Comments on "Anomalies" by Lu Zhang

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Q reminder

$$V(K_0,\{I_t\}) = E_0 \sum_{t=0}^{\infty} M_t D_t = E_0 \sum_{t=0}^{\infty} M_t \left\{\theta_t f(K_t) - \left[1+\frac{\alpha}{2}\left(\frac{I_t}{K_t}\right)\right] I_t\right\}$$

$$s.t.\ \ K_{t+1} = (1-\delta)K_t + I_{t+1}; \qquad K_0 = (1-\delta)K_{-1} + I_0$$

$$FOC:\ \frac{\partial V}{\partial I_0} + \frac{\partial V}{\partial K_0} = 0$$

$$1+\alpha\frac{I_0}{K_0} = \frac{\partial V}{\partial K_0} = \text{“marginal q”}$$

$$= E_0 \sum_{t=0}^{\infty} M_t (1-\delta)^t \left\{\theta_t f'(K_t) + \frac{\alpha}{2}\left(\frac{I_t}{K_t}\right)^2\right\}$$

constant returns to scale: $\dfrac{\partial V}{\partial K} = \dfrac{V}{K}$

$$1+\alpha\frac{I_t}{K_t} = \frac{V_t}{K_t} = Q_t = \frac{BE_t}{ME_t}$$

Implication: $1+\alpha\frac{I_t}{K_t}$ can substitute for $\frac{V_t}{K_t}$, Q_t, $\frac{BE_t}{ME_t}$ in any application.

Investment returns

$$
\begin{aligned}
R_{t+1} &= \frac{V_{t+1}+D_{t+1}}{V_t} \Leftarrow \frac{V_t}{K_t} = 1+\alpha\frac{I_t}{K_t} \\
R_{t+1} &= (1-\delta)\frac{\left(1+\alpha\frac{I_{t+1}}{K_{t+1}}+\frac{D_{t+1}}{K_{t+1}}\right)}{\left(1+\alpha\frac{I_t}{K_t}\right)\left(1-\frac{I_{t+1}}{K_{t+1}}\right)} \\
&\approx 1-\delta+\frac{D_{t+1}}{K_{t+1}}+(1+\alpha)\frac{I_{t+1}}{K_{t+1}}-\alpha\frac{I_t}{K_t} \\
\text{Stock return} &= \text{investment return}
\end{aligned}
$$

•*Ex post too.*

•A "first differenced" version of q theory.

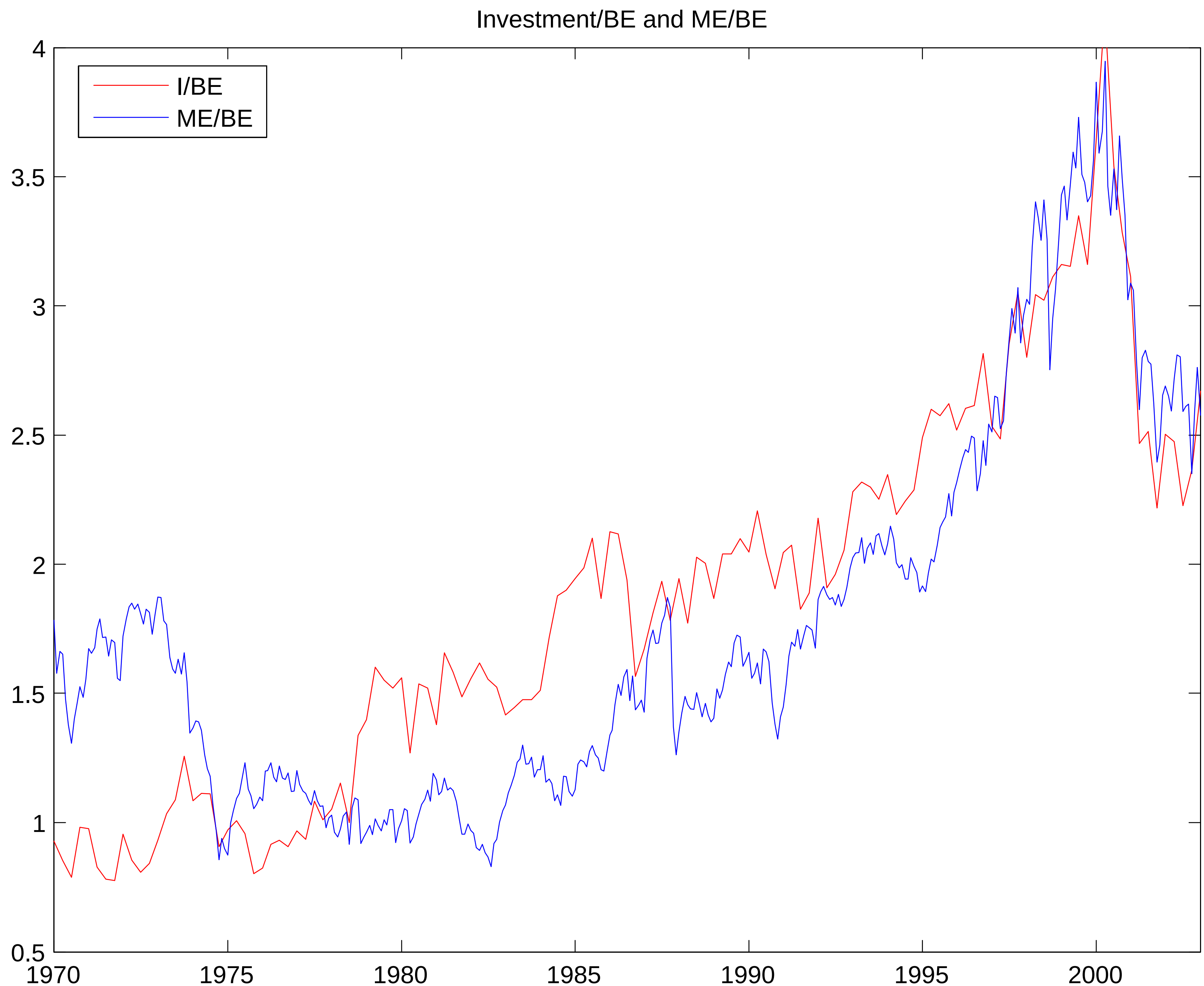
•Good: Emphasizes difference, where q theory works well. Good intuition for return anomalies.

