5 Week 3. Fama-French and the cross section of stock returns – overheads

5.1 Fama and French ”Multifactor Anomalies”

1. Big Questions
2. CAPM,

\[ E(R^{ei}) = \beta_i \lambda \quad (+\alpha_i) \]

(a) \( \beta_i \) are defined from time series regressions

\[ R^{ei}_t = \alpha_i + \beta_i R^{em}_t + \epsilon^i_t; \]

\( (R^{ei}_t = R^i_t - R^f_t) \)

(b) What we do: see if attractive opportunities \( E(R^{ei}) \) have higher \( \beta_i \).

3. Evidence: The CAPM worked great and still does on many assets.

(a) From “Discount Rates” The CAPM works great on size portfolios.

4. CAPM Example 2: industry portfolios
5. The Value Puzzle

(a) FF. Ok for size, industry, beta portfolios. What about book/market? Do low prices mean high returns across stocks?

(b) Facts: There is a big spread in average returns. But market beta is a disaster. **Puzzle depends on average returns and betas!** From “Discount rates”

(c) Also in “Discount Rates”

Value effect before and after 1963.
(d) Value From *Asset Pricing* 

Mean excess returns vs. market beta
Fama–French 25 Size and B/M sorted portfolios
Mean excess returns vs. market beta
lines connect changing SIZE within B/M categories
Mean excess returns vs. market beta
lines connect changing B/M within SIZE categories
6. Fama-French solution:

(a) Run *time series regressions* that include additional *factors* (portfolios of stocks) SMB, HML

\[ R_{it} = \alpha_i + b_i R_{it}^{em} + s_i SMB_t + h_i HML_t + \epsilon_{it}; \quad t = 1, 2, \ldots T \text{ for each } i = 1, 2, \ldots N. \]

(b) Look across stocks

\[ E(R_{it}) = \alpha_i + b_i E(R_{it}^{em}) + s_i E(SMB) + h_i E(HML) \]

(c) Result from “Discount rates.”

![Average returns and betas graph](image-url)
(d) 25 portfolios from *Asset Pricing*

Mean excess return vs. predicted by 3 factor model
Lines connect changing size within B/M categories

Actual mean excess return $E(R_i - R_f)$

Predicted, $\beta_{i,m}E(R^m - R^f) + \beta_{i,h}E(HML) + \beta_{i,s}E(SMB)$
Mean excess return vs. predicted by 3 factor model
lines connect changing BM within size categories

Actual mean excess return $E(R_p - R_f)$

Predicted, $\beta_{i,m}(R^m - R_f) + \beta_{i,h}E(HML) + \beta_{i,s}E(SMB)$
7. Fama-French paper:

(a) Run time series regressions that include additional factors (portfolios of stocks) SMB, HML

\[ R_{ti}^{e} = \alpha_i + b_i R_{t}^{em} + s_i SMB_t + h_i HML_t + \epsilon_{ti}^{i}; \quad t = 1, 2...T \text{ for each } i = 1, 2...N. \]

(b) Look across stocks at the cross-sectional implication of this time-series regression (Take \( E \) of both sides again):

\[ E(R_{ti}^{e}) = \alpha_i + b_i E(R_{t}^{em}) + s_i E(SMB) + h_i E(HML) \]

This works pretty well (\( \alpha \) not big) except for small growth.

(c) “Discount Rates” one stop summary again. Now look at the sum of red solid and red dashed lines. \( E(r) = b \times E(rmf) + h \times E(hml) \)

8. FF

(a) See FF Table 1. In depth!
(b) What’s wrong with \( E(R^{ci}) = (size_i) \lambda_s + (b/m_i) \lambda_{B/M} \)? “How you behave” vs. “who you are”
(c) Understand the difference between “explaining returns” (time-series regression) and “explaining average returns” (cross-sectional relation between average return and beta)!
(d) The main point is to produce a robust model that explains other anomalies. That is what the CAPM did for many years. See Sales, long term reversal. Not momentum

9. Do we really need the smb portfolio? Smb makes it a better model of returns, doesn’t help much on average returns, and improves precision.

(a) Example: Suppose the CAPM works add a beta-hedged industry portfolio.

\[
R_t^I = \alpha_I + \beta_I R_t^{em} + \varepsilon_{It}
\]

\[
R_t^{eI*} = R_t^I - \beta_I R_t^{em}
\]

Now run

\[
R_t^{ci} = \alpha_i + \beta_i R_t^{em} + \gamma_i R_t^{eI*} + \varepsilon_{it}
\]

i. \( \gamma_i > 0 \), \( R^2 \) improves, \( t \) statistics improve, \( \sigma(\varepsilon_i) \) decreases. The model of variance improves

ii. \[
E \left( R_t^{ci} \right) = \beta_i E (R_t^{em}) + \gamma_i E \left( R_t^{eI*} \right) = \beta_i E (R_t^{em}) + \gamma_i 0
\]

The model of mean is unchanged.
(b) This is roughly true. FF keep SMB because it is so useful to explain the \textit{variance} of size-sorted portfolios.

