

Cochrane Money as Stock

- Motivation.
 1. Financial innovation. What's the price level as $M^d \rightarrow 0$? Do we need $M^s \rightarrow 0$ too, so demand for the last dollar = supply for it determine the price level? Intuition: as M^d decreases, no reason the price level should explode.
 2. Fact: we see passive money, interest rate targets that should lead to unstable prices. What theory can account for this?
 3. Is there a *completely frictionless* benchmark for the price level in a fiat-money economy like ours?
- Analogy: What if microsoft stock were numeraire, medium of exchange, etc..Then the price level is determined by

$$\frac{\text{number of shares}}{\text{price level}} = \text{EPV}(\text{future dividends or earnings})$$

This is a perfectly coherent frictionless economy

- Insight: Nominal debt is the residual claim to government surpluses.

$$\frac{\text{nominal government debt}}{\text{price level}} = \text{EPV}(\text{future primary surpluses})$$

This works *exactly* the same way.

Notation: B_{t-1} show one day operation.

There are two equations for any model

$$M_t v(r) = p_t y \tag{2}$$

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \tilde{s}_{t+j} \tag{3}$$

$$\tilde{s} = s_t + \frac{M_t - M_{t-1}}{p_t}$$

Interpretation of the last one

Alternate form

$$\frac{B_{t-1} + M_{t-1}}{p_t} = \sum E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{r_{t+j}^f}{1 + r_{t+j}^f} \frac{M_{t+j}}{p_{t+j}} \right) \right] \tag{4}$$

2 equations, 1 unknown -> Regimes, games of chicken.

Definition of equilibrium: Govt picks $\{M, B, s\}$ s.t.... among other things, *both* (2) and (3). If not, no equilibrium, period. “Games” peer into this, but nothing hinges on game theory or stories.

1. *Monetarist*.

2 determines p , then treasury is “passive” adapting 3. But if the Fed deflates, will the treasury agree to finance a gift to bond holders?

2. *Fiscal* – 3 determines the price level, then 2 determines money, “passively” (“accommodate needs of trade”)

Note nobody feels passive.– adapting s or M is a big job!

Note Laffer – at some point the regime *must* be fiscal-dominant as there are only so many taxes that can be raised.

Note *observational equivalence theorem*. Very deep and misunderstood (Cumby Canzoneri Diba) – both equations hold in both regimes, so you can’t test them with any time series (Granger causality etc.) .

Analogy to supply and demand observational equivalence. Analogy to money supply and demand observational equivalence. Is the Fed moving a vertical curve out, or is the supply curve horizontal?

We can *ask* Fed what they do, and they say Passive!

3. *Cashless* – money as stock paper.

One day version of model, including intraday cash.

Off equilibrium, how does it feel? “Aggregate demand” (east coast) “too much money chasing goods” (Chicago). Yes, but relative to tax payments, not transaction demand.

Cashless version

“What does a bond mean?” a claim to a dollar makes sense even if in equilibrium nobody holds dollars.

Private moneys, circulating claims to maturing bonds make no difference.

Differences with $MV = PY$: All government debt on left, not just m ; no private money on the left; s on the right not transactions demand.

Microsoft story. It acts like the equity valuation formula.

Tax story.

“Ricardian regime” what if both equations are passive? Now no equilibrium.

Paper: outline CIA model, a multiple period version of what is on board. CIA: closed security markets. Reopen at end of day.

Look what we've done. 1) No frictions. 2) Fiat, looks like US (not commodity or gold standard, etc.). 3) No limits on private money, financial innovation. 4) no limits on money supply. 5) Composition of govt debt irrelevant. (OM ineffective) 6) No wise fed needed to keep things steady. 7) Inflation feels normal.

Obviously it's an extreme simplification. But start frictionless, as we add liquidity and default spreads in the term structure. Let the dog wag the tail.

Budget constraint issue.

wait....this is the govt budget constraint.