•Bad: Can't do this with time to build, irreversible investment. There, stick to investment = marginal q

Q theory is pretty good!

$$1 + \alpha \frac{I_t}{K_t} = \frac{V_t}{K_t}$$

1. I/K tracks V/K, (and P/X); investment returns track stock returns.
   (a) Time series (see graphs)
   (b) Cross section (see graphs)
2. Investment *plans* get timing better (Lamont). This suggests time-to plan is important for good empirical fits. (Of course).



Real nonresidential fixed investment (BEA), ME/BE (Pastor/Veronesi), ME(CRSP)

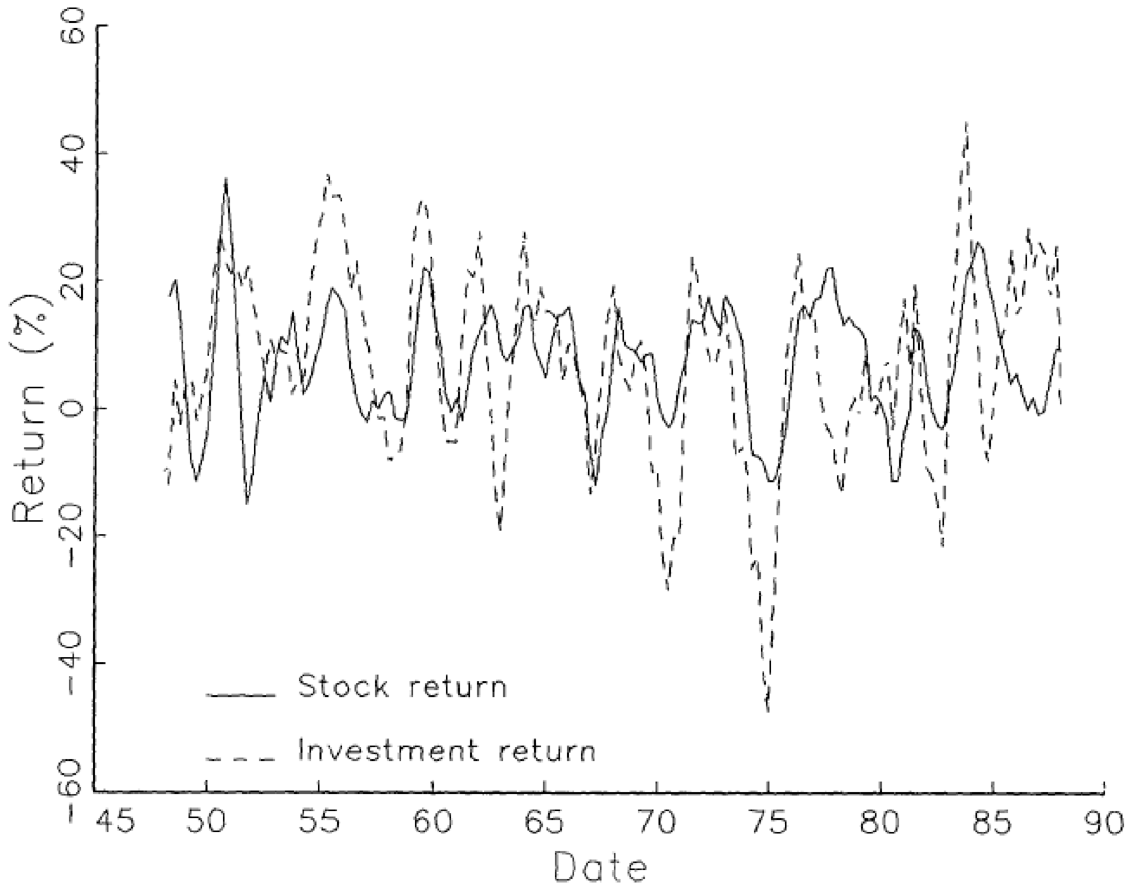


Figure 2. Quarterly observations of annual (from t − 4 to t) real returns on the value weighted NYSE portfolio, and annual investment returns.

