The New-Keynesian Liquidity Trap

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Abstract

Many new-Keynesian models produce a deep recession with deflation at the zero bound. These models also make unusual policy predictions: Useless government spending, technical regress, capital destruction, and forward guidance can raise output. Moreover, these predictions are larger as prices become less sticky and as changes are expected further in the future. I show that these predictions are strongly affected by equilibrium selection. For the same interest-rate path, equilibria that bound initial jumps predict mild inflation, small output variation, negative multipliers, small effects of far-off expectations and a smooth frictionless limit. Fiscal policy considerations suggest the latter equilibria.

Keywords: Zero bound, multiplier, multiple equilibria, fiscal theory

JEL: E5

1. Introduction

Many models in the new-Keynesian tradition predict a deep recession with deflation when the "natural" rate of interest is negative and the nominal rate is stuck at zero. Those models also produce unusual policy predictions. Forward guidance about central bank actions can strongly stimulate the current level of output. Fully-expected future inflation can raise output. Deliberate capital destruction or productivity reduction can raise output. Government spending, even if financed by current taxation, and even if completely wasted, can

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[**]I thank Tom Coleman, Behzad Diba, Bill Dupor, Martin Eichenbaum, Jesús Fernández-Villaverde, Miles Kimball, Narayana Kocherlakota, Olivier Loisel, Ed Nelson, Ricardo Reis, Mizuki Tsuboi, Iván Werning, Johannes Wieland, Michael Woodford, and seminar participants for many helpful comments. I especially thank nine referees and three editors for detailed comments. I thank CRSP and the Guggenheim Foundation for research support.

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have a large output multiplier. A given promise or expectation further in the future has larger effects today. As prices become more flexible, deflation and depression get worse and unusual policy prescriptions become stronger. Tiny price stickiness has unboundedly large effects, though all effects vanish when prices are fully flexible.

For a given path of expected interest rates, new-Keynesian models allow multiple stable equilibrium paths for inflation and output. Thus, to produce a prediction, a researcher must choose an equilibrium as well as a path for expected interest rates.

I show that these liquidity-trap predictions are sensitive to equilibrium choice. Choosing different equilibria, either directly as an additional modeling specification, or by different specifications of central bank equilibrium-selection policy, despite exactly the same path of interest-rate expectations, the same model can predict gentle inflation matching the negative natural rate, small output gaps, and normal signs and magnitudes of policies. Inflation, output and policy predictions are smaller for events expected further in the future, and smoothly approach the frictionless limit.

In the most general terms, the standard models choose equilibria by thinking about expectations of output and inflation when the economy exits the zero bound, and then working backwards. The alternative equilibria I study limit how much inflation and output can jump on the day that the economy learns of the natural rate shock. A variety of criteria suggest such a limitation, especially fiscal policy considerations. Since a sharp deflation raises the value of government bonds, a limitation on the government's ability or willingness to raise taxes limits initial deflation, and consequently all effects of the zero bound.

1.1. Literature

Werning (2012) shows clearly the predictions for a depression and deflation at the zero bound, and some policy paradoxes. I adopt his simple modeling framework. This paper is not a critique of Werning. Werning studies the properties of one equilibrium. He acknowledges multiple equilibria. I explore their nature.

Kiley (2016) and Wieland (2014) nicely summarize the puzzling predictions of new-Keynesian zero-bound analyses. Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011), Woodford (2011), and Carlstrom, Fuerst and Paustian (2014) all find large fiscal multipliers, and multipliers that increase with the duration of fiscal expansion.

Carlstrom, Fuerst, and Paustian (2015) study forward guidance, and show the backward explosions highlighted here, that inflation and output increase exponentially in the duration of forward guidance. They show the paradox is worse with inflation indexation in the Phillips curve, but lessened with a sticky-information curve following Mankiw and Reis (2002). Since I focus on equilibrium selection issues, I consider only the simple forward-looking Phillips curve.

Eggertsson (2010) and Wieland (2014) analyze the "paradox of toil" that negative productivity can be expansionary. Eggertsson, Ferrero and Raffo (2013) argue that structural reforms are contractionary. See also the discussion in Fernández-Villaverde (2013).

Werning's (2012) main point, as that of Eggertsson and Woodford (2003) and Woodford (2012), is to study optimal policy. These authors find a path of inflation, output, and interest rates that maximizes a planner's objective. This path typically involves keeping interest rates low for some time after the natural-rate shock ends. They then advocate "forward guidance," that central bank officials announce and somehow commit to such policies.

This paper makes no optimal policy calculations. I study outcomes for a variety of given policies, as in the above-cited literature. Some of those policies resemble optimal policies. For example, I study postponed rises in interest rates, which Werning (2012) finds are optimal. I focus on the "implementation" problem: To achieve optimal results, it is not enough for the central bank to specify the path of interest rates. The central bank must take some other action to select among multiple equilibria consistent with the optimal interest rate path. Looking at those equilibria, I find that this selection is far more important to the results than is the path of equilibrium interest rates.

2. Model

I use Werning's (2012) simple continuous-time specification of the standard new-Keynesian model:

$$\frac{dx_t}{dt} = \sigma\left(i_t - r_t - \pi_t\right) \tag{1}$$

$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa(x_t + g_t). \tag{2}$$

Here, x_t is the output gap, i_t is the nominal rate of interest, r_t is the "natural" real rate of interest, π_t is inflation, and g_t is a Phillips curve disturbance discussed below. I abstract from constants, so these are all deviations from steady state values.

Equation (1) expresses the intertemporal substitution of consumption, and consumption equals output. Equation (2) is the new-Keynesian Phillips curve. Solved forwards, it expresses inflation in terms of expected future output gaps.

Like Werning, I suppose that starting at $t = 0$, the economy suffers from a negative natural rate $r_t = r = -2\%$, which lasts until time $t = T = 5$ before returning to a positive value. Also following Werning, I complete the model by specifying that the path of equilibrium nominal interest rates is zero up to period T, and then rises back to the natural rate $i_t = r_t \geq 0$, for $t \geq T$. I use $\rho = 0.05$, $\sigma = 1$ and $\kappa = 1$.

Then, I find the set of output $\{x_t\}$ and inflation $\{\pi_t\}$ paths that, via (1) and (2), are consistent with this path of interest rates, and do not explode as time increases. It will turn out that there are many such paths.

Perfect foresight of a trap end date is unrealistic. However, it is simple and clear, and it provides a useful guide to the behavior of models with a stochastically ending trap or a slowly mean-reverting natural-rate processes.

Specifying directly the equilibrium path of interest rates does not mean that I assume a peg, that interest rates are exogenous, or that I ignore Taylor rules or other policy rules. Typically, one adds to (1)-(2) a policy rule of the form

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \tag{3}$$

or equivalently,

$$i_t = \hat{\imath}_t + \phi \pi_t \tag{4}$$

where i_t^*, π_t^*, or $\hat{\imath}_t = i_t^* - \phi \pi_t^*$ are set by the central bank as targets or policy disturbances. One then solves for the equilibrium path of inflation, output, and interest rates given the natural rate shock and the policy disturbances.

However, we can also work backwards. First, we can specify the desired equilibrium interest rate path $\{i_t\}$, find equilibrium output and inflation $\{\pi_t\}$, $\{x_t\}$, and then construct the underlying policy rule $\{i_t^* = i_t\}$, $\{x_t^* = x_t\}$ or $\{\hat{\imath}_t = i_t - \phi \pi_t\}$. Since we want to fix interest rate expectations rather than policy disturbances, this procedure saves us a search for the policy disturbances that produce the desired interest rate path. Werning (2012) innovated this clever solution strategy. Closing the model with a policy rule *per se* does nothing to prune multiple stable (non-exploding) equilibria for a given interest rate path.

2.1. The flexible-price case

The flexible-price case is an important benchmark. Prices become more flexible as κ increases, and $\kappa = \infty$ is the flexible-price case. At this value, the output gap is zero for any value of inflation.

Turning to (1), if $x_t = 0$ then $dx_t/dt = 0$ and we must have $i_t - r_t = \pi_t$. This is just the linearized Fisher relationship, which becomes the entire new-Keynesian model.

Thus, the flexible-price solution to our liquidity-trap scenario is $\pi_t = -r = 2\%,\ x_t = 0$ for $0 < t < T$, and $\pi_t = 0,\ x_t = 0$ for $t > T$. Inflation in the frictionless world rises to exactly equal to the negative natural rate, all on its own without extra prodding by the central bank, producing the required negative real rate at a zero nominal rate. There is no output gap.

In a perfect foresight model, this equilibrium is unique, up to the overall price level. If the natural rate change is unexpected, then there can be a price-level jump at the moment of that shock. In discrete time, $i_t = r_t + E_t\pi_{t+1}$ allows multiple equilibria $\pi_{t+1} - E_t\pi_{t+1}$ consistent with a given equilibrium nominal rate i_t. But then the path $\{E_t\pi_{t+j}\}$ is then unique. A price-level jump at 0 is the continuous-time counterpart.

2.2. Equilibria with price stickiness

Differentiate (2), and substitute from (1) for dx_t/dt to obtain

$$\frac{d^2\pi_t}{dt^2} - \rho\frac{d\pi_t}{dt} - \kappa\sigma\pi_t = -\kappa\sigma(i_t - r_t) - \kappa\frac{dg_t}{dt}. \tag{5}$$

The forward-stable solutions of this differential equation, described in the online appendix, are

$$\pi_t = Ce^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b}\left[\int_{s=-\infty}^{t} e^{-\lambda^b(t-s)}z_s ds + \int_{s=t}^{\infty} e^{-\lambda^f(s-t)}z_s ds\right], \tag{6}$$

where

$$\lambda^f \equiv \frac{1}{2}\left(\sqrt{\rho^2 + 4\kappa\sigma} + \rho\right);\ \lambda^b \equiv \frac{1}{2}\left(\sqrt{\rho^2 + 4\kappa\sigma} - \rho\right);\ z_t \equiv \kappa\sigma(i_t - r_t) + \kappa\frac{dg_t}{dt}. \tag{7}$$

From (2), then, the output gap follows

$$\kappa x_t = -\kappa g_t + \lambda^f C e^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b}\left[\lambda^f \int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} z_s ds - \lambda^b \int_{s=t}^{\infty} e^{-\lambda^f (s-t)} z_s ds\right]. \tag{8}$$

Inflation and output in the solutions (6) and (8) are two-sided moving averages of the driving processes. Inflation is a positive function of the driving disturbance in both directions. Output is a negative function of future disturbances, but a positive function of past disturbances. Since ρ is a small number, the forward λ^f and backward λ^b roots are nearly, but not quite the same, and the forward-looking weights λ^f are a bit smaller.

I set to zero multiple forward-explosive equilibria corresponding to a second free constant $C_f e^{\lambda_f t}$. But there remain multiple forward-stable equilibria indexed by the free constant C.

These formulas are perfect foresight solutions. As such, they capture the impulse-response function, and the path of expected values in a stochastic model. In the case of an unexpected shock, the economy jumps to these solutions on the date that the shock is known.

2.3. Inflation and output paths

From (6)-(8), it is straightforward to calculate the paths of inflation and output for the forcing variable $z_t = \kappa\sigma(i_t - r_t)$ that starts at zero, rises to $i_t - r_t = 2\%$ for $0 < t < T$, and then falls to zero again. The formulas are given in the online Appendix.

Figure 1 shows inflation in a range of such equilibria, generated by a range of values for the free constant C. These are all equilibria of the *same* model, with the *same* interest rate and natural rate path. The equilibria are all forward-stable, following the usual rules. They all use forward-looking solutions of the unstable eigenvalue. Unlike Cochrane (2011), this paper does not consider multiple explosive equilibria.

3. Three equilibria

I examine here the properties of three specific equilibria. Figure 1 includes their inflation paths. Section 5 discuss principles for choosing among equilibria. Since no principle is overwhelmingly preferred on ex-ante grounds, it is better to look at the results rather than to announce one principle and ignore the other possibilities.

3.1. The standard equilibrium

Werning (2012) chooses the equilibrium with zero inflation on the date that the trap ends, $\pi_T = 0$. To calculate this equilibrium, I find the value of C in (6) that yields $\pi_T = 0$.

