Asset Pricing Theory and the Equity Premium

John H. Cochrane
University of Chicago

October 7, 2004

Outline

1. What is the *equity premium*? How much should stocks outperform bonds over the long run?

   (a) What is the *unconditional mean* $E(R^{stock} - R^{bond})$? The very long run average

   (b) What is the *conditional mean* right now, and for the next 20-30 years?

   (c) Analogy: Temperature next week? Unconditional mean temp in Chicago: 50^o. Conditional mean if it's Jan 20: 20^o

2. Who cares?

   (a) Investors: put your money in stocks or bonds?

   (b) Public policy: should Social Security invest in stocks?

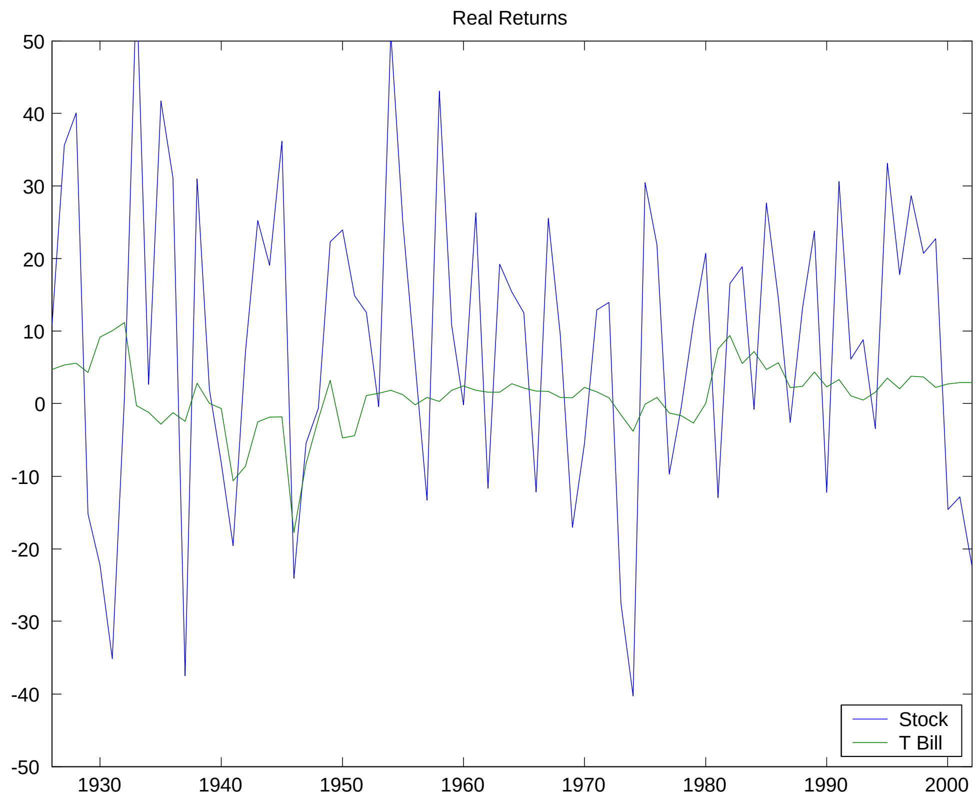
   (c) Corporate decision making. Do we build a factory? Well, how much would we earn if we put our money in stocks instead? "Cost of capital"

   (d) Macroeconomics: Are recessions important? Yes if *risk premium* is high; if people give up a lot of mean return to hold "safe" assets.

3. Approach: mix facts (statistical analysis) and economic theory.

4. Point: what is academic ("scientific") finance? (Why you should have paid attention in economics and statistics classes!)

Facts:



Real stock and bond returns 1927-2002

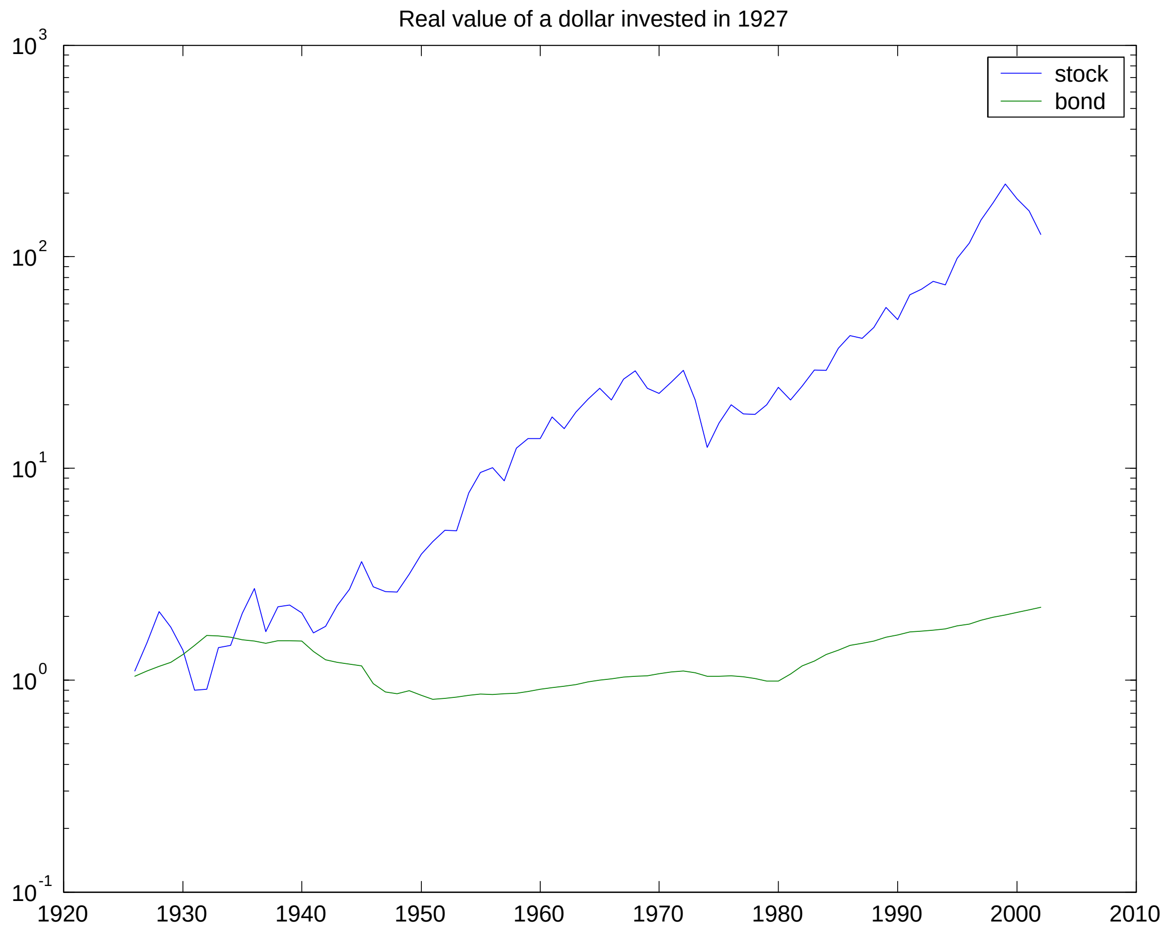
	Bond	(Stock-Bond)
Mean annual % return	1.1	7.5
Standard Deviation	4.4	20.8