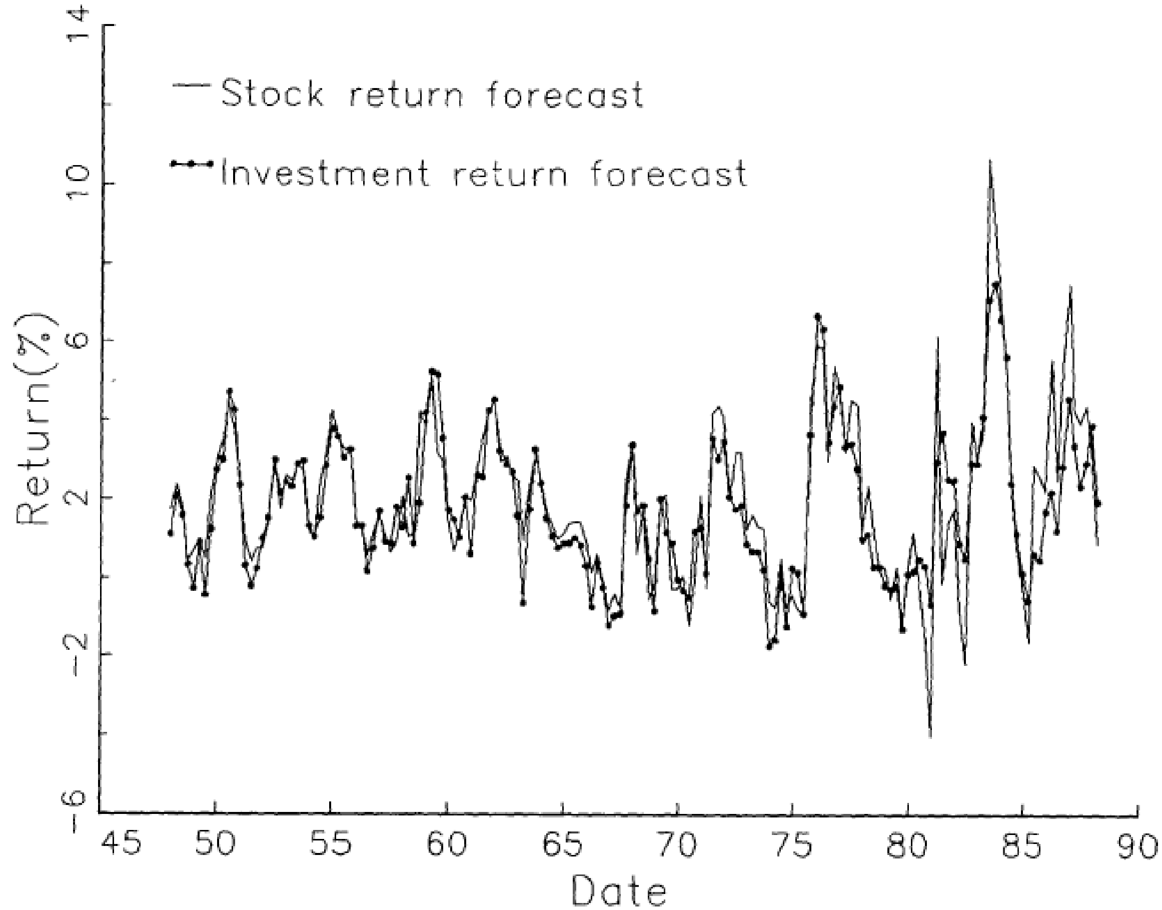