Figure 2 presents inflation and output in this standard equilibrium choice. It shows a large deflation and large output gaps during the liquidity-trap period $0 < t < T$.

We also see strong dynamics – deflation steadily improves, and expected output *growth* is strong. The forward-looking Phillips curve (2) produces a large output gap when inflation is lower today than in the future. If inflation is to end up at zero, that curve produces substantial, but improving, deflation.

This equilibrium does not show an unstable deflation "spiral," in which a small deflation grows bigger over time. Such a spiral is a feature of models with adaptive expectations. In this forward-looking model, inflation is stable, even in a liquidity trap. This equilibrium also does not produce a "slump," a large but steady output gap and steady but low inflation.

This equilibrium explodes *backward* in time. That observation helps to understand many of its predictions. In order to arrive at $\pi_T = 0$, the $t < T$ solution includes a nonzero $Ce^{-\lambda^b t}$ term. This backward explosion implies a large downward jump in inflation and output when the natural rate shock is revealed, at $t = 0$ here. A backward explosion also means that inflation and output deviations grow arbitrarily large as the period of the liquidity trap expands.

The dashed lines in figure 2 show how solutions with this equilibrium choice $\pi_T = 0$ behave as we reduce price frictions, raising κ. Deflation and (not shown) output gaps become *larger* as price stickiness is *reduced.* As pricing frictions decrease, dynamics happen faster, and both roots λ^f and λ^b increase in absolute value. Faster backward explosions, tethered to $\pi_T = 0$, imply lower inflation and output. It seems that, although price stickiness is the only friction in this economy, structural reform to reduce price stickiness would only make matters worse.

Despite this infinite limit, the limit *point* of the frictionless equilibrium is well-behaved at two percent inflation and no output gap. The model with $\pi_T = 0$ equilibrium selection thus displays a large discontinuity. Tiny price stickiness has huge effects, but zero price stickiness has no effect.

3.2. The backward-stable equilibrium

Figure 3 presents the equilibrium with $C = 0$ and thus no extra $Ce^{-\lambda^b t}$ term. Now inflation and output are each just two-sided moving averages of the driving shocks. The thick lines show the standard experiment, that the natural rate shock at $t = 0$ is unexpected.

In this equilibrium, inflation jumps up by 1% on the onset of the trap, rather than jump down by 132% as in the standard equilibrium. The small variation in inflation corresponds to small variation in output, with output low when inflation is low relative to the future and vice versa. At $t = T = 5$, inflation comes down smoothly to zero.

Figure 3 also shows how inflation behaves as we reduce the pricing friction with this equilibrium choice. The $C = 0$ equilibrium smoothly approaches the frictionless limit. I will call equilibria with this well-behaved limit "local-to-frictionless." With this property, small amounts of price stickiness gives inflation and output gaps close to frictionless values.

As the length of the trap episode widens, inflation just takes a similar hat-shaped path, approaching the negative of the natural rate $\pi_t = 2\%$ in the middle of the trap, and the output gap spends more time at zero. Unlike the standard solution, that gets exponentially worse for longer traps, this equilibrium is insensitive to trap length.

Figure 3 also shows what happens if the trap is expected ahead of time, in the thin lines marked "Expected" for $t < 0$. The solution is a two-sided moving average, so inflation and output gap smoothly move ahead of the trap. News of a trap further in the future has smaller impacts on inflation and output before the trap.

A natural description of these properties is that the solutions are "backward-stable." I use this property to give the equilibrium a more memorable name than "$C = 0$ equilibrium."

With a large $Ce^{-\lambda^b t}$ term, the standard $\pi_T = 0$ equilibrium shown in figure 2 explodes (even more) as we move back in time, if people know that the trap is coming ahead of time. (The backward explosion continues to the left of $t = 0$, not shown in figure 2.) Similarly, news of a trap further in the future has larger effects on inflation and output today, reversing standard intuition. Unless the probability of a future trap starting at date $t+s$ also declines at the rate $e^{-\lambda^b s}$, even a small probability of a trap further s in the future has a larger effect on inflation and output today. A constant small possibility of a trap at any date in the future produces an *infinitely* negative inflation and output today.

In sum, the backward-stable $C = 0$ equilibrium of figure 3 suggests that a negative natural rate and the zero bound is a mild event, associated with a mild inflation, which will emerge on its own without any additional policy, and little output variation. It suggests that longer traps are if anything less of a problem, because prices have more time to adjust, and that expectations of far off events have smaller and smaller effects today. If one does not like these predictions – the recession of 2008 was large – then one needs a different model. This paper is about what this model produces, not about what caused the 2008 recession.

By contrast the standard $\pi_T = 0$ equilibrium of figure 2 suggests that this liquidity trap produces a large output gap and deflation, that longer-lasting traps are exponentially (literally) worse, and that expectations of low-probability traps in the far-off future are worse still. At a minimum, we learn that the choice of equilibrium is not an innocuous technical detail, and instead that equilibrium choice is central to the model's economic predictions.

Figure 3 illuminates why the standard equilibrium choice has a discontinuous frictionless limit. The new-Keynesian Phillips curve (2) does not allow expected inflation jumps with a finite output gap. It does allow unexpected inflation jumps, such as at time 0. This backward-stable solution smooths the frictionless case naturally around the end of the trap with inflation following an S shape. But some of that smoothing must occur in the $t > T$ period. Insistence that inflation is zero *immediately* at $t = T$, for any value of price stickiness, drives the economic dislocation and puzzling limiting behavior of the standard solution.

3.3. The no-inflation-jump equilibrium

We can also index equilibria by their jump at time 0, π_0. The standard $\pi_T = 0$ equilibrium choice shown in figure 2 features a large downward jump in inflation and output when the trap is announced. The backward-stable solution in figure 3 has a much smaller, but nonzero, upward jump. The equilibrium in which inflation does not jump on news, $\pi_0 = 0$, is an interesting alternative case. To characterize this equilibrium, I find C such that $\pi_0 = 0$ in (6).

Figure 4 presents output and inflation in the no-inflation-jump $\pi_0 = 0$ equilibrium. The results are not terribly different from those of the backward-stable equilibrium, figure 3.

This equilibrium is also local-to-frictionless, in that the $\kappa \to \infty$ limit smoothly approaches

2% inflation for $0 < t < 5$ as in figure 3. The local-to-frictionless property is not unique to the backward-stable equilibrium.

In this $\pi_0 = 0$ equilibrium output still jumps at $t = 0$. A no-output-jump equilibrium choice $x_0 = 0$ gives a small jump in inflation and otherwise looks similar.

This equilibrium selection concept sets $\pi_t = 0$ on the date t that news of the trap arrives. As one moves the date of news of the trap back to $t = -1, t = -2$, etc., one must find a new C each time to ensure that $\pi_{-1} = 0$, $\pi_{-2} = 0$, etc. As a result, though individual choices of C here are not backward-stable, the equilibrium concept "pick $\pi_t = 0$ on the date people learn about the trap" is also backward-stable. News about traps further in the further have no effect on inflation today, and lower effects on output.

4. Large multipliers and paradoxical policies

Here, I investigate the effects of a disturbance g_t to the Phillips curve,

$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa\left(x_t + g_t\right), \tag{9}$$

Following Werning (2012) and Wieland (2014), the variable g_t can represent government spending. It also can represent deliberate destruction of capital or technological regress – changes that increase marginal costs and therefore shift the Phillips curve directly.

These policies increase inflation π_t for a given output gap. Then, in the IS curve, a rise in inflation reduces the real interest rate and consumption growth. Assuming a return to trend, reducing the consumption growth rate increases the current level of consumption. In this way government spending and adverse cost shifters, can be expansionary.

Solving (1) forward, we have

$$x_t = \int_{s=0}^{\infty} \frac{dx_{t+s}}{ds} ds = \int_{s=0}^{\infty} \sigma\left(i_{t+s} - r_{t+s} - \pi_{t+s}\right) ds. \tag{10}$$

Expected future inflation is the key for stimulus in this model, not current inflation, or unexpected current inflation. Similarly, since output is demand-determined, wealth or capital destruction does not directly affect output or consumption.

The new-Keynesian multiplier is utterly different from static Keynesian intuition. The static Keynesian multiplier results because more income generates more consumption which generates more income. $Y = C + I + G$, and $C = \bar{c} + mY$ imply $Y = (\bar{c} + I + G)/(1 - m)$ and thus a multiplier $1/(1 - m)$. In this new-Keynesian model, the marginal propensity to consume is effectively zero, as the consumer is intertemporally unconstrained and there are no permanent changes in the level of consumption. Fiscal policy acts entirely by creating future inflation, affecting the intertemporal allocation of consumption.

I specify that $g_t = g$ during the trap, for $0 < t < T$, and $g_t = 0$ thereafter. I examine how increasing g affects equilibrium output and employment by the multiplier $\partial x_t/\partial g$ evaluated at $g = 0$. To find the multipliers, I take the derivative with respect to g of the solution (8), including where needed the derivative of C with respect to g, evaluated at $g = 0$. The formulas are presented in the online Appendix.

Figure 5 presents these multipliers. Multipliers are large, and substantially greater than one, for the standard $\pi_T = 0$ equilibrium. The multipliers increase exponentially as the length of the liquidity trap increases, moving to the left. Multipliers *increase* as price stickiness is *reduced*. In the limit that price stickiness goes to zero, the multiplier goes to infinity. Very small amounts of price stickiness generate very large multipliers. The multiplier is -1 at the limit *point*, however, since $x_t = -g_t$.

This increase in multipliers presents an interesting policy paradox. Microeconomic efforts to reduce price stickiness make the depression worse, according to figure 2. But such efforts make multipliers larger, increasing the effectiveness of fiscal or broken-window stimulus.

By contrast, the multiplier in the no-jump $\pi_0 = 0$ equilibrium is small, and clustered around the frictionless value -1, as its output gaps are small. As price-stickiness is reduced or the period of the trap lengthens the no-jump equilibrium multipliers converge smoothly to -1. The multipliers in the backward-stable $C = 0$ equilibrium, not shown, are similar.

In sum, large multiplier predictions are direct results of equilibrium choice. The no-jump or backward-stable equilibria produce fiscal or productivity-reduction, cost-increase multipliers that are, if anything, lower than conventional wisdom, and more in line with the complete crowding-out or supply-limited results of equilibrium models.

4.1. Specification and Literature

The main point of this calculation is to see the core mechanism that produces large multipliers at the zero bound, to see the time-path of multiplier effects, and to see how they are affected by equilibrium selection, in the most transparent model. These calculations are too simplified to capture magnitudes, or complete evaluation of policies including all of their effects.

Modeling direct marginal cost increases such as capital destruction, or technical regress by a Phillips-curve shifter g is straightforward. However, the units are a bit tricky. Though g enters the Phillips curve together with x, both of them enter as they affect marginal costs. Thus, one unit of g is the amount of g that has the same effect on marginal cost as one unit of output.

The units of g are more complex when it is interpreted as government spending. Conventional multipliers would add g itself, so my -1 would be 0. I present private-expenditure $\partial x_t/\partial g$ multipliers so g can represent other Phillips curve shocks. Additionally, since the equations are log-linearized, $y = x + g$ does not hold. Additional scaling factors typically appear in front of g in most models, so g has to be interpreted with those scaling factors here.

For example, in Werning (2012) section 6, government spending enters the Phillips curve as I have specified, though multiplied by $1 - \Gamma$ where Γ is the flexible-price multiplier. (In flexible price models, government spending lowers wealth which induces labor supply, so the multiplier is between zero and one, not zero.) Government spending does not enter his IS curve.

In other specifications, however, government spending growth enters the IS curve. Christiano, Eichenbaum and Rebelo (2011) is a particularly clear treatment. The Phillips curve, their equation (2.13), includes government spending with a scaling factor,

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \left[\left(\frac{1}{1-g} + \frac{N}{1-N} \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right]. \tag{11}$$

(In this equation, $g \equiv G/Y$). However, the IS curve, their equation (2.14), is

$$\hat{Y}_t - g[\gamma(\sigma-1)+1]\hat{G}_t = E_t \left\{ -(1-g)[\beta(R_{t+1} - R) - \pi_{t+1}] + \hat{Y}_{t+1} - g[\gamma(\sigma-1)+1]\hat{G}_{t+1} \right\}. \tag{12}$$

So, with x interpreted as output, *growth* in G adds a disturbance isomorphic to the natural rate. Woodford (2011) equation (4.1) and (3.10), Eggertsson (2011) equations (10) and (13), and Kiley (2016) equations (5) and (6) have the same structure.