Equilibrium condition... $p(\text{cheeze whiz}) q(\text{cheeze whiz}) + p(\text{ferrari}) q(\text{ferrari}) \dots = Y$

Definition of equilibrium.....includes this equation. Can't I just solve that equation for $p(\text{ferrari})!$

In model, shouldn't it be "choose $\{B, s, M\}$ given $\{p_t\}$ subject to (*) just as it is for you and me?"

Must the follow a "Ricardian regime?"

Is the Government "special?" Is it an agent that can "threaten to violate its budget constraint at off equilibrium prices?"

No!

1. Share split/Currency reform. If equity is issued "subject to" prices, you'd issue a lot! B (number of shares) is a *definition of securities* (bundle of claims)

2. Respond to off equilibrium prices? Microsoft does not *have* to raise earnings in response to an "off equilibrium" bubble. The government is the same. It can honor the definition of securities –take dollars for taxes, sell bonds for dollars, redeem bonds for dollars – at any price level.

How about gold, indexed debt, foreign currency? Now the equation really *is* a constraint (or default). You cannot split/reform, you must respond to prices.

Nominal government debt is equity. real debt is debt.

More on Budget constraint debt explosions war and Maastricht:

Note *Raising s in response to lagged B/p* will result in a Ricardian regime – this forces the transversality condition to hold.

What about wars? In wars B increases a lot without (as much) inflation. After the war, we raise s to pay down the debt. Doesn't that mean we're Ricardian? What about the Maastricht treaty forcing such response?

Think about a bond sale.

$$\frac{B_{t-1}}{p_t} = s_t \rightarrow p_t = \frac{B_{t-1}}{s_t}$$

$$\text{real revenue raised} = \frac{1}{p_{t-1}} Q_{t-1} B_{t-1} = \frac{1}{p_{t-1}} \frac{1}{R} \frac{p_{t-1}}{p_t} B_{t-1}$$

If there is no change in s_t , then real revenue raised by a bond sale = s_t/R no matter what B / To raise revenue by extra bond sales, you must couple B with credible promise of more s .

Note very different marketing of split/ipo; currency reform/debt issue. We carefully market them differently to signal different “promises” of future s .

War *in fiscal regime*: Big debt issue, *with a* big future S promise. Big real value of debt. Future comes, s rises “in response to value of debt?” No. This is just keeping state-contingent promises.

War or recession lowers s_t . (Fact: s_t has a lot of short run variation) What to do ?

a) No change. Big burst of inflation (e.g. meet s by default on $B_{t-1}(t)$) Then back to normal at time $t + 1$.

b) Sell more $B_t(t + 1)$ w/o change in s_{t+1} ? Does not raise any revenue, no effect at all on p_t .

c) Sell more $B_t(t + 1)$ *with* change in s_{t+1} ? raises revenue to finance deficit. Can have *no* inflation this way. But time series looks “Ricardian”

d) raise distorting taxes at t , change s_t .

To do: some of each, depending on distortions. (Future: mix with optimal taxation, including explicit cost to inflation) But *a government that does not like surprise inflation will look very Ricardian. Most changes in debt come with changes in future surpluses precisely so they will raise revenue and not depress bond prices.* (More details: last section of “long term debt”)

But the issue is *off equilibrium* prices. How will the government respond to unexpected deflation that raises value of govt debt?

We never see the off equilibrium so time series data cannot tell us. *In* equilibrium the FT equation will always hold!

Note don’t expect off equilibrium economies to make sense! $S = D$ conditions are violated.

More regimes.

3a. *Peg, Currency board.* If you have no reserves, but lots of s , you can always borrow them. If you have reserves but no s , you’re due for a crash. (Someone will grab the reserves).

4. Unpleasant arithmetic.

$$v_t = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_{t+j}} \right]$$

Real debt. Treasury (or Laffer) fixes s , Fed chooses M . Think of bad news to future s . Fed can choose inflation now or inflation later. Less M now leads to bigger debt, more inflation later (at rate of interest) Criticism: seignorage very small for US.