7.5% is a huge rate of return Equivalently, *are stock prices really that* **low?**

Value of $1 invested

Horizon (Years)	Stock (1.1+7.5)%	Bond 1.1%
5	1.51	1.05
10	2.29	1.12
20	5.22	1.25
30	11.94	1.40
50	62.44	1.76

$$\$24 \times (1 + 0.075 + 0.011)^{(2004-1620)} = 1.38 \times 10^{15} = \$1,376 \text{ Trillion}$$



Real value of a dollar invested in 1927
10^3
10^2
10^1
10^0
10^-1
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
stock
bond

Implications? (Why is a 94 Yugo so cheap?)

1. Buy a lot of stocks, right now. Retire early and rich.

   - Doubt: Why haven't others figured this out and bid up the price of stocks so that returns from here on out will not be so good? Surely in 100 years they can see the pattern?

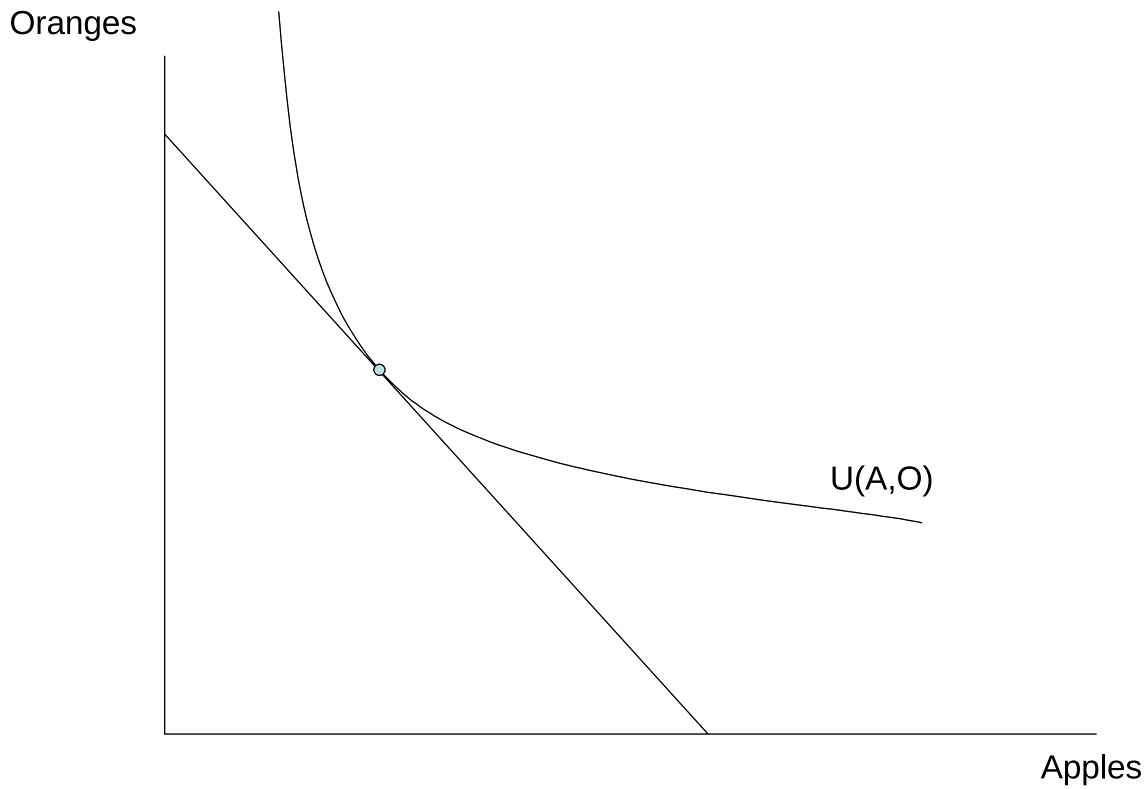
   - If so, 7.5% return will disappear once people do figure it out. (Market is not in equilibrium.) Social security, policy and macro should not use 7.5%

2. *Risk*. People know about the 7.5% extra average return, but they are afraid of stock risk. The attraction of 7.5% mean is just enough to get them to hold current stocks despite the risk.

   - If so, 7.5% will last; the market is in equilibrium. Beware the risk, it probably applies to you too.