Fortunately, adding government spending growth to the IS curve makes little difference for my calculations. Recalling equation (7), the driving force in this model is

$$z_t \equiv \kappa\sigma(i_t - r_t) + \kappa\frac{dg_t}{dt}. \tag{13}$$

Therefore, if we follow the latter authors and include another dg_t/dt term along with r_t, all that will do is to change the magnitude of the multiplier effect, not the time path. Since I have ignored constants in front of g and the magnitude of the multiplier, we don't lose anything.

In addition, to the extent that government spending induces a natural-rate shock r_t, we already have the effects of such a shock in the previous calculations. As a decline in r_t produces large deflation and output loss at the zero bound in the standard equilibrium, anything that raises r_t has the opposite effect. So to the extent that a government spending shock raises r_t, we already see its large multiplier in the standard equilibrium, its small multiplier in the other equilibria, and the sensitivity of that large multiplier to equilibrium selection. Since the model is linear, it makes sense here to examine the effects of a Phillips curve disturbance alone.

Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2014) challenge the common assumption of a return to trend, and no effect of government spending on potential output. They find that supply-side policies can raise potential enough that productivity and growth raise output.

Mertens and Ravn (2014) find that if we model the zero bound episode as an occurrence of multiple equilibria, as Benhabib, Schmitt-Grohé and Uribe (2001) analyze, rather than a natural rate shock, then fiscal multipliers are small.

4.2. Forward guidance

Many authors have advocated forward guidance policies to ameliorate a liquidity trap, in which the central bank announces a commitment to keep rates low for some time after the negative natural rate passes. The optimal policy in Werning (2012) and Woodford (2012) takes this form. A temporarily higher inflation target has a similar effect. In fact, a target $\pi_T^* > 0$ induces the gentle backward-stable or no-inflation-jump equilibria.

To address forward guidance, I assume that the interest rate remains zero for some time τ after T, even though the natural rate rises at T.

Figure 6 presents the standard equilibrium, selected by $\pi_{T+\tau} = 0$ for a variety of time intervals τ. Again, the formulas are presented in the online Appendix. The top left presents the previous solution with $\tau = 0$, which reminds us of the deep recession and deflation baseline. The remaining panels suppose that people expect the interest-rate rise to be delayed for $\tau = 0.6$, 0.703, and 0.8 years. This delay allows a little inflation to emerge between $t = T$ and $t = T + \tau$. Then, allowing small changes in the π_T terminal condition has large effects on inflation and output during the trap as before.

An 0.6 year delay, in the top right panel, raises inflation and output substantially. A 0.703 year delay in raising interest rates, bottom left panel, produces the benign results of the no-jump equilibrium. While not exactly the optimal policy of Werning (2012) and Woodford (2003), this choice carries their central message: by committing to a delay in raising rates after the trap is over, the central bank can dramatically improve an otherwise dismal outcome, even if it enforces the $\pi_{T+\tau} = 0$ equilibrium. An 0.8 year delay raises inflation π_T even further, and produces an upward jump at time $t = 0$, an inflationary boom.

This exercise is paradoxical in several ways, however. First, the vertical difference between the $\tau = 0$ and $\tau > 0$ solutions in each panel is larger as one moves back in time. Promises further in the future have larger effects today.

Second, I do not show the solutions for $t < 0$, but the backward-explosive eigenvalue continues to operate. Thus, a promise to hold rates low for half a year after a future trap ends has larger effects on output today, the further in the future that trap and promise occur.

Third, all the dynamics happen faster as prices become less sticky (also not shown). Forward guidance has larger effects for less sticky prices, and infinitely large effects in the flexible price limit – and then no effect at all at the flexible price limit point.

Fourth, the graphs reveal a strong sensitivity of forward guidance predictions to the length τ of the delayed rate rise. The $\tau = 0.703$ year delay produces a benign result. But get it just a little wrong – promise 0.6 years, or 0.8 years – and the economy still shows strong deflationary recession or a strong inflationary boom. (Carlstrom, Fuerst, and Paustian (2015) show this sensitivity of inflation and output to the duration of the bound and guidance period.)

Figure 7 presents the no-jump equilibrium in the same situation. The no-jump equilibrium shows very little effect of the delayed interest rate rise. It displays the normal economic prediction that promises about the further-off future have less effect today. Not shown, greater price-stickiness just brings the inflation and output paths closer to their frictionless values. The delayed rise's main effect here is to bring inflation down more quickly after then end of the trap than would occur otherwise.

The main point: equilibrium choice is centrally important to analyzing predictions of this model. The interest rate path makes almost no difference compared to the choice of equilibrium. For example, the benign $\tau = 0.703$ delay with the standard $\pi_{T+\tau} = 0$ equilibrium choice (bottom left, figure 6) is almost identical to the the no-jump equilibrium with no delay $\tau = 0$ (top left of figure 7). Within the no-jump equilibria of figure 7, the interest rate delay makes almost no difference. As far as improving outcomes during the trap, the $\tau = 0.703$ delay of figure 6 is just a way to raise inflation π_T and thus to choose the no-jump equilibrium for $0 < t < T$.

Since in optimal policy exercises the central bank can choose any of these equilibria, why not just choose the no-jump equilibrium, by a suitable inflation target π_T? Why do Werning (2012) and Woodford (2012) find that delay is an optimal policy? Figure 7 reveals the answer: Once one chooses a (nearly) optimal equilibrium, such as the no-jump equilibrium shown here, outcomes during the trap are basically unaffected by delay or no delay. But this model is Fisherian: inflation is a positive function of nominal interest rates. (See (6): Inflation is a two-sided moving average of interest rates with positive weights). So keeping interest rates low for a while after the trap brings inflation down faster than it otherwise would fall, and that slightly faster disinflation slightly improves the central bank's objective in the post-trap world. Delay is a small part of optimality. Equilibrium choice – allowing a slightly positive inflation $\pi_T > 0$ – is the main part of the story, and does not need a delayed interest rate rise.

4.3. Jumps and limits

The paradoxical policies – increasingly large multipliers, large effects of far-off promises, less price stickiness makes matters worse – all have their roots in backward-explosive solutions, and thus large jumps at time 0.

Therefore, reversing these predictions is not unique to the specific no-jump $\pi_0 = 0$ and $C = 0$ backward-stable equilibria. Any limit on the size of the initial jump produces a local-to-frictionless result, and declining effects of expectations of events further in the future.

Specifically, consider equilibria in which the initial jump is limited, $\|\pi_t\| < \Pi$ where people learn the shock at t. In this set of equilibria, 1) Expectations of future events have smaller effects the farther in the future the event lies; 2) As price stickiness decreases $\kappa \to \infty$, inflation and output smoothly approach the frictionless limit point; 3) The Phillips-disturbance multiplier $\partial x/\partial g$ smoothly approaches -1 as price stickiness declines.

5. Choosing equilibria

With an understanding of the effects of equilibrium choices, we can now consider how we ought to make that choice.

5.1. Policy rules

In the standard new-Keynesian approach, the central bank chooses the desired equilibrium interest rate path $\{i_t^*\}$. It then *also and additionally* conducts an equilibrium-selection or implementation policy to select which of the many possible equilibria $\{\pi_t\}$ and $\{x_t\}$ consistent with that $\{i_t^*\}$ will emerge as the equilibrium $\{\pi_t^*\}$ and $\{x_t^*\}$. Finally, people know about all this, as it is their expectations of central bank equilibrium-selection policy in the future that determines which equilibrium emerges today.

To be specific, after choosing the equilibrium interest rate path $\{i_t^*\}$, the central bank selects the equilibrium π_t^* from the set $\{\pi_t\}$ consistent with $\{i_t^*\}$ (for example, the set graphed in figure 1) by following for $t > T$ a Taylor-rule inspired policy of the form

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) = \hat{\imath}_t + \phi\pi_t. \tag{14}$$

This policy de-stabilizes the economy. With $\|\phi\| > 1$, all the equilibria $\{i_t, \pi_t, x_t\}$ other than $\{i_t^*, \pi_t^*, x_t^*\}$ now explode forward as $t \to \infty$. The new-Keynesian tradition adopts as an equilibrium-selection principle that the economy will not choose non-locally-bounded equilibria, and thus predicts that π_t^* is the unique observed equilibrium.[1]

For example, Werning (2012) writes "I assume that the central bank can guarantee... $\pi(t), x(t) = (0,0)$ for $t \geq T$," and this "presumes that the central bank somehow overcomes the indeterminacy of equilibria that plagues these models. A few ideas have been advanced to accomplish this, such as adhering to a Taylor rule with appropriate coefficients..."

A policy rule like (14) by itself does nothing to select equilibria. Equation (14) shows how to construct $\{\pi_t^*\}$ or $\{\hat{\imath}_t\}$ that deliver any equilibrium shown in figure 1. To use policy rules for equilibrium selection, we have to think about *which* policy rule, why the central bank might insist on $\pi_t^* = 0$ or $\hat{\imath}_t = 0$ for $t > T$ – and why people expect this choice.

Many papers just assume a rule with $\pi_T = 0$, $\pi_t^* = 0$, or $\hat{\imath}_t = 0$ for $t > T$. But given large historical deviations from Taylor rules ($R^2 < 1$); given the strong persistence in empirical Taylor rules (lagged interest rate terms); given much Federal Reserve talk of temporary deviations from "normal" policy, and "glidepath" and "soft landing" inflation goals; given that optimal policy recommends intercepts $\hat{\imath}_t$ that vary with shocks (Woodford (2003), Svensson and Woodford (2005)), and given the advantages for the central bank not to insist on $\pi_T = 0$, it is a questionable assumption on which to hang such dramatic predictions.

Werning (2012) offers a principled reason for people to expect $\pi_T = 0$: People expect that the central bank is fully discretionary. It will do ex-post what looks best going forward no matter what last year's forward-guidance speeches said. At time T, the equilibrium-selection policy $\pi_t^* = x_t^* = 0$ $t \geq T$ is forward-looking optimal. The delayed rise in Werning and Woodford (2012) proposals requires pre-commitment as well as guidance, which both authors emphasize.

That central banks are not expected by people to pre-commit to things they will regret ex-post, is a sensible assumption, buttressed by fairly explicit statements from Federal Reserve

[1] A technical note: Equation (14) is simplified to make the point transparently and to remind the reader of more common discrete-time treatments. In continuous time one must specify a rule with some persistence, such as $d(i_t - i_t^*)/dt = \theta\left[\phi(\pi_t - \pi_t^*) - (i_t - i_t^*)\right]$, or the same rule generalized to respect the zero bound, allowing only a positive derivative when $i_t = 0$. See Sims (2004), Fernández-Villaverde, Posch and Rubio-Ramírez (2012) and simulations in Cochrane (2013).

officials and the FOMC defending discretionary policy. But there is a deep contradiction in this view about what central banks can and cannot commit to. Under an "active" $\phi > 1$ policy, all equilibria except the selected x_t^*, π_t^* are disastrous for the central bank's objective – output and inflation explode. So people must believe that the central bank cannot commit at all to interest rate and inflation targets, $\{\pi_t^*, i_t^*\}$, but the same central bank *completely* pre-commits to a doomsday-machine equilibrium-selection threat which, ex-post, is disastrous for its objectives.

The whole idea of policy-rule equilibrium selection is not unassailable. Taylor (for example Taylor (1993)) advanced the $\phi > 1$ rule in the context of an adaptive-expectations, backward-looking model. In that case, $\phi > 1$ brings *stability* to an economy that is *unstable* under an interest rate peg. If inflation rises, interest rates rise more, real interest rates rise, demand decreases and expected future inflation decreases.

But this conventional intuition does not apply to forward-looking new-Keynesian models, such as (1)-(2). Here, the economy is already stable under an interest rate peg; by $\phi > 1$ the central bank *destabilizes* the economy, in order to select from multiple equilibria. If inflation rises, interest rates rise more, but this leads to *more* subsequent inflation, spiraling off to infinity, so inflation had better not rise in the first place.