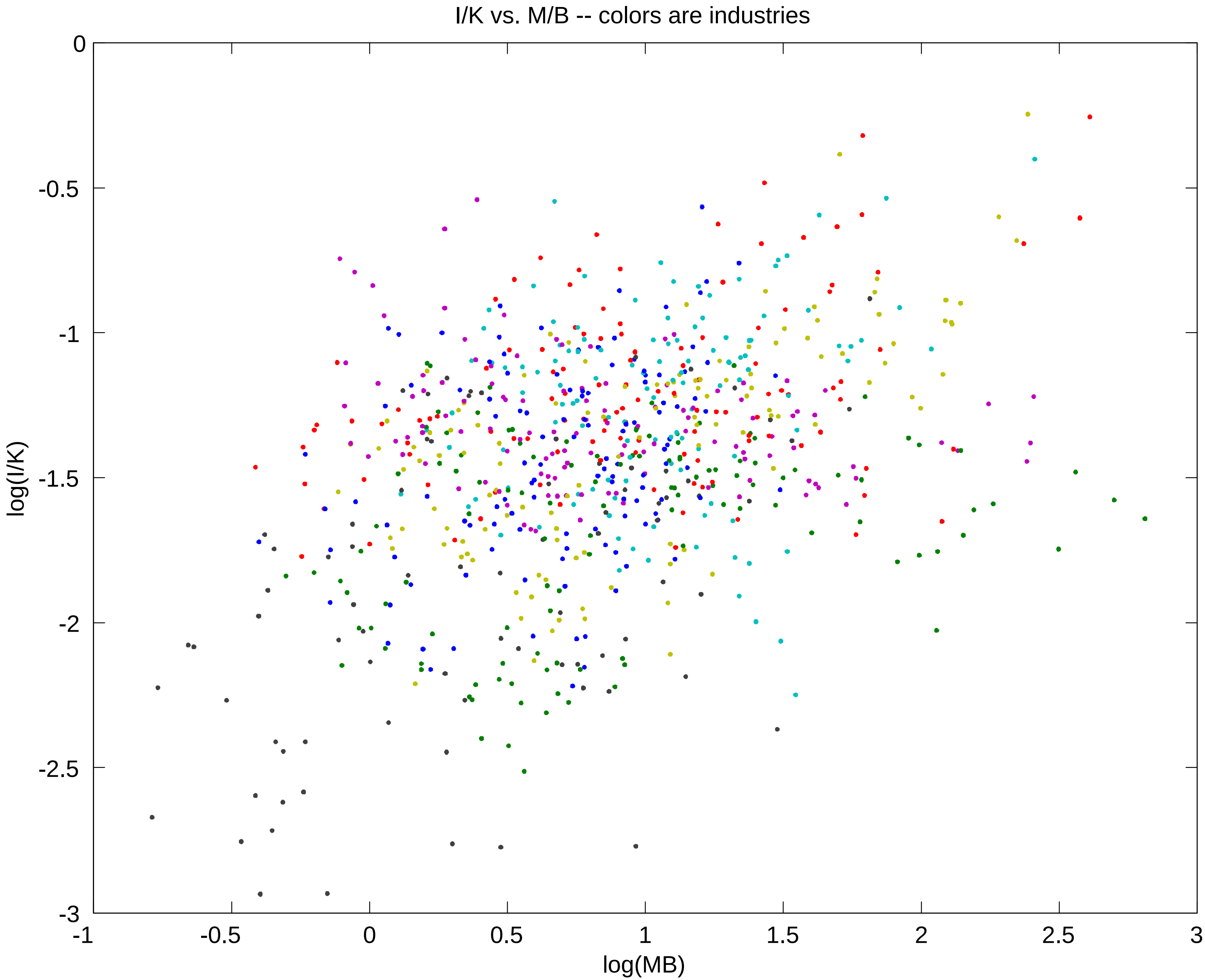
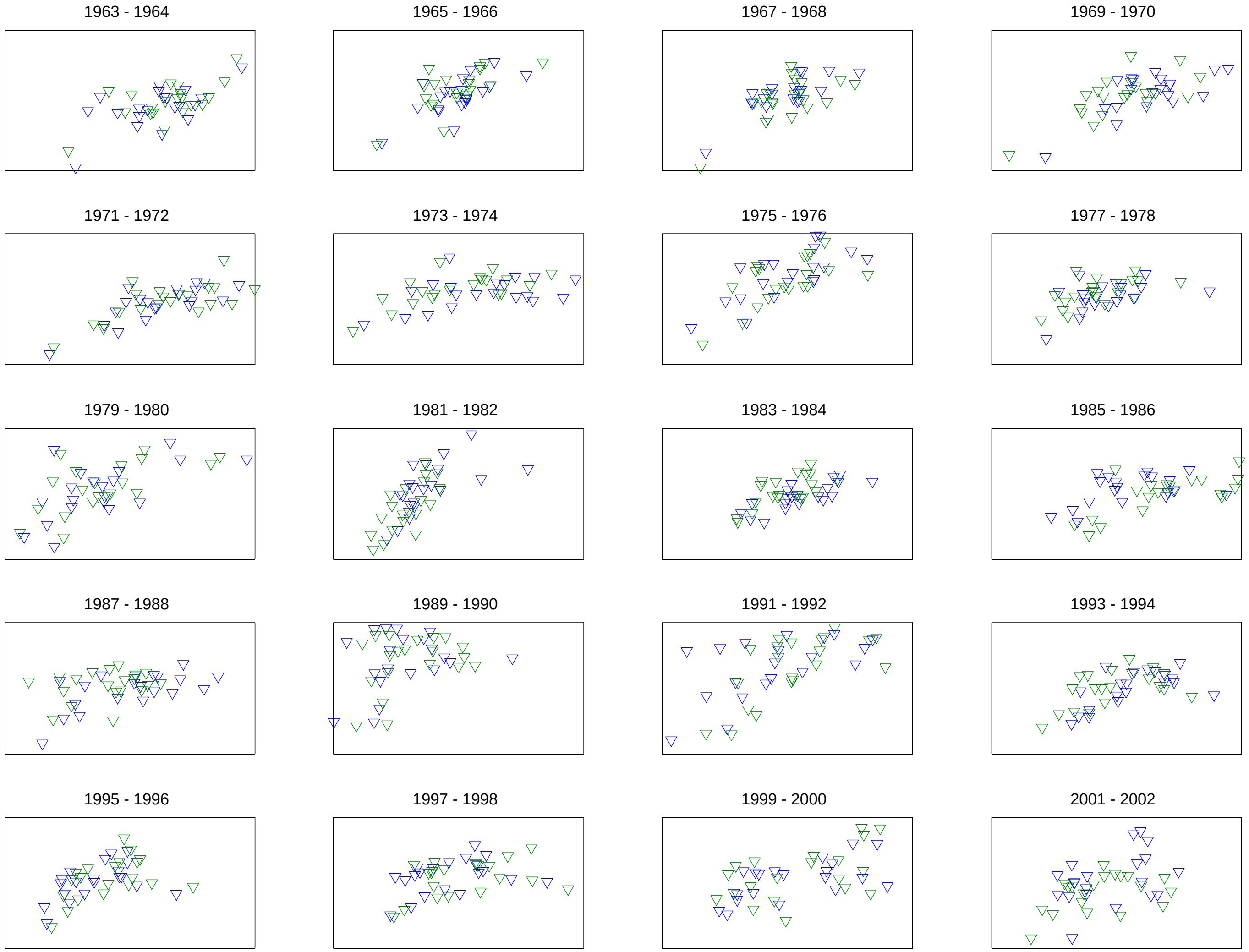
