For sure

\[ u(c) \]

\[ u(\text{sure}) \]

\[ u(\text{bet}) \]

\[ +/ - 50/50 \text{ bet} \]

\[ c \]

---

Power utility functions \( c^{1-\gamma} \)

Marginal utility of power utility \( c^{-\gamma} \)
Jagannathan and Wang 2005 “Consumption Risk and the Cost of Equity Capital” (NBER working paper)

Figure 2.4. Mean excess returns of 10 CRSP size portfolios versus predictions of the power utility consumption-based model. The predictions are generated by $-R_t^i \text{cov}(m, R_t^i)$ with $m = \beta(c_{i+1}/c_i)^{-\gamma}$. $\beta = 0.98$ and $\gamma = 241$ are picked by first-stage GMM to minimize the sum of squared pricing errors (deviation from 45° line). Source: Cochrane (1996).

Figure 1: Annual Excess Returns and Consumption Betas

Plot figure of average annual excess returns on Fama-French 25 portfolios and their consumption betas. Each two digit number represents one portfolio. The first digit refers to the size quintiles (1 smallest, 5 largest), and the second digit refers to the book-to-market quintiles (1 lowest, 5 highest). Annual excess returns and consumption betas are reported in previous table.
Figure 2: Realized vs. Fitted Excess Returns: FF25 Portfolios

This figure compares realized returns and fitted returns of Fama-French 25 portfolios 1954-2003. Each two digit number represents one portfolio. The first digit refers to the size quintiles (1 smallest, 5 largest), and the second digit refers to the book-to-market quintiles (1 lowest, 5 highest). Three models are compared: CCAPM, CAPM and Fama-French 3 factor model. Models are estimated by using Fama-MacBeth cross-sectional regression procedure. Estimation results are reported in previous table.
8 Week 3 Asset Pricing Theory Outline/Review

1. Value question: Payoff $x_{t+1}$. What is its value today $p_t$?

2. Value to who? Utility function captures aversion to risk and delay

$$U(c_t, c_{t+1}) = u(c_t) + \mathbb{E}_t [u(c_{t+1})]$$

3. $u(c)$ shape. Concavity: people dislike risk. $\beta < 1$ people dislike delay. Power example, $\gamma$ allows you to vary curvature / risk aversion

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}; \quad u'(c) = c^{-\gamma}$$

$$u(c) = \log(c); \quad u'(c) = c^{-1}$$

4. What is $x_{t+1}$ worth (willingness to pay) to a typical investor? Marginal cost = marginal benefit led to.

$$p_t = \mathbb{E}_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]$$

5. We separated this to

$$p_t = \mathbb{E}_t [m_{t+1} x_{t+1}]$$

$$m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \approx 1 - \delta - \gamma \Delta c_{t+1}$$

$m, u'(c)$ measure “hunger” – higher $c$ means lower $m$

6. Classic issues in finance

(a) Risk free rate

$$1 = \mathbb{E}(m R^f); \quad R^f = 1/\mathbb{E}(m)$$

$$R^f \approx 1 + \delta + \gamma \mathbb{E}_t (\Delta c_{t+1})$$

(b) Expected returns and covariance/beta

$$E(R^e) = -R^f \text{cov}(m, R) = \beta R^e, m \times \lambda_m$$

$$E(R^e) \approx \gamma \text{cov}(R^e, \Delta c) = \beta R^e, \Delta c \times \lambda_{\Delta c}$$

(c) Variance $\sigma^2(R^e)$ does not matter

(d) *Discount for risky assets/projects

$$p = \mathbb{E}(mx) = \mathbb{E}(m) E(x) + \text{cov}(m, x)$$

$$p = \frac{E(x)}{R^f} - \gamma \text{cov}(x, \Delta c)$$

(e) Mean- variance frontier

$$\frac{E(R^{ei})}{\sigma(R^{ei})} \leq \frac{\sigma(m)}{\mathbb{E}(m)}$$
(f) Frontier returns carry all pricing information.

\[ E(R^c) = \beta R^e R^{mv} \lambda_{mv} \leftrightarrow R^{emv} \] is on the mvf.

(g) Predictable returns

\[ E_t(R_{t+1}) - R_t^f \approx \sigma_t(R_{t+1}) \sigma_t(m_{t+1}) \rho_t(R, m_{t+1}) \]

\[ \approx \gamma \sigma_t(R_{t+1}) \sigma_t(c_{t+1}) \rho_t(R, \Delta c) \]

(h) *Long lived securities

\[ p_t = E_t \sum_{j=1}^{\infty} \beta_j u'(c_{t+j}) d_{t+j} = E_t \sum_{j=1}^{\infty} m_{t+j} d_{t+j} \]

7. Asset pricing models. Big picture

\[ m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a - b' f_{t+1} = a - b_1 f_{t+1} - b_2 f_{t+1}^2 \]

is equivalent to

\[ E(R^{ei}) = \beta' \lambda = \beta_{1,1} \lambda_1 + \beta_{1,2} \lambda_2 + ... \]

8. What do we use for \( f \)? Idea 1: measures of good/bad times, determinants of consumption (absolute pricing)

(a) CAPM: market return

\[ m_{t+1} = a - b R^m_{t+1} \]

(b) CAPM also says this is the only factor. Multifactor models say “other things matter too.”

(c) Macro: \( f = \) labor, other income; investment, unemployment, etc.

(d) ICAPM: \( f \) give news of future investment opportunities (shocks to d/p, interest rates)

(e) Mimicking portfolios.“For state variables of concern to investors.”

(f) Fama “Fishing license” Almost! Momentum?

(g) Derivations: what can’t be a factor? .

9. Portfolio logic for multifactor models. If the average investor wants to get rid of stocks that fall when \( f \) falls, independent of what the market is doing, then \( E(R^e) = ... + \beta R^e f \lambda_f \)

10. What do we use for \( f \)? Idea 2: APT: Portfolios that explain comovement of asset returns should be factors to explain Average returns. “Relative pricing” to those portfolios

(a) “Small” residuals should give “small” alphas.

(b) Central result:

\[ R^{ei}_{t+1} = \alpha_i + \beta_{i,1} f^1_{t+1} + \beta_{i,2} f^2_{t+1} + \varepsilon^i_{t+1}. \]

\[ E(R^{ei}) = \alpha_i + = \beta_{i,1} E(f^1) + \beta_{i,2} E(f^2) \]

\[ \alpha_i < (\max \text{ surviving Sharpe}) \times \sigma(\varepsilon^i) \]

Small \( \varepsilon \), large \( R^2 \) implies small \( \alpha \) or huge Sharpe ratio.
(c) Only works when $\varepsilon$ is small: for portfolios not individual stocks.

11. Comments.

(a) What model you use depends on what you’re going to use it for.
   i. “Explanation,” behavioral vs. rational debate.
   ii. Manager/strategy evaluation
   iii. Risk management

(b) So, is FF3F a “ICAPM” or an “APT?”