Do central banks really have, and do people believe that they have, an "equilibrium-selection" policy, that *de*stabilizes the economy for inflation not equal to its target, distinct from its "interest rate policy?" The Federal Reserve resolutely describes its behavior as stabilizing, reacting to unexpected inflation in a way to bring inflation back down again – as it does, under adaptive expectations. Furthermore, the $\phi > 1$ reaction is unobservable and hence unlearnable from time series. If the model is right, we only see the equilibrium $\pi_t = \pi_t^*$, and hence neither we nor people in the economy can learn the value of ϕ or the existence of equilibrium selection policy. (For more on these doubts, see Cochrane (2011))

Moreover, the zero-bound literature makes an important innovation here, by substituting expectations of future active ($\phi > 1,\ t > T$) monetary policy in place of current ($\phi > 1,\ t < T$) policy to select equilibria. But are multiple equilibria really ruled out by expectations of how the central bank will react to inflation, should it emerge in the far future, even though the central bank does not react to inflation today? How far in the future can reaction really be postponed in order to successfully prune equilibria?

The point: Equilibrium selection by active $\phi > 1$ policy rules may not be as rock-solid as it appears. We can at least contemplate other equilibria and other ways of choosing equilibria.

5.2. Rules with a zero bound

When considering a zero bound, we most often consider bound-limited policy rules, i.e.

$$i_t = \max\left[i_t^* + \phi(\pi_t - \pi_t^*), 0\right] \tag{15}$$

or equivalently,

$$i_t = \max\left(\hat{\imath}_t + \phi\pi_t, 0\right). \tag{16}$$

Allowing such a rule does not substantially change the analysis. During the trap, the standard new-Keynesian deflationary $\pi_t < 0$, $i_t = 0$ equilibria are unchanged. Some of the alternative equilibria with positive inflation are potentially affected, with more inflation leading to interest rates rising above the zero bound and potentially leading to still higher inflation, ruling out those equilibria by the rule against inflationary explosions. But this outcome also depends on our assumptions about policy disturbances $\hat{\imath}_t$ or π_t^*. A sufficiently low $\hat{\imath}_t$, or significantly high π_t^* response to the negative natural rate – a willingness to tolerate extra inflation during the negative natural rate episode, or equivalently a negative deviation from the usual Taylor rule – would leave interest rates at zero despite inflation. There always remains a zero-bounded policy rule that produces any of the equilibria, and as usual the question is merely which π_t^* or $\hat{\imath}_t$ one assumes the central bank to follow.

Imposing the lower bound after the trap means means that even with locally active policy, $\phi > 1$ at $\pi = \pi^*$, there are still multiple equilibria that converge to the zero bound. Therefore, the policy rule no longer guarantees global determinacy, as pointed out by Benhabib, Schmitt-Grohé and Uribe (2001) point out that there really are still multiple globally-bounded equilibria, since the policy rule must respect the zero bound even after the natural rate shock passes. One must either strengthen the equilibrium selection criterion to rule all but the locally-bounded equilibria near π^*, even those that remain globally bounded, or one must invoke some other off-equilibrium threats people believe that central banks make, as discussed by Atkeson, Chari, and Kehoe (2010).

5.3. Fiscal theory

The fiscal theory of the price level offers a clean approach to equilibrium selection. Even a lite version, just looking at the fiscal implications of the various equilibria, is helpful.

Consider the simplest case, one-period nominal government debt in discrete time. Then, the equilibrium condition that the real value of nominal government debt equals the present value of primary surpluses reads

$$\frac{B_{t-1}}{P_t} = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \right], \tag{17}$$

where B_{t-1} is the face value of debt outstanding at period $t-1$ and due at t, P_t is the price level, and s_t is the real primary surplus.

Multiply and divide by P_{t-1} and take innovations, yielding

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \right]. \tag{18}$$

This equation tells us that unexpected inflation – the jump at $t = 0$ – corresponds entirely to innovations in the present value of future surpluses.

Multiplying by marginal utility and taking expected values, we have

$$\frac{B_{t-1}}{P_{t-1}} E_{t-1} \left(\beta \frac{u'(c_t)}{u'(c_{t-1})} \frac{P_{t-1}}{P_t} \right) = \frac{B_{t-1}}{P_{t-1}} \frac{1}{1+i_{t-1}} = E_{t-1} \left[\sum_{j=0}^{\infty} \beta^{j+1} \frac{u'(c_{t+j})}{u'(c_{t-1})} s_{t+j} \right]. \tag{19}$$

This equation shows how the government can follow an interest rate target. Since P_{t-1} is determined by fiscal expectations at time $t-1$, selling more or less debt B_{t-1} with no change in surpluses changes the interest rate i_{t-1}. Conversely, by fixing the interest rate on government debt i_{t-1} with constant surpluses, and selling any amount of debt at that price, this equation it describes the number of bonds B_{t-1} that will be sold, and verifies that number is determined, positive, and finite. Cochrane (2017) shows how this arrangement is consistent with current Federal Reserve and Treasury operating procedures.

Together, (17)-(19) challenge standard monetary doctrines: Under an interest rate peg (static or time-varying), or a passive $\phi < 1$ interest rate rule, the price level and inflation rate are stable and uniquely determined. In the Leeper (1991) terminology, "active" fiscal policy can substitute for "active" monetary policy.

In the continuous-time perfect-foresight simplified setup of this paper, with a single unexpected jump at time 0, the government debt valuation equation (17) reads

$$\frac{B_t}{P_t} = \int_{j=t}^{\infty} e^{-\int_{\tau=t}^{j} r_\tau d\tau} s_j dj \tag{20}$$

and the continuous time version of (18) describes a price level jump,

$$\frac{B_0}{P_{0-}} \left(P_{0-} \left(d\frac{1}{P_0} \right) \right) = d \left(\int_{j=0}^{\infty} e^{-\int_{\tau=0}^{j} r_\tau d\tau} s_j dj \right). \tag{21}$$

Here B_0 is predetermined and can't jump, P_{0-} is the value just before a jump, and d is the forward differential operator.

(For response to standard objections see Cochrane (2005). For an explicit integration of fiscal theory with a sticky-price model and interest rate targets, see Cochrane (2014, 2017). Cochrane (2017) also describes the generalization to long-term debt, and shows how continuous time models with price stickiness can smoothly approach this price-level jump.)

Equations (18) or (21) apply immediately to our equilibrium-selection problem. They tell us to pick equilibria by inflation π_0 at time $t = 0$, or when people learn of the negative natural rate shock, not by expectations of inflation at time T. They tell us to pick equilibria by understanding *fiscal* policy responses to the natural rate shock, rather than expectations of central bank equilibrium-selection policy after the shock ends. (Changes in the discount rate are also a potentially important influence on the price level, but not quantitatively important in these models.) In this simple framework, fiscal considerations do not otherwise change the dynamics of output, inflation and interest rates. They have no other effect than to choose the initial jump, and thus the equilibrium.

Even if one does not wish to use the fiscal theory to select equilibria, it is useful to examine the fiscal implications of different equilibria. Equations (18) and (21) are present in all models. In the standard new-Keynesian model, one assumes that these equations describes the behavior of the Congress and the Treasury: they adjust taxes and spending "passively" ex-post to validate any price level. If, for example, the price level falls by half, then the government will double fiscal surpluses to pay off an unexpected windfall to bond holders. That assumption is worth questioning, not just sweeping under the rug with a footnote about "passive" fiscal policy. If people do not seamlessly expect that reaction, the deflation can't happen. Therefore, it remains useful to index equilibria by the time-zero jump, and to examine the magnitude and plausibility of the required "passive" fiscal policy reaction.

The fiscal theory by itself doesn't help, as the assumption of an active interest rate policy rule (3) or (4) by itself does not help. Fiscal theory requires us to specify expectations of fiscal policy, as monetary policy requires us to specify the central bank's equilibrium selection choices π^*. Assumptions about those expectations select equilibria, the theory only provides the framework by which the assumptions take effect.

The no-jump equilibrium $\pi_0 = 0$ occurs if there is no change in present value of future surpluses coincident with or in response to the negative natural rate shock or the monetary policy response. This is not an obvious choice. Given the large deficits and fiscal stimulus in the 2008-2009 recession and beyond, the assumption of looser fiscal policy seems initially more plausible. That line of thought pushes us to a positive inflation jump at time zero, such as the backward-stable equilibrium of figure 3 or even more. And that line of thought suggests that the zero bound is even less of a problem than the no-jump $\pi_0 = 0$ equilibrium suggests.

However, equation (18) directs us to examine the innovation to the present value of *all* future surpluses. If the government reacts to the negative natural rate shock with large deficits during the trap, $s_t < 0$ for $0 < t < T$, but also credibly promises to pay back the resulting debt by future tax increases or spending cuts, $s_t > 0$ for $t > T$, stimulus now but austerity later, then there is no innovation to the present value of future deficits. This is a plausible assumption. Increases in debt usually convey expectations that the debt will be paid back, as governments finance wars with current deficits but future surpluses. Even in the middle of the stimulus debates of 2009, the US administration promised to follow current stimulus with future debt reduction, not implicit default via inflation.

Furthermore, discount rates matter to present values. In 2008, real interest rates on government bonds dropped suddenly. A plausible way therefore to make sense of the small but sharp disinflation in 2008-9 via (18), is that the larger value of government debt corresponded to sharply lower real interest rates, not to a tightening of current or expected future surpluses. This is the "flight to quality." Again, (18) holds in every model, and as an identity using ex-post returns. Thus the question is *how* it holds, not *if* it holds, and is relevant to interpreting any model's prediction.

By contrast, the $\pi_T = 0$ standard solution graphed in figure 2 includes a -132% deflation at $t = 0$, corresponding to a jump of the price level down to $100 \times e^{-1.32} = 27\%$ of its initial level, and $100 \times e^{1.32} = 376\%$ increase in the value of government debt. Raising taxes or cutting spending that much would surely strain the "passive" assumption. Large unexpected deflations require large ex-post taxes or spending cuts. The fiscal theory offers a reason why large unexpected deflations don't happen.

Moreover, the increasingly large time-0 deflations that occur as we reduce price stickiness with the standard $\pi_T = 0$ selection require increasingly large and eventually unbounded fiscal responses. Merely bounding the fiscal response, say at taxes equal to 100% of GDP, eliminates the frictionless limit puzzles, the forward guidance puzzle, and any other result of backwards-explosive equilibria. Any fiscal equilibrium selection that imposes a bound on time zero deflation is local-to-frictionless.

The point here is not to advocate a particular fiscal assumption as the right one. The point is that we can think about equilibrium selection this way. The jump at π_0 corresponds to expectations about fiscal policy and discount rates, no matter whether "actively" or "passively" achieved. To figure out which is the right equilibrium, we have to think as hard about fiscal policy and discount rates as we think hard about monetary policy, expected interest rate paths, equilibrium selection policies, pre-commitment, and so forth.

But even without picking a specific value, fiscal considerations at least suggest that one place a limit on the allowable jumps in inflation at time 0. Per section 4.3, such a limit cures the strange limiting behavior of the model and its policy predictions. Conversely, the paradoxical limits resulting from the standard equilibrium choice require that "passive" fiscal policy validate *unbounded* increases in the value of government debt.

5.4. Other equilibrium-selection principles

In models with multiple equilibria, a wide range of principles extending the standard definition of equilibrium have been advocated to select equilibria.

The basic new-Keynesian selection procedure has such an element as well. It rests primarily on the principle that expectations should "coordinate" on particular equilibria. (See Woodford (2003) p.128 and King (2000) p. 58-59.) A long list of efforts to uniquely select equilibria using completely economic criteria in passive-fiscal models fail on closer examination, especially if one focuses on beliefs people might currently have about central bank actions rather than proposals for additional policies that future central banks might adopt. (See Cochrane (2011), and Cochrane (2015) response to Sims (2013).) A long literature refines rational expectations by adding various learning criteria, in an effort to prune equilibria.

One could make a similar case for equilibrium selection here, by turning properties of various equilibria into criteria for their selection. Rules that eliminate sets of equilibria are also useful even if they do not deliver a unique result. If we can bound initial jumps, we resolve most of the issues.