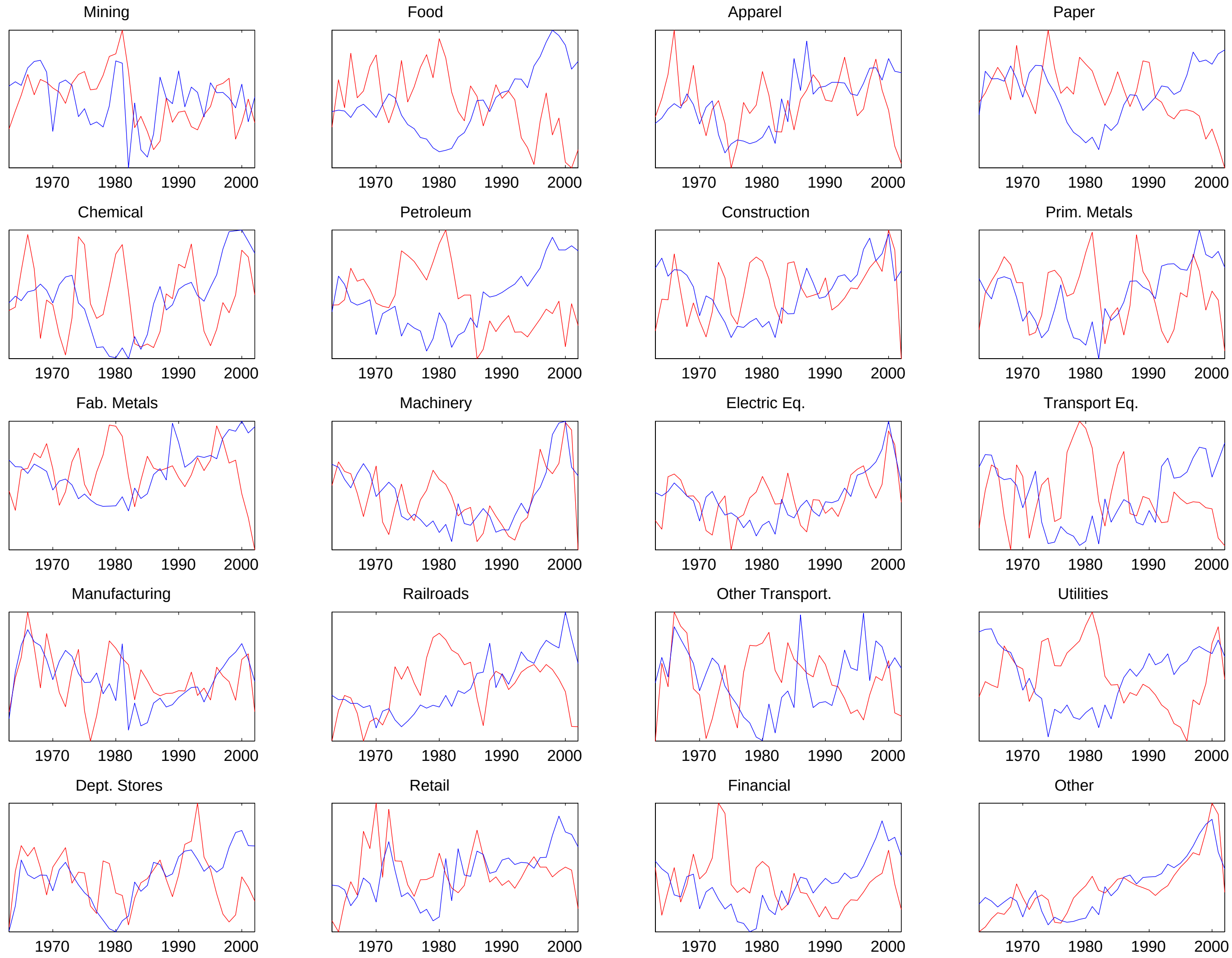


