Final Exam

YOUR NAME: _____________________________.

Your mail folder location _________________
(Economics, GSB PhD/MBA mailfolders, elsewhere)

INSTRUCTIONS
DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. Please don’t rip the exam apart—keep it stapled.

Put your answers in the spaces provided. There is extra paper stapled to the end of the exam. You can also use the paper at the end of the exam for scratch. Show your work – quote what equation you start with, and explain the logic you use to get your answer. Keep it short; I’m only looking for the really obvious big point, and don’t give more credit for long winded answers. Make sure you try every question – I can’t give partial credit for a blank answer! Each question has a suggested time; the times add up to 2:50 hours.

I will place your graded exam and course grade in your mailfolder. Good luck!

GSB rules require that the following is placed on all exams, and that you sign it.

I pledge my honor that I have not violated the Honor Code during this examination.

Signature: _____________________________
Formula Sheet

Discount factors

\[ p = E(mx); \quad E(mR) = 1; \quad E(m^e) = 0 \]

\[ E(R^e) = -\frac{1}{E(m)} E(m, R^e) \]

\[ E(dR) - r^f dt = -E \left( \frac{d\Lambda}{\Lambda} dR \right) \]

\[ x^* = \frac{p^2 E(xx^*)^{-1} x}{x^*} \]

\[ x^* = \frac{1}{R^f} - \frac{1}{R^f} \left[ E(R) - R^f \right] \Sigma^{-1} [R - E(R)] \]

\[ \frac{d\Lambda^*}{\Lambda^*} = -r^f dt - \left[ \mu - r^f dt \right] \Sigma^{-1} \sigma dz \text{ if } dR = r^f dt + \mu dt + \sigma dz \]

\[ R^* = x^*/p(x^*); \quad E(R^{e2}) = E(R^e R) \forall R \in R \]

\[ R^{e*} = proj(1|R^e), \quad E(R^{e*} R^e) = E(R^e) \forall R^e \in R^e \]

GMM

\[ g_T(b) = E_T [u_t(b)] = E_T [f(x_t, b)] \]

\[ a_T g_T(b) = 0 \]

\[ \sqrt{T}(b - b) \rightarrow N(0, \left( ad \right)^{-1} a S a^T (ad)^{-1}) \]

\[ \sqrt{T} g_T(b) \rightarrow N(0, \left[ I - d(ad)^{-1} a \right] S \left[ I - d(ad)^{-1} a \right]') \]

\[ d = \frac{\partial g_T(b)}{\partial b} \]

\[ S = \sum_{j=-\infty}^{\infty} E \left[ f(x_t, b) f(x_{t-j}, b) \right] = \sum_{j=-\infty}^{\infty} E_t [u_t(b) u_{t-j}(b)] \]

Efficient GMM

\[ a = d' S^{-1} \]

\[ \text{var}(\hat{b}_2) = \frac{1}{T} (d' S^{-1} d)^{-1} \]

\[ \text{cov} \left[ g_T(b_2) \right] = \frac{1}{T} \left( S - d(d' S^{-1} d)^{-1} d' \right) \]

\[ g_T' \text{cov} \left( g_T \right)^+ g_T = T g_T' S^{-1} g_T' \chi^2_{#mom - #par} \]

Fun facts

\[ \sigma^2(x_{t+1}) = E(\sigma^2(x_{t+1})) + \sigma^2(E_t(x_{t+1})) \]

\[ E x^e = E E(x) + \frac{1}{2} \sigma^2(x) \text{ if } x \text{ is normal} \]

\[ \int_0^\infty e^{-\rho^t} dt = \frac{1}{\rho} \text{ if } \rho > 0 \]
1. If returns are predictable over time, investors ought to be able to do better by dynamic portfolio strategies than they can with fixed-weight portfolios, buying more when expected returns are high and less when low. You’ll derive a little identity here to quantify this idea.

Suppose there is one excess return \( R^e_{t+1} \) and a constant riskfree rate \( R_f^t = R_f \). The one return is predictable i.e. \( E_t(R^e_{t+1}) \) varies over time \( t \), but its conditional variance \( \sigma_t(R^e_{t+1}) \) does not vary over time. (For example, you might find \( b > 0 \) in \( R^e_{t+1} = a + bx_{t+1} + \varepsilon_{t+1} \) with \( \sigma_t^2(\varepsilon_{t+1}) \) constant. However, it will be easier to work the problem in terms of \( E_t(R^e_{t+1}) \) and \( \sigma_t(R^e_{t+1}) = \sigma(R^e_{t+1}) \) rather than the regression parameters.)