The local-to-frictionless property is attractive – pick equilibria in which small frictions have small effects. That principle does not pick a unique equilibrium here, as any equilibrium that limits the initial response is local-to-frictionless. But that principle can serve to rule out equilibria, and the standard equilibrium choice in particular.

The property that news about further-off events should have smaller effects today, or that equilibria should not explode backward, are properties that one could use for equilibrium selection. They have some of the same flavor as the views in Woodford (2003) and King (2000) about sensible expectations and coordination mechanisms. They also bound initial jumps and thus solve most of the issues. One could also bound initial jumps directly as an equilibrium-selection principle.

5.5. Empirical equilibrium selection

Equilibrium selection can be an empirical project as well as a theoretical one. The equilibrium choice centrally matters to how the model fits the data, just like preferences and technology. So, one can ask the data which equilibrium choice fits best. The present model is not rich enough, nor have I calibrated or estimated parameters and shocks, to do a serious job of such estimation. But I can point to the general issues.

First, we can ask which equilibrium choice produces a better fit with the data. In this simple model, the stability of zero bound experience would be a key observation. The US economy 2009-2014 featured steady slow growth, a level of output stuck about 7% below the previous trendline, and steady positive 2% or so inflation. European and Japanese experience has been similar.

The backward-stable and no-inflation-jump equilibria shown in figures 3 and 4 can produce this steady outcome. However, they do not produce a big output gap. Thus, they only account for disappointing output if one thinks that growth has been limited by "supply" rather than "demand," that calculations of potential output were optimistic. Substantial ex-post downward revision in potential output calculations lends support to this view.

The standard equilibrium choice as in figure 2 cannot produce stagnation. Here and in more general models, the standard equilibrium choice counterfactually predicts large and time-varying deflation (Hall (2011), Ball and Mazumder (2011), King and Watson (2012), Coibion and Gorodnichenko (2013)), which did not happen, and it counterfactually predicts strong growth. To generate stagnation, one has to imagine a stream of unexpected negative shocks. (Failure of a Poisson exit shock to appear is a negative shock relative to expectations.)

Alternatively, one can fundamentally modify the model itself, as do Del Negro, Giannoni and Schorfheide (2015) and Eggertsson and Mehrotra (2014). But the latter course strengthens the case that *this* model doesn't produce a slump, so within the context of this model the data are likely to choose something like the no-jump equilibrium.

Second, for an exercise such as the one in this paper, one could condition on the observed downward jump in inflation to select equilibrium. The standard equilibrium produced a sharp -132% downward jump in the price level. In the data, core inflation decreased from about 2.5% in mid 2008 to just a bit below 1% in 2011 before rebounding. We could pick the equilibrium with (say) -1.5% deflation at $\pi_0 = 0$.

Third, for model simulations, one could measure the typical downward jump in inflation and output in response to shocks. We can measure equilibrium selection by the correlation of shocks in the impulse-response function. Yes, identifying shocks is hard, but this is a regular task of empirical macroeconomics, not a special task that must be relegated to theory or philosophy alone.

Finally, the equilibrium choice, along with the rest of the model, can be evaluated by its policy predictions and the historical record. We have the recent past and the Great Depression at zero interest rates. Similar predictions also emerge in this model when interest rates respond less than one for one $\phi < 1$ to inflation, so early postwar interest rate targets and the 1970s are informative. At a casual level, deliberate inflation, output destruction, technical regress, more price stickiness (wage and price controls), useless government spending, and central banker promises do not seem to have had in those periods the large effects claimed for them now. For example, Dupor and Li (2013) find that stimulus spending was not associated with a rise in expected inflation and thus no multiplier by this mechanism; Wieland (2014) shows that several cases of endowment destruction and adverse supply shocks did not induce inflation or stimulus at the zero bound; and Del Negro, Giannoni and Patterson (2015) measure the effects of forward guidance, finding that "standard medium-scale DSGE models tend to grossly overestimate the impact of forward guidance."

The point here is not to settle the case, but to outline the methodological possibility. Whether by matching data directly, by conditioning on an observation like 2008, by matching impulse-response functions, or by matching policy experience, equilibrium selection rules are identifiable and measurable parts of a model. They do not have perpetually to remain a theoretical or philosophical controversy.

6. Concluding comments

I examine a standard new-Keynesian analysis of the zero bound, following Werning's (2012) elegantly simple example: A negative natural rate lasts from time 0 to time T, and the nominal rate is stuck at zero. I find there are many equilibria, each bounded, forward-stable, and nonexplosive going forward in time.

The conclusion that the zero bound is a big economic problem, and that counterintuitive policies can have dramatic curative effects, follows from selecting equilibria by setting expected inflation at the end of the trap to zero, $\pi_T = 0$. This equilibrium features a deep recession with deflation. It also features strong expected output growth, which is why the level of output is so low, and rapidly declining deflation. It predicts large multipliers to wasted government spending, and to wealth or productivity destruction. It predicts that announcements about far-off future policies have large effects. These predictions grow larger the longer the period of the liquidity trap, and as price stickiness is reduced. It predicts a large downward jump in inflation and output at time 0, when people learn of the negative natural rate shock.

Indexing equilibria by the initial jump in inflation π_0, and limiting such jumps overturns all of these results. In particular, the "backward stable" and "no-inflation-jump" equilibria of the same model, with the same interest rate path, instead predict mild *in*flation during the liquidity trap, little if any reduction in output relative to potential, small or negative multipliers, and little effects of promises of far-off policies or other events.Their predictions smoothly approach the frictionless limit as pricing frictions are reduced.

At a minimum, this analysis shows that equilibrium selection, rather than just the path of expected interest rates, is vitally important for understanding these models' predictions. In usual interpretations of new-Keynesian model results, authors feel that interest rate policy is central, and equilibrium-selection policy by the central bank or by the author are "implementation" details relegated to technical footnotes (as in Werning (2012)), game-theoretic foundations, or philosophical debates, which can all safely be ignored in applied research.

My most concrete suggestion for addressing multiple equilibria is to marry new-Keynesian models with the fiscal theory of the price level. That approach transparently produces a single (globally determinate) equilibrium price level as well as inflation rate, it results in a backward-stable equilibrium choice that solves all the puzzles, and it gives a smooth frictionless limit. Even if one does not wish to fully embrace the fiscal theory, fiscal considerations – the large "passive" tax increases needed to finance a deflation-induced rise in the value of government debt – can help to weed out the puzzling equilibria of new-Keynesian zero-bound predictions.

The new-Keynesian structure plus fiscal theory – or with another limitation on the size of jumps in endogenous state variables – produces an attractive and tractable model of nominal stickiness and interest rate targets. But it eliminates the puzzle, or the promise, depending on your reaction to earlier work, of some new-Keynesian models' diagnoses of and their policy prescriptions for the zero bound.

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Online Appendix to "The new-Keynesian Liquidity Trap"

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August 2017

This Appendix collects derivations and formulas for "The new-Keynesian Liquidity Trap." Computer programs are also available here (the JME website). These materials, and any updates and corrections are also available on my personal website, `http://faculty.chicagobooth.edu/john.cochrane/`

7.1. General solution

Here I derive the general solution (6), (7), (8) . To recap, the model (1), (2) is

$$\frac{dx_t}{dt} = \sigma\left(i_t - r_t - \pi_t\right) \tag{22}$$

$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa(x_t + g_t). \tag{23}$$

I proceed by analogy to discrete-time lag operator methods.

Differentiate (23), and substitute from (22) for dx_t/dt to obtain

$$\frac{d^2\pi_t}{dt^2} - \rho\frac{d\pi_t}{dt} - \kappa\sigma\pi_t = -z_t \equiv -\kappa\sigma(i_t - r_t) - \kappa\frac{dg_t}{dt}. \tag{24}$$

Write this differential equation in the operator form

$$\left(\frac{d}{dt} - \lambda^f\right)\left(\frac{d}{dt} + \lambda^b\right)\pi_t = -z_t. \tag{25}$$

To invert the differential operator (25), note that

$$\left(\frac{d}{dt} - \lambda^f\right)\pi_t = y_t \tag{26}$$

has solution

$$\pi_t = Ce^{\lambda^f t} - \int_{s=t}^{\infty} e^{-\lambda^f(s-t)} y_s ds, \tag{27}$$

while

$$\left(\frac{d}{dt} + \lambda^b\right)\pi_t = y_t \tag{28}$$

has solution

$$\pi_t = Ce^{-\lambda^b t} + \int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} y_s ds. \tag{29}$$

Therefore, write (25) as

$$\pi_t = Ce^{-\lambda^b t} + C_f e^{\lambda^f t} + \frac{1}{\left(\frac{d}{dt} - \lambda^f\right)\left(\frac{d}{dt} + \lambda^b\right)} z_t \tag{30}$$

$$= Ce^{-\lambda^b t} + C_f e^{\lambda^f t} + \frac{1}{\lambda^f + \lambda^b}\left[\frac{1}{\frac{d}{dt} + \lambda^b} - \frac{1}{\frac{d}{dt} - \lambda^f}\right] z_t. \tag{31}$$

Set to zero the forward-explosive solutions $C_f e^{\lambda^f t}$, and we immediately have the solution (6),

$$\pi_t = Ce^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b}\left[\int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} z_s ds + \int_{s=t}^{\infty} e^{-\lambda^f (s-t)} z_s ds\right]. \tag{32}$$

We can find the solutions for x_t similarly, or more easily by solving (23) for x_t and differentiating (32). The result is (8), i.e.

$$\kappa x_t = -\kappa g_t + \lambda^f C e^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b}\left[\lambda^f \int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} z_s ds - \lambda^b \int_{s=t}^{\infty} e^{-\lambda^f (s-t)} z_s ds\right]. \tag{33}$$

7.2. Formulas for step function impulses

For $r_t = -r$, $g_t = g$, $i_t = 0$, $T_l < t < T_h$ and $r_t = r$, $g_t = 0$, $i_t = r$ otherwise, evaluating the integrals in (6) and (8), repeated above as (32) and (33), yields

$t \le T_l$:

$$\pi_t = Ce^{-\lambda^b t} + \frac{\kappa}{\lambda^f + \lambda^b}\left[e^{-\lambda^f (T_l - t)} - e^{-\lambda^f (T_h - t)}\right]\left(\frac{\sigma r}{\lambda^f} + g\right) \tag{34}$$

$$\kappa x_t = \lambda^f C e^{-\lambda^b t} + \frac{\kappa \lambda^b}{\lambda^f + \lambda^b}\left[e^{-\lambda^f (T_h - t)} - e^{-\lambda^f (T_l - t)}\right]\left(\frac{\sigma r}{\lambda^f} + g\right) \tag{35}$$

$t \ge T_h$:

$$\pi_t = Ce^{-\lambda^b t} + \frac{\kappa}{\lambda^f + \lambda^b}\left[e^{-\lambda^b (t - T_h)} - e^{-\lambda^b (t - T_l)}\right]\left(\frac{\sigma r}{\lambda^b} - g\right) \tag{36}$$

$$\kappa x_t = \lambda^f C e^{-\lambda^b t} + \frac{\kappa \lambda^f}{\lambda^f + \lambda^b}\left[e^{-\lambda^b (t - T_h)} - e^{-\lambda^b (t - T_l)}\right]\left(\frac{\sigma r}{\lambda^b} - g\right) \tag{37}$$

(38)

$T_l \le t \le T_h$:

$$\pi_t = Ce^{-\lambda^b t} + \frac{\kappa}{\lambda^f + \lambda^b} \times \tag{39}$$
$$\left[\left(\frac{1 - e^{-\lambda^b (t-T_l)}}{\lambda^b} + \frac{1 - e^{-\lambda^f (T_h - t)}}{\lambda^f} \right) \sigma r + \left(e^{-\lambda^b (t-T_l)} - e^{-\lambda^f (T_h - t)} \right) g \right]$$

$$\kappa x_t = -\kappa g + \lambda^f C e^{-\lambda^b t} + \frac{\kappa}{\lambda^f + \lambda^b} \times \tag{40}$$
$$\left[\left(\frac{\lambda^f}{\lambda^b} (1 - e^{-\lambda^b (t-T_l)}) - \frac{\lambda^b}{\lambda^f} (1 - e^{-\lambda^f (T_h - t)}) \right) \sigma r + \left(\lambda^f e^{-\lambda^b (t-T_l)} + \lambda^b e^{-\lambda^f (T_h - t)} \right) g \right].$$

Figures 1 through 3 plot the case $T_l = 0$, $T_h = T$, and $g = 0$.

