - T_{t+j} \right). \tag{7}$$

(The Appendix contains a somewhat more careful, but also somewhat more lengthy, treatment of budget constraints.)

This is a closed economy with no capital, so the equilibrium condition states that equilibrium consumption must equal the constant endowment,

$$C_t = Y.$$

3.2 Equilibria

The consumer's first order conditions state that marginal rates of substitution equal contingent claims price ratios

$$\beta \frac{u_c(C_{t+1})}{u_c(C_t)} = m_{t+1}. \tag{8}$$

Equilibrium $C_t = Y$ implies a constant real discount factor

$$m_{t+1} = \beta \frac{u_c(Y)}{u_c(Y)} = \beta.$$

Therefore, the real interest rate is constant,

$$\frac{1}{1+r} = E_t \left(m_{t+1} \right) = \beta$$

and the nominal discount factor is

$$Q_{t,t+1} = \frac{P_t}{P_{t+1}} m_{t+1} = \beta \frac{P_t}{P_{t+1}}. \tag{9}$$

The one period interest rate then follows a Fisher relation,

$$\frac{1}{1+i_t} = E_t(Q_{t,t+1}) = \beta E_t \left(\frac{P_t}{P_{t+1}} \right) = \frac{1}{1+r} E_t \left(\frac{1}{\Pi_{t+1}} \right) \tag{10}$$

The usual relation

$$i_t \approx r + E_t \pi_{t+1}$$

follows by linearization.

From the consumer's present value budget constraint (6), equilibrium $C_t = Y$ also requires

$$B_{t-1}(t) = \sum_{j=0}^{\infty} E_t \left(Q_{t,t+j} P_{t+j} T_{t+j} \right)$$

and using contingent claim prices from (9),

$$\frac{B_{t-1}(t)}{P_t} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t\left(T_{t+j}\right). \tag{11}$$

This "government debt valuation equation" says that in equilibrium, the value of government debt is the present value of future tax payments.

The Fisher equation (10) and the government debt valuation equation (11) are the *only* two conditions that need to be satisfied for the price sequence $\{P_t\}$ to represent an equilibrium. If they hold, then the allocation $C_t = Y$ and the resulting contingent claims prices (9) imply that markets clear and the consumer has maximized subject to his budget constraint. Obviously and hardly surprisingly, the equilibrium is not yet *unique*, in that many different price or inflation paths will work. We need *some* specification of monetary and fiscal policy to determine the price level.

The new-Keynesian/Taylor rule analysis maintains a "Ricardian" fiscal regime; net taxes T_{t+j} are assumed to adjust so that the government debt valuation equation (11) holds given any price level. It adds a Taylor rule, for example $i_t = r + \phi\pi_t$ to this analysis; the government simply announces the nominal interest rate and stands ready to buy and sell government debt at this rate. The only equilibrium condition is then the Fisher equation (10). Solutions of the simple model consisting of a Fisher equation and a Taylor rule (1)-(2), as I studied above, do then represent the full set of equilibrium conditions of this explicit model.

3.3 Explosive equilibria?

Nothing in the economics of this system rules out explosive inflation or deflation or "non-local" equilibria. For example, let us rule out uncertainty by studying perfect-foresight equilibria. (Including uncertainty can only further expand the set of equilibria.) Suppose the central bank follows an interest rate rule of the form

$$1 + i_t = (1+r)\left[1 + \phi\left(\Pi_t - 1\right)\right]. \tag{12}$$

with $\phi > 1$. Substituting this interest rate rule in the exact Fisher equation (10), we find that equilibrium inflation must follow

$$\Pi_{t+1} = 1 + \phi\left(\Pi_t - 1\right). \tag{13}$$

One equilibrium is of course $\Pi_t = 1$, but let's try an explosive equilibrium

$$\left(\Pi_t - 1\right) = \phi^t\left(\Pi_0 - 1\right),\ \Pi_0 > 1$$

as well. Is this an equilibrium? The Fisher equation (10) implies

$$1 + i_t = (1+r)\,\Pi_{t+1} = (1+r)\,\phi^{t+1}\left(\Pi_0 - 1\right).$$

Once the Fisher equation is satisfied, the real discount factor is constant, so the consumer's first order conditions are satisfied with goods market equilibrium $C = Y$ as before.

Now all we need to check is the government debt valuation equation (11). In the new-Keynesian tradition, we just assume that taxes adjust so that (11) holds. Let's check that this is possible and sensible. Start with any P_0 (the new-Keynesian model never claimed to determine the price *level*, only the inflation *rate*). In each period, the government has to redeem old debt, tax, and sell new debt. Mechanically, these choices must satisfy the government flow budget constraint: proceeds from new debt sale + taxes = cost to redeem old debt,

$$\frac{1}{1+r}\frac{B_t(t+1)}{P_{t+1}} + T_t = \frac{B_{t-1}(t)}{P_t}. \tag{14}$$

The "Ricardian Regime" assumption further limits these choices: *given* the path of the price level, the government chooses $\{T_t, B_t(t+1)\}$ so that the real value of government debt does not explode,

$$\lim_{j\to\infty} E_t\left(m_{t,t+j}\frac{B_{t+j}(t+j+1)}{P_{t+j}}\right) = \lim_{j\to\infty}\frac{1}{(1+r)^j}\left(\frac{B_{t+j}(t+j+1)}{P_{t+j}}\right) = 0 \tag{15}$$

(The second equality specializes to the constant real rate and perfect foresight of this example.) The combination of (15) and (14) implies that the government debt valuation equation (11) holds.

There are lots of ways the government can implement such a policy. We only need to exhibit one to verify that explosive inflation really is a valid equilibrium. As a simple example, the government can plan a constant level of real debt,

$$\frac{B_{t-1}(t)}{P_t} = \bar{B}$$

This policy certainly satisfies the condition (15). Now, to satisfy (14), the government has to charge lump sum taxes to service the debt,

$$T_t = \left(1 - \frac{1}{1+r}\right)\bar{B}$$

each period.

In sum, we have verified that this hyperinflationary equilibrium corresponding to a Taylor rule is in fact a completely valid equilibrium, globally, of this explicit model. Nothing real explodes, all budget constraints and first order conditions are satisfied. Nothing in economics rules out this equilibrium.

3.4 Non-Ricardian Regimes

We can determine inflation in this frictionless model by strengthening the government valuation equation – specifying a non-Ricardian regime. Since it figures so much in the following discussion, I quickly review how such a regime can work.

As the simplest example, suppose the government simply commits to a path of real net taxes $\{T_t\}$. The initial stock of government debt $B_{t-1}(t)$ is given. Then, (11) by itself determines the initial price level,

$$\frac{B_{-1}(0)}{P_0} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^t} T_t. \tag{16}$$

This is exactly the same as the mechanism by which the market value of shares of stock are determined as the present value of dividends. (See Cochrane 2005 for a detailed discussion of the analogy, a demonstration that a non-Ricardian regime is possible, does not violate government budget constraints, and is even plausible.)

By varying the path of nominal debt sold at each date, the government can then control the price level date by date. For example, the government flow identity at the first date is, proceeds from bond sales + taxes = cash required to redeem bonds,

$$\frac{1}{1+r}\frac{B_0(1)}{P_1} + T_0 = \frac{B_{-1}(0)}{P_0}.$$

$B_{-1}(0)$ is given; we just determined P_0 from (16), and the tax sequence including T_0 is given in this regime. Thus, by changing how much nominal debt it sells $B_0(1)$, the government can ensure any price level P_1 that it wants. The real value of government debt is fixed by the real present value of taxes. As the government auctions more nominal debt $B_0(1)$, the future price level P_1 rises, the interest rate rises $1 + i_0 = (1 + r)\, P_1/P_0$, and the bond price declines by exactly enough that the real value of the debt $B_0(1)/P_1$ and the revenue raised from its sale is unchanged. Analogously, if a corporation does a 2 for 1 split, the stock price halves; the corporation can set the stock price to any value it desires by an appropriate split.

Rather than set the *quantity* of new debt $B_t(t+1)$, and let the price adjust at each date, the government can just as easily set the *price* of new debt, i_t, at each date, and let the quantity adjust. The "monetary authority" can set interest rates $\{i_t\}$, by "open market operations" in which it freely exchanges bonds $B_t(t + 1)$ for cash (or maturing bonds $B_{t-1}(t)$) at the posted price $1/(1+i_t)$. By exactly the same logic, $1+i_t = (1+r)P_{t+1}/P_t$, will now determine P_{t+1}. If a corporation repurchased its shares, and then announced that it will issue new shares in any quantity demanded at half the initial market price, it would sell double the quantity, effecting a 2-1 split by price rather than quantity.

In sum, if the government fixes (or is forced to fix) the path of taxes $\{T_t\}$, rather than commit to changing them ex-post in response to inflation and deflation, then we have a unique equilibrium inflation rate and a unique equilibrium price level, despite having a fixed nominal interest rate target. The extension to uncertainty and more complex models does not change the basic picture.

One may wonder, how is it that I accept explosive solutions in the new-Keynesian model, while I deny them in the non-Ricardian regime? The fundamental difference is that there is a transversality condition forcing the consumer to avoid *real* explosions, but no corresponding condition forcing anyone to avoid nominal explosions. Correspondingly, there is an economic mechanism forcing (11) to hold in a non-Ricardian regime. If the price level is below the value specified by (11), nominal government bonds appear as net wealth to consumers. They will try to increase consumption. Collectively, they can't do so, so this increase in "demand" will push prices back to the equilibrium level. There is no corresponding mechanism to push inflation to the value (5). In the new-Keynesian model we are choosing *among* equilibria; supply equals demand for any of the alternative paths. In the non-Ricardian regime, we are finding the unique equilibrium itself; supply equals demand only at the unique equilibrium.

This regime is the heart of Leeper (1991), Sims (1994), and Woodford's (1995) demonstration that a non-Ricardian fiscal rule can uniquely determine the price level, even with a pure interest rate target. Woodford (2001) applies this regime to the "bond price support" period of US monetary policy in the late 1940s and early 1950s, in which the US did experience stable inflation and an interest rate peg. Fixed taxes are a simple example, but weaker restrictions on taxes give rise to a Ricardian vs. non-Ricardian regime, and are obviously a complication one would adopt for realistic models. All that is necessary for a Ricardian regime is that the present value of terminal government debt converges to zero. In this model, that means real debt must grow at less than the real interest rate. Thus, for example, a Ricardian regime results if taxes respond at all to the real value of debt. See Canzoneri, Cumby and Diba (2001). "Ricardian" and "non-Ricardian" are in many ways unfortunate labels. For example, the government pays back its debts in equilibria from both regimes, unlike the "non-Ricardian" overlapping generations models. However, this language has become standard.

3.5 Non-Ricardian regimes and new-Keynesian models

This is a lovely regime, and a potential baseline for understanding US monetary policy and price level determination. However, it is not a simple footnote to add to a standard new-Keynesian analysis in order to solve multiple-equilibrium and global-determinacy issues.

First, the Taylor principle has completely vanished. This regime produces a unique equilibrium for *any* interest rate target, including simple fixed nominal interest rates. Whether interest rates react more or less than one for one with inflation is completely irrelevant to price-level determination in this regime.

Second, this regime does not rule out hyperinflationary solutions; it does not deliver a magic bullet (or magic footnote) that allows us to focus on the unique locally bounded equilibrium when the Fed does follow a Taylor rule. To see this, suppose the Fed follows the Taylor rule (12), so inflation dynamics follow $\Pi_{t+1} = 1+\phi\left(\Pi_t - 1\right)$. The initial stock of nominal debt $B_{-1}(0)$ is given, so the initial price level P_0 solves the time-zero government valuation equation (16). This determines $\Pi_0 = P_0/P_{-1}$, and all further inflation follows. *Which* solution the economy follows depends entirely on the stock of nominal debt $B_{-1}(1)$ vs. the time-zero present value of net taxes; *any* of them are possible[2].

[2] The appearance of P_{-1} is a bit strange here. We only need it so we know what interest rate the Fed chooses at time zero. Arguments about time zero are always confusing, but that's not the way to think of

A new-Keynesian hopes for something else, that the non-Ricardian assumption could act as the equilibrium-selection device and give us just the unique local equilibrium of the new-Keynesian system (and perhaps the initial price level), no more, no less. Perhaps one can construct a regime that is locally Ricardian, but non-Ricardian considerations rule out the multiple equilibria. After all, governments can and do change taxes in response to small variations in debt, while we know there is a limit to taxing ability in the tails. I review two suggestions along these lines in the next section.

4 New-Keynesians agree (if you read closely)

The central theoretical question is, again, why we should restrict attention to local, non-explosive equilibria of a new-Keynesian Taylor rule model? The only way to answer this question is, as above, to consider equilibria *globally*, and then search for some *economic* reason to rule out the explosive paths.

Woodford (2003) treats this issue in Ch. 2.4, starting in p. 123, and continues in Ch. 4.4 starting on p. 311. Since Woodford summarizes the literature, and is the standard reference, it is useful to focus on his analysis. Benhabib Schmitt-Grohé and Uribe (2002) nicely formalize some of Woodford's suggestions, and apply them to ruling out "liquidity traps."