Figure 3. Forecasts of quarterly stock returns and investment returns. Forecasts are from linear regressions of returns on the term premium, corporate premium, lagged return and investment to capital ratio.



20 Industry I/K and B/M, 1963-2002. (Santos-Veronesi)


1963 - 1964
1965 - 1966
1967 - 1968
1969 - 1970
1971 - 1972
1973 - 1974
1975 - 1976
1977 - 1978
1979 - 1980
1981 - 1982
1983 - 1984
1985 - 1986
1987 - 1988
1989 - 1990
1991 - 1992
1993 - 1994
1995 - 1996
1997 - 1998
1999 - 2000
2001 - 2002


Mining
Food
Apparel
Paper
Chemical
Petroleum
Construction
Prim. Metals
Fab. Metals
Machinery
Electric Eq.
Transport Eq.
Manufacturing
Railroads
Other Transport.
Utilities
Dept. Stores
Retail
Financial
Other
1970
1980
1990
2000

Why do people think q is bad?

$$1 + \alpha \frac{I_t}{K_t} = \frac{V_t}{K_t}$$

1. Theory is rejected. No error! Predicts $R^2 = 1$! "Errors" are specification errors, unsurprisingly correlated with right hand variable, cashflow, etc.

2. Works better at high frequencies. Low frequency movement in p_I, α, etc.?

3. Tried to line I up with *interest rates.* We now know that most variation in cost of capital is in the *risk premium*.

4. From 2, 3, found "too high" adjustment costs. Now more reasonable numbers.

Anomalies

1. Time series predictability, value/growth in the cross-section

$$1 + \alpha \frac{I_t}{K_t} = \frac{V_t}{K_t} = \frac{1}{K_t} E_t \sum_{j=1}^{\infty} M_{t+j} D_{t+j}$$

$$\alpha \frac{I_t}{K_t} \approx \ln \frac{V_t}{K_t} \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \left(d_{t+j} - r_{t+j} \right) \quad (d = \ln(1 + \frac{D}{K}))$$

   (a) If $E_t r_{t+j}$ varies (TS or XS) $\rightarrow V_t/K_t$ varies $\rightarrow V_t/K_t$ predicts returns

$$r_{t+j} = a + b \frac{V_t}{K_t} + \varepsilon_{t+j}$$

   (b) If $E_t d_{t+j}$ varies $\rightarrow V_t/K_t$ varies $\rightarrow V_t/K_t$ predicts profitability

$$d_{t+j} = a + b \frac{V_t}{K_t} + \varepsilon_{t+j}$$

   (c) Time series: mostly effect a. Cross section: both a and b.

$$\alpha \frac{I_t}{K_t} \approx \ln \frac{V_t}{K_t} \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \left(d_{t+j} - r_{t+j} \right)$$

2. What do V/K predictions of returns r and profits d have to do with I/K?

   (a) Zip.