To select equilibria with $\pi_0 = 0$ or by $\pi_T = 0$, we solve for the corresponding C, giving

$$\pi_0 = 0: \; Ce^{-\lambda^b t} = -\frac{\kappa}{\lambda^f + \lambda^b} e^{-\lambda^b t} \left(1 - e^{-\lambda^f T} \right) \left(\frac{\sigma r}{\lambda^f} + g \right) \tag{41}$$

$$\pi_T = 0: \; Ce^{-\lambda^b t} = \frac{\kappa}{\lambda^f + \lambda^b} e^{-\lambda^b t} \left(1 - e^{\lambda^b T} \right) \left(\frac{\sigma r}{\lambda^b} - g \right). \tag{42}$$

To plot equilibria, I use these values in (34)-(39).

7.3. Formulas for multipliers

To find the multipliers, I take the derivative with respect to g of the formulas for x_t, (35)-(40), and derivatives of C with respect to g from (41) and (42), evaluated at $g = 0$.

Defining x_{2t} by

$$\kappa x_t = \lambda^f C e^{-\lambda^b t} + \kappa x_{2t}, \tag{43}$$

we have

$$\left. \frac{\partial x_t}{\partial g} \right|_{g=0} = \left. \frac{\partial}{\partial g} \left(\frac{\lambda^f C e^{-\lambda^b t}}{k} \right) \right|_{g=0} + \left. \frac{\partial x_{2t}}{\partial g} \right|_{g=0}. \tag{44}$$

The parts are

$$\pi_0 = 0: \; \left. \frac{\partial}{\partial g} \left(\frac{\lambda^f C e^{-\lambda^b t}}{\kappa} \right) \right|_{g=0} = -\frac{\lambda^f}{\lambda^f + \lambda^b} e^{-\lambda^b t} \left(1 - e^{-\lambda^f T} \right) \tag{45}$$

$$\pi_T = 0: \; \left. \frac{\partial}{\partial g} \left(\frac{\lambda^f C e^{-\lambda^b t}}{\kappa} \right) \right|_{g=0} = -\frac{\lambda^f}{\lambda^f + \lambda^b} e^{-\lambda^b t} \left(1 - e^{\lambda^b T} \right) \tag{46}$$

and

$$t \leq 0: \left.\frac{\partial x_{2t}}{\partial g}\right|_{g=0} = \frac{\lambda^b}{\lambda^f + \lambda^b}\left(e^{-\lambda^f(T-t)} - e^{\lambda^f t}\right) \tag{47}$$

$$t \geq T: \left.\frac{\partial x_{2t}}{\partial g}\right|_{g=0} = -\frac{\lambda^f}{\lambda^f + \lambda^b}\left(e^{-\lambda^b(t-T)} - e^{-\lambda^b t}\right) \tag{48}$$

$$0 \leq t \leq T: \left.\frac{\partial x_{2t}}{\partial g}\right|_{g=0} = -1 + \frac{1}{\lambda^f + \lambda^b}\left(\lambda^f e^{-\lambda^b t} + \lambda^b e^{-\lambda^f(T-t)}\right) \tag{49}$$

Equation (46) holds the key to large multipliers. The term $e^{\lambda^b T}$ is the only exponent of a positive number in these formulas. As T grows, this term grows without bound.

7.4. Formulas for forward guidance

The postponed interest rate rise solution comes from adding up two cases of (34)-(40), $T_l = 0$, $T_h = T$ with $z_1 = \kappa\sigma(i - r) = 2\%$ and $T_l = T$, $T_h = T + \tau$ using $z_2 = -2\%$. We obtain:

$$\pi_t = Ce^{-\lambda^b t} + \frac{w_t}{\lambda^f + \lambda^b} \tag{50}$$

$$\kappa x_t = -\kappa g_t + \lambda^f Ce^{-\lambda^b t} + \frac{v_t}{\lambda^f + \lambda^b} \tag{51}$$

where

$$t < 0: w_t = \frac{z_1}{\lambda^f}\left(1 - e^{-\lambda^f T}\right)e^{\lambda^f t} + \frac{z_2}{\lambda^f}\left(1 - e^{-\lambda^f \tau}\right) \tag{52}$$

$$0 < t < T: w_t = \frac{z_1}{\lambda^b}\left(1 - e^{-\lambda^b t}\right) + \frac{z_1}{\lambda^f}\left(1 - e^{-\lambda^f(T-t)}\right) + \frac{z_2}{\lambda^f}\left(1 - e^{-\lambda^f \tau}\right)e^{\lambda^f(t-T)} \tag{53}$$

$$T < t < T + \tau: w_t = \frac{z_1}{\lambda^b}\left(e^{\lambda^b T} - 1\right)e^{-\lambda^b t} + \frac{z_2}{\lambda^b}\left(1 - e^{-\lambda^b(t-T)}\right) + \frac{z_2}{\lambda^f}\left(1 - e^{-\lambda^f(T+\tau-t)}\right) \tag{54}$$

$$t > T + \tau: w_t = \frac{z_1}{\lambda^b}\left(e^{\lambda^b T} - 1\right)e^{-\lambda^b t} + \frac{z_2}{\lambda^b}\left(e^{\lambda^b \tau} - 1\right)e^{-\lambda^b(t-T)} \tag{55}$$

$$t < 0 : v_t = -\frac{\lambda^b z_1}{\lambda^f}\left(1 - e^{-\lambda^f T}\right) e^{\lambda^f t} - \frac{\lambda^b z_2}{\lambda^f}\left(1 - e^{-\lambda^f \tau}\right) e^{\lambda^f (t-T)} \tag{56}$$

$$0 < t < T : v_t = \frac{\lambda^f z_1}{\lambda^b}\left(1 - e^{-\lambda^b t}\right) - \frac{\lambda^b z_1}{\lambda^f}\left(1 - e^{-\lambda^f (T-t)}\right) - \frac{\lambda^b z_2}{\lambda^f}\left(1 - e^{-\lambda^f \tau}\right) e^{\lambda^f (t-T)} \tag{57}$$

$$T < t < T+\tau : v_t = \frac{\lambda^f z_1}{\lambda^b}\left(e^{\lambda^b T} - 1\right) e^{-\lambda^b t} + \frac{\lambda^f z_2}{\lambda^b}\left(1 - e^{-\lambda^b (t-T)}\right) - \frac{\lambda^b z_2}{\lambda^f}\left(1 - e^{-\lambda^f (T+\tau-t)}\right) \tag{58}$$

$$t > T+\tau : v_t = \frac{\lambda^f z_1}{\lambda^b}\left(e^{\lambda^b T} - 1\right) e^{-\lambda^b t} + \frac{\lambda^f z_2}{\lambda^b}\left(e^{\lambda^b \tau} - 1\right) e^{-\lambda^b (t-T)} \tag{59}$$

I then pick $C = 0$, the C that delivers $\pi_{T+\tau} = 0$ and the C that delivers $\pi_0 = 0$.

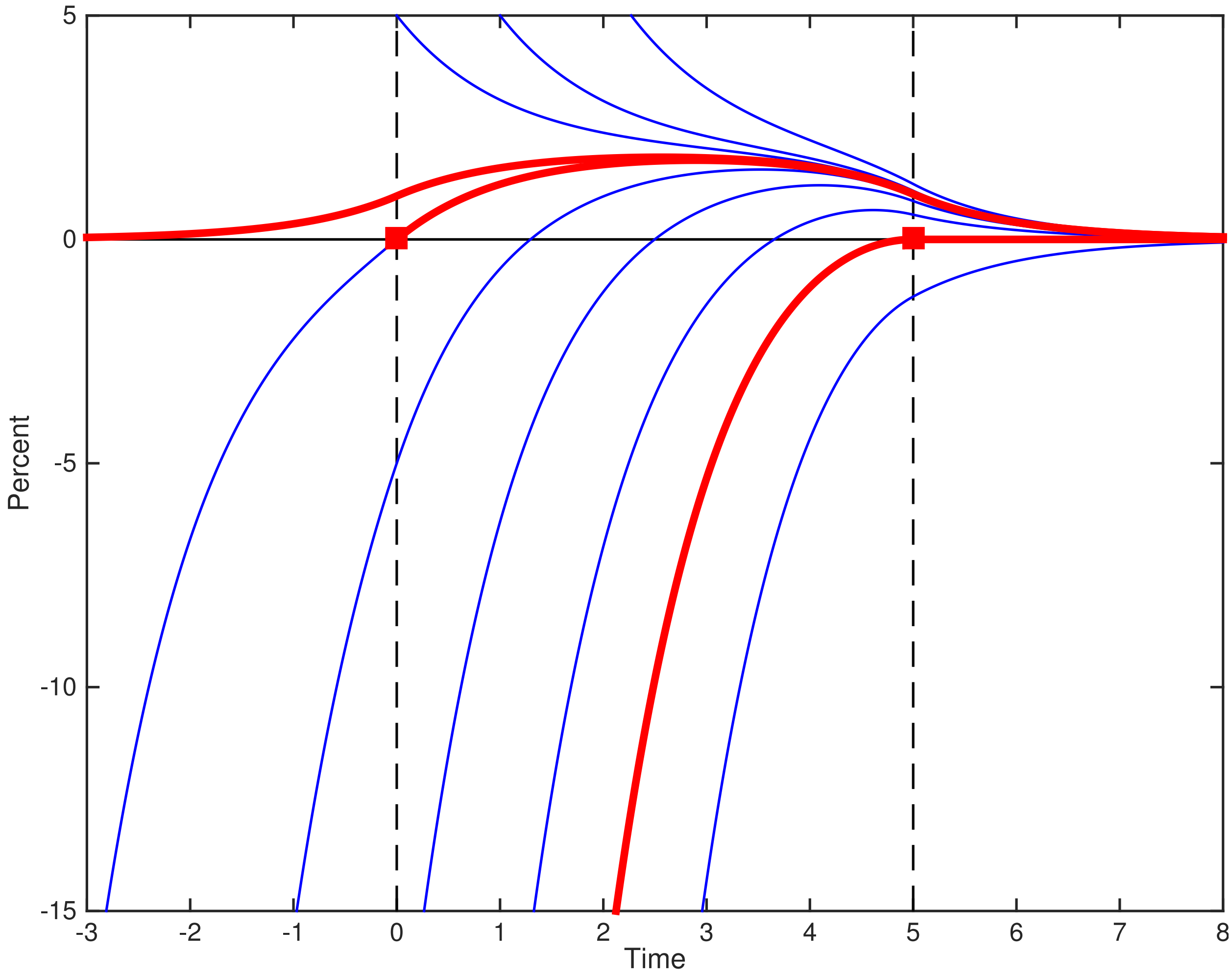

Figure 1: Inflation in a range of multiple equilibria. $i_t - r_t = -2\%$ between $t = 0$ and $t = 5$, shown by vertical dashed lines, and $i_t = r_t$ otherwise. The thick lines show the backward-stable equilibrium, the no-jump equilibrium, and the standard equilibrium discussed below. Thinner lines show a range of additional possible equilibria.