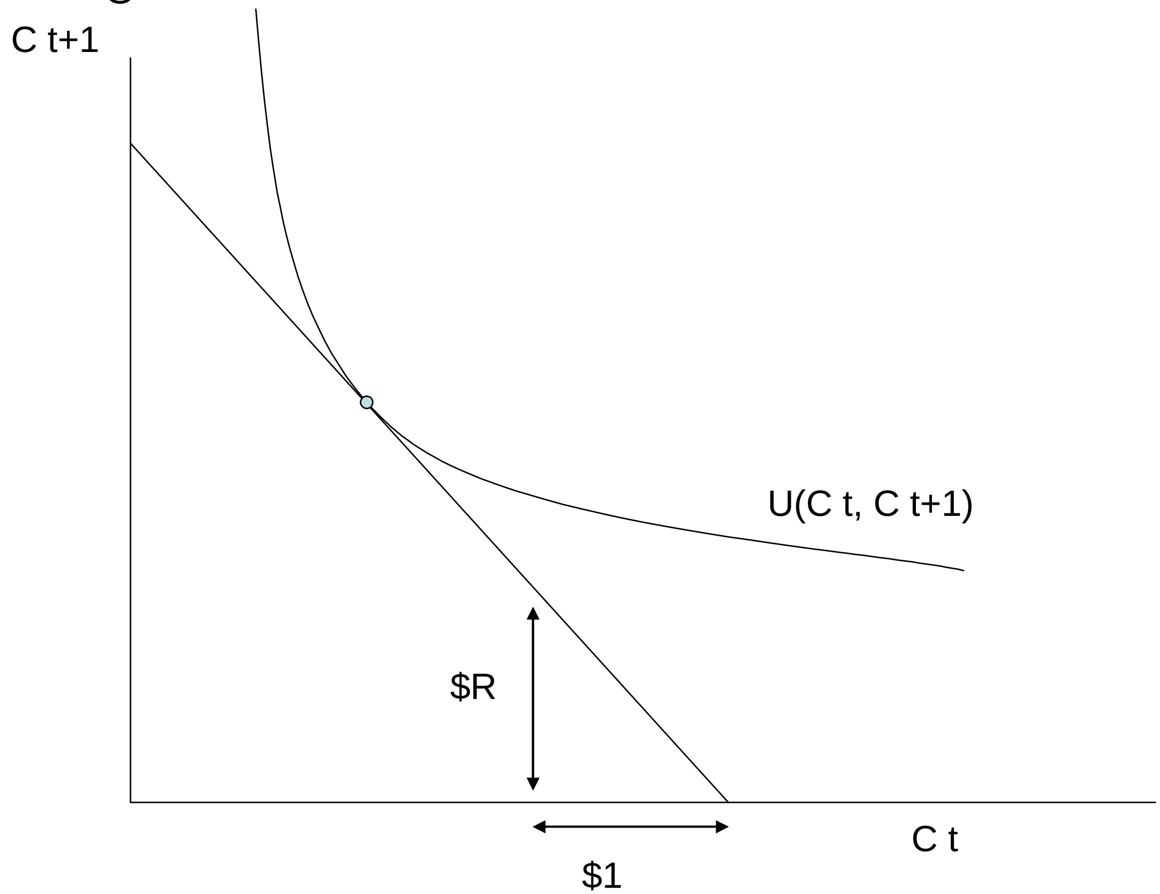
   - →Understand the *risk premiums. This is the basic question of asset pricing theory.* To Economics.

maximize $U(A, O)$ s.t. $p_A A + p_O O = Y$



Marginal rate of substitution $\mathrm{O/A} =$ price ratio

Let's to the same thing.



Marginal rate of substitution today / tomorrow = rate of return

$$
\begin{aligned}
\text{utility cost of \$1 less today} &= \text{utility benefit of } R \text{ more \$ tomorrow} \\
u'(c_t) \times 1 &= E\left[\beta u'(c_{t+1}) R_{t+1}\right]
\end{aligned}
$$

$$
1 = E\left[\beta \frac{u'(c_{t+1})}{u(c_t)} R_{t+1}\right] = E\left[m_{t+1} R_{t+1}\right]
$$

A typical form:

$$
\begin{aligned}
u(c) &= c^{1-\gamma} \\
\gamma &= \text{coefficient of risk aversion}
\end{aligned}
$$

Use 1: understand interest rates.

