Schmitt Grohe and Uribe

- Background: in BER, Sims, we see in some sense devaluation as a choice – we want some cost of inflation plus some cost of distorting taxes, and the government will choose halfway in between
- In my long-term debt, you match US time series data well only by thinking the government really wants smooth prices. Thus it meets a current deficit largely by increasing long run taxes, and borrowing to cover the cyclical deficit, with little (no in the model) inflation.

1. Try an AR(1) surplus model.

$$\begin{aligned} v_t &= \frac{B_{t-1}}{p_t} = E_t \sum_j \beta^j s_{t+j} \\ s_t &= \rho s_{t-1} + \varepsilon_t \\ v_t &= \sum_j \beta^j \rho^j s_t = \frac{1}{1-\beta\rho} s_t \end{aligned}$$

   Then, if B is constant, we expect

$$\frac{B}{p_t} = \frac{1}{1-\beta\rho} s_t$$

   Or, use B_{t-1} data,

$$B_{t-1}\left(\frac{1}{p_t} - E_{t-1}\frac{1}{p_t}\right) = \frac{1}{1-\beta\rho}\left(s_t - E_{t-1}s_t\right) = \frac{1}{1-\beta\rho}\left(\varepsilon_t\right)$$

2. None of this is *remotely* true (see graphs from section 1 of Long term debt)
3. What's going on? High *current* s must come with lower *future s* in just such a way that v actually declines when s rises. "Just as if" the government is borrowing and plans to pay off the debt.
4. This is exactly what a fiscal theory government does that wants to smooth price fluctuations. Read p. 98 of long term debt
5. My model in long term debt:
   (a) Objective: minimize volatility of inflation.
   (b) AR(1) cyclical surplus unavoidable. Random walk long run surplus (Barro) you can change with taxes

$$\begin{aligned} c_t &= \rho c_{t-1} + \varepsilon_t \\ z_t &= z_{t-1} + \xi_t \\ s_t &= c_t + z_t \end{aligned}$$

   (c) Choose z,B

(d) Result: Can nail inflation (so act "as if" facing a fixed price level)
(e) See Figure 6,7; matches the character of the data
(f) We always had a theorem "you can't tell fiscal from monetary". But is the fiscal story *plausible*? If it had to be "surplus follows a crazy process with long run going the other way from short run", then no. But here I show that the government *chooses* a "crazy process" in a circumstance much like our own

- Needs to be much more sophisticated; a real optimal tax problem; inlcude the distortions of the z taxes along with the advantages of inflation smoothing.
- Let's quantify the costs of distorting taxes vs inflation distortions. Why does the US largely choose smooth inflation and not depreciating the debt? SGU!

Read intro – very clear.

p.201 why does a small bit of stickiness mean a lot of price smoothness? It's a Lucas welfare cost story. Price sticky costs are first order. Reducing risk is second order.

Model

Note: First "sticky price" model. Lots of microfoundations to deliver quite simple aggregates. If firms choose prices and aren't charging the same ones, we need differentiated products/monopolistic competition so that the high price ones don't sell zero. This means we lose the welfare theorems, and the models are much more complex. Hello, NK economics.

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$$E\sum \beta^t U(c_t, h_t)$$

h =labor. Money from a transaction cost,

$$s(v_t);\ v_t = \frac{P_t c_t}{M_t}$$

(I think we could get rid of money completely in this model, at a vast simplification! This model has *two* inflation distortions, one that people have too much transactions costs under inflation, the other the product distortions of price stickiness. One is enough!)

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c is a composite. Each household makes one good, using $z_t\tilde{h}_t$. Demand $Y_t d(p_t)$; $P_t = \tilde{P}_t/P_t$ = local price / price level. Set a price, then satisfy demand $z_t\tilde{h}_t = Y_t d(p_t)$

Adjustment cost

$$\frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2$$

Flow budget constraint (4). r =s.d.f what we call m

203-205 standard first order conditions

205 government with a standard "constraint" Interesting that they take R, τ as policy levers, then let M and B follow.

205. First order conditions lead to NK "Phillips curve" (12) Note π_t on the left, $E_t \pi_{t+1}$ on the rhs, and something like output. All the monopolistic firm etc. stuff is to get here.

206 Definition of equilibrium, as usual.

206. Ramsey problem: maximize utility by choice of tax and interest rate subject to market clearing conditions as a constraint on government policy.

210 calibration Is price stickiness small? 9 months average between price changes! How big are labor distortions??? Debt/GDP of 0.44 means a small price change can give a pretty large effect.

212 no analytical solution; numerical approximation around steady state.

212 results;

1) Flex prices, perfect competition (but still transaction cost / labor income tax distortion. Note though that transactions costs depend on expected inflation so they won't play a part here)

$R = 0$. mean π negative (Friedman rule) but large $\sigma(\pi)$. π is nearly iid; τ is very autocorrelated and small variance. This is the "Sims" solution. (Why aren't taxes constant then?)