FTPL innovations: Nominal debt on left,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_{t+j}} \right]$$

“Inflation now” devalues outstanding nominal debt, makes the situation better. In fact, it can work even with zero seignorage (interest on money) *inflation is = to default on nominal debt.*

5. Interest rate targets

Fed sets $\{M_t\}$ to generate r . Here, $p_{t+1}/p_t = \pi$. thus for each $p_0 M_t = yp_0\pi^t/v$ $p_t = p_0\pi^t$. “Price level” p_0 indeterminate. Here the fiscal equation can determines $p_0!$

Note “initial period” confusion. trouble of perfect foresight models – a bad parable. With randomness, $\{M_t\}$ targets $r = E\pi = E_t(p_{t+1}/p_t)$. $p_{t+1}/p_t = \pi + \varepsilon_{t+1}$ passive does not determine ε_{t+1} , thus price level can change (if permanently) every day! “M accommodates any increase in p ” is exactly the charge of the 70s. “self-fulfilling expectations”

6. Hyperinflation dynamics and indeterminacies

$$M_t \bar{v} (p_{t+1}/p_t)^b = p_t y \tag{5}$$

Then, (5) implies

$$\ln p_{t+1} - \ln (M_t \bar{v}/y) = \frac{1+b}{b} [\ln p_t - \ln (M_t \bar{v}/y)]$$

usual use:

$$\ln p_t = \frac{b}{1+b} \ln p_{t+1} - \left(\frac{b}{1+b} - 1 \right) \ln (M_t \bar{v}/y)$$

$$\ln p_t = \frac{b}{1+b} \ln p_{t+1} + \left(\frac{1}{1+b} \right) \ln (M_t \bar{v}/y)$$

$$\ln p_t = \left(\frac{1}{1+b} \right) \sum_{j=0}^{\infty} \left(\frac{b}{1+b} \right)^j \ln (M_{t+j} \bar{v}/y) + \lim_{j \rightarrow \infty} \left(\frac{b}{1+b} \right)^j p_{t+j}$$

$$\text{note } \frac{\left(\frac{1}{1+b} \right)}{1 - \frac{b}{1+b}} = 1$$

Note discount is interest elasticity, not marginal utility. Can be much bigger/smaller than usual. I used to think small,

$$b = 1/2 \rightarrow \frac{b}{1+b} = \frac{1}{3}$$

$$b = 0.1 \rightarrow \frac{b}{1+b} \approx 0.1$$

But this is an error. We want the elasticity with respect to $1+r$, not with respect to r .

$$\frac{\partial \ln M}{\partial \ln(1+r)} = \frac{\partial \ln(r)}{\partial \ln(1+r)} \frac{\partial \ln M}{\partial \ln r}$$

$$\ln(r) = \ln(e^{\ln(1+r)} - 1)$$

$$\frac{\partial \ln(r)}{\partial \ln(1+r)} = \frac{1+r}{r}$$

Thus the appropriate b is 20 times larger, and the discount factor is near one. *This has been ignored for 30 years in macro.*

Back to fiscal theory. Problem: convergence term? Simple: money M is constant. There is a family of solutions,

$$\ln p_t - \ln(M\bar{v}/y) = \left(\frac{1+b}{b}\right)^t [\ln p_0 - \ln(M\bar{v}/y)], \quad (6)$$

indexed by the initial price level p_0 . The price level is, again, indeterminate. Draw possibilities. Note $(1+b)/b$ likely much bigger than $1+r$.

“Minimum state variable” or “forward-looking” or “non-explosive” solution, *this is a big theoretical problem – is this right or not!* No agent TV condition rules out explosive p . If we do it anyway, we have

$$\ln p_0 = \ln(M\bar{v}/y).$$

Fiscal to the rescue. Now we don’t need the philosophy to pick one solution, the fiscal equation does it.

$$\frac{B_{-1}}{p_0} = \sum_{t=0}^{\infty} \frac{1}{r^t} s = \frac{r}{r-1} s.$$

If $p_0 > M\bar{v}/y$, this picks an explosive price level solution, So what? If we do not see explosive prices, this means that governments do not pig-headedly follow policies such as this one (constant M , constant surplus) that give rise to explosive price levels. Nothing in economics rules out explosive prices.