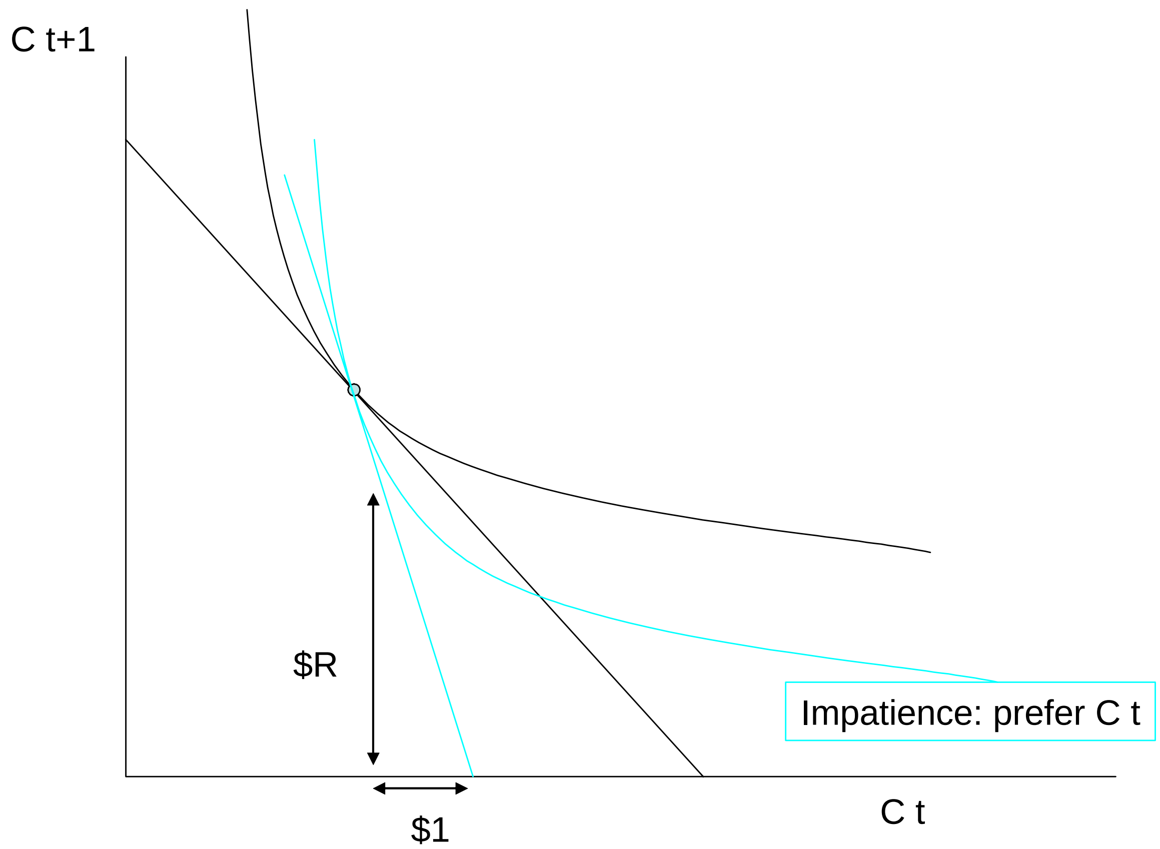
$$R^f \approx 1 + \delta + \gamma E\left(\Delta c_{t+1}\right) - \frac{1}{2}\gamma(\gamma - 1)\sigma^2(\Delta c_{t+1})$$

When are interest rates high?

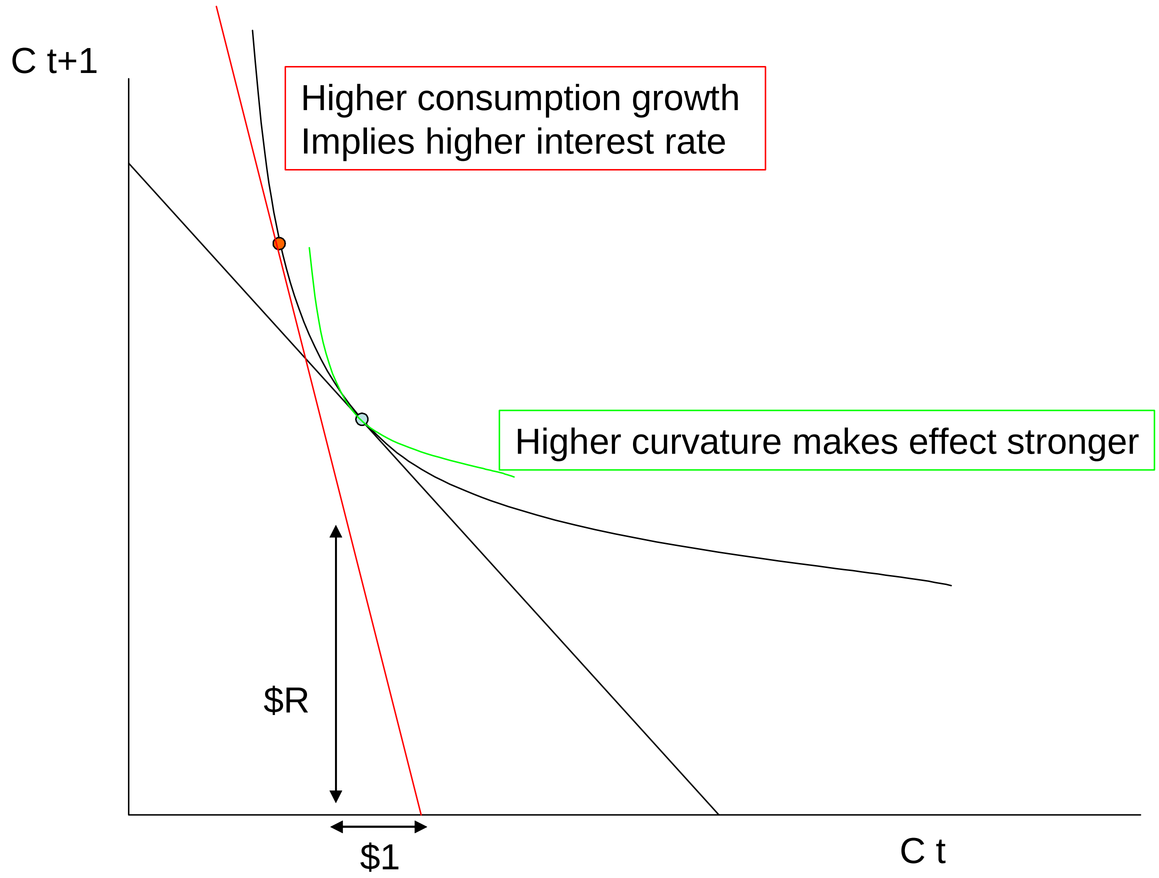
1. When people are more *impatient*, δ is high. Everyone wants to borrow, driving up rates.

2. In *good times*, $E_t\left(\Delta c_{t+1}\right)$ is high. No one wants to save, must offer them high rates. γ controls the effect – "intertemporal substitution elasticity"

3. In *safe* times. $\sigma^2(\Delta c_{t+1})$ is low. Less demand to "save for a rainy day". γ controls the effect, "risk aversion coefficient."