Consider the perfect foresight version of the above model. Consider an interest rate rule

$$i_t = \Phi(\Pi_t);\ \Pi_t = P_t/P_{t-1}. \tag{17}$$

$\Phi(\cdot)$ is a function, so we allow nonlinear policy rules. The Fisher relation, equivalently the consumer's first order condition (10), is

$$\Pi_{t+1} = \beta(1 + i_t). \tag{18}$$

As in my linearized examples, we are looking for solutions to the pair (17) and (18). We can find solutions by substituting out the interest rate and studying directly the equation

$$\Pi_{t+1} = \beta\left[1 + \Phi(\Pi_t)\right]. \tag{19}$$

Benhabib Schmitt-Grohé and Uribe's first point is that a Taylor rule with slope greater than one cannot apply *globally*, because nominal interest rates should[3] not be less than zero. Thus, we should think about the situation as illustrated in Figure 1. The equilibrium at Π^* satisfies the Taylor principle, and is a "unique *local* equilibrium" of the model. Any value of Π_0 other than Π^* leads away from the neighborhood of Π^* as shown. With a lower bound on nominal interest rates, however, the function $\Phi(\Pi)$ must also have another stationary point, labeled Π_L. This stationary point must violate the Taylor

the issue. We use perfect foresight models to think through how the economy reacts to shocks at time t, and then every time t acts like a new time zero. Thus, it is appropriate here to think of P_{-1} as existing.

[3] I say "should," not "must" because negative nominal rates are theoretically possible if money vanishes. See Bassetto (2004).

principle, cutting at less than 45° as shown. Therefore, many paths lead to Π_L and there are "multiple local equilibria" near this point. Schmitt-Grohé and Uribe's second point is that even restricting attention to "locally bounded" or "nonexplosive" equilibria (for whatever reason) is not enough to ensure global determinacy.

In sum, at a minimum, we all agree that all of these are perfectly valid equilibria, and we need something beyond just the "non-local" property to rule them out.

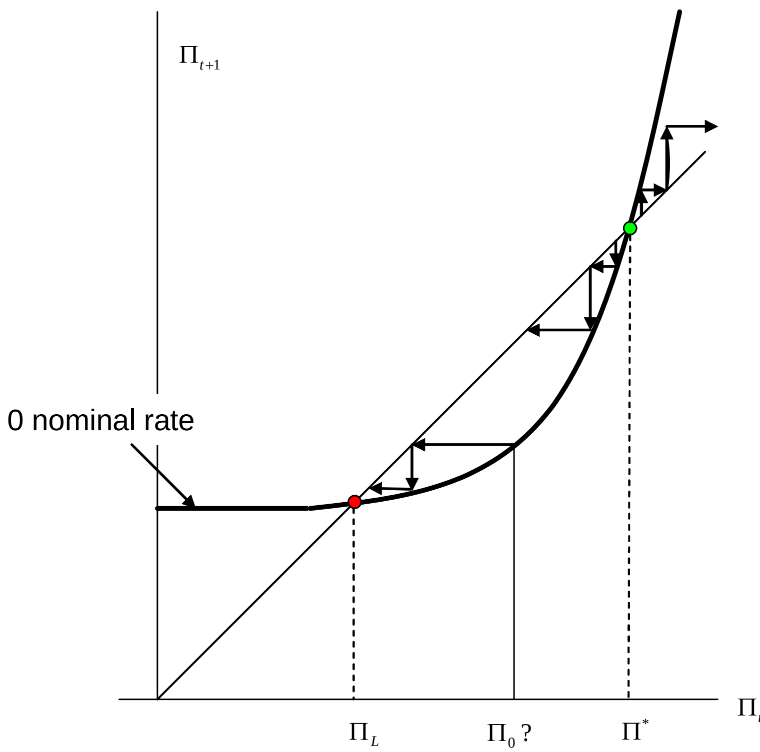

Figure 1: Dynamics in a perfect foresight Taylor-rule model.

4.1 Reasonable expectations?

Now, how can we rule out the multiple equilibria near Π_L, as well as the explosive equilibria to the right of Π^*? First, (p.128) Woodford argues that

> "The equilibrium ..[Π^*].. is nonetheless *locally* unique, which may be enough to allow expectations to coordinate upon that equilibrium rather than on one of the others."

Similarly, King (2000, p. 58-59) writes

> "By specifying [$\phi > 1$] then, the monetary authority would be saying, 'if inflation deviates from the neutral level, then the nominal interest rate will be

> increased relative to the level which it would be at under a neutral monetary policy.' If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.

Needless to say, these are rather weak arguments on which to hang the first question of macroeconomics. Again, notice the absence of any *economic* force (supply, demand) pushing inflation to the desired value.

Importantly for judging the reasonableness of expectations, Woodford argues that we should not think of an economy making an ε "mistake" and therefore slipping from Π^* into an explosive equilibrium; instead we should think of expectations of future inflation driving inflation today: (p. 128)

> Indeed it is often said that .. the steady state with inflation rate Π^* is "unstable" implying that an economy should be expected almost inevitably to experience either a self-fulfilling inflation or a self-fulfilling deflation under such a regime.
>
> Such reasoning involves a serious misunderstanding of the causal logic of the difference equation [(19)]. This equation does not indicate how the equilibrium inflation rate in period $t+1$ is determined by the inflation that happens to have occurred in the previous period. If it did it would be correct to call Π^* an unstable fixed point of the dynamics–even if that point were fortuitously reached, any small perturbation would result in divergence from it. But instead, the equation indicates how the equilibrium inflation rate in period t is determined by *expectations* regarding inflation in the following period... The equilibria that involve initial inflation rates near (but not equal to) Π^* can only occur as a result of expectations of *future* inflation rates (at least in some states) that are even *further* from the target inflation rate. Thus the economy can only move to one of these alternative paths if expectations about the future change significantly, something that one may suppose would not easily occur."

Now, a "serious misunderstanding of causal logic" is a strong charge, and I think unwarranted here. The equations of the model do not specify a causal ordering. They are just equilibrium conditions. And I think Woodford has slipped in a small misunderstanding of his own, by implicitly assuming that expectations must be formed exogenously. If you are observing an unstable dynamic system, and you see a small change today, the fact of that change causes a large change in your expectations of the future. If you see the waiter trip, it's a good bet the stack of plates he is carrying will crash. In particular, in the system of section 2 in which the policy rule contained a small disturbance, there is nothing that prevents agents from seeing the disturbance, knowing the Fed will feed back on its own past mistakes, thinking "oh no, here we go," and radically changing their expectations of the future. They don't need to wake up and think "gee, I think there will be a hyperinflation" before reading the morning paper. In fact, the forward-looking solutions rely exactly on this mechanism; they rely on a fortuitous jump in near-term expectations in response to a shock, to put the economy back on the saddle path that has no change in asymptotic expectations.

Still, there is some appeal to the argument that expectations of hyperinflations or liquidity traps seem far-fetched. But expectations that are far-fetched in our intuitive understanding of our own world are not necessarily so far-fetched for agents in this model. In this model, the Fed is absolutely committed to raising interest rates more than one for one with inflation, for all values of inflation. In this model, real rates are constant, so the Fed is committed to raising future inflation in response to higher past inflation – precisely the opposite of the stabilizing language that pours out of the real-world Federal Reserve's account of its actions. If we really lived in such a world, I, for one, would confidently expect hyperinflation. If we think that forecast is unreasonable, it means we don't believe the model. In particular, we don't really think the Fed ever would respond to past inflation by *increasing* future inflation, or at least that it is so pig-headed as to keep doing so forever as inflation explodes.

4.2 Fiscal solutions

Recognizing, I think, the weakness of these arguments about what is or is not "reasonable" for people to expect (if not, we would not need to go on), and how centrally important the question of inflation-determination is, Woodford's real answer lies in the analysis of section 4.2 "Policies to prevent a deflationary trap" (i.e. to cut off equilibria to the left of Π^*) and 4.3 "Policies to prevent an inflationary panic" (i.e. to cut off equilibria to the right of Π^*). In both of these sections, Woodford fundamentally argues for price-level determinacy by moving to a non-Ricardian regime, and having *fiscal* policy prune equilibria. (Woodford notes on p. 124 that the model we have discussed so far is completed by an explicitly Ricardian fiscal regime.)

This fact is clearer and more explicit in Benhabib, Schmitt-Grohé and Uribe's (2002) treatment. The main point of their paper is explicitly to introduce non-Ricardian regimes in order to eliminate the liquidity trap equilibrium. (They credit an early draft of Woodford's analysis for that basic idea, and the constant debt growth implementation.) Their first proposal is simply to pair the Taylor rule for interest rates with a commitment by the government that, in low-inflation states, it will lower taxes so much that real debt grows explosively.

In particular, (their equations (18)-(20)) they specify that in a neighborhood of Π_L, the government will start making net lump-sum transfers funded by borrowing. In my notation, they specify that tax rates will follow $T_t = \alpha(\Pi_t)\,(B_{t-1}/P_t)$ with $\alpha(\Pi_L) < 0$. The transversality condition holds, and the regime is Ricardian, if real taxes respond positively at all to the real value of debt, if $\alpha > 0$ and vice versa. Therefore, with this specification, if we get to Π in the region near Π_L, real debt explodes, the limiting condition (15) is violated, and the government debt valuation equation cannot hold. Ergo, this region is ruled out as an equilibrium. As they note, this policy sounds like common advice to "spend your way out of a liquidity trap."

They show that this kind of policy can also be implemented if the government to commits to a fixed growth rate for nominal liabilities. If the growth rate of total nominal liabilities (debt in my simplification) is greater than the interest rate at Π_L but less than the interest rate at Π^*, then the required transfer at Π_L emerges naturally from the government's flow constraint. If we rearrange the flow constraint (14) as

$$\frac{1}{1+r}\frac{P_t}{P_{t+1}}\frac{B_t(t+1)}{B_{t-1}(t)} = 1 - \frac{T_t}{B_{t-1}(t)/P_t} = 1 - \alpha \tag{20}$$

we can see that a growth rate $B_t(t+1)/B_{t-1}(t) > \Pi_{t+1}(1+r) = 1 + i_t$ means negative net taxes $\alpha < 0$ and vice versa. If you commit to a large nominal growth rate at low inflation, you're going to borrow; if you commit to a small nominal growth rate at a large inflation, you're going to pay back debt. As they note, this policy rings of proposals to escape liquidity traps by printing unbacked money.

Woodford suggests this possibility as well (p. 132): "let total nominal government liabilities D_t be specified to grow at a constant rate $\bar{\mu} > 1$ while monetary policy is described by the Taylor rule (4.1). [My (17)]." "Thus, in the case of an appropriate *fiscal* [my emphasis] policy rule, a deflationary trap is not a possible rational expectations equilibrium."

The extra adjective "fiscal" covers a lot. "Let total nominal Government liabilities D_t be specified.." is an *additional* assumption, one that violates the explicitly Ricardian assumptions of the model described so far. Growing nominal debt in a deflation means that real debt grows explosively. We are pruning equilibria and determining the price level by changing the Ricardian *fiscal* regime.

Woodford tries a different approach in section 4.3, p. 135 "Policies to prevent an inflationary panic," i.e. to rule out equilibria to the right of Π^*. After two minor suggestions, reviewed below, Woodford's main suggestion is (p. 138):

> ...self-fulfilling inflations may be excluded through the addition of policy provisions that apply only in the case of hyperinflation. For example, Obstfeld and Rogoff (1986) propose that the central bank commit itself to peg the value of the monetary unit in terms of some real commodity by standing ready to exchange the commodity for money in event that the real value of the total money supply ever shrinks to a certain very low level. If it is assumed that this level of real balances is one that would never be reached except in the case of a self-fulfilling inflation, the commitment has no effect except to exclude such paths as possible equilibria. ...[This proposal could] well be added as a hyperinflation provision in a regime that otherwise follows a Taylor rule.

This proposal is inherently *fiscal* as well. In order for the government to exchange the money stock (nominal liabilities) for some real commodity, it has to have sufficient stocks of that commodity on hand, or a commitment to raise enough tax revenue to obtain the commodity. A purely Ricardian fiscal regime cannot defend a commodity standard. Governments facing hyperinflations, in fact, notoriously do not have the resources to redeem even their small money stocks, a point emphasized by Sims (1994, 2003).

Benhabib, Schmitt-Grohé and Uribe also acknowledge (footnote 10) that the explosive "non-local" equilibria to the right of Π^* are perfectly valid equilibria of the model, but they do not say anything about how to rule them out.

In sum, then, I read Woodford and Benhabib, Schmitt-Grohé and Uribe's analysis as an agreement on the central points. First, nothing in *economics* rules out non-local equilibria, since nothing in economics rules out nominal hyperinflation or deflation. Hence, they agree that we cannot jump from "unique *local* equilibrium" to "unique equilibrium" without further analysis. Second, the central ingredient both authors add in an attempt to rule out the undesired equilibrium is a non-Ricardian fiscal policy, in which the price level is determined by the valuation equation for government debt.

4.3 Do these solutions work?

Uncoordinated policy

An equilibrium requires coordination between fiscal and monetary policy. Woodford and Benhabib, Schmitt-Grohé and Uribe do *not* suggest that the government move to well-specified, fiscally-dominant or non-Ricardian but coordinated regimes. If they did so, these events would still count as equilibria, the paths to them would count as equilibria, and we would not have pruned anything. They suggest instead that in certain states the government will adopt an *uncoordinated* policy that rules out *any* equilibrium, and thus rules out the paths leading to those equilibria.

The proposals to rule out deflation with nominal debt printing or by government spending sound like sensible and time-honored prescriptions to inflate the economy, i.e., to head back to the desired equilibrium Π^*. Benhabib, Schmitt-Grohé and Uribe even describe their proposal to increase government spending this way (p. 548):

> Interestingly, this type of policy prescription is what the U.S. Treasury and a large number of academic and professional economists are advocating as a way for Japan to lift itself out of its current deflationary trap...A decline in taxes increases the household's after-tax wealth, which induces an aggregate excess demand for goods. With aggregate supply fixed, price level must increase in order to reestablish equilibrium in the goods market.