If $p_0 < M\bar{v}/y$ this violates TV condition. (if $\frac{1+b}{b} > r$.) So what? This just means there is no equilibrium. *It is not a requirement that one to produce* equilibrium for all parameters in R . We already know need coordinated policy, so parameters must be on a lower dimensional subspace. Traditional theory can’t produce equilibrium for $M^s < 0$..

Moral: fiscal theory **can** be used to solve indeterminacy problems in monetary economics, but that's not all it can do!

Long term debt

$$\frac{\text{nominal debt}}{\text{price level}} = \text{pv of surpluses}$$

With long term debt, the relative price can change (bond prices down.) That's future inflation not current. Which is it?

Example. Perpetuity.or no new debt sale

$$\frac{B_{t-1}(t)}{p_t} = s_t$$

just like one period model! *The tie of p today to long future s comes from rolling over debt stock. (short term debt implies long term present values).*

Issue: is the coupon or the bond numeraire? The price of the *bond* is still pv and moves all over the place. In this case, news about future surpluses *entirely* lowers long term bond prices – future inflation. *The effect of future surplus shocks on current price level depends on maturity structure of outstanding debt. (paper: plus state-contingent future debt!)* Intermediate maturities give intermediate effects.

Effects of long term debt.

Consol example.

$$\frac{B_{t-1}}{p_t} = s_t$$

Note: the tie of current price level to future surpluses is not generic. It depends crucially on rolled over one period debt. *Long* term debt means a *Shorter* link between surplus and price level. Each day is like the end of world.

This is essentially $\partial p / \partial s$ holding B constant. What about $\partial p / \partial B$ holding s constant? Notation

$$B_{t-1}(t+j) = \text{outstanding at } t-1, \text{ due at } j$$

Total amounts, so a given bond becomes part of $B_{t-1}(t+j), B_t(t+j), \dots$

Two period example. $t+1$ is easy

$$t+1 : \frac{B_t(t+1)}{p_{t+1}} = s_{t+1}$$

t : “Flow”, source and use of funds

$$B_{t-1}(t) = Q_t [B_t(t+1) - B_{t-1}(t+1)] + p_t s_t \quad (7)$$

$$Q_t = E_t \left(m_{t,t+1} \frac{p_t}{p_{t+1}} \right) = \frac{1}{R} \frac{p_t}{p_{t+1}}$$

$$\frac{B_{t-1}(t)}{p_t} = \frac{1}{R} \left[\frac{B_t(t+1) - B_{t-1}(t+1)}{p_{t+1}} \right] + s_t \quad (8)$$

value of bonds coming due = revenue from selling new bonds + surplus

“Stock” substitute $B_{t-1}(t+1)$ out so only preexisting debt in equation (like old pv formula)

$$B_t(t+1) = p_{t+1}s_{t+1}$$

From 7,

$$B_{t-1}(t) + Q_t B_{t-1}(t+1) = Q_t p_{t+1} s_{t+1} + p_t s_t$$

$$\frac{B_{t-1}(t) + Q_t B_{t-1}(t+1)}{p_t} = \frac{1}{R} s_{t+1} + s_t$$

real value of nominal debt = pv of surpluses

Note the new channel: Q can decline. Substituting in Q again

$$\frac{B_{t-1}(t)}{p_t} + \frac{1}{R} \frac{B_{t-1}(t+1)}{p_{t+1}} = \frac{1}{R} s_{t+1} + s_t$$

(you can also get here from 8 by substituting $B_{t-1}(t+1)$, but then don't get to show last one on the way.)

A “Solution” is $\{p_t\}$ in terms of $\{B, s\}$. In either stock or flow equations, substitute out p_{t+1}

Flow:

$$\frac{B_{t-1}(t)}{p_t} = \frac{1}{R} \left[s_{t+1} \frac{B_t(t+1) - B_{t-1}(t+1)}{B_t(t+1)} \right] + s_t$$

Note the new term.

Now, the point of all this, what is $\partial/\partial B$, constant s ?