2) Imperfect competition and flex prices. Output is lower because of monopoly. Inflation level is higher as a partial tax on monopoly (inducement to produce more). Other features are the same

3) price stickiness. Now inflation volatility drops like a stone. Taxes std dev goes up dramatically – using state-contingent taxation rather than ex-post devaluation. Taxes still highly autocorrelated.

Figure 1 only a "little" price stickiness will do. How much is much??

JC: Suppose we could introduce "government equity" that was *not* tied to the numeraire of every private contract. For example, suppose we pass a law that prices are posted in dollars, then paid in domestic currency. (I.e. eliminate price change costs). Now the government might choose much more volatile processes. *We might expect much greater volatility of "government equity" value when it is invented.*

JC: Too complex a model! Having resorted to numerical solutions means we miss all the intuition.

New Keynesian and Taylor rules

Previews

- King:
  1. 'Theory', presentation of the model.
  2. Interest rate rule and determinacy, $R = a + \phi\pi$, $\phi > 1$ for "determinacy"
- Clarida gali and Gertler
  1. Evidence that $\phi < 1$ for the 70s, $\phi > 1$ for the 80s. Thus, claim that this change in the rule was the critical act that stopped US inflation.
- Athanasoulis:
  1. One empirical doubt about CGG. $R = a + b(y - \bar{y}) + \phi\pi$. y and π are correlated. If the Fed thinks $\bar{y}$ is higher than it is during a time of high inflation, it may leave policy "loose" despite ϕ high. Documents that this did in fact happen in the 70s. Maybe there was no change in ϕ after all.
- JC: Many other doubts on this "standard story"

King, The New ISLM Model, Language, Logic and Limits

- Background; why do we care.
  1. For us, interested in the price level: This is the third kind of model that can (supposedly) determine the price level in an economy with fiat money.
     (a) Claim: (Ricardian regime) if the fed pegs interest rates, but raises rates more than 1-1 with inflation, this will result in a determinate price level $r_t = (\alpha_t + \beta_t y_t) + \phi\pi_t + \varepsilon_t$, with $\phi > 1$.
     (b) Old intuition: π rises $\rightarrow$ r rises more $\rightarrow$ rr rises $\rightarrow$ "demand" is less $\rightarrow$ π goes down again (phillips). Eliminates the old problem of interest rate targets that a rise in p gives rise to more m demand which is automatically accommodated.
     (c) This is not the way the models work, as we will see.
     (d) If so, note it applies to a fiat money economy with an interest rate target. Since no (little) mention of money demand, it apparently applies with unstable or vanishing money demand – as long as the central bank can control one nominal interest rate.
     (e) Thus, the only competitor to fiscal theory for describing current institutions.
  2. More generally
     (a) This is *the* kind of model used by academics and policy people who want to think about monetary policy these days. Period. You may disagree (I do) but you have to learn French to convert the French.
     (b) For example, it gives a model in which the Fed has to look at the economy and carefully reset interest rates each period. As they do.
  3. Other issues, "New Keynesian Doctrines"
     (a) (45) "Inflation targeting." Here, a description of optimal monetary policy.
        i. Low inflation is good.
        ii. When there are "inflation" (supply?) shocks, some inflation is ok.
        iii. "Inflation should *not* respond to many economic disturbances" (45)
     (b) (In other contexts, "inflation targeting is about a political contract between government and central bank to allow the central bank to avoid fighting recessions and funding government deficits.)
     (c) To produce this optimal inflation, $r_t = \phi\pi_t$ with $\phi > 1$ is not enough. The Fed must spy and accommodate changes in the "natural" real rate of interest. $r_t = rr_t + \phi\pi_t$ justifies $r_t = (\alpha_t + \beta_t y_t) + \phi\pi_t + \varepsilon_t$.
     (d) (46) $\phi > 1$ for determinacy. – The *rule* is as (more) important than the *shocks*.
     (e) (46) Fit the data? Money shocks can lead to serially correlated effects. Prices are sticky for a long time, where expectations are only sticky for one period (Lucas, $y_t = \phi(\pi_t - E_{t-1}\pi_t)$)

(f) (Not mentioned in the intro) The importance of "Policy credibility" "commitment" "shaping expectations." etc.; The importance of *rules* in policy. (Woodford)

(g) *All of these conclusions* (esp. $\phi > 1$) *are model-dependent.*

4. "New" – exquisite "rational microfoundations". No "behavioral relations"; consumption functions or empirical policy-invariant Phillips curve. (Really? - JC)

- Model (50, 53)

$$\begin{aligned} y_t &= E_t y_{t+1} - s\left[r_t - r\right] + x_{dt} \;\; \text{IS} \\ \pi_t &= \beta E_t \pi_{t+1} + \phi\left[y_t - \bar{y}_t\right] + x_{\pi t} \; \text{Phillips} \end{aligned}$$

$$R_t = r_t + E_t \pi_{t+1} = f(\cdot) \text{ Policy rule}$$

Point: Solve for $\{y, \pi\}$ as a function of shocks. Find the effects of different policy rules.