That is, indeed, how a coordinated fiscally-dominant regime works, good intuition for operation of the fiscal theory of the price level, and undoubtedly what real-world proponents of the policy had in mind. But that's *not* the equilibrium-selection proposal. The proposal does *not* steer the economy back to Π^*. The proposal sits at Π_L with an uncoordinated policy and explodes government debt. Inflation dynamics are still determined as in Figure 1 and $\Phi(\Pi)$ is unchanged. This is a good example of the dangers of mixing new-Keynesian equations, which rely on de-stabilizing dynamics and ruling out equilibria, with old-Keynesian intuition, which relies on stabilizing dynamics.

The point may be even clearer if we recognize that the same logic can be used to cut off equilibria to the right of Π^*: Just specify that $\alpha(\Pi) < 0$ for $\Pi > \Pi^*$ as well. Now, "spend your way out of a hyperinflation" does not have quite the ring that "spend your way out of a liquidity trap" does, but the equations work just as well.

Similarly, we might ask exactly how a contingent commodity standard rules out a hyperinflation. Why can't we just hyperinflate, hit the bound, redeem the currency, and then continue on our merry way with a new currency? If so, the hyperinflation is a perfectly valid equilibrium. Again, the key is that Woodford keeps the Taylor rule interest rate policy alive along with the redemption to a new currency. Again, we have an "uncoordinated" policy and no equilibrium.

It would be no different[4], and a lot simpler, if the government were simply to say "if inflation gets to 500% or 1% we commit to blow up the world." More seriously, there are plenty of monetary-fiscal policies that preclude equilibrium more transparently than a subtle threat to let the real value of debt grow too fast. Tax everything. Burn the money stock. Explode government debt today. Better yet, go back to a $\Phi(\Pi)$ function that includes negative nominal interest rates. Why go to all the effort to specify a monetary policy that can lead to equilibrium at low inflation, and then write an impossible fiscal policy to rule out that equilibrium? Threatening negative nominal interest rates would do just as well and would be a lot simpler. An old-Keynesian doctor tries to cure a sick economy. A new-Keynesian doctor threatens to kill it, so effectively that it makes sure not to get sick in the first place.

Reasonable expectations?

The trouble with any of these threats is that they are not credible – they are not at all a plausible description of people's expectations. The most sensible expectation regarding deflation or hyperinflation is that some muddle-through policy will emerge. Even in times of economic calamity and government chaos, *some* equilibrium has emerged, even if we had to use cigarettes as a medium of exchange. How can people sensibly believe that an undesired path – hyperinflation or deflation – would lead to *no* equilibrium, whatever that means?

Keep in mind, these proposals are supposed to describe people's expectations *now*, not a possible set of future threats the government could make and people might or might not believe, or a set of threats a theorist can write down to prune unpleasant equilibria of a model.

Economic analysis mirrors sensible expectations. In these proposals, both fiscal (spending, nominal debt printing, commodity standard) and monetary (interest rate rule) policies pig-headedly continue, and the theorist gives up, saying "no equilibrium is possible." In most economic analysis of such "uncoordinated" policy, however, one or the other policy soon gives way so the policy is "coordinated" after all. Sargent and Wallace (1981) famously described uncoordinated policy as a "game of chicken." Their point was that one side would swerve, not that the cars would crash.

[4] Bassetto (2005) would point out that there is a difference between an expectation that the government will "blow up the world" and an expectation that the government will follow an impossible course, such as an inconsistent policy. I won't be that careful about language or about game-theoretic underpinnings of rational expectations equilibria.

One reason such threats are not very credible is that, once inflation or deflation happens, carrying through on the threat is disastrous policy, i.e., ex-post very welfare-reducing. I do not argue that policy is always optimal, nor that commitment to ex-post unpleasant acts is impossible, but surely severely self-destructive threats are less likely to be carried out ex-post than milder threats, and thus less likely to be believed ex-ante.

Bottom line

In sum, one's natural response is that the above quote *is* right, the central bank *will* eventually give in, and the economy *will* head back to Π^*, rather than sit at "no equilibrium," whatever that means. I agree, but this view means that all of the multiple equilibria are still valid, and indeterminacy has not been cured. If we resist proposals that rely on Strangelovian threats, especially as a description of current expectations, because such threats are not credible, then we cannot use the idea that policy is locally Ricardian but globally non-Ricardian, and eventually uncoordinated, as a simple wand to eliminate the non-local equilibria of new-Keynesian models. Finally, if we accept these proposals, we must state that fiscal underpinnings are a crucial element to the story; interest rate rules alone are not enough to determine the inflation rate.

Isn't the fiscal theory equally guilty?

One might object that non-Ricardian regimes also rely on interesting-looking threats and expectations of off-equilibrium government behavior. I hope by now we have settled that the government *can* follow a "supply curve" in which the real present value of nominal debt explodes at off-equilibrium prices. Cochrane (2005) is a paper-length defense of this possibility. Most simply, nominal debt is simply a promise to print money, which can always be honored. More deeply, we understand that if a bubble lifts Microsoft's stock price, Microsoft need not respond by raising earnings. In fact, we usually state that a company has already maximized earnings, so that such a response would be impossible. A non-Ricardian regime just says that the government behaves the same way.

Less obviously, *will* the government let real debt explode at off-equilibrium prices, and, mirroring my criticisms above, is it sensible for people to believe that it will do so? Like a firm, governments at some point *must* be non-Ricardian, as taxes cannot rise arbitrarily. But before this limit is reached, will a government facing an "off-equilibrium" deflation choose to raise taxes to pay off this surprise gift to bond holders? Or will it roll over the debt, and wait for the deflation to end? The latter is exactly what the US did in the Great Depression and Japan did recently, so it's not so silly an expectation.

There is also a fundamental difference between expectations of what an agent will do *out of* equilibrium, and what it will do *in* an alternative equilibrium. The underpinnings of the non-Ricardian regime concern the former, the proposals to rule out equilibria of the new-Keynesian model concern the latter. Sensible expectations about off-equilibrium behavior do not have to respect market-clearing conditions – in fact, they can't. Sensible expectations of behavior *in* an equilibrium do need coherently to describe the entire economy.

In sum, the expectations about off-equilibrium behavior that underlie a non-Ricardian regime are not at all necessarily that unreasonable, and they accord with some historical experience.

Benigno and Woodford (2007) argue that a non-Ricardian regime is undesirable, that outcomes under the locally-Ricardian new-Keynesian equilibrium are more desirable, since they insulate the economy from fiscal shocks. This may be true, but following standard practice they simply choose the unique locally bounded equilibrium. If there is no way to ensure that this is the equilibrium the economy actually follows – if the new-Keynesian equilibrium simply does not determine the inflation rate – then the comparison is not valid. Also, the Ricardian regime endogenously generates a fiscal policy consistent with a stable price level. There is no reason the government in a non-Ricardian regime cannot choose the same fiscal policy, and generate the same stable price level. Again, Ricardian and non-Ricardian regimes produce observationally equivalent time series.

4.4 Weird rules and economic explosions

A number of other, non-fiscal, suggestions have been made to rule out inflationary and deflationary equilibria.

Weird rules

Woodford starts "Policies to prevent an inflationary panic" with a strengthening of the Taylor principle (p.136). He suggests that the Fed commit to a policy in which the graph in Figure 1 becomes *vertical* at some finite inflation above Π^*. Then, there is no rational-expectations equilibrium with exploding inflation; at least if "equilibrium" requires a finite price level. Similarly, Alstadheim and Henderson (2006) remove the Π_L equilibrium by introducing discontinuous policy rules, or V-shaped rules that only touch the 45° line at the Π^* point.

At one level, these proposals are not as extreme as they sound. After all, the Taylor principle in new-Keynesian models amounts to the Fed making unpleasant threats about behavior in alternative equilibria. The more unpleasant the threat (if it is believed), the more effective, so a threat to instantly hyperinflate away the entire monetary system certainly removes this branch as an equilibrium, and perhaps more effectively than threatening a hyperinflation or fiscal imbalance that will slowly gain steam. On the other hand, the real-world economy would presumably substitute to other moneys before the value of money reached zero, making this hyperinflationary path possible, if admittedly unpleasant. Again, I think the reasonable expectation the Fed would abandon such a $\Phi(\Pi)$ long before we actually reverted to a barter economy. And again, the point is to describe *current* expectations, and a vertical $\Phi(\Pi)$ seems far-fetched.

Letting the economy blow up

Schmitt-Grohé and Uribe (2000) offer a similar way to rule out hyperinflations, without assuming the Fed directly blows up the economy, by adding a little money. This idea is also reviewed by Woodford (2003 p. 137).

Schmitt-Grohé and Uribe's idea is easiest to express with real balances in the utility function, though a cash in advance formulation, or transaction cost formulation as in Sims (1994) can give the same result. I give a self-contained presentation of the model in the Appendix. With money, the Fisher equation contains monetary distortions:

$$1 + i_t = \beta^{-1} \Pi_{t+1} \frac{u_c(Y, M_t/P_t)}{u_c(Y, M_{t+1}/P_{t+1})}. \tag{21}$$

In other words, the real interest rate is not constant, despite the constant endowment, but is instead affected by real money balances

$$1 + r_t = \frac{u_c(Y, M_t/P_t)}{\beta u_c(Y, M_{t+1}/P_{t+1})}. \tag{22}$$

Substituting out i_t from the Taylor rule (17), inflation dynamics follow

$$\Pi_{t+1} = \beta \left[1 + \Phi(\Pi_t)\right] \frac{u_c(Y, M_{t+1}/P_{t+1})}{u_c(Y, M_t/P_t)}. \tag{23}$$

Now, equilibrium money balances can be found from the usual rearrangement of the first order condition for money vs. bonds, $M_t/P_t = L(Y, i_t)$, and of course i_t follows the Taylor rule. So, plugging all this in, we have another difference equation for inflation,

$$\frac{\Pi_{t+1}}{u_c\left[Y, L(Y, \Phi(\Pi_{t+1}))\right]} = \beta \frac{1 + \Phi(\Pi_t)}{u_c\left[Y, L(Y, \Phi(\Pi_t))\right]} \tag{24}$$

instead of (19).

The idea, then, is that *this* difference equation may rise to require $\Pi_{t+1} = \infty$ above some bound $\bar{\Pi}$, even if the Taylor rule $\Phi(\Pi_t)$ remains bounded for all Π. Woodford and Schmitt-Grohé and Uribe give examples of specifications of $u(C, M/P)$ in which this can happen, essentially in which the left side of (24) remains bounded at high inflation while the right side increases without bound.

As an additional perspective on this proposal, write from (22) and (23)

$$\Pi_{t+1} = \left[1 + \Phi(\Pi_t)\right](1 + r_t)$$

Thus, in this suggestion, Π_{t+1} is driven to infinity because the Fed pushes us to a region in which monetary distortions are so extreme that the *real* interest rate rate rises to infinity.

This is an example of a more general idea, familiar from similar proposals such as Obstfeld and Rogoff (1983), and Sims (1994) to rule out multiple equilibria under money supply targets rather than interest rates: perhaps the demand for money at extremes of hyperinflation and deflation behaves in such a way as to rule these paths out as economic equilibria, even if arrived at with coordinated government policy.

Is this the answer? I think not. First and most importantly, if we do not regard the government's threat to directly blow up the economy as a reasonable characterization of expectations, why is the threat by the government to take the economy to a configuration (hyperinflation or deflation) in which the we all know the economy will blow up all on its own any more credible? Surely the Fed would notice that real interest rates are infinite! Really, this is not much different than Woodford's suggestion of a vertical $\Phi(\Pi)$.

Second, it is delicate. In general, this approach relies on particular behavior of the utility function or the cash-credit goods specification at very low real balances. This particular proposal also needs the policy function $\Phi(\Pi)$ to rise sufficiently quickly on the right hand side of (24).

More deeply, are monetary frictions really important enough to rule out any hyperinflation above a certain limit, sending real rates to infinity, or deflation below another limit? We have seen some astounding hyperinflations, and real rates did not seem all that affected. (The real rate in (22) is not just the real rate on nominal contracts, it is also real rate for real contracts, i.e., for gold, foreign-currency, or goods-denominated transactions.) Is this, an explosive non-neutrality for hyperinflations greater than ever observed, after all, the central piece of economics that determines the price level? Once again, the point is not to find a specification of money demand that an economic theorist could adopt to avoid unpleasant equilibria. The point is a reasonable description of people's expectations *today*, that eliminates multiple equilibria in a class of models that are applied to data from our economy.

At a minimum, if we accept this as the answer, we negate the proposition that interest rate rules can be studied, and determine prices all by themselves, in frictionless economies. We *must* include money, study monetary frictions, and include at least a large footnote to a statement that interest rate rules are the key ingredient for inflation determination.

5 Determinacy in the three-equation model

One may, and perhaps should, be uncomfortable with the analysis so far. Most of the analysis introduced no frictions at all. The stories one hears about Taylor rules and inflation usually involve at least the Phillips curve and Fed control of real rates of interest: nominal rates rise, gaps appear, these gaps drive down inflation. Maybe it is simply a mistake to think one can analyze inflation determination in a frictionless model. Let's analyze inflation determinacy in a real new-Keynesian model.

5.1 The new-Keynesian three-equation model

Throughout, I will base the analysis on standard New-Keynesian IS-LM models, for example as in the excellent expositions in King (2000) and Woodford (2003). For determinacy questions, we can work with a stripped-down model without constants or shocks

$$y_t = E_t y_{t+1} - \sigma r_t \tag{25}$$

$$i_t = r_t + E_t \pi_{t+1} \tag{26}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t \tag{27}$$

where y = output, r = real interest rate, i =nominal interest rate, π = inflation. This representation can represent deviations from a specific equilibrium of a model with shocks. See King (2000) and Cochrane (2007).