   (b) I_t/K_t should also predict returns (TS, XS) and profitability (XS).

      i. This works pretty well. TS: Cochrane 1991 I/K forecasts returns. Lamont 2000: Investment plans work even better (see table) . Direct XS estimate?

      ii. $\rightarrow$ "investment anomaly" "SEO anomaly"

   (c) If $\alpha = 0$ then $Q = 0$. There must be some investment friction! (Investment is endogenous.)

$$r_{t+1} = a + bx_t + \varepsilon$$

x	coeff. b	s.e.
I_{t-1}/K_{t-1}	-3.55	(2.74)
$P_{t-1}(I_t)/K_{t-1}$	-8.36	(2.49)
I_t/K_{t-1}	-8.75	(2.43)
I_{t-1}/I_{t-2}	-0.33	(0.29)
$P_{t-1}(I_t)/I_{t-1}$	-1.36	(0.29)
I_t/I_{t-1}	-1.07	(0.25)

Lamont, Investment Plans and Stock Returns. $P_{t-1}(I_t)$ is investment in year t planned in year $t-1$ Sample 1948-1993: it should work even better now!

Profitability Anomalies

$$\alpha \frac{I_t}{K_t} \approx \ln \frac{V_t}{K_t} \approx E_t \sum_{j}^{\infty} \rho^j \left(d_{t+j} - r_{t+j} \right)$$

1. Why should high $E(d)$ have anything to do with high $E(r)$? High $E(d)$ should just mean high V/K and high I/K and that's it.

2. Zhang's partial is $\left. \frac{\partial}{\partial d_{t+j}} \right|_{\{I_t\}}$ *Holding investment (or V/K) contsant* we should see high $E(d)$ correlate with high $E(r)$. High $E(r)$ must *offset* the high $E(d)$ if I/K and V/K are constant. Is that the empirical work?

An "Explanation" ?

- "The Q framework vs. the beta framework" "The Q framework vs. the SDF framework" (beta $=$ SDF!)

- Does the Q framework give a "rational explanation" for anomalies?

- No, alas.

  1. Shiller might say: "irrational exuberance" raises $V/K \rightarrow$ *Firms* act optimally, I/K is too high. It would be better if firms *did not* react. Q is *how* "irrational" prices cause bad allocations.

  2. (JC: well, if people are rational at work, why irrational at home? But let's not argue about the coherence of behavioral stories.)

  3. We can't really write a coherent endowment economy with fixed *investment*, prices must adjust.

- Q does provide a *connection to macroeconomic events*. Market is not completely off on its own disconnected from economics. It's nice that *one side* of the market works well!

- Success of a (consumption) beta model would not be a full "explanation" either. Where do the betas come from? We don't live in an endowment economy.

- Only general equilibrium is an "explanation" alas

The Future

1. *Quantitative* explanation of puzzles. It would be lovely to see the same, low adjustment cost parameter in each case!

2. Time to build so we don't need to see only investment *expenditures*. Lamont and plans again.

3. Marginal q $\neq$ average q. We will study price vs. present values in asset pricing, and investment vs. marginal present value.

4. Classic Q (Summers) had detailed and sophisticated calculation of taxes, "replacement cost." Now we just use accounting book value. Worth improving X in V/X?

5. Where are the shocks?

$$\alpha \frac{I_t}{K_t} \approx \log\left(\frac{V_t}{K_t}\right) \approx E_t \sum_{j=1}^{\infty} \rho^j \left(d_{t+j} - r_{t+j}\right)$$

$$1 + \alpha \frac{I_t}{K_t} = \frac{1}{K_t} E_t \sum M_{t,t+j} D_{t+j}$$

$$D_t = \theta_t f(K_t) - \left[1 + \frac{\alpha}{2}\left(\frac{I_t}{K_t}\right)\right] I_t$$

(a) If technology/marginal productivity shocks, then shouldn't we see I/K and V/K forecasting profitability d but not returns r?

(b) Does this mean that much variation in V/K, I/K must be due to "preference shocks" $M, E(r)$?