C t+1
$R
$1
C t
Impatience: prefer C t


C t+1
Higher consumption growth
Implies higher interest rate
Higher curvature makes effect stronger
$R
$1
C t

Use 2: Understand *risk premia* – why do stocks pay *more* than bonds?

1. Central equation

$$E(R_{t+1} - R_t^f) \approx \gamma \times cov(\Delta c_{t+1}, R_{t+1}^e)$$

2. *Must pay a risk premium if the return is bad in "bad times."*

3. Football bets pay 0% (50/50 odds). What makes you fear stocks so much more than (say) a football bet, that stocks must offer 7.5% to get you in? Stocks tend to fall *when times are bad* (low consumption)

4. *Volatility*, per se, *does not matter.* **Co***variance* with consumption growth matters. (And you thought economics was all obvious!)

5. Is a stock with a high expected return a "good stock" which you should buy?

   - No. A stock with a high expected return is *in equilibrium* like a low-priced car. The high return is there to *compensate you for risk*. Covariance with consumption growth measures the risk.

- Digression: This is *the* central starting point for the *entire theory of asset pricing.* All models do two things

1. Manipulate $1 = E(mR)$ to address the problem at hand

2. Find different ways of constructing m that work for the problem at hand. Less "pure" but work better.

- Examples:

1. CAPM. Use the market portfolio return to proxy for consumption

$$\begin{aligned} \Delta c_t &\approx -kR_t^{\text{market}} \\ m_t &= a - bR_t^{\text{market}} \iff E(R^{ei}) = \beta_i E(R^{e\text{market}}) \end{aligned}$$

2. "Multifactor asset pricing models" (Fama-French 3 factor)

$$m_t = a - b_1 R_t^{\text{market}} - b_2(\text{other portfolios}) - b_3(\text{other macro indicators})$$

3. "Arbitrage pricing"

$$m_t = a - b'(\text{portfolios that do a good job of tracking returns})$$

4. Black-Scholes option pricing

$$m_t = f(\text{Stock}_t,\ \text{Bond}_t)$$

Construct m to price S, B, use it to price option

5. Models of the term structure of interest rates (bond prices)

$$R = \frac{1}{p}$$

$$p_t^{(1)} = E(m_{t+1} \times 1)$$

$$p_t^{(2)} = E(m_{t+1} m_{t+2} \times 1)$$

Back to the equity premium. Can it really be 7.5%? Can the risk be so high to require this much compensation?

- Theory reminder

$$E(R_{t+1} - R^f) \approx \gamma cov(\Delta c_{t+1}, R_{t+1})$$

$$R^f \approx 1 + \delta + \gamma E\left(\Delta c_{t+1}\right) - \frac{1}{2}\gamma(\gamma - 1)\sigma^2(\Delta c_{t+1})$$

- Facts

Annual data 1948-2002, percent

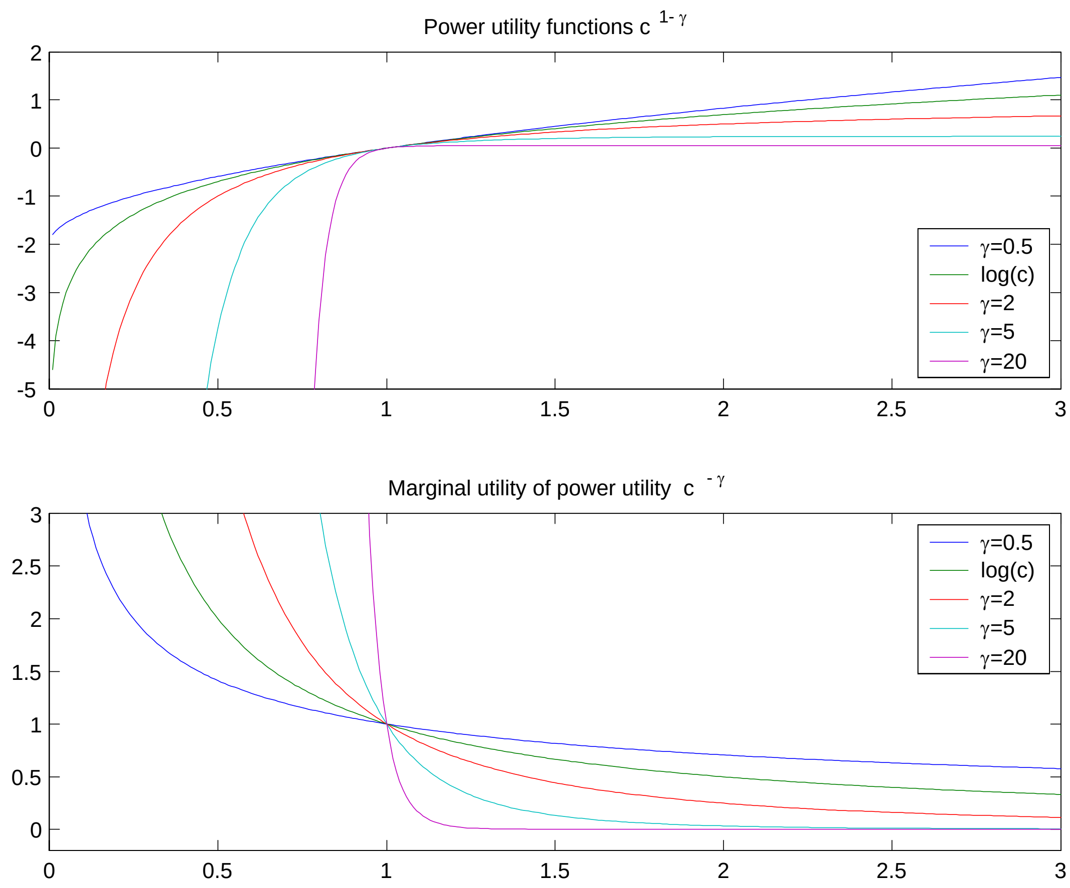
$E(\Delta c)$	$\sigma(\Delta c)$	$E(R - R^f)$	$\sigma(R)$	$E(R^{\text{bond}})$	corr$(\Delta c, R)$	cov$(\Delta c, R)$
1.31	1.93	7.21	18.0	0.39	0.39	0.135

- Theory is a *qualitative* (sign, story telling) success. Stock go down in bad times.