While seemingly ad-hoc, the point of the entire literature is that this structure has exquisite micro-foundations, which are laid out in Woodford (2003) and King (2000). The first two equations derive from consumer first order conditions for consumption today vs. consumption tomorrow. The last equation is the "new-Keynesian Phillips curve." It is derived from the first order conditions of intertemporally-optimizing firms that set prices subject to costs. What makes it "new" is the steady advance of the timing of inflation on the right hand side. Phillips would have had a constant: output is higher when inflation is higher. An "accelerationist" would put π_{t-1} on the right hand side: output is higher when inflation is *increasing*. Friedman (1968) and Lucas (1972) would put $E_{t-1}(\pi_t)$ on the right hand side: output is higher if inflation is higher than expected, Lucas' expectations being "rational" and Friedman's "adaptive." "New-Keynesians" put $E_t(\pi_{t+1})$ on the right hand side. This change in timing has dramatic effects on the dynamic properties of the model.

There is an active debate on the right specification of (27). Some authors including Furher and Moore (1995) and Mankiw and Reis (2002) point out that (27) specifies high output when inflation is high relative to *future* inflation, i.e. when inflation is *declining*, and that this prediction is contrary to fact. (This is an instance of the point that the slight change in timing has dramatic implications). Mankiw and Reis argue for a return to mechanical or adaptive expectations, i.e. π_{t-1} on the right hand side, though this means throwing out economic microfoundations. Others such as Galí (2003), Sbordone (2005) and Cogley and Sbordone (2005) respond that marginal cost should really appear in place of output on the right hand side of (27). This change can save the estimate of the sign of γ, but the cost is that the "gap" series has nothing to do with recessions as conventionally understood. Galí's Figure 2 is particularly dramatic on this point – the implied "gap" in this figure has essentially no correlation with detrended GDP or recessions. In deference to this debate we might consider values $\gamma < 0$. Bilbiie (2006) presents a model with limited participation which also predicts an equation of the form (27) with $\gamma < 0$. Some models add lagged inflation π_{t-1} to (27). However, this change adds another state variable, raising the algebraic complexity considerably without changing the basic picture. All such models still look for the same number of unstable eigenvalues.

Now it is clearly true that if the Fed sets $i_t = 0$, then $\pi_t = 0$, $y_t = 0$ are *an* equilibrium. But setting $i_t = 0$ does not determine that this is the *only* equilibrium. To see this point, I find it useful to write (25)-(26) with $i_t = 0$ in a standard form as

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \tag{28}$$

Since the model only restricts the dynamics of *expected* future output and inflation, we have multiple equilibria. Any

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \delta_{y,t+1} \\ \delta_{\pi,t+1} \end{bmatrix} \tag{29}$$

with $E_t \delta_{y,t+1} = 0,\ E_t \delta_{\pi,t+1} = 0$ is valid, not just $\delta_{y,t+1} = \delta_{\pi,t+1} = 0$ and hence $y_t = \pi_t = 0$ for all t.

Perhaps however the dynamics of (28) are explosive, so at least $y = \pi = 0$ is the only *locally-bounded* equilibrium; the only one in which $E_t (y_{t+j})$ and $E_t (\pi_{t+j})$ stay near zero. Alas, this hope is dashed as well: the eigenvalues of the transition matrix in (28) are less than one. This fact should not surprise us. We have just verified the usual doctrine holds that an interest rate peg does not determine inflation.

To determine inflation, then, it is not enough to say what nominal interest rates will be in the desired equilibrium $i_t = y_t = \pi_t = 0$. We must say that policy will do something else in the other equilibria, in order to rule them out. For example, let us specify the simplest sort of Taylor rule,

$$i_t = \phi \pi_t. \tag{30}$$

The dynamics generalize from (28) to

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma (1 - \beta\phi) \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}. \tag{31}$$

The eigenvalues of this transition matrix are

$$\lambda = \frac{1}{2\beta} \left[(1 + \beta + \sigma\gamma) \pm \sqrt{(1 + \beta + \sigma\gamma)^2 - 4\beta (1 + \sigma\gamma\phi)} \right]. \tag{32}$$

Figure 2 presents the parameter regions in which both eigenvalues are greater than one in absolute value. (The algebra is in the Appendix.) For the usual parameter values $\sigma > 0, \gamma > 0$, we see that $\phi > 1$ guarantees both eigenvalues are greater than one. In this case, $E_t (y_{t+j})$ and $E_t (\pi_{t+j})$ explode in any equilibrium other than $y = 0$, $\pi = 0$. Ruling out such "non-local" equilibria, the new-Keynesian tradition concludes that output and inflation are again "determinate."

With the argument before us, you can see it is exactly the same as in the frictionless case. No economic consideration rules out the explosive solutions. With $\phi > 1$, the explosive solutions are legitimate solutions of the model, just as the multiple solutions are legitimate with $\phi < 1$.

One might complain that I have not shown the full, nonlinear model in this case, as I did for the frictionless model. Perhaps one can rule out the non-local equilibria by looking at the full statements of these models. Perhaps the unique local equilibrium is also the unique global equilibrium as well. This is a valid question. I do not pursue it here, as I find no claim in any new-Keynesian writing that this route can rule out the non-local equilibria. And there is no reason, really, to suspect that this route will work either. Equations (25)-(27) are linear approximations, so true output explosions are unlikely in the non-local equilibria of the true nonlinear model. The non-local equilibria may feature hyperinflation or deflation, but sensible economic models work in hyperinflation or deflation. If they don't, it usually reveals something wrong with the model rather than the price level. The two ingredients here are the "forward - looking IS curve" (25) and the "new-Keynesian Phillips curve" (27). The IS curve is simply derived from the

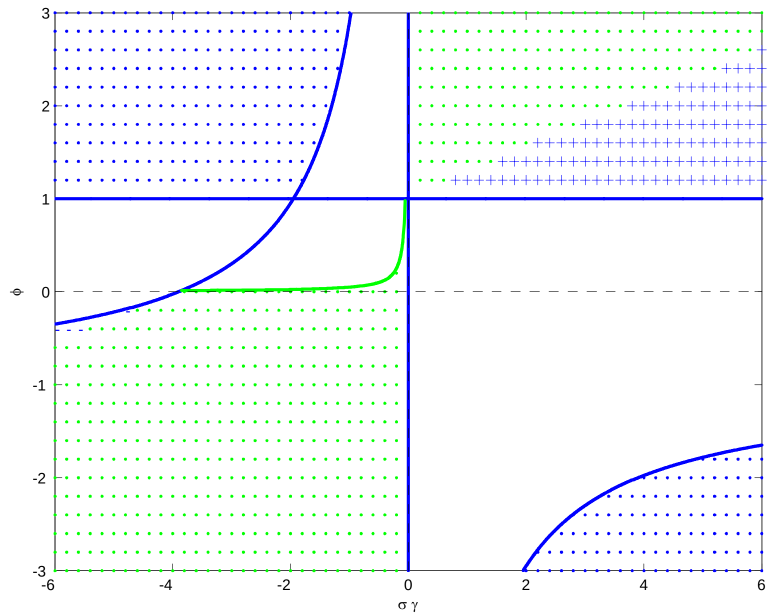

Figure 2: Regions of unique local equilibrium (both roots greater than one) for the three-equation new Keynesian model, in which the Fed follows a Taylor rule $i_t = \phi\pi_t$. $\beta = 0.95$. Blue indicates real roots, green indicates complex roots; +, - and $\cdot$ indicate positive, negative, and mixed roots respectively. Lines denote regions in which one root is equal to one.

consumer's intertemporal first order condition, $u_c(C_t, \cdot) = \beta E_t \left[u_c(C_{t+1}, \cdot)R_{t+1}\right]$. This first order condition does not break down in a hyperinflation or deflation. The only way to get the transversality condition to be violated is to specify unusual money demand functions or an inconsistent policy regime, as I reviewed in the last section. The Phillips curve is derived from optimal price-setting by monopolistically-competitive firms facing price adjustment costs. One can imagine specifications of this problem that do break down at high levels of inflation. However, all such models recognize that some of their simple fictions, such as fixed intervals between price changes, are useful approximations for low inflation. If they do break down at high inflation rates, one would not want to use that fact to decide that high inflation equilibria and paths leading to them are impossible.

Some of the additional suspicious behavior of the simple model is reflected in this three-equation new-Keynesian model. Figure 2 shows that with $\sigma\gamma > 0$ there is a second region,

$$\phi < -\left(1 + 2\frac{1+\beta}{\sigma\gamma}\right) \tag{33}$$

in which both eigenvalues are also greater than one in absolute value. As pointed out by King (2000), p. 78, the latter region works as well as the conventional former region to determine inflation. This is the counterpart of the $\phi < -1$ region studied above in which the Fed threatens oscillating hyperinflation and deflation. This region reminds us that threatened explosions, not stabilization, are behind the dynamics.

In fact, looking at the logic of what we are doing, there is no reason to limit ourselves to such mild equilibrium-selection devices. We want the model to produce one equilibrium, $y_t = 0, \pi_t = 0$, and we find that interest rate policy $i_t = 0$ will produce this equilibrium. The trouble is, if the Fed simply sets $i = 0$, there will be lots of other equilibria too. So the Fed needs to commit to doing something unpleasant in the other equilibria, to rule them out. Setting a different interest rate $i_t = \phi\pi_t$ with $\phi > 1$ so that inflation slowly explodes in alternative equilibria is one possibility. But one can imagine far more effective threats by the Fed, say $i_t = 5000\%$, "we'll blow up the banks," etc. The only reason to specify the mild behavior $i_t = \phi\pi_t$ is that this has a historical connection with Taylor's (1993) observation that *in* equilibrium interest rates seem to rise with inflation. But that was an empirical observation, and as made explicit in Taylor (1999), one connected to a totally different kind of model in which higher interest rates *stabilize* the economy, not the other way around.

Determinacy also depends on other parameters of the model, as seen in Figure 2. The region 33 depends on β, σ, and γ. For $\sigma\gamma < 0$, there is a region with $\phi > 1$ in which both eigenvalues are *not* greater than 1, so we have indeterminacy despite an "active" Taylor rule. There is another region in which both eigenvalues *are* greater than one despite $0 < \phi < 1$, so we have local determinacy despite a "passive" Taylor rule. One can argue on a-priori grounds against $\sigma < 0$ or $\gamma < 0$, of course, but the point remains that local determinacy is a property of the whole system, not just of the policy rule, and more complex models can feature determinacy regions bounded by structural parameters that are less easy to dismiss.

5.2 Dynamic responses

Again, the "Old-Keynesian" intuition is that the Fed will react to inflation by raising real rates; this action will lower output and via the Phillips curve, lower future inflation. To emphasize that the "new-Keynesian" model works in a fundamentally different way, I graph in Figure 3 the path of output, inflation, and interest rates following a one percent innovation to inflation together with no unexpected change in output. This is the response of the system (30) to an expectational shock, δ_π in (29), so that $\pi_1 = 1$, $y_1 = 0$. I use parameters $\beta = 0.95, \sigma = 1, \gamma = 1, \phi = 1.3$. What is, exactly, the Fed's threat that rules out this change?

At period 1, we see the shock to inflation, $\pi_1 = 1$, $y_1 = 0$, and the Fed responds by setting interest rates $i_1 = 1.3\times\pi_1 = 1.3$. Real interest rates rise throughout the simulation, as one might have hoped. However, output *increases* uniformly and eventually explodes in the *positive* direction, while inflation explodes in the *negative* direction, precisely the opposite of what we might have expected from standard old-Keynesian intuition that rising interest rates should lower output. In the context of the model, however, this behavior makes sense. In the new-Keynesian IS curve (25), a high real rate lowers current output *relative to future output*, not on its own. Given no change in current output, a higher real rate must correspond to higher future output. Similarly, if output is increasing then the new-Keynesian Phillips curve (27) means that current inflation must be large

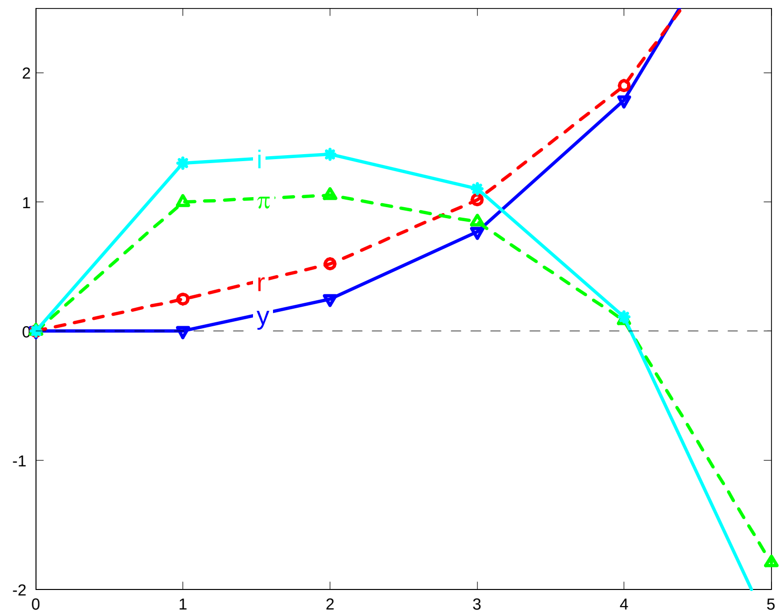

Figure 3: Response of the three-equation new-Keynesian model to a one-percent off-equilibrium inflation innovation, with no change in output. Parameters are $\beta = 0.95, \sigma = 1, \gamma = 1, \phi = 1.3$

relative to future inflation. Given current inflation, then, we must have declining future inflation.

In both ways, then, the surprising dynamics of Figure 3 emphasize that the expected future terms in new-Keynesian models essentially change the sign of all the familiar dynamic relationships, and that the heart of "stabilization" in new Keynesian models is instability, i.e. hoping that inflation and output will jump to the unique values each period that rules out explosive paths such as these.