1. Most treatments take a long time to "derive" 1 and 2.
2. IS.

(a) Old: "higher real rates suppress demand" New: $E_t y_{t+1}$. Thus, lower demand *relative to the future* which will really change the dynamics! (Introduces a growth rate where before there was a level.)

(b) Derivation of IS (72) basically just loglinearize the consumption first order condition and then substitute $c = y$. Note $c_t = E_t c_{t+1}$ the first term, and higher interest rates lead to higher consumption growth in the second term.

$$\begin{aligned} 1 &= E\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] R^f = E\left[e^{-\delta} e^{-\gamma \Delta c_{t+1}}\right] e^{r_t} = e^{-\delta - \gamma E(\Delta c_t) + \frac{1}{2}\gamma^2\sigma^2 + r_t} \\ 0 &= -\delta - \gamma E\left(\Delta c_t\right) + \frac{1}{2}\gamma^2\sigma^2 + r_t \\ 0 &= -\frac{\delta}{\gamma} - \left(E_t c_{t+1} - c_t\right) + \frac{1}{2}\gamma\sigma^2 + \frac{1}{\gamma} r_t \\ c_t &= \left(\frac{\delta}{\gamma} - \frac{1}{2}\gamma\sigma^2\right) + E_t c_{t+1} - \frac{1}{\gamma} r_t \end{aligned}$$

(Note $c = y$ is probably a terrible idea in models that think a lot about investment!)

3. Phillips:

$$\pi_t = \beta E_t \pi_{t+1} + \phi\left[y_t - \bar{y}_t\right] + x_{\pi t}$$

(a) Old was $\pi_t = \phi\left[y_t - \bar{y}_t\right]$. Friedman/Lucas $\pi_t = E_{t-1}\pi_t + \phi\left[y_t - \bar{y}_t\right]$. Now *future* inflation. Again, this can (and does) dramatically change the dynamics; "stable" roots become "unstable"

(b) Derivation of Phillips: Sticky prices, forward looking firms.

   i. With price-setting, you need monopolistic competition – downward sloping demand curves for differentiated goods.

   ii. To get to Phillips, "a herioc assumption" p. 62. The models give price in terms of marginal cost. To get to Phillips, we need marginal cost rising in output.

(c) Current problems with the Phillips curve. Mankiw: look, a boom when inflation is *declining!* This is the wrong sign!

   i. Mankiw fix: Go back to adaptive expectations $\pi_t = \pi_{t-1} + \phi [y_t - \bar{y}_t] + x_{\pi t}$

   ii. Christiano fix: No, output is negative with marginal cost, so the right equation is $\pi_t = \beta E_t \pi_{t+1} - \phi [y_t - \bar{y}_t] + x_{\pi t}$

   iii. JC: Whoa, does changing the sign screw everything up?

(d) Expectations preserve some properties of Lucas. For example, with

$$\pi_t = \beta E_t \pi_{t+1} + \phi [y_t - \bar{y}_t] + x_{\pi t}$$

A sudden, permanent unexpected change in inflation has no output effects. (70) (JC: what about the firms who got stuck with old prices?)

4. Is this it? King – results here only useful to understand "real models."

   (a) Most people CGG skim the "microfoundations" and stop with this model though.

   (b) Are "real models" (complicated) robust?

- "Neutral' policy (55)

1. Without utility functions to do proper welfare, we can ask what it takes to set $y = \bar{y}$. From Phillips,

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \phi [y_t - \bar{y}_t] + x_{\pi t} \\ \pi_t &= \beta E_t \pi_{t+1} + x_{\pi t} \\ \bar{\pi}_t &= E_t \sum_{j=1}^{\infty} \beta^j x_{\pi t} \end{aligned}$$

2. First King conclusion: with $x_\pi = 0$, $\pi = 0$ is optimal.
3. Second King conclusion: Other shocks don't enter optimal inflation
4. Third king conclusion: x_π can be serially correlated; so can optimal inflation.
5. JC: What's an "inflation shock?"
6. 56. These models as parables for "fully articulated." But do we believe those? Are the results there special to assumed frictions?

7. Example: in "real" models, monopolistic competition leads to another set of welfare headaches – can the government trick firms into producing more? If not on average, how about in response to shocks? (56)

- Interest rate rules, 57, 58-59 and 74 ff. This is the most important part for us.

1. Interest rates to support "neutral" policy. (57)

$$\begin{aligned}
\bar{y}_t &= E_t\bar{y}_{t+1} - s\left[r_t - r\right] + x_{dt} \\
r_t &= r + \frac{1}{s}\left[E_t\bar{y}_{t+1} - \bar{y}_t\right] + \frac{1}{s}x_{dt} \\
R_t &= r_t + E_t\pi_{t+1} = r + \frac{1}{s}\left[E_t\bar{y}_{t+1} - \bar{y}_t\right] + \frac{1}{s}x_{dt} + E_t\pi_{t+1} = \\
R_t &= r + \frac{1}{s}\left[E_t\bar{y}_{t+1} - \bar{y}_t\right] + \frac{1}{s}x_{dt} + E_t\sum_{j=1}^{\infty}\beta^j x_{\pi t+1} \equiv \bar{R}_t
\end{aligned}$$

Note: r to set y to $\bar{y}$, then $E_t\bar{\pi}_{t+1}$ to satisfy Phillips curve.