- Good "scientific" theory needs *quantitative* success.

$$\begin{aligned} E(R^e_{t+1}) &\approx \gamma \times cov(\Delta c_{t+1}, R^e_{t+1}) = \gamma \times \sigma(\Delta c)\sigma(R)\rho \\ 7.2 &= \gamma \times 0.135 \\ \gamma &= 53! \end{aligned}$$

- 53 is *HUGE!*


Power utility functions c 1-γ
2
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
2
2.5
3
γ=0.5
log(c)
γ=2
γ=5
γ=20
Marginal utility of power utility c -γ
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
γ=0.5
log(c)
γ=2
γ=5
γ=20

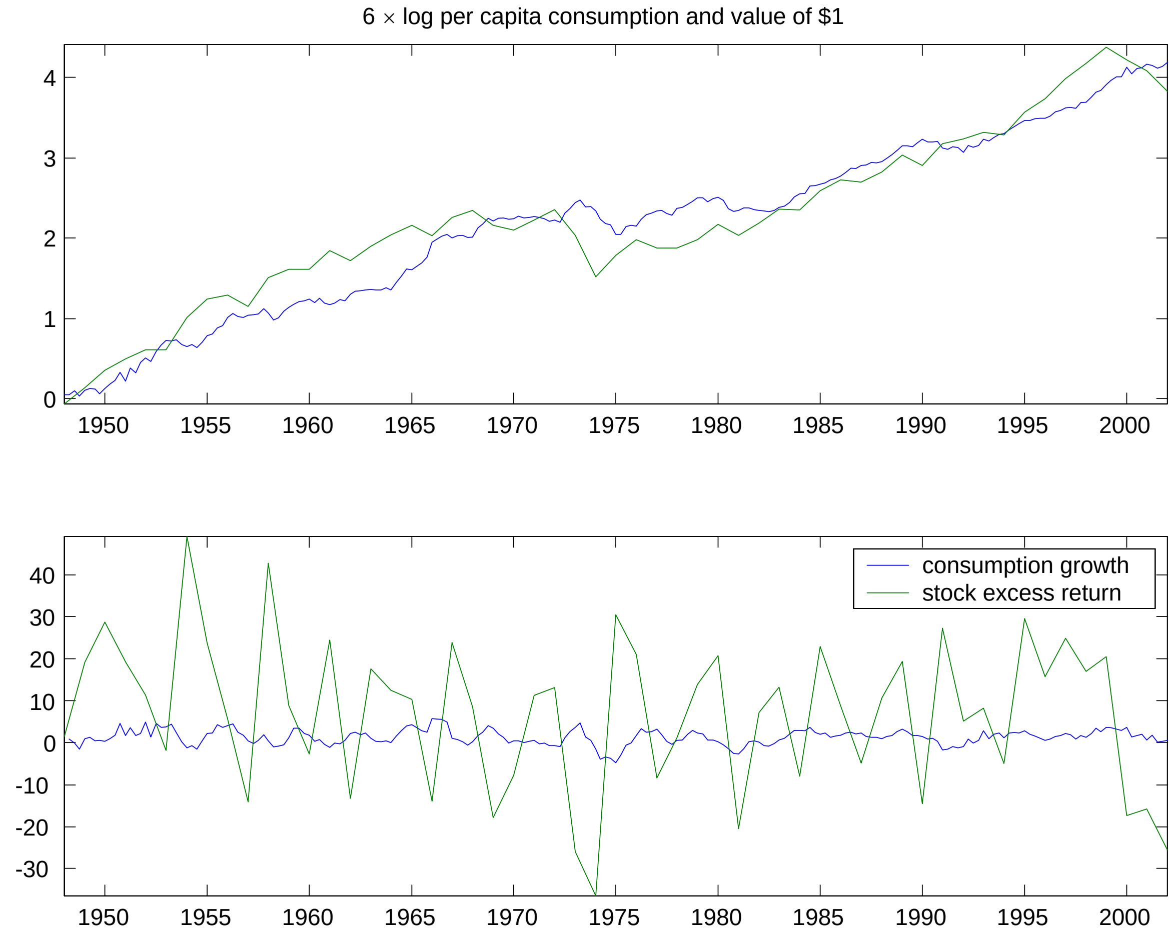
$\gamma = 53$ is huge

How much would you pay to avoid a 50/50 bet? (Assume consumption = 30k/.year)

	Risk aversion γ			
bet	2	10	50	100
\$10	\$0.00	\$0.01	\$0.05	\$0.10
\$100	\$0.20	\$1.00	\$4.99	\$9.94
\$1,000	\$20	\$99	\$435	\$665
\$10,000	\$2,000	\$6,921	\$9,430	\$9,718.00

$$\frac{\text{amount willing to pay to avoid bet}}{\text{size of bet}} = \gamma \frac{\text{size of bet}}{\text{consumption}}$$

Source of the problem: Consumption is much smoother than stock returns. If we had σ(consumption) $= 25\%$, like stocks, there would be no problem – $cov(R, \Delta c)$ would be huge. But consumption says we live in a quite safe economy.



- Even if we accept $\gamma = 53$, it predicts crazy interest rates. (*Two* facts hinge on *one* parameter).