As a related example, consider how we calculate the response to a monetary policy shock in a new-Keynesian model. To this end, I append a shocks to the policy rule $i_t = \phi\pi_t + x_t$. The standard form (31) then generalizes to

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & \sigma(\beta\phi - 1) \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma \\ 0 \end{bmatrix} x_t. \tag{34}$$

Now, let's calculate the response to an unexpected shock, $x_t = 1$, $x_{t+j} = 0$. One's first instinct might be to hold y_t, π_t constant and simulate forwards, which will result in an explosion similar to that in Figure 3. However, only $E_{t-1}y_t$ and $E_{t-1}\pi_t$ were similarly constrained in the previous period, so there can be an expectational error $\delta_{\pi t}$ and δ_{yt} by which we switch to a different equilibrium. The unique locally-bounded equilibrium will be the one in which $E_t y_{t+1}$ $E_t \pi_{t+1}$ and all subsequent variables are zero, since the expected values of the shocks are zero past t. Therefore, we can set the left hand side of (34) to zero and solve for the jump in y_t and π_t required to head off the explosion. The answer is

$$y_t = -\frac{\sigma}{1+\sigma\gamma\phi}, \quad \pi_t = -\frac{\sigma\gamma}{1+\sigma\gamma\phi}.$$

A one-period surprise interest rate rise lowers both output and inflation contemporaneously, with no effect on future variables.

Having seen how the response is calculated though, we see there is no economic force at work, no sense in which "slack demand" pushes down prices or output as it does in every other theory (MV=PY, fiscal, etc.) of the effects of monetary policy. We simply count on the economy jumping to one of an infinite number of possible equilibria, the one that happens not to lead to equally valid explosive paths.

6 Taylor on Taylor rules; the old-Keynesian model

A natural question is, "how does Taylor think Taylor rules work?" Taylor (1999) is an answer to this question, and this paper highlights the deep tensions between how many economists write about inflation determinacy and what the new-Keynesian models actually do.

Taylor adopts a "simple model" (p. 662, using my notation),

$$y_t = -\sigma(i_t - \pi_t - r) + u_t \tag{35}$$

$$\pi_t = \pi_{t-1} + \gamma y_{t-1} + e_t \tag{36}$$

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t \tag{37}$$

We see a striking difference – all the forward-looking terms are absent. The "IS" curve (35) is missing $E_t(y_{t+1})$; the "Fisher equation" implicit in (35) relates real rates to nominal rates and current rather than expected future inflation, the "Phillips" curve (36) has *past* rather than current or expected future inflation in it. This is an old, not "new" Keynesian model. Of course Taylor is perfectly aware of these differences. He writes (p. 662)

> In general $[\gamma, \sigma]$ and r are reduced form parameters that will depend on the policy parameters $[\phi_\pi$, ϕ_y, and $\phi_0]$. For example, [Equation (35)] could be the reduced form of an optimizing 'IS' curve with future values of the real interest rates... [Equation (36)] ... could be the reduced form of a rational expectations model with staggered wage and price setting, in which expectations of future wages and prices have been solved out. If the parameters do not change very much when the policy parameters change, then treating Eqs. (35) and (36) as policy invariant... will be a good approximation. ... Nevertheless, when viewed as a reduced form, these equations summarize more complex forward-looking models and are useful for illustrating key points.

However, the difference between this model and the new-Keynesian model (25)-(27) has little to do with "policy invariance." We want to analyze dynamics for *given* "policy parameters" ϕ_π, ϕ_y, and ϕ_0. Even if γ, σ and r change with different ϕ, they are constant for a given ϕ. The central difference is that the *dynamics* of this system are fundamentally different from those of the forward-looking new-Keynesian model. Equations (35)-(37) are not a "simpler" or "reduced form" version of (25)-(27). They are the same model with different t subscripts. (King 2000 p. 72 also details a number of fundamental differences between "new" and "old" Keynesian models of this sort.)

Taylor states (p. 663) that "it is crucial to have the interest rate response coefficient on the inflation rate (or a suitable inflation forecast or smoothed inflation rate) above a critical 'stability threshold' of one," (p. 664)

> The case on the left [$\phi_\pi > 1$] is the stable case...The case on the right [$\phi_\pi < 1$] is unstable... This relationship between the stability of inflation and the size of the interest rate coefficient in the policy rule is a basic prediction of monetary models used for policy evaluation research. In fact, because many models are dynamically unstable when ϕ_π is less than one... the simulations of the models usually assume that g_π is greater than one.

This is exactly the *opposite* philosophy from the new-Keynesian models. In new-Keynesian models, $\phi_\pi > 1$ is the condition for *unstable* dynamics. These models want unstable dynamics, in order to rule out multiple equilibria and force forward-looking solutions. In Taylor's model, $\phi_\pi > 1$ is the condition for *stable* dynamics, in which we solve for endogenous variables (including inflation) by *backward-looking* solutions.

For example, take $\phi_y = 0$, and assume $\phi_\pi > 1$. Then if π_t rises in (37), $i_t - \pi_t$ rises as well. In (35) this lowers y_t and in (36) this lowers $E_t(\pi_{t+1})$. This model thus embodies the classic idea that more inflation makes the Fed raise interest rates which lowers future inflation, exactly the opposite of the new-Keynesian dynamics.

A little more formally, substituting (37) into (35) and simplifying to $\bar{\imath} = r$, the standard form of this model is

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \sigma\gamma\frac{1-\phi_\pi}{1+\sigma\phi_y} & \sigma\frac{1-\phi_\pi}{1+\sigma\phi_y} \\ \gamma & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{1+\sigma\phi_y} & \sigma\frac{1-\phi_\pi}{1+\sigma\phi_y} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ e_t \end{bmatrix}.$$

The eigenvalues of the transition matrix are

$$\lambda_1 = 1 + \sigma\gamma\frac{1-\phi_\pi}{1+\sigma\phi_y}; \quad \lambda_2 = 0.$$

Therefore, $\phi_\pi > 1$ and $\phi_y > 0$ generate values of the first eigenvalue *less* than one. Following the usual decomposition, we can then write the unique solution of the model as a backward-looking average of its shocks,

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{1+\sigma\phi_y+\sigma\gamma(1-\phi_\pi)} \begin{bmatrix} \lambda_1 - 1 & \lambda_1 \\ 1+\sigma\phi_y & 0 \end{bmatrix} \sum_{j=0}^{\infty} \lambda_1^j \begin{bmatrix} u_{t-j} \\ e_{t-j} \end{bmatrix}.$$

Why do the two models disagree so much on the desired kind of dynamics? First, because Taylor's model has no expected future terms on the right hand side. Hence, there are no expectational errors. The model completely determines actual values of endogenous variables, not just their expectations, and all the shocks (u_t, e_t) driving the system are exogenous economic disturbances. By contrast, the new-Keynesian model has expected future values in its "structural" equations (25)-(27), so the shocks in its standard representation such as (29) contain expectational errors. If we are to avoid multiple equilibria, we have to find some excuse to solve *forward* to remove these as shocks to the economy. Second, the dynamics are fundamentally reversed. In the new-Keynesian model a higher real interest rate $r_t = i_t - E_t\pi_{t+1}$ means *higher* output growth as illustrated in Figure 3, by the usual intertemporal substitution mechanism. In the new-Keynesian model, higher output means *declining* inflation, since price-setting firms are forward-looking.

In sum, Taylor's model is no simple "reduced form," no rough guide to give intuition formalized by the fancier new-Keynesian effort. This old-Keynesian model and analysis is in most respects exactly the opposite of micro-founded new-Keynesian models. It is a deep mistake to use new-Keynesian models to understand old-Keynesian stories, either verbal or as expressed in "simple" models such as this one.

Since it easily delivers a unique equilibrium, and thus inflation determinacy, why not conclude that Taylor's model is the right one to use? Alas, our quest is for *economic* models that deliver price determinacy from an interest rate rule. This model fails on the crucial qualification – as Taylor's apologetic discussion makes very clear. If in fact inflation has nothing do to with expected future inflation (the heart of the new-Keynesian optimizing Phillips curve), so inflation is mechanistically *caused* by output gaps which are directly under the Fed's control; if in fact the output gap driving inflation has nothing to do with expected future output, then, yes, the Taylor rule does lead to inflation determinacy. But despite a half-century of looking for it, *economic* models do not deliver the "if" part of these statements. If we follow this model, we are giving up on an *economic* model of price-level determination, in favor of (at best) a mechanistic description.

7 Rules with leads and lags

So far, I have used only very simple Taylor rules that relate interest rates to the current inflation rate. The literature contains a wide variety of specifications, however, and in particular specifications in which the central bank reacts to expected *future* inflation, and to leads and lags of output. A policy rule that reacts to expected future inflation is particularly sensible, as this ensures that the Fed really does raise real interest rates, equal to the difference between nominal rates and expected future inflation. It is often claimed that the principle "raise interest rates more than one for one with inflation" is quite robust to details of model and rule specification. (Taylor (1999) and Woodford (2003) among many others.) I find that this claim is not true in new-Keynesian models. I also verify that simple changes to timing do not solve the determinacy problems.

7.1 The frictionless model

To address this question, start with our simple Fisher equation model (1),but allow the Fed to respond to expected future inflation rather than current inflation,

$$i_t = r + \phi E_t \pi_{t+j},$$

Again, we find equilibria by eliminating i_t between these two equations.

For $j = 0$ (contemporaneous inflation), the equilibrium condition is

$$E_t \pi_{t+1} = \phi \pi_t$$

as we have seen, the condition for a unique local equilibrium is $\|\phi\| > 1$.

For $j = 1$, a reaction to expected future inflation, the equilibrium condition becomes

$$E_t \pi_{t+1} = \phi E_t \pi_{t+1}.$$

If $\phi = 1$, anything is a solution. For any $\phi \neq 1$ (both $\phi > 1$ and $\phi < 1$), solutions must obey

$$E_t \pi_{t+1} = 0; \;\; \pi_t = \delta_{t+1}. \tag{38}$$

We conclude that inflation must be white noise – real rates are constant. But that's all we can conclude. No value of ϕ gives even local determinacy.

For $j = 2$, we have

$$E_t \pi_{t+1} = \phi E_t \pi_{t+2}$$

Now a necessary condition for "unstable" or "forward-looking" equilibrium is reversed, $\|\phi\| < 1$. Since interest rates react to inflation *two* periods ahead, and interest rates control expected inflation *one* period ahead, the interest rate and *one*-period ahead inflation must move less than *two* period ahead inflation if we want an explosive root. And even this specification is now not enough to give us a unique local equilibrium, since there is an E_t on both sides of the equation. $\pi_{t+1} = \delta_{t+1}$, $E_t(\delta_{t+1}) = 0$ is a solution for *any* value of ϕ.

In sum, in this simple model, Taylor determinacy disappears as soon as the Fed reacts to expected future rather than current inflation, and the solution is extremely sensitive to the timing convention. All the dynamics of the model, which are crucial to the idea of using forward-looking solutions to determine expectational errors, rely entirely on the assumed dynamics by which the Fed reacts to inflation.

7.2 Continuous time and dynamics

The same timing issue drives the apparently strange modifications one needs to make in order to take the continuous-time limit. Benhabib, Schmitt-Grohé and Uribe (2002) present one such model. If we eliminate money from their perfect foresight model, the Fisher equation is simply

$$i_t = r + \pi_t.$$

In continuous time (and with continuos sample paths) the distinction between past and expected future inflation vanishes. If we write a Taylor rule

$$i_t = \phi(\pi_t),$$

as they do, we see that this system behaves exactly as the $i_t = r + \phi E_t \pi_{t+1}$ or $j = 1$ case above: If $\phi(\pi) = r + \pi_t$ anything is an equilibrium; otherwise there is a unique equilibrium in perfect foresight, but the same multiplicity once we allow expectational errors. It seems there are no dynamics (or the dynamics happen infinitely quickly), so the forward-looking trick to determine expectational errors disappears.

Benhabib, Schmitt-Grohé and Uribe do have dynamics which look a lot like Figure 1 (see their Figure 1). However, these dynamics come from an entirely different source. Their model has money in the utility function. The dynamics of inflation in their model come from the standard interest-elasticity of money demand, much like Cagan (1956) hyperinflation dynamics under a money target.

Here is their argument in continuous time: With money in the utility function and a constant endowment, the first-order condition for money M_t vs. consumption C_t (Equation (47) in the Appendix) implies a "money demand" curve (my notation) $M_t/P_t = L(Y, i_t)$. Thus, we can write the marginal utility of consumption in equilibrium as

$$u_c(C_t, M_t/P_t) = u_c\left[Y, L(Y, i_t)\right] = \lambda(i_t) = \lambda\left[\phi(\pi_t)\right]$$

where the last equalities define the function λ. Differentiating, and using the continuous-time first order condition

$$\frac{\dot{u}_c}{u_c} = i - \pi - \rho$$

where ρ is the rate of time preference, we have

$$\dot{\pi}_t = \frac{\lambda\left[\phi(\pi_t)\right]\left[\phi(\pi_t) - \pi_t - \rho\right]}{\lambda'\left[\phi(\pi_t)\right]\phi'(\pi_t)}.$$

This differential equation in π_t turns out to look just like Figure 1.