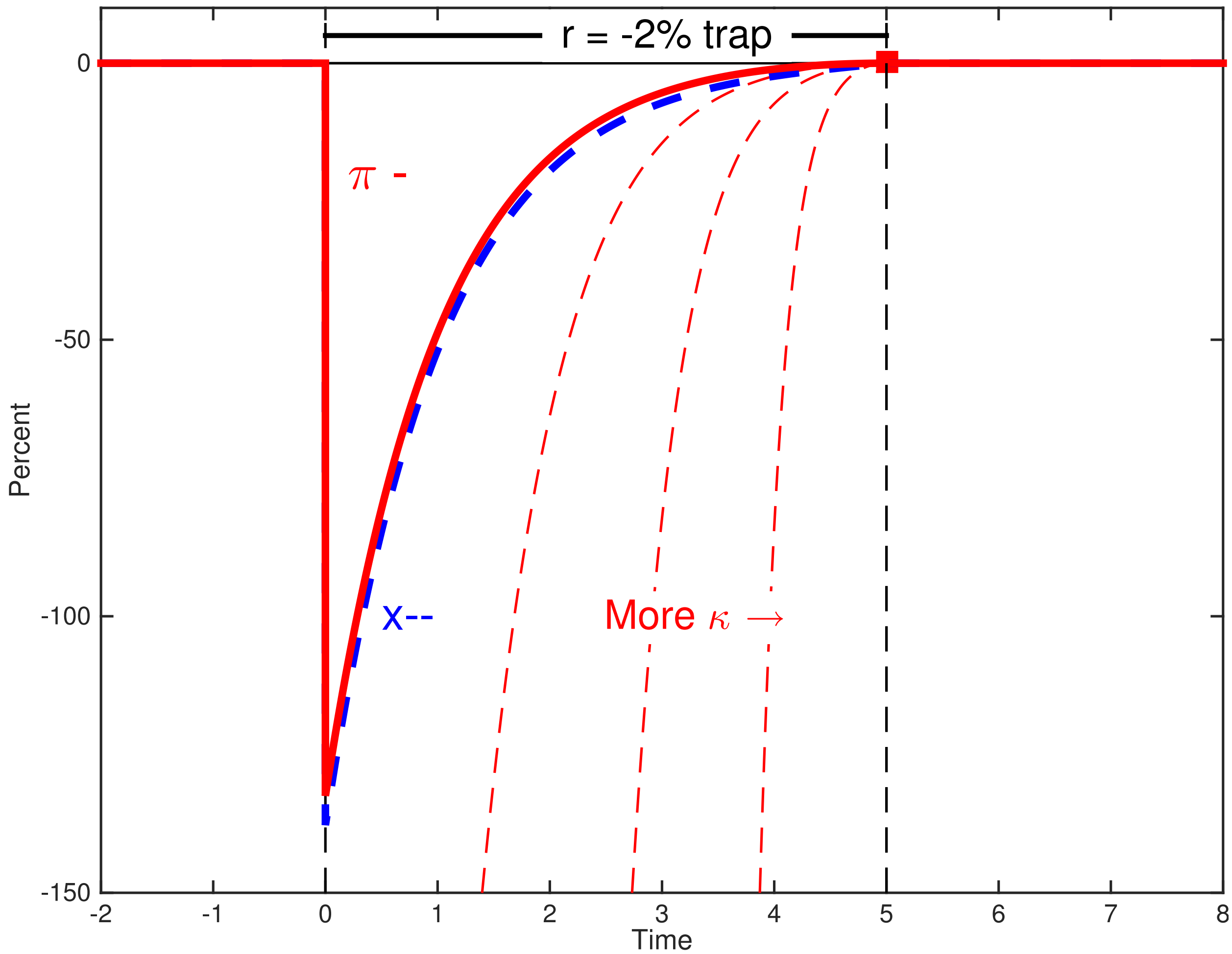

Figure 2: Output and inflation in the standard $\pi_T = 0$ equilibrium. The thick lines show $\kappa = 1$. The thin dashed lines plot inflation as the price-stickiness parameter κ increases from 1 to 2, 5, and 20. The natural rate shock is unexpected at time $t = 0$, and then lasts until $t = T = 5$. The square at $t = 5$ indicates the selection assumption $\pi_5 = 0$.

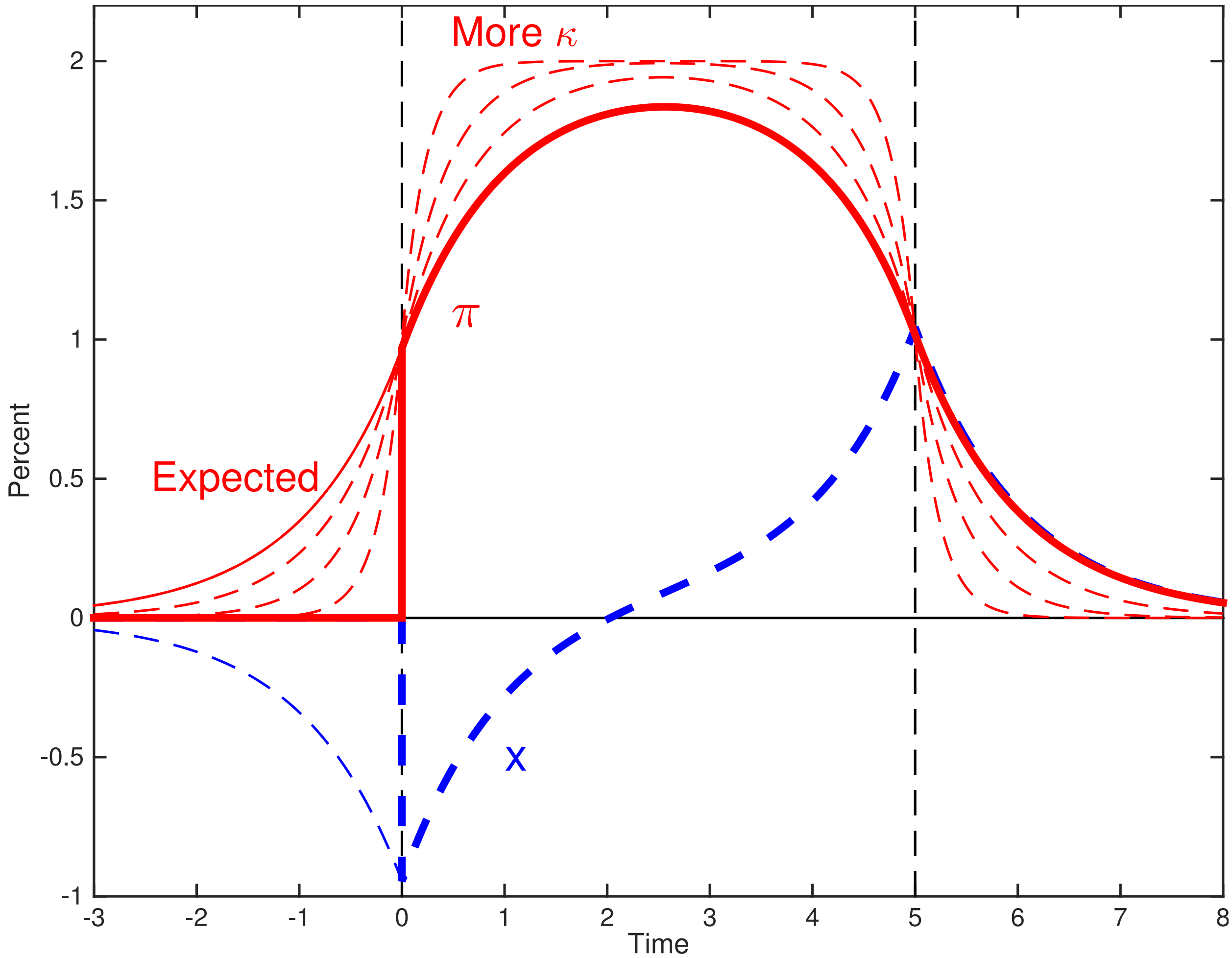

Figure 3: Output and inflation in the backward-stable $C = 0$ equilibrium. $i_t = 0$, $r_t = -2\%$ between $t = 0$ and $t = 5$. Thick lines show inflation and output when the trap is unexpected at $t = 0$. Lines to the left of $t = 0$ show inflation and output when the event is expected. Thin dashed lines show inflation as price-stickiness diminishes from κ =1 to 2, 5, 20.

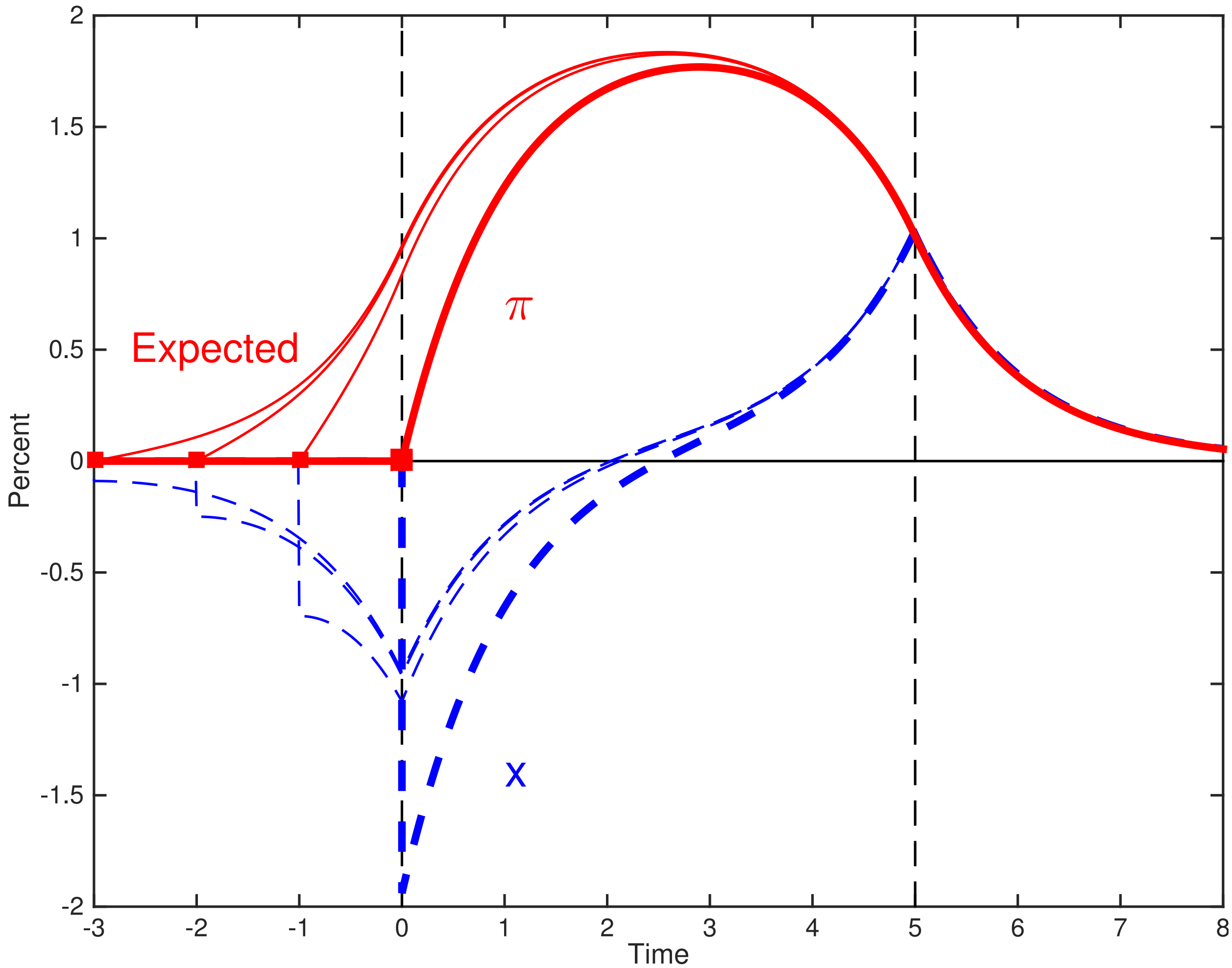

Figure 4: Output and inflation in the no-inflation-jump $\pi_0 = 0$ equilibrium. The thin lines give equilibria with no inflation jump at time $t = -1, -2$, and -3, corresponding to news of the trap arriving on those dates. $\kappa = 1$ throughout. The solid squares remind us visually of the equilibrium selection by $\pi_t = 0$.