- R^f with $\gamma = 53$,

$$\begin{aligned} R^f &\approx 1 + \delta + \gamma E\left(\Delta c_{t+1}\right) - \frac{1}{2}\gamma(\gamma - 1)\sigma^2(\Delta c_{t+1}) \\ &\approx 1 + \delta + 53 \times 0.0131 - \frac{1}{2}53 \times 52 \times 0.0193^2 \\ &= 1 + \delta + 0.18 \end{aligned}$$

  Either $\delta = -18\%$ (prefer future) or model predicts $18 + \delta +$ inflation $\approx 25\%$ interest rate!

- Worse: a 1% increase in consumption growth (consumption growth from 1% to 2%) implies interest rates rise by 53%!!

- $\Rightarrow$*Existing economic theory does not deliver anything like a 7.5% equity premium. "Equity Premium /Risk free rate Puzzle."*

- Like "speed of light puzzle" in 19th century physics, or "position of the planets" puzzle in 16th century, this sort of thing is good!

Responses:

1. LOTS (me included)! Different utility not $u(c) = c^{1-\gamma}$? How about $u(c_t, c_{t-1})$? How about $u(c_t, l_t)$? Different consumption data? Individual risks $\sigma(\Delta c)$ larger than economy average? Not everyone holds stocks? Care about job loss, etc., not just consumption? Disentangle risk aversion (stocks) from substitution over time (interest rates)? Fear of occasional large meltdowns?

2. Result: Extremely high risk aversion has not yet been avoided if you want 7.5%.

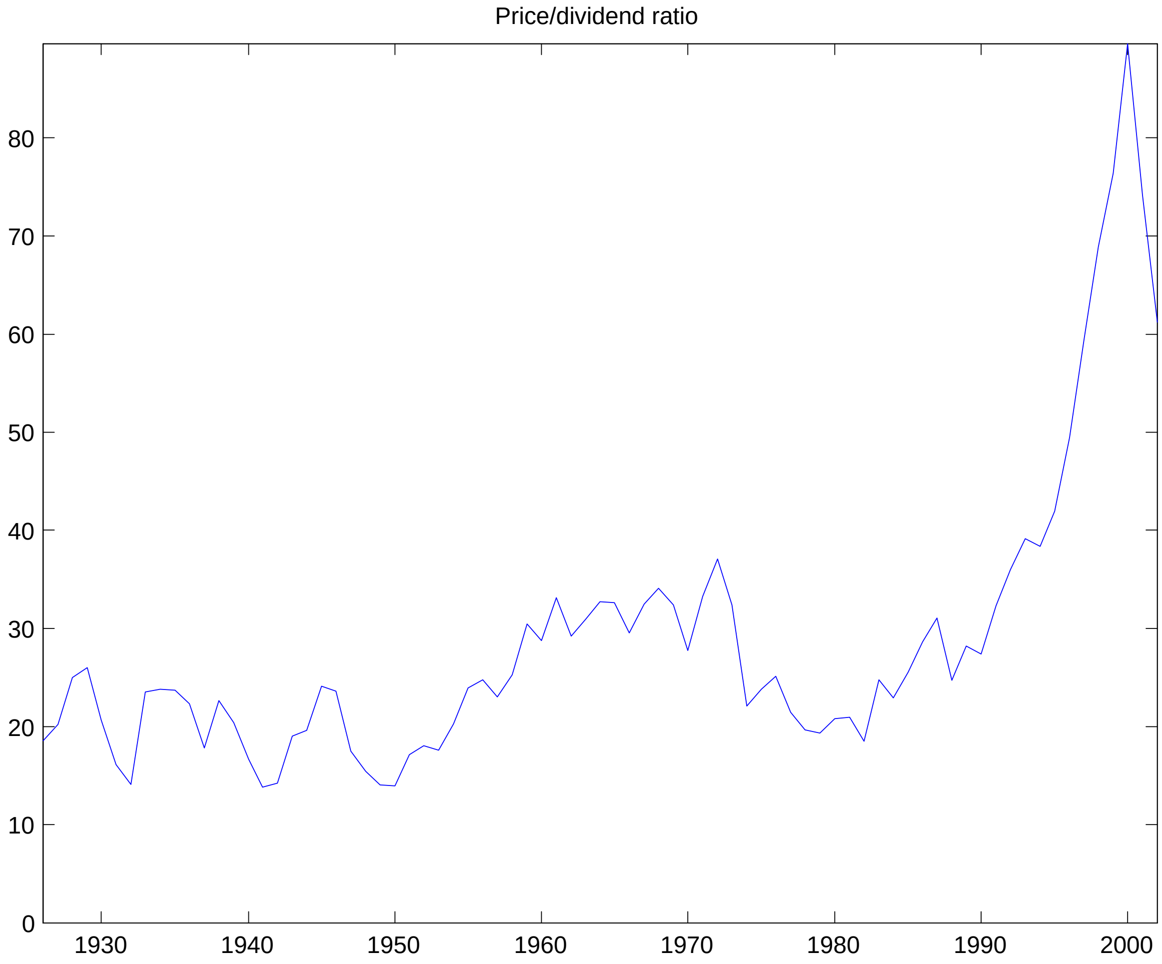
3. If it makes no economic sense, is it really there?

	1927-2002 Stock-TB	TB
Mean $E(R)$	7.49	1.13
Std dev $\sigma(R)$	20.9	4.40
Std. error $\sigma/\sqrt{T}$	2.38	0.50
Mean +/- 1 σ (66%)	5.11 – 9.87	
Mean +/- 2 σ (95%)	2.73– 12.25	

Stocks are so volatile that even close to a century of data we don't know the mean stock return for sure!