10. Is it a tautology to “explain” 25 B/M, size portfolios by 2 B/M, size portfolios? (No, why?) \rightarrow Other sorts.

11. Where does FF come from?

(a) ICAPM: “State variables of concern to investors” Suppose people don’t want stocks that fall especially (more than others) in recessions.
(b) APT: “Minimalist interpretation.” Suppose

\begin{align*}
R_t^{ei} = b_t r_m f_t + h_t h_{ml} + s_t s_m b_t + 0
\end{align*}

\rightarrow

\begin{align*}
E \left( R_t^{ei} \right) = b_t E (r_m f_t) + h_t E (h_{ml}) + s_t E (s_m b_t)
\end{align*}

(c) Practice: like the CAPM for digesting anomalies.

12. A big picture for “dissecting anomalies” and the whole question of multivariate forecasts:

\begin{align*}
dp_t \approx E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} - E_t \sum_{j=1}^{\infty} \rho^j \Delta d_{t+j}
\end{align*}

\textit{dp reveals} \textit{to us} market expectations.

(a) How can $z_t$ help?
(b) $z_t$ can predict both $r$ and $\Delta d$. $z_t$ can predict $r_{t+1}$ and $r_{t+j}$ in opposite directions.
(c) Fama and French “Dissecting anomalies:" This is why additional “cashflow forecast” anomaly variables help to forecast returns.
(d) \textquote{“Discount rates”} the cay experiment turns out to forecast the time path of returns.
13. Regressions summary.

(a) Forecasting

\[ R_{t+1}^{em} = a + bx_t + \epsilon_{t+1}; \ t = 1, 2, ..., T \]

(b) The “market model” of returns (return variance)

\[ R_{t}^{ii} = \alpha_i + \beta_i R_{t}^{em} + \epsilon_{t}^{i}; \ t = 1, 2, ...T \text{ for each } i \]

(c) FF’s three-factor model of returns (return variance)

\[ R_{t}^{ei} = \alpha_i + b_i \text{rmrf}_t + h_i \text{hml}_t + s_i \text{smb}_t + \epsilon_{t}^{i}; \ t = 1, 2, ...T \text{ for each } i \]

(d) The CAPM model of mean returns. (We implicitly run this when we look at expected return vs. beta. We will run this “cross-sectional regression” explicitly soon.)

\[ E \left( R_{t}^{ei} \right) = \beta_i \lambda_m + \alpha_i; \ i = 1, 2, ...N \]

(e) The slope coefficient in d should equal the mean market return (since its beta is one) \( \lambda_m \) should = \( E(R^{em}) \), so we sometimes force that in the implicit cross sectional “regression”.

\[ E \left( R_{t}^{ei} \right) = \beta_i E \left( R^{em} \right) + \alpha_i; \ i = 1, 2, ...N \]

(f) Fama and French. They do option e. They are implicitly running a cross sectional regression with the slopes equal to means of the factors. Table 1 is just data for this regression

\[ E \left( R_{t}^{ei} \right) = b_i E \left( \text{rmrf}_t \right) + h_i E \left( \text{hml}_t \right) + s_i E \left( \text{smb}_t \right) + \alpha_i; \ i = 1, 2, ...N \]

(g) The cross-sectional characteristic regression. Rather than Table 1A, FF dissecting anomalies and discount rates describe mean returns by a characteristic regression

\[ E \left( R_{t}^{ei} \right) = a + bE \left[ \log(B/M_{it}) \right] + cE \left[ \log(ME_{it}) \right] + \epsilon_i; \ i = 1, 2, ...N \]

more generally with \( C_i \) a vector of characteristics

\[ E \left( R_{t}^{ei} \right) = a + bC_i; \ \epsilon_i; \ i = 1, 2, ...N \]

(h) The characteristic regression is the same thing as a forecasting regression. (Note sometimes there are fixed effects, \( a_t \) or \( a_t \))

\[ R_{t+1}^{ei} = a + b \log(B/M_{it}) + c \log(ME_{it}) + \epsilon_{t+1}; \ t = 1, 2, ...T \ i = 1, 2, ...N \]

\[ R_{t+1}^{ei} = a + bC_{it} + \epsilon_{t+1} \]
5.2 Fama and French “Dissecting Anomalies”

1. The relationship between portfolios and cross-sectional regressions

Sorted portfolios and cross-sectional regressions.

A warning on OLS equally-weighted cross-sectional regressions