2. The equation doesn't matter. What matters is $\bar{R}_t$ depends on external shocks to the economy in a precise way. The Fed does have to set interest rates actively, as it does now.
3. Claim (58). This is not enough. $y_t = \bar{y}_t$ $\pi_t = \bar{\pi}_t$ is one equilibrium, but there are others. (Indeterminacy with interest rate targets)
4. Claim (58). You can fix this with

$$R_t = \bar{R}_t + \tau(\pi_t - \bar{\pi}_t)$$

What does this mean? *In* equilibrium, $\pi = \bar{\pi}$, so no change. *Out* of equilibrium, the Fed threatens to do something else. Thus, "Threats of out of equilibrium behavior that coordinate expectations on the desired equilibrium." See (59), top.

5. Preview to JC criticism of CGG: *If this works, you never see $\pi \neq \bar{\pi}$. Therefore you cannot measure τ from time series data.*
6. Simple model to show the multiple equilibrium problem (76)

$$\text{Fed: } R_t = r + \tau\pi_t + x_{Rt}$$

x_R = monetary policy shocks.

$$r + E_t\pi_{t+1} = r + \tau\pi_t + x_{Rt}$$

$$E_t\pi_{t+1} = \tau\pi_t + x_{Rt}$$

(In more general models we can solve out for the other variables to get a inflation equation. It has two roots, and the key will be that one is explosive)

(a) $\tau > 1$ (Taylor case) "Solve unstable roots forward"

$$\pi_t = \frac{1}{\tau}E_t\pi_{t+1} - \frac{1}{\tau}x_{Rt}$$

$$\pi_t = -E_t\sum_{j=0}^{\infty}\frac{1}{\tau^{j+1}}x_{Rt+j}$$

"unique stable rational expectations solution"

(b) If $x_R = 0$, then $\pi = 0$, stable! Optimal! Determinate! FTPL competitor for frictionless economy!

(c) (Note footnote 43 – this is weird. If x_R is iid, then $\pi_t = -\frac{1}{\tau}x_{Rt}$. A positive shock, but inflation exactly moves to offset it so R_t is unchanged. If you get weird stuff maybe something is really wrong!)

(d) $0 < \tau < 1$. "multiple stable rational expectations solutions." (77)

$$\pi_{t+1} = \tau\pi_t + x_{Rt} + \xi_{t+1};\ E_t\xi_{t+1} = 0$$

ξ are "sunspots" or "animal spirits" or "self-confirming expectations" "multiple equilibria"

(e) Stability: to make it even simpler ignore the x_R terms and uncertainty.

$$\pi_{t+1} = \tau\pi_t$$

Then the general solution is

$$\pi_t = \tau^t\pi_0.$$

If $\tau < 1$ then all π_0 lead to "stable" (really "bounded") sequences. Again, an interest rate target cannot determine the price level. If $\tau > 1$ then any π_0 but $\pi_0 = 0$ leads to "unbounded" "unstable" sequence. Thus $\pi_0 = 0$ is the "unique stable equilibrium."

(f) 77. Note $\tau < -1$ would work too. (This should really make you wonder!)

7. JC comments

(a) Nothing in *economics* rules out explosive hyperinflation. (Ask someone from Argentina or Brazil). The transversality condition says that *real* quantities can't explode, but nothing says that *nominal* quantities can't explode.

(b) Thus the choice $\pi_0 = 0$ or the restriction to bounded solutions, the unique stable forward solution must come from outside economics. McCallum: "Occam's razor" "minimum state variable criterion."

(c) JC: This is looney. Inflation can and does explode. What will happen if the Fed commits to increase inflation by 2 % tomorrow for every 1% increase in inflation today? *Hyperinflation, Duh.* King says this commitment will never be tested and leads to zero inflation.

(d) JC conclusion the solutions to this model

$$E_t\pi_{t+1} = \tau\pi_t + x_{R,t}$$

are

$$\pi_{t+1} = \tau\pi_t + x_{Rt} + \xi_{t+1};\ E_t\xi_{t+1} = 0$$

Inflation is *not* pinned down, and sunspots exist, *for every* value of τ

(e) NK: "you have to choose $\tau > 1$ regime or the price level is indeterminate" JC: "It's indeterminate even if you do choose $\tau > 1$ " NK: What determines the price level, then? JC: You threw out an equation. These are all *Ricardian* regimes (explicitly in Woodford, next week). FTPL still can determine price level. Making the regime Ricardian is the step at which price level determinacy was lost.

(f) Note *timing* will make huge difference to this analysis. What if the Fed responds to expected future inflation in this little model?

$$\text{Fed: } R_t = r + \tau E_t\pi_{t+1} + x_{Rt}$$

$$r + E_t\pi_{t+1} = R_t = r + \tau E_t\pi_{t+1} + x_{Rt}$$

$$E_t\pi_{t+1} = \frac{1}{1-\tau}x_{Rt}$$

Solutions

$$\pi_{t+1} = \frac{1}{1-\tau}x_{Rt} + \xi_{t+1}$$

for any value of τ. Determinacy got lost! Thus, little issues like where π goes in the Phillips curve make a huge difference.