My own view: A lot of the last century 7.5% was good luck and many more people joining the stock market. Prices are high, so the mean return going forward is a lot smaller – 2-3% over bonds at most. (That's still a lot!)

What is the *conditional* equity premium? Is it January or July? P/D is like temperature:



$$
\begin{aligned}
\frac{P}{D} &= \frac{1}{r-g} \\
r &= \text{return} \\
g &= \text{dividend growth rate}
\end{aligned}
$$

1. High price/dividend means either low return r (price goes down) or high dividend growth g. (dividend goes up)

2. *In the past, rises in p/d have always signaled times of low subsequent returns, not times of high subsequent growth.*

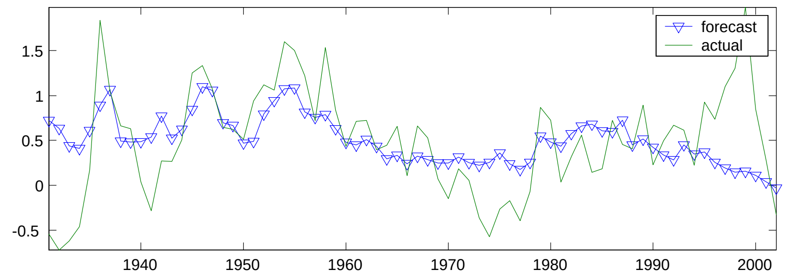
3. Economics: high expected returns and low prices in the depths of recessions, and vice versa. ("Random walk is long dead!")

4. How do we document this sort of thing? *Run a regression of returns, dividend growth on today's PD ratio.*

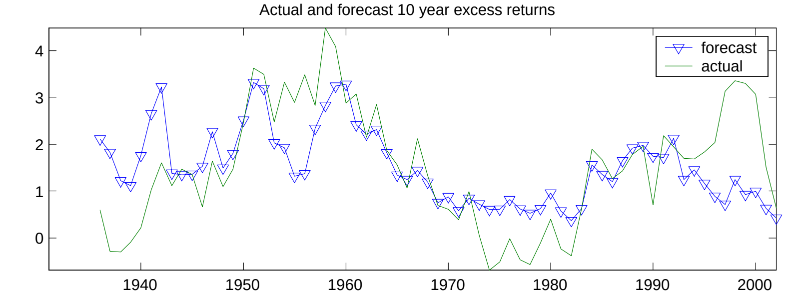
5. Analogy: run a regression of Saturday's temperature on Friday's forecast. If the forecast is good, you should see a positive coefficient.

	1 year	5 year	10 year
	$R_{t+1} = a + bD_t/P_t + \varepsilon_{t+1}$		
b	4.8	20.3	64.8
t	2.9	2.1	3.0
R^2	0.11	0.19	0.37
	$D_{t+1}/D_t = a + bD_t/P_t + \varepsilon_{t+1}$		
b	0.8	2.3	3.06
t	0.9	1.5	0.98
R^2	0.01	0.01	0.01

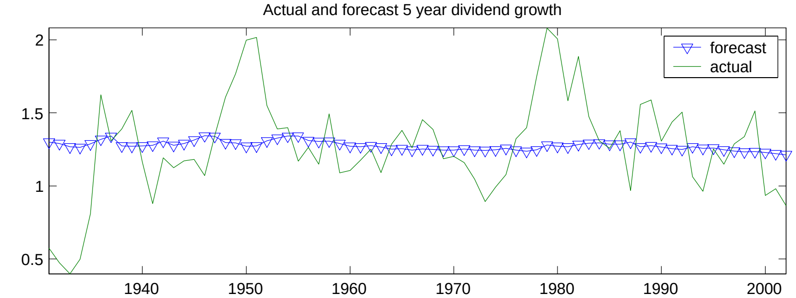
Actual and forecast 5 year excess returns



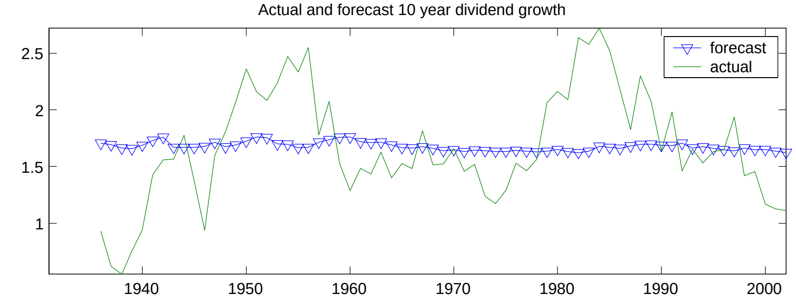
Actual and forecast 10 year excess returns



Actual and forecast 5 year dividend growth



Actual and forecast 10 year dividend growth



1. So, what does high p/d now mean? Some possibilities:

   (a) *This time,* P/D will stay high (on average) forever. "Permanently higher plateau" (Irving Fisher 1929). If so, and if g does not change, then returns will be lower.

$$r = g + \frac{d}{p}$$

$$\begin{aligned} \text{past:} \quad & 7.5\% = 3.5\% + (\frac{1}{20} = 4\%) \\ \text{high p/d} \quad : \quad & 5.5\% = 3.5\% + (\frac{1}{50} = 2\%) \end{aligned}$$

   (b) P/D will revert back to its historical norm, as it has every time in the past.

      i. As in the past, via low r. $\longrightarrow$ D/P forecast of 1-2% premium

      ii. *This time* via exceptionally high g.

2. Answer? Who knows. We do know the logical possibilities!.

3. Learn more?

   (a) This talk: www.gsb.uchicago.edu, faculty, cochrane, research unrestricted. Also, "Where is the market going?" "New Facts in Finance" "Portfolio advice for a multifactor world?"

   (b) *Asset Pricing* Princeton University Press (This is Ch.1)

   (c) Do an MBA or Ph.D. in finance!