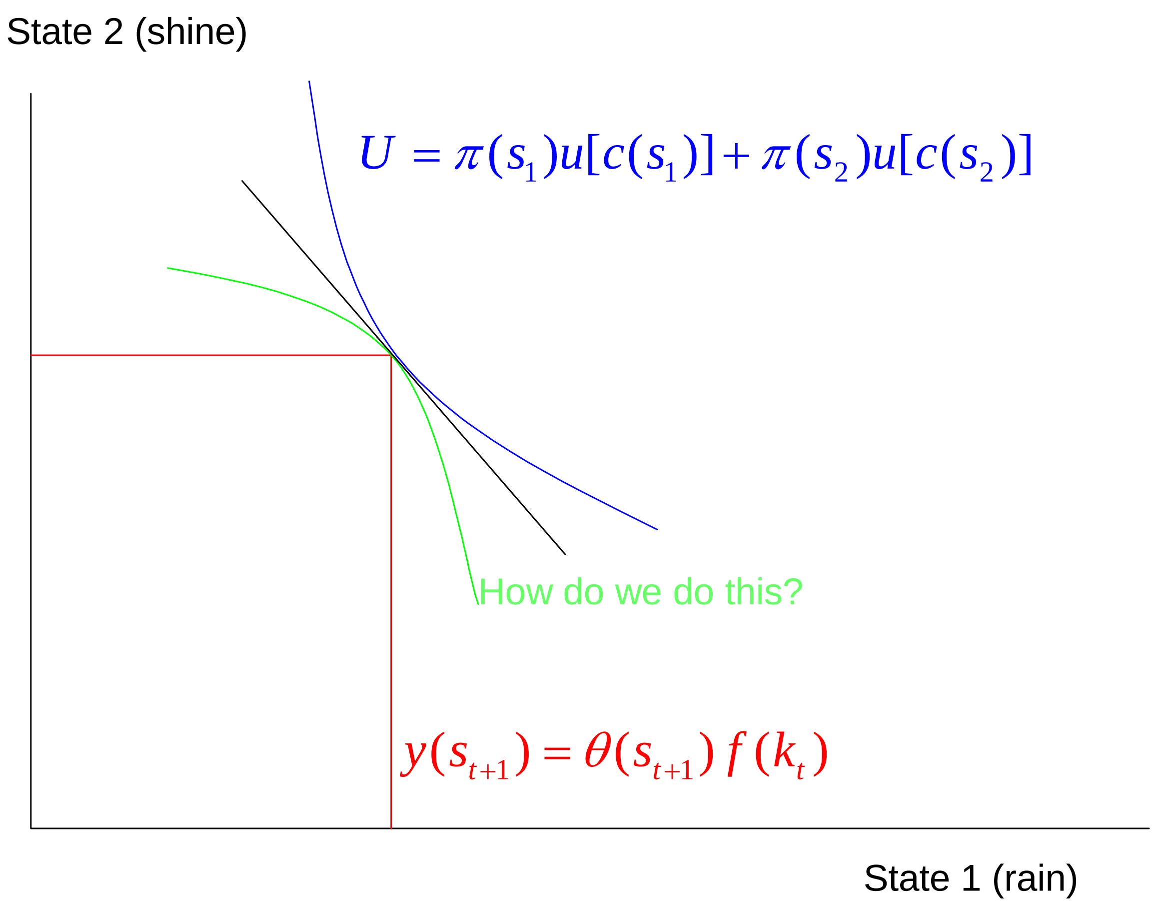
(c) Not necessarily. $E(d_{t+j}) \uparrow \Rightarrow$ $c_t \uparrow \Rightarrow$ risk aversion $\downarrow$ (e.g. habits) $\Rightarrow$ $E(r_{t+j}) \downarrow$ Then in an economy driven entirely by technology shocks, we see endogenous change in risk aversion, V/K, I/K forecast returns. Long-ahead D can result in high *current* Er.

(d) Similar, more complex mechanisms in the cross section. (Menzly, Santos and Veronesi, Kogan Zhang, Gourio, Gala, etc.)

6. Why no "production-based asset pricing"? Why do we lose the symmetry we had in micro?

$$y_t(s) = \theta(s) f(k_{t-1})$$

The models we write down are Leontief *across states*. This is just an accident of history; we added shocks to $f = f(k)$ from nonstochastic models. Firms *do* have actions to transform goods across states. We need a model of this!



The algebra:

$$R^s_{t+1} = \frac{V_{t+1} + D_{t+1}}{V_t}$$

$$R^s_{t+1} = \frac{1 + \alpha \frac{I_{t+1}}{K_{t+1}} + \frac{D_{t+1}}{K_{t+1}}}{1 + \alpha \frac{I_t}{K_t}} \frac{K_{t+1}}{K_t}$$

$$\frac{K_{t+1}}{K_t} = \frac{(1-\delta)K_t + I_{t+1}}{K_t} = (1-\delta) + \frac{I_{t+1}}{K_{t+1}} \frac{K_{t+1}}{K_t}$$

$$\left(1 - \frac{I_{t+1}}{K_{t+1}}\right) \frac{K_{t+1}}{K_t} = (1-\delta)$$

$$\frac{K_{t+1}}{K_t} = \frac{(1-\delta)}{\left(1 - \frac{I_{t+1}}{K_{t+1}}\right)}$$