(g) Note it's the pervasive *expected future* stuff in the NK model that drives the result. The dynamics here are

$$E_t\pi_{t+1} = \tau\pi_t$$

If the model was of the form

$$\pi_{t+1} = \tau\pi_t + \varepsilon_{t+1}$$

i.e. if ε_{t+1} *are shocks, not forecast errors*, then we can solve it to

$$\pi_t = \sum_{j=0}^{\infty} \tau^j \varepsilon_{t-j}$$

Here we want $\tau < 1$–sensibly the economy becomes unstable if the fed *does* commit to hyperinflation.

i. "Old keynesian" models without expectations do not generate any of this stuff. But they aren't economic models in a meaningful sense.

8. Ok for a model with constant real rates. Does the NK model work the same way?
   (a) Yes, Figure 3
   (b) King considers (75)

$$R_t = \bar{R}_t + \tau_1 \left(E_t \pi_{t+1} - E_t \bar{\pi}_{t+1} \right) + \tau_0 \left(\pi_t - \bar{\pi}_t \right)$$

   This is a clever trick that makes the algebra much easier.
   (c) To get to Figure 3. Plug this in to the model and solve.

$$\begin{aligned} y_t &= E_t y_{t+1} - s\left[r_t - r \right] + x_{dt} \text{ IS} \\ \pi_t &= \beta E_t \pi_{t+1} + \phi \left[y_t - \bar{y}_t \right] + x_{\pi t} \text{ Phillips} \end{aligned}$$

   The bar equilibrium is where $y = \bar{y}$

$$\begin{aligned} \bar{y}_t &= E_t \bar{y}_{t+1} - s\left[\bar{r}_t - r \right] + x_{dt} \text{ IS} \\ \bar{\pi}_t &= \beta E_t \bar{\pi}_{t+1} + x_{\pi t} \text{ Phillips} \end{aligned}$$

   Subtract the bar equilibrium from the model,

$$\begin{aligned} y_t - \bar{y}_t &= E_t y_{t+1} - E_t \bar{y}_{t+1} - s\left[r_t - \bar{r}_t \right] \\ \pi_t - \bar{\pi}_t &= \beta E_t \pi_{t+1} - \beta E_t \bar{\pi}_{t+1} + \phi \left[y_t - \bar{y}_t \right] \end{aligned}$$

   Thus, we can write the model as

$$\begin{aligned} \tilde{y}_t &= E_t \tilde{y}_{t+1} - s \tilde{r}_t \\ \tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \phi \tilde{y}_t \end{aligned}$$

   with $\tilde{y} = y - \bar{y}$ etc.
   Now, the policy rule

$$R_t = \bar{R}_t + \tau_1 \left(E_t \pi_{t+1} - E_t \bar{\pi}_{t+1} \right) + \tau_0 \left(\pi_t - \bar{\pi}_t \right)$$

$$\tilde{R}_t = \tau_1 E_t \tilde{\pi}_{t+1} + \tau_0 \tilde{\pi}_t + x_{Rt}$$

$$\tilde{r}_t = (\tau_1 - 1) E_t \tilde{\pi}_{t+1} + \tau_0 \tilde{\pi}_t$$

   Plugging in, we have

$$\begin{aligned} \tilde{y}_t &= E_t \tilde{y}_{t+1} - s\left[(\tau_1 - 1) E_t \tilde{\pi}_{t+1} + \tau_0 \tilde{\pi}_t \right] \\ \tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \phi \tilde{y}_t \end{aligned}$$

   Now solve.
      i. Matrix form

$$\begin{aligned} E_t \tilde{y}_{t+1} &= s(\tau_1 - 1) E_t \tilde{\pi}_{t+1} + \tilde{y}_t + s\tau_0 \tilde{\pi}_t \\ \beta E_t \tilde{\pi}_{t+1} &= \tilde{\pi}_t - \phi \tilde{y}_t \end{aligned}$$

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \tilde{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 - \frac{s\phi(\tau_1 - 1)}{\beta} & \frac{s(\tau_1 - 1)}{\beta} + s\tau_0 \\ -\frac{\phi}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \end{bmatrix}$$

      Find eigenvalues. We need both > 1 to determine $\tilde{y}_t, \tilde{\pi}_t$. (This is a vector version of the model we solved above.)

ii. Get rid of y, solve π equation. Solve Phillips for y,

$$\begin{aligned}\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \phi \tilde{y}_t \\ \tilde{y}_t &= \frac{1}{\phi}\tilde{\pi}_t - \frac{\beta}{\phi} E_t \tilde{\pi}_{t+1}\end{aligned}$$

Plug in IS (with rule)

$$\frac{1}{\phi}\tilde{\pi}_t - \frac{\beta}{\phi}E_t\tilde{\pi}_{t+1} = E_t\left(\frac{1}{\phi}\tilde{\pi}_{t+1} - \frac{\beta}{\phi}\tilde{\pi}_{t+2}\right) - s\left[(\tau_1 - 1)E_t\tilde{\pi}_{t+1} + \tau_0\tilde{\pi}_t\right]$$

A second order difference equation in π_t Check that both roots are >1.