The idea may be clearer in the discrete-time formulation of equations (21)-(24). Using a Taylor rule $i_t = \phi\left(\Pi_{t+1}\right)$, Equation (23) becomes

$$\Pi_{t+1} = \beta\left[1 + \phi(\Pi_{t+1})\right]\frac{u_c(Y, M_{t+1}/P_{t+1})}{u_c(Y, M_t/P_t)}.$$

If we had no money in the utility function, you can see how once again we are stuck. There are no dynamics. However, the interest-elasticity of money demand offers hope. Substituting for money demand, we have

$$\Pi_{t+1} = \beta\left[1 + \phi(\Pi_{t+1})\right]\frac{u_c\left[Y, L(Y, i_{t+1})\right]}{u_c\left[Y, L(Y, i_t)\right]}.$$

$$\Pi_{t+1} = \beta\left[1 + \phi(\Pi_{t+1})\right]\frac{u_c\left\{Y, L\left[Y, \phi(\Pi_{t+2})\right]\right\}}{u_c\left\{Y, L\left[Y, \phi\left(\Pi_{t+1}\right)\right]\right\}}.$$

Now we again have a difference equation that can look like Figure 1.

In sum, despite the superficially similar behavior of Benhabib, Schmitt-Grohé and Uribe's (2002) model to the frictionless models studied above, we see they are fundamentally different. Benhabib, Schmitt-Grohé and Uribe's model cannot work in a frictionless economy; it relies on the dynamics induced by interest-elastic money demand rather than dynamics induced by the Policy rule.

To mirror the sort of dynamics we have seen from $i_t = \phi(\pi_t)$ rules in continuos time, one must specify some explicit time lag between inflation and the interest rate, $i_t = \phi(\pi_{t-k})$, or $i_t = \int_{k=0}^{\infty} f(k)\pi(t-k)dk$. For example, Sims (2003) models a Taylor rule in continuous time in this way, as

$$\frac{di}{dt} = \theta_0 + \theta_1 \frac{1}{P}\frac{dP}{dt} - \theta_2 i. \tag{39}$$

Here, more inflation causes the Fed to raise the *rate of change* of interest rates. Sims also has a Fisher equation

$$i = \rho + \frac{1}{P}\frac{dP}{dt}$$

Sims solves for the interest rate,

$$\frac{di}{dt} = \theta_0 + (\theta_1 - \theta_2)\, i_t - \theta_1 \rho$$

thus wanting $\theta_1 < \theta_2$ for forward looking solutions. The specification (39) isn't an ad-hoc peculiarity, it is exactly the sort of modification we must make for Taylor-rule dynamics to work in continuous time.

7.3 Interest rate rules that seem to work

Loisel (2007) proposes a rule that responds to both current and future inflation (simplified to this setting)

$$i_t = r + E_t\pi_{t+1} + \psi(\pi_t - z_t)$$

where ψ is any nonzero constant, and z_t is any exogenous random variable. If we merge this with the usual Fisher equation

$$i_t = r + E_t\pi_{t+1} \tag{40}$$

we obtain a unique equilibrium

$$\pi_t = z_t.$$

This seems to be the Holy Grail: a nominal interest rate rule that delivers a unique equilibrium inflation rate in a frictionless economy. The trick, as Loisel explains, is to have the interest rate rule exactly cancel the troublesome forward-looking terms of the model.

To digest this proposal, write it as a special case of a rule that responds to current and expected future inflation,

$$i_t = r + \phi_0\pi_t + \phi_1 E_t\pi_{t+1} - \phi_0 z_t.$$

Merging this rule with the usual Fisher equation (40), we obtain

$$\begin{aligned} E_t\pi_{t+1} &= \phi_0\pi_t + \phi_1 E_t\pi_{t+1} - \phi_0 z_t \\ E_t\pi_{t+1} &= \frac{\phi_0}{1-\phi_1}\pi_t - \frac{\phi_0}{1-\phi_1} z_t \end{aligned}$$

As usual, this system displays multiple equilibria

$$\pi_{t+1} = \frac{\phi_0}{1-\phi_1}\pi_t - \frac{\phi_0}{1-\phi_1} z_t + \delta_{t+1}; \quad E_t\delta_{t+1} = 0. \tag{41}$$

The eigenvalue or root is $\phi_0/(1-\phi_1)$. Thus, if

$$\frac{\phi_0}{1-\phi_1} > 1, \tag{42}$$

We have at least a unique locally-bounded equilibrium,

$$\pi_t = E_t \sum_{j=0}^{\infty} \left(\frac{1-\phi_1}{\phi_0}\right)^j z_{t+j}.$$

Condition (42) is equivalent to

$$\phi_0 + \phi_1 > 1$$

which has the familiar Taylor-rule ring to it, that overall interest rates must rise more than one-for-one with inflation.

Now, we can study not only the *point* $\phi_1 = 1$, but the *limit* $\phi_1 \to 1$. By studying that limit, we see what's going on. As $\phi_1 \to 1$, the eigenvalue or root (42) of the difference equation (41) rises to infinity. For ϕ_1 near 1, the Fed is saying "if inflation doesn't come out to the desired value, we'll hyperinflate *very fast,*" and in the limit multiple equilibria are ruled out by a threat to hyperinflate with *infinite* speed. Thus, this proposal is not really anything new and different, it is an extremely accelerated version of the usual logic. That is not a criticism: much analysis of Taylor rules finds that larger responses are better, and you can't get larger than infinite. The point is just that we can digest this proposal as a limit of the usual logic rather than have to think of it as a fundamentally new type of interest rate target.

Adão, Correia and Teles (2007) advance a similar proposal. Simplified to the linearized, constant real-rate, frictionless environment, they propose the target

$$i_t = r + E_t p_{t+1} - z_t \tag{43}$$

where z_t is any exogenous random variable. If we merge this rule with the usual Fisher equation expressed as

$$i_t = r + E_t\left(p_{t+1} - p_t\right) \tag{44}$$

we obtain a unique equilibrium

$$p_t = z_t.$$

Thus we have an interest rate target that delivers a unique, determinate, price level in a frictionless economy, with no multiple equilibria. Again, the key is that the interest rate rule exactly cancels the forward-looking term of the model, in this case the price level rather than the inflation rate. Adão, Correia and Teles' analysis is in fact conducted in the full nonlinear version of a cash in advance model with labor supply, so linearization, local approximation, and a frictionless economy are not central to the result[5].

We can understand this proposal as a similar infinitely-explosive limit of the sort of "Wicksellian" price-level-stabilizing interest rate rules studied by Woodford (2003, p. 81). Generalize the rule to

$$i_t = r + \phi_1 E_t p_{t+1} + \phi_0 p_t - z_t$$

Equate to the Fisher equation (44), and we find the equilibrium condition

$$E_t p_{t+1} = \frac{1+\phi_0}{1-\phi_1} p_t + \frac{1}{1-\phi_1} z_t$$

The eigenvalue is

$$\lambda = \frac{1+\phi_0}{1-\phi_1},$$

and we have a unique locally-bounded equilibrium if $\|\lambda\| > 1$. Woodford studies the case $\phi_1 = 0$, so obtains the condition $\phi_0 > 0$. Adão, Correia and Teles specify $\phi_0 = 0$, and study the limit as $\phi_1 \to 1$. Again, you can see that this is the limit of an infinite eigenvalue, in which the threatened explosion happens infinitely fast.

Loisel's (2007) actual example (p.11) of an interest-rate rule that seems to avoid multiple equilibria is given in the context of the three-equation model; in my notation

$$i_t = E_t \pi_{t+1} + \phi_{\pi,0} \pi_t + \frac{1}{\sigma} \left(E_t y_{t+1} - y_t \right)$$

If we place this rule in the standard model (25)-(27),

$$\begin{aligned} y_t &= E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) \\ \pi_t &= \beta E_t \pi_{t+1} + \gamma y_t \end{aligned}$$

[5] Here's the nonlinear version: Start with the consumer's first order condition

$$\frac{u_C(t)}{P_t} = (1+i_t) E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right]$$

Assume a constant endowment $C_t = Y$, so the u_C terms cancel. Then, we can write

$$1 + i_t = \frac{1}{P_t E_t \left[\frac{\beta}{P_{t+1}} \right]}$$

Write the policy rule

$$1 + i_t = \frac{1}{z_t E_t \left[\frac{\beta}{P_{t+1}} \right]}$$

The globally-unique equilibrium is $P = z_t$.

we can quickly see that $\pi_t = 0$, $y_t = 0$ is the only equilibrium so long as $\phi_{\pi,0} \neq 0$. In this context, the variables are deviations from a desired equilibrium, so we have shown that the interest rate rule implements the desired equilibrium uniquely.

We can understand this rule as the limit of a standard rule of the form

$$i_t = \phi_{\pi,0}\pi_t + \phi_{\pi,1}E_t\pi_{t+1} + \phi_{y,0}y_t + \phi_{y,1}y_{t+1}.$$

When $\phi_{y,1} = 1/\sigma$, the (single, repeated) eigenvalue of the three-equation model, derived in the Appendix to Cochrane (2007) is

$$\lambda = \frac{1 + \sigma\left(\phi_{y,0} + \gamma\phi_{\pi,0}\right)}{\beta + \sigma\left[\gamma(1 - \phi_{\pi,1}) + \beta\phi_{y,0}\right]}$$

Taking the limit, $\phi_{\pi,1} \rightarrow 1$, $\phi_{y,1} = 1/\sigma, \phi_{y,0} = -1/\sigma$, the denominator goes to zero, so again this is a limit with an infinite eigenvalue.

All forward-looking rules, and these ones in particular, require the central bank to respond to private expectations. *In* a rational expectations equilibrium, expectations $E_t(P_{t+1})$ or $E_t\left(\pi_{t+1}\right)$are just given by $E_t(z_{t+1})$. However, if people develop a sunspot expectation for more inflation, the Fed must increase interest rates to match it. One can question the informational and off-equilibrium or game-theoretic foundations of such a response.

7.4 Timing in the three-equation model

The familiar three-equation model is also sensitive to timing. The Taylor-rule parameter regions required to produce a forward-looking solution vary considerably whether the central bank reacts to current or expected future inflation, and whether the central bank reacts to output. The Appendix derives the eigenvalues and provides analytical chacterization of the determinacy regions. The equations aren't that revealing: since we're studying roots of quadratic equations there are multiple special cases for real roots, imaginary roots, and roots equal to one or to negative one, and one must check the smaller of two roots. Therefore, I focus here on a graphical analysis of some special cases.

Figure 4 presents the regions of local determinacy in the three-equation model, for a policy rule that responds to expected future inflation $i_t = \phi_{\pi,1}E_t\pi_{t+1}-$ again, perhaps the most interesting case. In the usual parameter region $\sigma\gamma > 0$, we see a comforting region $\phi_{\pi,1} > 1$. The rest of the parameter space is quite different from the case $i_t = \phi_{\pi,0}\pi_t$ of Figure 2, however. In particular, as King (2000, Figure 3b) notices, there is a second region of *large* $\phi_{\pi,1}$ that again leads to local indeterminacy. In this region, both eigenvalues are negative, but one is less than one in absolute value. "Sunspots" that combine movements in output and inflation, essentially offsetting in the Phillips curve so that expected inflation doesn't move much, lead to stable dynamics.

Figure 4 plots regions of local determinacy combining responses to current and future inflation $\phi_{\pi,0}$ and $\phi_{\pi,1}$. We see the expected condition $\phi_{\pi,0} + \phi_{\pi,1} > 1$ in the downward-sloping line of the left hand part of the plot. However, this line disappears when $\phi_{\pi,0} <$

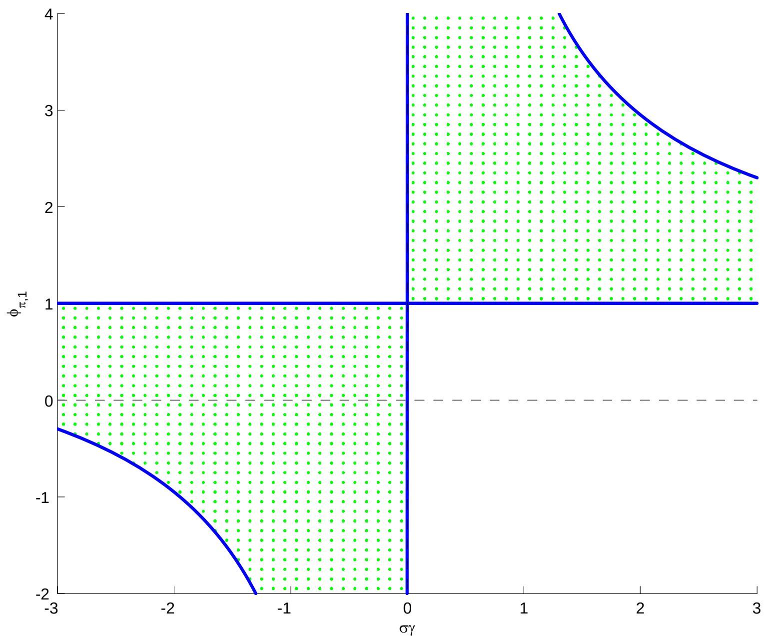

Figure 4: Zones of local determinacy – both eigenvalues greater than one in absolute value – when the policy rule responds to expected future inflation $i_t = \phi_{\pi,1} E_t \pi_{t+1}$. The plotted boundaries are $\phi_{\pi,1} = 1$, $\phi_{\pi,1} = 1 + 2(1+\beta)/(\sigma\gamma)$.

$-(1-\beta)/\sigma\gamma = -0.025$. A greater $\phi_{\pi,1}$ cannot make up for an even slightly negative $\phi_{\pi,0}$. In fact, we see that the sum $\phi_{\pi,0} + \phi_{\pi,1}$ must be *less than one* for local determinacy when $\phi_{\pi,0} < 0$, an excellent counterexample to the view that $\phi_{\pi,0} + \phi_{\pi,1} > 1$ is a robust result. In both cases we see again that $\phi_{\pi,1}$ cannot get too big or again we lose determinacy for any value of $\phi_{\pi,0}$.