Find the maximum Sharpe ratio \( SR \) of the unconditional mean-variance frontier in this situation – the largest value \( E/\sigma \) that the investor can get including all possible dynamic strategies for going in and out depending on \( x_t \). Express your answer in terms of the Sharpe ratio \( SR_F = E(R^e)/\sigma(R^e) \) that the investor receives if he ignores conditioning information and the \( R^2 = \sigma^2(bx_t)/\sigma^2(R^e_{t+1}) \) of the regression. There are two steps:

(a) Show that \( SR^2 = E(SR^2_t) \) where \( SR_t \) is the conditional Sharpe ratio \( SR_t = E_t(R^e_{t+1})/\sigma_t(R^e_{t+1}) \)

(b) Link \( SR \) to \( SR_F \) and \( R^2 = \sigma^2[E_t(R^e_{t+1})]/\sigma^2(R^e_{t+1}) \)

(c) Suppose that this return is on average \( E(R^e) = 0 \), like a lot of long-short or bond strategies. How much predictability do you need \( (R^2) \) before the market-timing strategy gives the same Sharpe ratio as just investing in the equity premium?

Hints: Part a: No surprise, the key is linking Sharpe ratios to the properties of discount factors \( m \). First, link the unconditional Sharpe ratio \( SR \) to the unconditional mean and variance of a discount factor \( m \). Then figure out the unconditional mean and variance of the discount factor from what you know about the conditional mean and variance of the discount factor, given \( E_t(R^e_{t+1}) \) and \( \sigma_t(R^e_{t+1}) \). Part b is just math. The problem does not take a lot of algebra, so if you’re going on and on you’re headed the wrong way. If you can’t get part a, you can do part b using the result in part a. You will use the assumptions of a constant risk free rate and constant conditional variance of the return to pull things out of \( E_t \). Useful formula: \( \sigma^2(x) = E[\sigma^2_t(x)] + \sigma^2[E_t(x)]; \)
More space for problem 1
2. (30) Suppose consumption and dividends follow simple geometric Brownian Motions,

\[
\begin{align*}
\frac{dC}{C} &= \mu_C dt + \sigma_C dz \\
\frac{dD}{D} &= \mu_D dt + \sigma_D dw \\
E(dz^2) &= E(dw^2) = 1; \ E(dzdw) = \rho.
\end{align*}
\]

The investor maximizes

\[
E \int e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma} dt.
\]

(a) Find the price/dividend ratio at time \(t\)

(b) Find the stock’s return

(c) Find the instantaneous interest rate and the stock’s expected excess return

(d) Check that the answer in c changes in the right ways as you change \(\delta, \gamma, \mu_C \) and \(\mu_D\) (I.e. should stocks with higher growth in dividends get higher or lower expected excess returns?)
More space for problem 2
3. (15) If the “Conditional CAPM” holds, a discount factor of the form

\[ m_{t+1} = a_t - b_tR_{t+1}^W. \]

You’re interested in seeing if a conditional CAPM prices a set of excess returns \( R_{t+1}^e \) as well, of course, as pricing \( R^W \) itself. (Excess only – no interest rate). Furthermore, you think that all the conditioning information in the market proxy can be summed up by a linear regression

\[ R_{t+1}^W = \alpha + \delta x_t + \varepsilon_{t+1} \]

where \( x_t \) is the dividend/price ratio. For example,

\[ E_t \left( R_{t+1}^W \right) = \alpha + \delta x_t. \]

(a) What can we say about \( a_t \) and \( b_t \)?

(b) How would you estimate and test this model?

Hints: It’s enough to give the moment conditions you would use, or the factors you would use in running average returns on betas. Note you can do better than just “scaled factors” here, though you can say that for partial credit if you can’t think of something better to do. Obviously, you want to use the structure of the forecasting regression.
More space for problem 3
4. (15) Consider a standard expected return-beta model with a single factor $f$ that is also an excess return $E(R_{ei}) = \beta_i \lambda$, estimated on a set of expected returns $R^e$ such as the Fama French 25.

(a) A researcher reports with puzzlement that pricing errors $\alpha$ are correlated with betas, large beta assets seem to have large pricing errors. I’ve said that most results are not sensitive to estimation technique. Is this one of them, or is this likely to change with different estimation techniques?

(b) Show that if you fit the model by a time-series regression, the implied cross-sectional relation $E(R_{ei}) = \beta_i \lambda$ fits the factor and the riskless rate perfectly ignoring the other assets.