(d) Figure 3. Yes, the same intuition goes through. Much fun though, other parameter restrictions (beyond $\tau > 1$), many special cases depending on details of the timing.

9. Even if it did work, you can never measure $\tau > 1$. (I think!)

   (a) Simple model

$$E_t \pi_{t+1} = \tau \pi_t + x_{Rt}$$

For example,

$$\begin{aligned}E_t\pi_{t+1} &= 1.1\pi_t + x_{Rt} \\ x_{Rt+1} &= 0.9 x_{Rt} + \varepsilon_{t+1}.\end{aligned}$$

Following the Taylor-rule philosophy, we solve this model by solving the unstable root forward

$$\pi_t = -E_t \sum_{j=0}^{\infty} \frac{1}{1.1^{j+1}} x_{R,t+j} = -\sum_{j=0}^{\infty} \frac{0.9^j}{1.1^{j+1}} x_{R,t} = -\frac{1}{1.1}\frac{1}{1-\frac{0.9}{1.1}} x_{R,t} = -\frac{1}{0.2} x_{R,t} = -5x_{R,t}$$

then, the *reduced form* or *solution* is

$$\pi_{t+1} = 0.9\pi_t - 5\varepsilon_{t+1}.$$

A regression in artificial data, of course, recovers the reduced form, (**??**). Thus, we *measure* the stable root, not the "structural" unstable root.

   (b) Does the point extend to (iv) regressions of τ in

$$R_t = a + \tau E_t \pi_{t+1} + \varepsilon_t$$

as in CGG? I think so, but not proved yet.

- JC bottom line:

1. Nice try, but in fact this "unique bounded solution stuff" means that the price level is *not* determined by a Taylor rule in a NK model. All NK models are indeterminate.

2. How did anyone ever get themselves so lost in equations that they started to believe this: If the Fed credibly commits to hyperinflate if inflation misses its target by one percentage point, the result is *stability*, not *hyperinflation*
3. Price level can be determined by a Taylor rule in an "old keynesian model" – but there are no economics in that, so you might as well say it's determined by assumption. ("institutions") (Problem set)
4. The FTPL is the only coherent theory that determines the price level with fiat money and an interest rate rule.
5. And even if the Taylor stuff works in theory, you cannot measure $\tau > 1$ in data. CGG regressions are internally inconsistent.
6. Note: These are guesses for my *next* two papers. None of this is proved yet.

- Basic idea: if r in creases more than 1-1 with π, price level might be stable under an interest rate target.
- Explanation: 70s r increased less than 1-1; 80's to now it increases more than 1-1 – victory over inflation.
- Note about *systematic* part of policy not shocks.
- The Fed responds to output as well as inflation and does try to stabilize output.

$$r = \beta\pi + \gamma y_{gap}$$

Thus, regressions of this in pre- and post 80

- Bottom line: TABLE II
- p. 150 This paper: a bit more sophisticated. Fed reacts to *future* output and inflation

$$r_t^* = r^* + \beta\left(E\left\{\pi_{t,k}|\Omega_t\right\} - \pi^*\right) + \gamma E\left\{x_{t,q}|\Omega_t\right\} \quad (10)$$

$x_{t,q}$ = percent deviation between GDP and corresponding target.

- Important implication – why does the Fed react to so many variables (including interest rates)? – Maybe because those variables forecast the thing the Fed really cares about.
- 153 Fed smooths. (Note Woodford has paper where smoothing is optimal.) Here ad hoc.

$$\begin{aligned} r_t &= \rho(L)r_{t-1} + (1-\rho)r_t^* \qquad (3) \\ \rho &= \rho(1) \end{aligned} \quad (11)$$

- (11) in (10) gives[1]

$$r_t = (1-\rho)\left[r^* - \beta\pi^* + \beta E_t\left(\pi_{t,k}\right) + \gamma E_t\left(x_{t,q}\right)\right] + \rho(L)r_{t-1}$$

- What to do about $E_t\pi_{t+k}$? Forecast them using some variables z_t. First stage,

$$\pi_{t,k} = a + bz_t + \varepsilon_{t+k}$$

then use $a + bz_t$ for $\beta E_t\left(\pi_{t,k}\right)$.