Case 1) old. No long term debt $B_{t-1}(t+1)$. Then real revenue from selling $B_t(t+1)$ is independent of amount sold (shares in s_t).

$$\frac{1}{p_t} Q_t = \frac{1}{p_t} \frac{1}{R} \frac{p_t}{p_{t+1}} = \frac{1}{R} \frac{s_{t+1}}{B_t(t+1)}$$

As you sell more $B_t(t+1)$, bond price goes down one for one (unit elastic); future p_{t+1} goes up. No effect on p_t

Case 2) More $B_t(t+1)$ raises real revenue, lowers p_t ! Less inflation today, more tomorrow! Unexpectedly devalue outstanding bonds as claims on tomorrow's surplus.

$$\frac{1}{p_t} Q_t = \frac{1}{R} \frac{s_{t+1}}{B_t(t+1)} = \frac{1}{R} \frac{s_{t+1}}{[B_t(t+1) - B_{t-1}(t+1)] + B_{t-1}(t+1)}$$

is less than unit elastic in amount sold at t (though the same overall). *Debt operations can change the path of inflation.* p_t now depends on actions taken at time t too. (Note it always was possible to inflate ex post)

How much? *The stock equation gives us a "budget constraint" for price levels achievable by changing $B_t(t+1)$*

proof: These equations are all there is, flow and stock are equivalent.

$$\frac{B_{t-1}(t)}{p_t} + \frac{1}{R} \frac{B_{t-1}(t+1)}{p_{t+1}} = \frac{1}{R} s_{t+1} + s_t$$

No $B_{t-1}(t+1)$: no tradeoff, no way to change p_t . (remember these are fixed. Thus, the equation is a vertical line) More B_{t-1} means its easier to get more effect on p_t by accepting some p_{t+1} .

As we go to $t-1$, expectations of state contingent time t debt sales will matter!

Don't expect to see these comparative statics in data.

At first blush, like $Mv = py$. Friedman interprets much history as comparative statics, change M or y , (implicitly holding the other constant), see p answer.

We don't see fiscal comparative statics in data because *Almost all debt sales come with changes in future surpluses. As above with wars, maastricht.* See simulations in paper.

Why does B come with s ?. Go back to flow,

$$\frac{B_{t-1}(t)}{p_t} = \frac{1}{R} \left[\frac{B_t(t+1) - B_{t-1}(t+1)}{p_{t+1}} \right] + s_t$$

Recall earlier (should still be on board) War or recession lowers s_t . (Fact: s_t has a lot of short run variation) What to do ?

a) No change. Big burst of inflation (e.g. meet s by default on $B_{t-1}(t)$) Then back to normal at time $t+1$.

b) Sell more $B_t(t+1)$ w/o change in s_{t+1} ? if no $B_{t-1}(t+1)$ outstanding, does not raise any revenue, no effect at all on p_t . NEW: With some outstanding debt, this pushes inflation (usually more inflation) to p_{t+1} "default" on long-term bonds only at t .

c) Sell more $B_t(t+1)$ *with* change in s_{t+1} ? Raises revenue to finance deficit. Can have *no* inflation this way. But time series looks "Ricardian"

d) raise distorting taxes at t , change s_t .

In "long term debt" paper: state contingent future B matter. Will the govt devalue my debt ex post?

1156 What they're arguing against: self fulfilling prophecy; speculators. contagion, sunspots, multiple equilibrium. Yuk

1156 Alternative: rising deficits force abandonment of a peg. "first generation models" But there were no deficits

Note NOONE is saying open market operations, central banks need spine!

"speculative attack" = abandon peg

p. 1156 This paper: *prospective* deficits: bad news on $E_t \sum \beta^j s_{t+j}$ "inflation tax on outstanding nominal debt" is B_{t-1}/p_t

p.1156: a choice – huge distorting tax or "default" by devaluation. The deficits could be met by future tax increases. Default is a *choice*. Thus the next step is clearly integration into the optimal tax literature: when is a "crash" (default) worse than a rise in distorting taxes?