(c) I motivated cross-sectional regressions of average returns on betas

$$E(R_{ei}) = \gamma + \beta_i \lambda + \alpha_i$$

by the idea that they would pick the parameter $\lambda$ to try to fit all assets well. Give two circumstances under which this technique will nonetheless ignore the information in other assets and just fit the cross-section through the mean of the factor.
More space for problem 4
5. (15) A researcher is evaluating the standard power utility consumption-based model. To get going, he looks only at one asset return, the excess return of the market portfolio rmrf. Thus, he looks at one moment condition

\[ g_T(\gamma) = E \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1}^e \right] \]

(\(\beta\) is not identified so we might as well set it to one.) He follows standard procedure of the two-step minimization, i.e.

1st stage: \( \min_{\gamma} g_T(\gamma)'I g_T(\gamma) \)

2nd stage: \( \min_{\gamma} g_T(\gamma)'S^{-1} g_T(\gamma) \)

He reports the following result of the 2nd state estimate:

<table>
<thead>
<tr>
<th>(\gamma) Estimate</th>
<th>112.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard error</td>
<td>5.011e+07</td>
</tr>
<tr>
<td>(J_T) statistic</td>
<td>1.766</td>
</tr>
<tr>
<td>% p-value ((\chi^2_1))</td>
<td>18.39</td>
</tr>
<tr>
<td>Minimized (gt \times 400)</td>
<td>3.163</td>
</tr>
</tbody>
</table>

He says, “The high estimate is what we expect from the equity premium literature. It’s too bad the standard errors are so large, but that’s what happens when you start raising things to the 112 power. Still the JT statistic doesn’t reject at a 20% probability value. Again, though, the economic size of the pricing errors is pretty large at 3.2% annual mean return, about half of the equity premium. Again, standard errors are big I guess.” You disagree. What is in fact the right econometric conclusion to draw from the table? (I’m only looking for one point and a few sentences here, not a dissertation on the consumption model.)
More space for problem 5
6. (10) Suppose we construct a discount factor from a set of returns by $x^* = 1'E(R'R)^{-1}R$ This discount factor can be negative in some states of the world. Does that mean that $R$ has arbitrage opportunities, or that this is not a valid discount factor?
7. (20) You have a $N \times 1$ vector of returns $R_t$ and you know the mean and second moment matrices $E(R)$ and $E( RR')$.

(a) Show how to construct $R^*$ and $R^c*$

(b) Find the mean and second moments of $R^*$, $R^c*$

(c) Find the mean-variance frontier. You can leave the answer in terms of a parameter $w$ that sweeps out the frontier, and in terms of the mean and second moment of frontier returns as a function of $w$. I.e. you can leave the answer as $E(R^{mv}) = \text{function of } (w, E(R), E( RR'))$ and $E(R^{mv2}) = \text{function of } (w, E(R), E( RR'))$.

The ideal answer will be expressed only in terms of $E(R)$, $E( RR')$ and other quantities derived from those. If you can’t do that though, feel free to take other moments of returns $R$ that you need.
More space for problem 7
8. (15)

(a) Why is the 8% postwar average excess return of stocks over bonds difficult to reconcile with the consumption-based asset pricing model? Use some equations from the course to explain the quantitative problem posed by the equity premium.

(b) Why not “solve” the equity premium puzzle by simply using very high risk aversion?
9. (10) The conditional mean of the discount factor is related to the risk free rate. Suppose that a real risk free rate is traded, but that its value varies over time: 3% on some days, 5% on other days, and so on, with equal probability. In this case

(a) The unconditional mean of the discount factor $E(m)$ is not tied down by the risk free rate since the latter is not constant. $E(m)$ is a free parameter, like a zero-beta rate. You treat the risk free rate data like any other risky return.

(b) The unconditional mean of the discount factor $E(m)$ is determined by the average value of the risk free rate $E(R^f_t)$. Show how this works.

(c) The unconditional mean of the discount factor $E(m)$ is determined, but by something else. Show what determines $E(m)$. 


10. (10) A fund manager says that his fund evaluate risks by the Fama and French size and book to market model, i.e.

\[ E(R_{ei}) = \alpha_i + \beta_i E(R_{em}) + s_i \lambda_{size} + h_i \lambda_{hml} \]

where \( s_i \) is the size of the company and \( h_i \) is the company’s book to market ratio. They then pick companies with high alphas. Is this a good strategy? Is this necessarily a bad strategy? What else do you need to know.
11. More space for any problem
More space for any problem