[1]

$$\begin{aligned} r_t &= \rho(L)r_{t-1} + (1-\rho)\left[r^* + \beta\left(E\left\{\pi_{t,k}|\Omega_t\right\} - \pi^*\right) + \gamma E\left\{x_{t,q}|\Omega_t\right\}\right] \\ r_t &= (1-\rho)\left[r^* - \pi^* + (1-\beta)\pi^* + \beta E\left\{\pi_{t,k}|\Omega_t\right\} + \gamma E\left\{x_{t,q}|\Omega_t\right\}\right] + \rho(L)r_{t-1} \\ r_t &= (1-\rho)\left[r^* - \pi^* + (1-\beta)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q}\right] + \rho(L)r_{t-1} + (1-\rho)\left[\beta\left(\pi_{t,k} - E\left\{\pi_{t,k}|\Omega_t\right\}\right) + \gamma\left(x_{t,q} - E\left\{x_{t,q}|\Omega_t\right\}\right)\right] \\ r_t &= (1-\rho)\left[r^* - \pi^* + (1-\beta)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q}\right] + \rho(L)r_{t-1} + \varepsilon_t \end{aligned}$$

- This is IV! Equivalent: Estimate

$$r_t = (1-\rho)\left[r^* - \beta\pi^* + \beta E_t \pi_{t,k} + \gamma E_t x_{t,q}\right] + \rho(L) r_{t-1} + \varepsilon_{t,k}$$

  by IV.

- *IF* there are no policy shocks, $\varepsilon_{t,k}$ is a forecast error, orthogonal to time t information. Note: They are assuming *no* policy shocks, and especially not serially correlated shocks. This turns out to be crucial.

- If your z is a subset of the Fed's z, no problem – it just introduces a larger forecast error uncorrelated with the right hand variables[2]

- (Digression on OLS/GMM

$$E\left[(y_t - bx_t)x_t\right] = 0$$

  Implies $b = E\left(xx'\right)^{-1} E(xy)$. Thus, you use this moment condition to do OLS by GMM and get distribution theory that corrects for everything. Similarly the moment

$$E\left[(y_t - bx_t)z_t\right] = 0$$

  implies IV so you can map IV to GMM and get a good distribution theory)

- "overidendification tests" check if you get the same number using different zs. (Power? Weak instruments will not reject.)

- 155 Data:

  1. Avg federal funds – not target; GDP deflator. Also CPI; CBO output gap; Instuments lags of ff, π, y, comodity prices, M2, spread. (JC why not level of 30 year yield to forecast inflation?)
  2. 156. Horizon one quarter
  3. 157. 4 lags of instruments!
  4. Should show first stage estimate, and forecasts. Are they reasonable? Is everything from a few data points? Episodes? Is there marginal information in the 4th lag of M2?

- TII estimates. **Big point of the paper** is β changing from < 1 to > 1.

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[2]

$$\begin{aligned} r_t &= (1-\rho)\left[r^* - \pi^* + (1-\beta)\pi^* + \beta E\left(\pi_{t,k}|z_t\right) + \gamma E\left(x_{t,q}|z_t\right)\right] + \rho(L) r_{t-1} \\ &\quad + (1-\rho)\left[\beta\left(E\left(\pi_{t,k}|z_t\right) - E\left\{\pi_{t,k}|\Omega_t\right\}\right) + \gamma\left(E\left(x_{t,q}|z_t\right) - E\left\{x_{t,q}|\Omega_t\right\}\right)\right] \end{aligned}$$

if $z \subset \Omega$, the right hand variable is uncorrelated with the error.

- Note: including output is *crucial* to this result.