King concludes from experiments such as these: "Forward-looking rules, then, suggest a very different pattern of restrictions are necessary to assure that there is a neutral level of output." He also takes seriously the regions with local indeterminacy (one eigenvalue less than one) despite large $\phi_{\pi,1}$ as important policy advice, writing (p.80) "It is important, though, that it [monetary policy] not be too aggressive, since the figure shows that some larger values are also ruled out because these lead to indeterminacies."

As a slightly novel example, consider what happens if the Fed responds to expected inflation two periods ahead as well as one period ahead, i.e. consider a Taylor rule of the form

$$i_t = \phi_{\pi,0}\pi_t + \phi_{\pi,1} E_t \pi_{t+1} + \phi_{\pi,2} E_t \pi_{t+2}.$$

Figure 6 presents the region of determinacy (both roots greater than one in absolute value) for this case. As you can see, there is some sense in the view that $\phi_{\pi,0} + \phi_{\pi,1} + \phi_{\pi,2} > 1$ is important for determinacy. (As one raises $\phi_{\pi,0}$, the region of local determinacy descends as you would expect.) However, there's more to it than that. We *must* have $\phi_{\pi,2} \leq 1$ – the Fed must respond negatively if at all to expected inflation two periods out! Furthermore, we see another example in which too large $\phi_{\pi,1}$ leads to indeterminacy for any $\phi_{\pi,2}$.

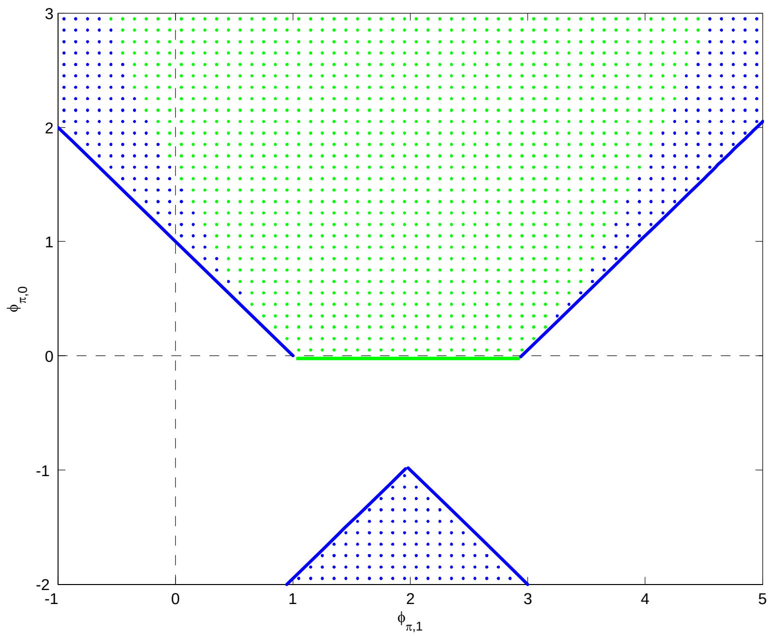

Figure 5: Zones of local determinacy when the policy rule responds to both current and expected future inflation, $i_t = \phi_{\pi,0}\pi_t + \phi_{\pi,1}\pi_{t+1}$, using $\beta = 0.95$ and $\sigma\gamma = 2$. The plotted boundaries are $\phi_{\pi,0} + \phi_{\pi,1} = 1$, $\phi_{\pi,1} - \phi_{\pi,0} = 1 + 2(1+\beta)/\sigma\gamma$ and $\phi_{\pi,0} = -(1-\beta)/\sigma\gamma$.

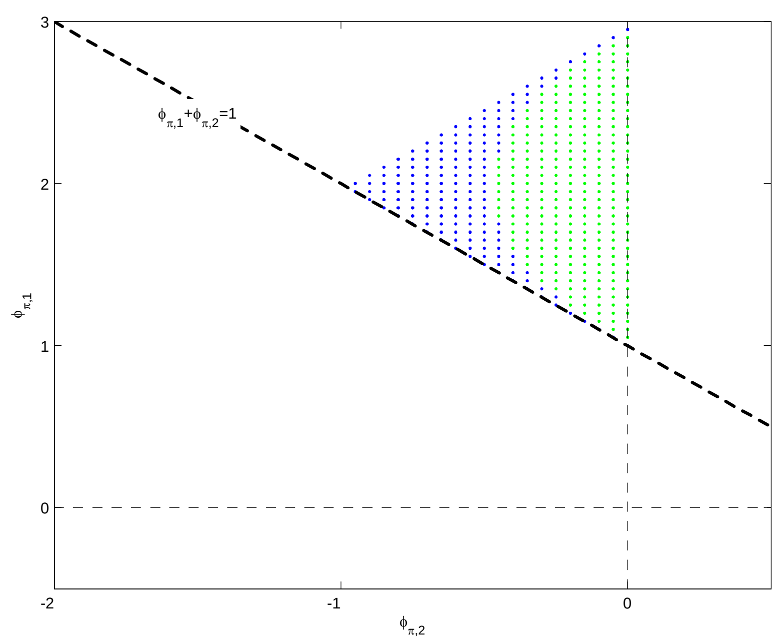

Figure 6: Region of local determinacy when the policy rule responds to expected future inflation one and two periods ahead, $i_t = \phi_1 E_t\pi_{t+1} + \phi_2 E_t\pi_{t+2}$, using $\beta = 0.95, \sigma\gamma = 2$.

Allowing responses to output adds a whole interesting new range of possibilities. Figure 7 presents the determinacy region for rules limited to current output and inflation, $i_t = \phi_{\pi,0}\pi_t + \phi_{y,0}y_t$, for $\beta = 0.95, \sigma = 2, \gamma = 1$. Figure 8 presents the same region for rules limited to expected future output and inflation, $i_t = \phi_{\pi,0}E_t\pi_{t+1} + \phi_{y,0}E_ty_{t+1}$.

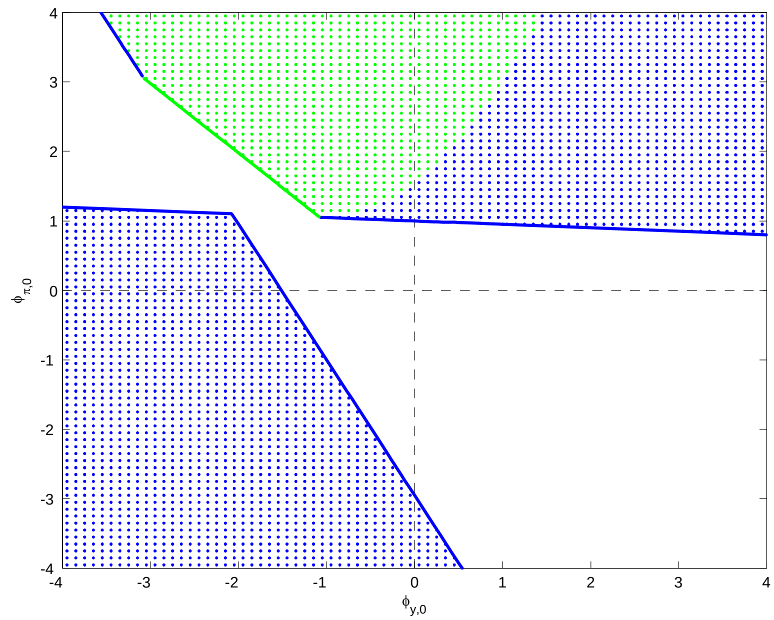

Figure 7: Region of determinacy in the three-equation model for a rule $i_t = \phi_{\pi,0}\pi_t + \phi_{y,0}y_t$, using $\beta = 0.95, \sigma = 2, \gamma = 1$.

In Figure 7, you can see that output responses can substitute for inflation responses. Rules are possible that use *only* output responses, ignoring inflation all together. Depending on ϕ_y and the other parameters of the model, almost any value of ϕ_π can be consistent or inconsistent with determinacy.

Figure 8 shows that the region of determinacy using expected *future* output and inflation is radically different than that which uses *current* output and inflation. In particular, almost the whole range $\phi_y < 0$ is wiped out, and there are severe constraints on how strong the inflation and output responses can be.

Two-dimensional graphs can only do so much justice to this 7-dimensional space ($\phi_{\pi,0}$, $\phi_{\pi,1}$, $\phi_{y,0}$, $\phi_{y,1}$, β, γ, σ), of course. The determinacy $\|\lambda\| = 1$ boundaries in this case are as follows. (These conditions are derived in the Appendix and included in the plots. To turn them into boundaries, one has to also check that the other eigenvalue is greater than one.)

1. $\sigma\phi_{y,1} \neq 1$, real roots, $\lambda = 1$

$$\left(\phi_{\pi,0} + \phi_{\pi,1} - 1\right) + \frac{1-\beta}{\gamma}\left(\phi_{y,1} + \phi_{y,0}\right) = 0 \tag{45}$$

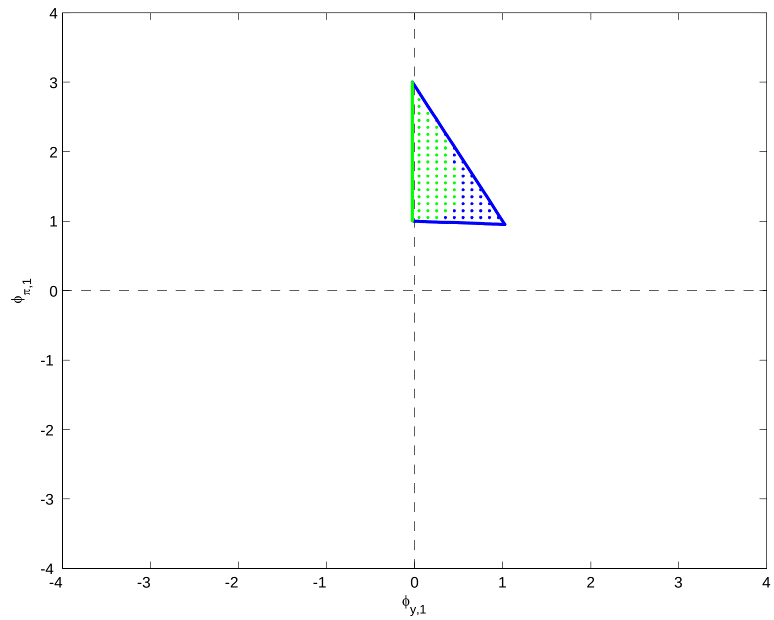

Figure 8: Region of determinacy for the three-equation model, for an interest rate rule $i_t = \phi_{\pi,1} E_t \pi_{t+1} + \phi_{y,1} E_t y_{t+1}$, using $\beta = 0.95, \sigma = 2, \gamma = 1$.

2. $\sigma\phi_{y,1} \neq 1$, real roots, $\lambda = -1$

$$\left(1 + \phi_{\pi,0} - \phi_{\pi,1}\right) - \frac{1+\beta}{\gamma}\left(\phi_{y,1} - \phi_{y,0}\right) = -2\frac{(1+\beta)}{\sigma\gamma}$$

3. $\sigma\phi_{y,1} \neq 1$, Complex roots

$$\gamma\phi_{\pi,0} + \phi_{y,0} + \beta\phi_{y,1} = \frac{\beta - 1}{\sigma}.$$

4. $\sigma\phi_{y,1} = 1$, $\lambda = 1$,

$$\left(\phi_{\pi,0} + \phi_{\pi,1}\right) + \frac{(1-\beta)}{\sigma\gamma}\left(1 + \sigma\phi_{y,0}\right) = 1$$

5. $\sigma\phi_{y,1} = 1$, $\lambda = -1$:

$$\phi_{\pi,0} - \phi_{\pi,1} + \frac{(1+\beta)}{\sigma\gamma}\left(1 + \sigma\phi_{y,0}\right) = -1 \tag{46}$$

Equations (45)-(46) show a variety of interesting additional interactions. Four of the five conditions do not involve $\phi_{\pi,0}+\phi_{\pi,1}$ and $\phi_{y,0}+\phi_{y,1}$ – we see analytically that responding to current vs. future output and inflation are not the same thing. In fact the second and last conditions includes $\phi_0 - \phi_1$: The two responses have opposite effects. (Unsurprisingly, these are conditions in which the eigenvalue is equal to negative one.) The other conditions trade off responses with coefficients that depend on structural parameters.

The point of all these examples is to emphasize that the Taylor rule does not "stabilize inflation" in new-Keynesian models; rather it threatens explosive equilibria. The examples emphasize that the regions of determinacy depend on the entire system, not just the policy rule. The regions are sensitive functions of policy rule parameters and timing, as well as economic parameters. The "robustness" may be a feature of old-Keynesian models and stories, but not of these.

8 Conclusions and implications

Practically all *verbal* explanations for the wisdom of the Taylor principle – the Fed should increase interest rates more than one for one with inflation – use old-Keynesian, stabilizing, logic: This action will raise real interest rates, which will dampen demand, which will lower future inflation. New-Keynesian *models* operate in an entirely different manner: by raising interest rates in response to inflation, the Fed threatens hyperinflation or deflation, or at a minimum a large "non-local" movement, unless inflation jumps to one particular value.

Alas, there is no economic reason why the economy should pick this unique initial value, as inflation and deflation are valid economic equilibria. No supply/demand force acts to move inflation to this value. The attempts to rule out multiple equilibria basically state that the government will blow up the economy should a hyperinflation or deflation occur. This is not a credible threat, or a reasonable characterization of anyone's expectations. Inflation is just as indeterminate, in microfounded new-Keynesian models, when the central bank follows a Taylor rule, as it is under fixed interest rate targets.

If not this, then what?

The contribution of this paper is negative, establishing that one popular theory does not, in the end, determine the price level or the inflation rate. So what theory *can* determine the price level, in an economy like ours? It seems sensible to at least point to logical possibilities and give an opinion.