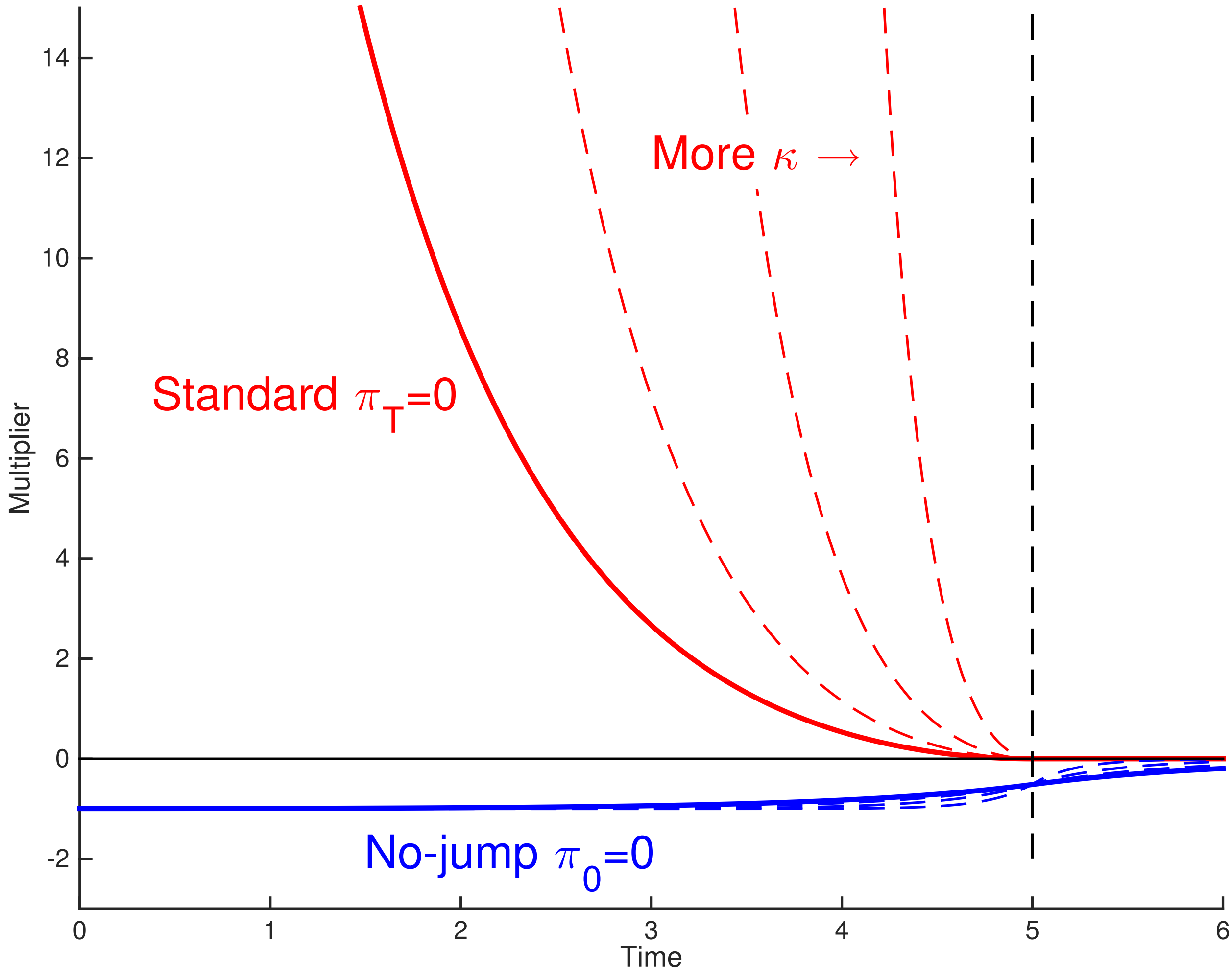

Figure 5: Multipliers with respect to a Phillips curve disturbance. I modify the Phillips curve to $d\pi_t/dt = \rho\pi_t - \kappa(x_t + g_t)$. The graph plots the multiplier $\partial x_t/\partial g$ for an increase in g through the trap episode from $t = 0$ to $t = T = 5$. The thin lines show multipliers as price stickiness is reduced to $\kappa = 2$, 5, 20.

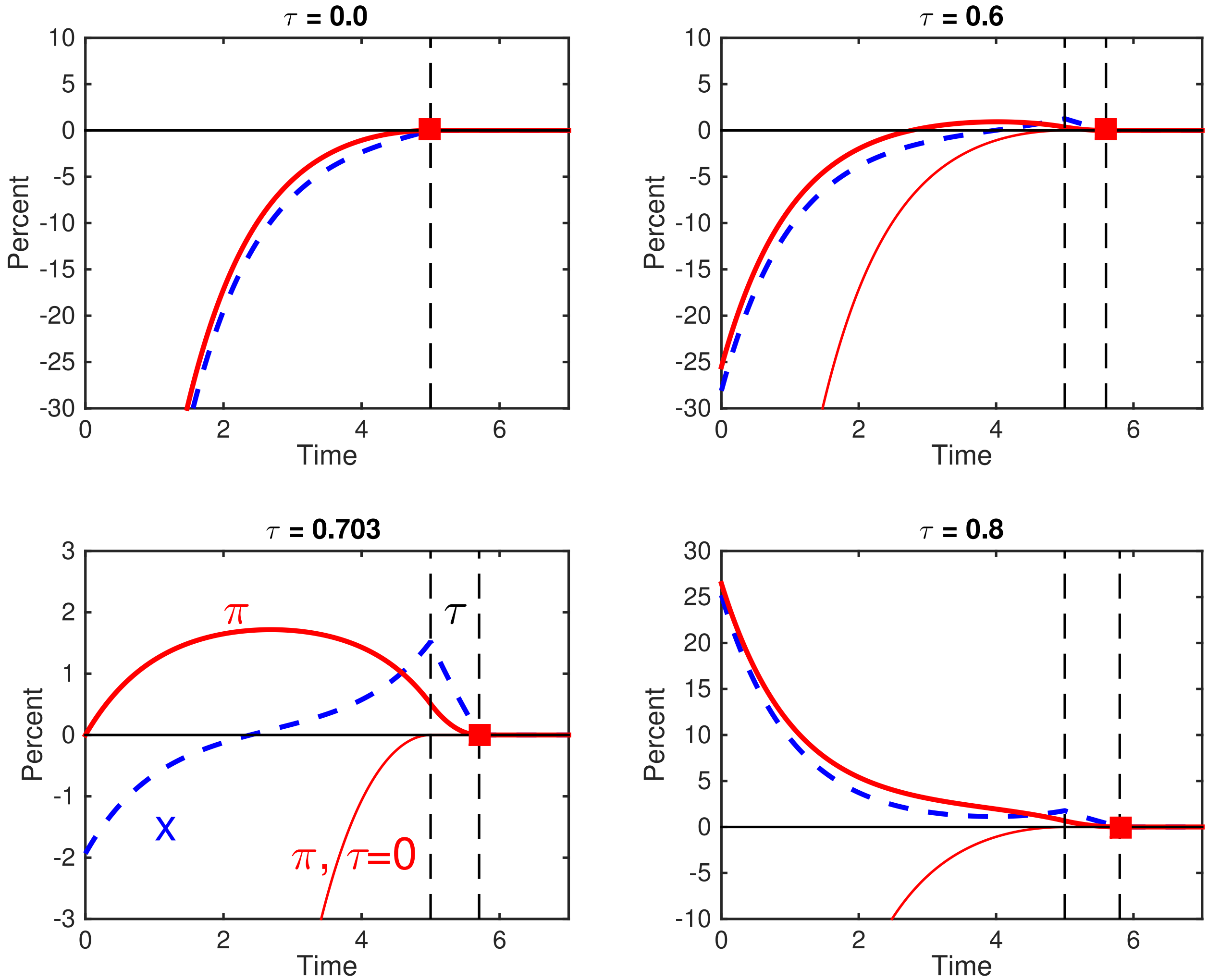

Figure 6: Output gap x and inflation π in the standard equilibrium choice $\pi_{T+\tau} = 0$, when the interest-rate rise is delayed. At $t = T = 5$, marked by the left dashed line, the natural rate changes from -2% to +2%. At $t = T + \tau$, marked by the right dashed line, people expect the nominal interest rate to rise from $i = 0$ to $i = 2\%$, for τ as indicated. The thin line marked "π, $\tau = 0$" repeats the $\tau = 0$ inflation line for comparison. The symbol τ marks the period in which the natural rate has risen but the interest rate remains at zero.

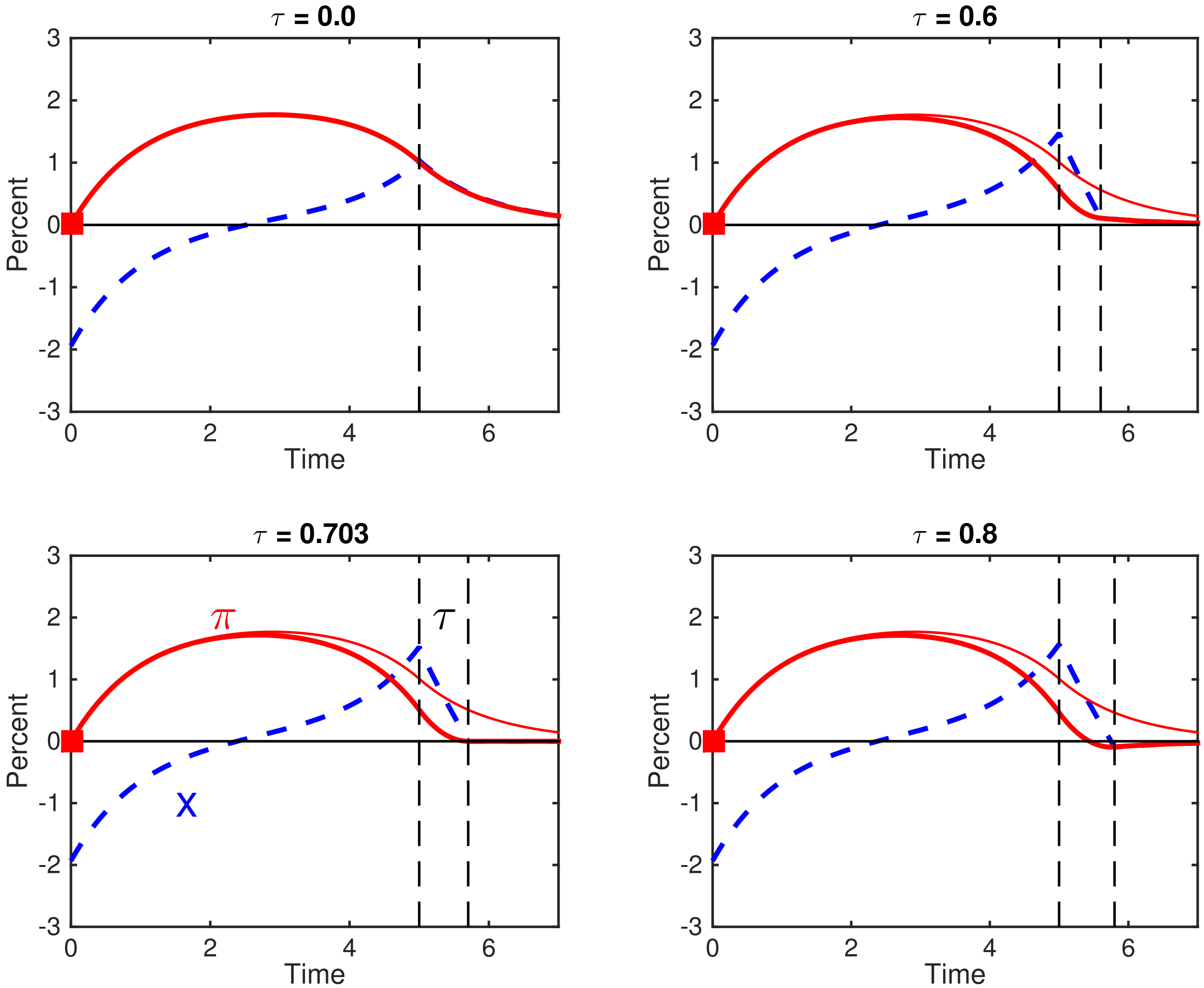

Figure 7: Output gap x and inflation π in the no-jump equilibrium choice $\pi_0 = 0$, when the interest-rate rise is delayed. At $t = T = 5$, marked by the left dashed line, the natural rate changes from -2% to +2%. At $t = T + \tau$, marked by the right dashed line, people expect the nominal interest rate from $i = 0$ to $i = 2\%$, for τ as indicated. The thin line presents the $\tau = 0$ inflation value for comparison.