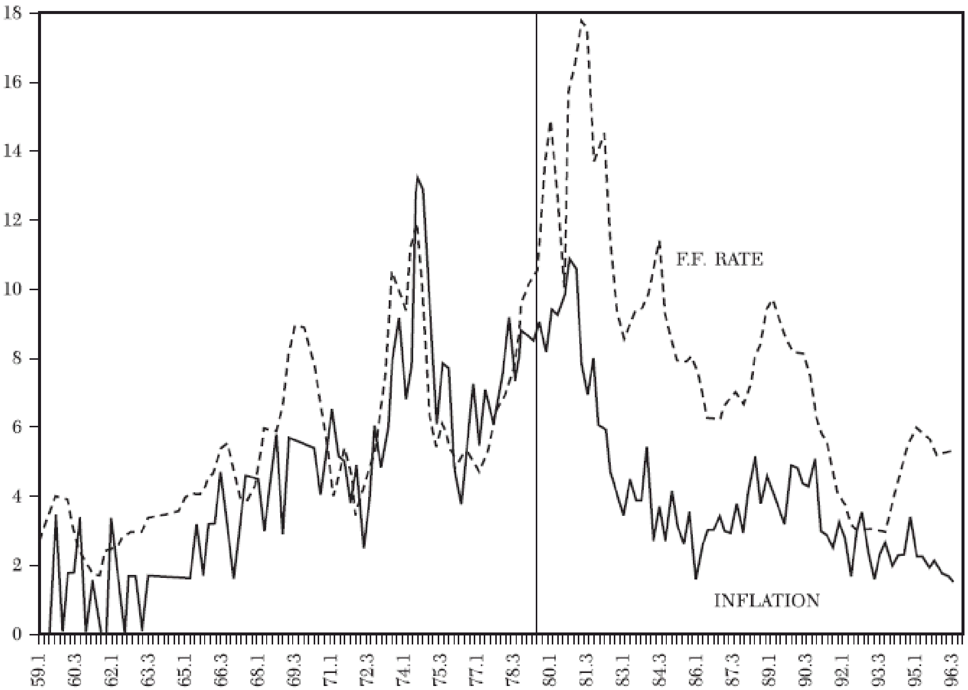

Figure 4. The Federal Funds Rate and the Inflation Rate

Where is the evidence for $\beta = 2$ post but not pre? If you run $r = \beta\pi$ you get $\beta \approx 1$ because of Fisher (real rates vary much less than inflation). $\beta = 2$ as found here is impossible – we don't have 12% interest rates at 6% inflation. Thus, the output gap and correlation of output and inflation are vital to get β far from one. At 6% inflation you have 6% rates, but if output is low you can say "The fed *would have raised* rates to 12% if this recession weren't going on."

- Why should γ rise? Thought pre was more concerned with stabilization? Different γ make a difference on β!

- FI Don't be too impressed. Lagged r is a predictor, AR(1)s look pretty good.

- TIII TIV robust. This is important. Always put the regression equation in the table caption

- TV subsamples. Conclusion 164 of pure inflation targeting. But it's weird that you add two subsamples $\gamma = 1$ and get 0.

- T VI 164 "backward looking" works too – the simple regression. But now what are instruments? Should be a pure regression.

- 165.top. "volatile?" what models???

- Is the Fed really so smart? *Would the Fed really raise interest rates to 20% if we replayed the 73 oil shock? Or would all this "inflation targeting" business go right out the window? Was the Fed tough or lucky in the post period.* Note its' more about the level Would the Fed really leave interest rates that high for a decade?

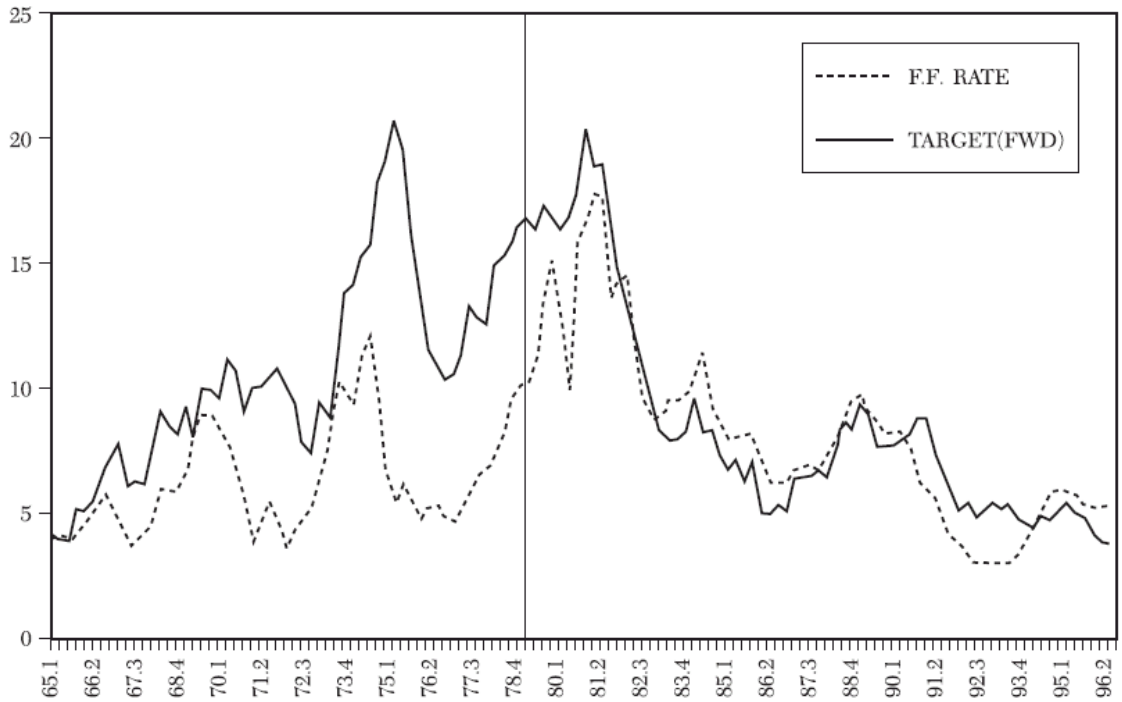

Figure 5. Target Based On Estimated Post-October '79 Rule vs. Actual Funds Rate

Orphanides A view From the Trenches

153, Figure 1. Real interest rates are high now.

CGG conclusion that Fed may have thought $\bar{y}$ was higher than it was is inconsistent. We're modeling the Fed's behavior, so we should use their $\bar{y}$

156 Estimate just like CGG

Fig 2 and 3. Note how deep the output gap seemed in 1974. .

Table 1 161. Big point. Use the right gap, you get the same β in both periods.

163 Basic story: there was a productivity slowdown in 1973, and nobody caught on until much later. they were using 60s trends to calibrate 70s "potential"

164. what they thought at the time vs. current "detrended" (using ex-post information) output.