One standard theoretical possibility does not apply to modern economies. A completely free commodity (gold) standard, an exchange rate peg, or other explicit backing can determine the price level, at least relative to the chosen standard. But modern central banks do not make any redemption promises, so this possibility cannot apply.

The quantity theory MV=PY can determine the price level. This mechanism requires three crucial ingredients. First, there must be a "special" medium of exchange, unique in that capacity by law or by custom. Second, that medium must be held in artificially low supply. If people can transact with bonds, foreign currency, freely created "inside money" (banknotes, iou's) or if the central bank leaves the money supply passive, MV=PY does not determine the price level. Third, one must solve the parallel issues of price level indeterminacy with interest-elastic money demand. The first requirement is tenuous in modern economies. The second requirement seems to be blatantly violated. Central banks target interest rates, following indicators of economic activity and inflation, and largely ignoring monetary aggregates.

The caveat "that ignore aggregates" is important. Central banks might use interest rates to implement monetary targets. Regressions linking interest rates to output and inflation do not immediately contradict this view. With constant money supply, we see higher interest rates with higher output and inflation, tracing out the money demand function. (Minford, Perugini and Srinivasan 2001, 2002 argue for this interpretation of the evidence.)

However, central banks at best *say* they use aggregates as one of many indicators of economic activity, but certainly not as the key nominal anchor. For example, Ben Bernanke (2006) said

> It would be fair to say that monetary and credit aggregates have not played a central role in the formulation of U.S. monetary policy since [1982], although policymakers continue to use monetary data as a source of information about the state of the economy.
>
> ...Although a heavy reliance on monetary aggregates as a guide to policy would seem to be unwise in the U.S. context, money growth may still contain important information about future economic developments. Attention to money growth is thus sensible as part of the eclectic modeling and forecasting framework used by the U.S. central bank.

The European central bank says it pays a bit more attention to M3 growth, but the as one of many indicators of long term inflation rather than a central target and nominal anchor (see, for example, European Central Bank 2004). Many regressions have been run to characterize what central banks actually *do*, finding at best weak evidence that they pay any attention to monetary aggregates. Of course, this is dicey empirical work – if the aggregate were perfectly stabilized, then the regression would fail to see influence from money to interest rates. Still, the overwhelming evidence from central bank statements and the state of the art in empirical work is that they are not targeting aggregates.

In sum, if central banks really are secretly targeting aggregates as the central nominal anchor, it is a closely-guarded and carefully-disguised secret. Thus, even if the quantity theory *could* determine the price level in a modern economy– if money demand were stable and interest-inelastic enough – interest rate targets that ignore aggregates mean that the quantity theory *does* not do so. And the third requirement – solving the parallel indeterminacy issues with interest-elastic money demand (see the first footnote)– still has not been solved.

There is one currently-available economic theory remaining that can determine the price level in modern economies: the fiscal theory. New-Keynesian models explicitly specify "Ricardian" regimes, in which the government stands ready to change taxes and spending to accommodate any inflation-induced changes in the value of government debt. By doing so, they throw away an equilibrium condition, that the real value of nominal debt must equal the present value of future real net tax payments. The price level *can* be determined in models that change this assumption, and adopting – or, perhaps, recognizing – that governments follow a fiscal regime that is at least partially non-Ricardian. As we have seen, a non-Ricardian regime gives a unique price level and inflation when the central bank targets interest rates, solving all the determinacy and uniqueness problems in one fell swoop. As in Sims (1994, 2003) and Woodford (1995, 2003), a non-Ricardian regime also solves the parallel indeterminacy under monetary targets. The fiscal theory *can* apply to our economy, as argued extensively in Cochrane (1998, 2005); time-series tests of the response of taxes to debt are invalid.

Since none of the alternatives can determine the price level or inflation rate in a fiat-money economy with the sort of interest rate targets that we observe central banks to follow, I conclude that the fiscal theory is the only currently-available economic model that can do so. The fiscal theory does not yet have a compelling *empirical* counterpart, especially a story for the rise of inflation in the 1970s and its decline in the 1980s, but that may be because nobody has really looked yet.

"Economic" is an important qualifier. Most of the case for Taylor rules in popular and central bank writing, and often in academic contexts, emphasizes the old-Keynesian stabilizing story. This is a pleasant and intuitively pleasing story to many. However, it throws out the edifice of theoretical coherence – explicit underpinnings of optimizing agents, clearing markets etc. – that is the hallmark achievement of the new-Keynesian effort. If inflation is, in fact, stabilized in modern economies by interest rate targets interacted with *backward*-looking IS and Phillips curves, economists really have no idea *why* this is so.

"Economic" is also an important qualifier, because a wide variety of almost philosophical principles have been advocated to prune equilibria. For example, Evans and Honkapohja (2001) advocate criteria based on least-squares learnability, and McCallum (2003) advocates a "minimum state variable criterion," which he relates to learnability. These refinements go beyond the standard definitions of economic equilibria. One may argue that when a model gives multiple equilibria, we need additional selection criteria. I argue instead that we need a different model.

Sims (1994, p. 381) neatly summarized a long history of monetary wisdom with

> "The existence and uniqueness of the equilibrium price level cannot be determined from knowledge of monetary policy alone; fiscal policy plays an equally important role."

Sims was talking about money-targeting regimes and fixed interest rate targets. I really have only said the same thing in the case of active interest rate targets following Taylor rules, and new-Keynesian models. Sims also said "these points are not new," which is surely correct, though old wisdom occasionally needs restating.

9 References

Adão, Bernardino, Isabel Correia and Pedro Teles, 2007, "Unique Monetary Equilibria with Interest Rate Rules," Manuscript, Banco de Portugal, Universidade Catolica Portuguesa and CEPR.

Alstadheim, Ragna, and Dale W. Henderson, 2006, "Price-Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps," *The B.E. Journal of Macroeconomics* 6:1 (Contributions), Article 12.
http://www.bepress.com/bejm/contributions/vol6/iss1/art12

Bassetto, Marco, 2004, "Negative Nominal Interest Rates," *American Economic Review* 94 (2), 104-108.

Bassetto, Marco, 2005, "Equilibrium and Government Commitment," *Journal of Economic Theory* 124, (1), 79-105.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe, 2002, "Avoiding Liquidity Traps," *Journal of Political Economy* 110, 535-563

Benigno, Pierpaolo and Michael Woodford, 2007, "Optimal Inflation Targeting under Alternative Fiscal Regimes," in F. Mishkin and K. Schmidt- Hebbel, eds., *Monetary Policy under Inflation Targeting*, Santiago: Central Bank of Chile, forthcoming.

Bernanke, Ben S., 2006, "Monetary Aggregates and Monetary Policy at the Federal Reserve: A Historical Perspective," Speech At the Fourth ECB Central Banking Conference, Frankfurt, Germany Nov 2006.
http://www.federalreserve.gov/boarddocs/speeches/2006/20061110/default.htm

Bilbiie, Floring O., 2006, "Limited Asset Markets Participation, Monetary Policy and (Inverted) Keynesian Logic," Manuscript, University of Oxford, Nuffeld College and CEP, London School of Economics.
http://www.nuff.ox.ac.uk/Users/Bilbiie/index_files/lamp_theory0106.pdf.

Cagan, Phillip, 1956, "The Monetary Dynamics of Hyperinflation," in *Studies in the Quantity Theory of Money*, Milton Friedman, ed., Chicago: University of Chicago Press, 25- 117.

Canzoneri, Matthew, Robert Cumby, and Behzad Diba, 2001, "Is the Price Level Determined by the Needs of Fiscal Solvency," *American Economic Review* 91, 1221 - 1238.

Clarida, Richard, Jordi Galí, and Mark Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115 (1), 147-180.

Cochrane, John H., 2007, "Identification with Taylor Rules: a Critical Review," Manuscript, University of Chicago.

Cochrane, John H., 2005, "Money as Stock," *Journal of Monetary Economics* 52, 501-528.

Cochrane, John H., 1998, "A Frictionless model of U.S. Inflation," in Ben S. Bernanke and Julio J. Rotemberg, eds., *NBER Macroeconomics Annual 1998,* Cambridge MA: MIT press, 323-384.

Cogley, Timothy, and Argia Sbordone, 2005, "A Search for a Structural Phillips Curve," *Federal Reserve Bank of New York Staff Report* 203.

Davig, Troy and Eric Leeper, 2005, "Generalizing the Taylor Principle," Manuscript, Indiana University at Bloomington.

Eagle, David, 2006, "The Eventual Failure and Price Indeterminacy of Inflation Targeting," MPRA Paper 1240, University Library of Munich, Germany, http://ideas.repec.org/p/pra/mprapa/1240.html

Eagle, David, 2007, "Revealing the Naked Truth Behind Price Determinacy, Infinite-Horizon Rational Expectations, and Inflation Targeting" http://www.cbpa.ewu.edu/~deagle/banking/Nonexplosive.doc

European Central Bank, 2004, *The Monetary Policy of the ECB* http://www.ecb.int/pub/pdf/other/monetarypolicy2004en.pdf

Evans, George W. and Seppo Honkapohja, 2001, *Learning and Expectations in Macroeconomics*, Princeton: Princeton University Press.

Friedman, Milton, 1968, "The Role of Monetary Policy," *The American Economic Review*, 58 (1), 1-19.

Galí, Jordi, 2003, "New Perspectives on Monetary Policy, Inflation, and the Business Cycle," in *Advances in Economic Theory*, M. Dewatripont, L. Hansen, and S. Turnovsky Eds., vol. III, p. 151-197, Cambridge University Press. http://www.econ.upf.edu/crei/people/Galí/pdf_files/wcpaper.pdf

King, Robert G., 2000, "The New IS-LM Model: Language, Logic, and Limits," Federal Reserve Bank of Richmond *Economic Quarterly* Volume 86/3 Summer 2000, 45-103.

Leeper, Eric , 1991, "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies, *Journal of Monetary Economics* 27, 129-147.

Loisel, Olivier, 2007, "Bubble-Free Interest-Rate Rules," Manuscript, Banque de France and CEPREMAP.

Lucas, Robert E., Jr., 1972, "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4, 103-124.

Mankiw, N. Gregory and Ricardo Reis, 2002, "Sticky Information Versus Sticky Prices: A Proposal to Replace the new-Keynesian Phillips Curve," *Quarterly Journal of Economics* 117, 1295-1328.

McCallum, Bennett T., 1981, “Price Level Determinacy With an Interest Rate Policy Rule and Rational Expectations,” *Journal of Monetary Economics* 8, 319-329.

McCallum, Bennett T., 2003, “Multiple-Solution Indeterminacies in Monetary Policy Analysis,” *Journal of Monetary Economics*, 50, 1153-75.

Minford, Patrick, Francesco Perugini and Naveen Srinivasan, 2001,“The Observational Equivalence of Taylor Rule and Taylor-Type Rules” Forthcoming in *Progress in Macroeconomics Research,* by Columbus F.(Ed.), Nova Science Publishers, Inc, NY, USA, also Centre for Economic Policy Research Discussion Paper No. 2959, September 2001.

Minford, Patrick Francesco Perugini and Naveen Srinivasan, 2002, “Are Interest Rate Regressions Evidence for a Taylor Rule?” *Economics Letters,* 76 (1), 145-150.

Obstfeld, Maurice and Kenneth Rogoff, 1983, “Speculative Hyperinflations in Maximizing Models: Can we Rule them Out?,” *Journal of Political Economy* 91, 675-687.

Obstfeld, Maurice and Kenneth Rogoff, 1986,“Ruling out Divergent Speculative Bubbles,” *Journal of Monetary Economics* 17, 349-362.

Patinkin, Don, 1949, “The Indeterminacy of Absolute Prices in Classical Economic Theory,” *Econometrica* 17, 1–27.

Patinkin, Don, 1965, *Money, Interest, and Prices*, 2nd Edition. New York: Harper and Row.

Sargent, Thomas J., and Neil Wallace, 1975, “‘Rational’ Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy* 83, 241-254.

Sargent, Thomas J., and Neil Wallace, 1981, “Some Unpleasant Monetarist Arithmetic,” *Quarterly Review*, Federal Reserve Bank of Minneapolis, (Fall).

Sbordone, Argia M., 2005, “Do Expected Future Marginal Costs Drive Inflation Dynamics?,” *Journal of Monetary Economics*, 52 (6),1183-1197.

Schmitt-Grohé, Stephanie and Martin Uribe, 2000, “Price Level Determinacy and Monetary Policy Under a Balanced Budget Requirement,” *Journal of Monetary Economics* 45, 211-246.

Sims, Christopher A., 1994, “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory* 4, 381-99.

Sims, Christopher, 2003, “Limits to Inflation Targeting,” in Bernanke, Ben S., and Michael Woodford, eds., *The Inflation-Targeting Debate*, NBER Studies in Business Cycles Volume 32, Ch. 7, 283-310. Chicago: University of Chicago Press.
http://sims.princeton.edu/yftp/Targeting/TargetingFiscalPaper.pdf.

Taylor, John B. 1993, "Discretion Versus Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Taylor, John B. 1999, "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank," *Journal of Monetary Economics* 43, 655-679.

Woodford, Michael, 1994, "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *Economic Theory* 4, 345-380.

Woodford, Michael, 1995, "Price Level Determinacy Without Control of a Monetary Aggregate," *Carnegie Rochester Conference Series on Public Policy,* 43, 1-46.

Woodford, Michael, 2001, "Monetary Policy in the Information Economy," in *Economic Policy for the Information Economy*, Kansas City: Federal Reserve Bank of Kansas City.

Woodford, Michael, 2003, *Interest and Prices,* Princeton: Princeton University Press.