1157 Empirical

1) No standard macro indicators. No big deficits before the crash.

2) Crashes related to banking crises; bank stocks declining fast before crisis. Here: evidence that banks were in trouble and were going to need government bailouts. The correlation between currency crashes and bank problems leads to lots of bankish confusion!

3) Evidence that banks will be bailed out

4) No tax for bailout. See actual deficits, IMF money

1157 basic model: bad news about future deficit. It will be met by a mix of seignorage (Sargent-Wallace) and devaluation (abandoning the peg).

When does the government abandon peg? a) when debt reaches a bound b) "only constraint is intertemporal"?

1158:

Model: look for "attack" – the decision to float – before lots of money is printed or deficits observed.

i) This leads you to think about sunspots, etc.

ii) Burst of inflation but no steady state inflation. Seignorage can be small if you can devalue outstanding debt.

Fig 1. Indoensia malaysia thailand, korea and philippines. Thus an international comparison – why did the others not have crises (no "contagion")?

Facts: 1160 1) macro does not predict 2) you can see the banking crisis coming 3) the

government will bail out the banks 4) it won't raise taxes to do so.

(This is about what expectations were rational)

T1 NO big deficits before 1997 but big deficits afterwards. But ex post deficits seem pretty small. Is the story quantitatively consistent?

T2 bank problems in crisis countries.

Fig 2, T 3, stock in banking sectors on the way down, even relative, before the crisis (well....)

1163. Stock goes down if you think government will bail out depositors, but not stockholders..

T4 T5 T6. BIG deficits. 1164 10 x bigger than US S&L or Great depression bank problems. But...size of crash does not line up with size of nonperforming loans...But again..Maybe B is bigger; how is s related to size of nonperforming loans.

T7 got big aid too.

1166 model

Now standard news-of-bad-surplus scenario.

1166. Three things to do: 1) fiscal reform, 2) print money for seignorage 3) deflate outstanding nominal debt.

1167.

$$\begin{aligned}g &= \text{output} \\ \tau &= \text{taxes} \\ v &= \text{spending (transfers)} \\ m &= M/P\end{aligned}$$

start with constant v , then at 0 learn that at T' v will rise.

$$\phi = \int_{T'}^{\infty} e^{-rt} (v_t - v) dt$$

Nominal consols with value B , pay r , dollar debt with value b_0

1168 Note consols with B units of local currency. Long term debt so there's something to devalue

$$pv \text{ new transfers } \phi = \int_0^{\infty} (\dot{m} + \pi_t m_t) e^{-rt} dt + \sum_i e^{-ri} \Delta m_i + \left(\frac{B}{S}\right) - \int_0^{\infty} \frac{rB}{S_t} e^{-rt} dt$$

Why? $\dot{m} + \pi m = \dot{M}/P$ =seignorage for steady inflation, Δm =seignorage for jumps in M,P . S =exchange rate, i.e. B/S =real (dollar) value of nominal debt. Last term is the reduction in real value of nominal debt due to exchange rate changes.

Read 1169: choices the government faces (like “inflation now or inflation later” in Sargent-Wallace. This adds “devalue now or seignorage later”) To make an artificial time series, they have to take a stand on which options the government chooses.

CIA model for the money part. 1170

$$c_t \leq m_t$$

1171 CIA model.

$$c_t^{-\sigma} = \lambda(1 + r + \pi)$$

Important! Larger π makes consumption more expensive so you do less of it in order to hold less real money. *In CIA models, the intertemporal elasticity of substitution becomes the money demand elasticity. This means future M can give price effects today (recall the “hyperinflation dynamics”. In reality what might be a good intertemporal substitution elasticity for people might not be a good money demand elasticity! They use $\sigma^{-1} = 1$.*

1172 Idea: Debt grows. At a certain threshold, a one-time big M at time T and then growth afterwards (SW solution)

1173 1: until t^* info has come, but no collapse yet $\bar{c}\bar{m}\bar{M}$

2 t^* to T collapse to new steady state flex rate

3 T to T' new money, but no new transfers

JC comment: all this timing of attack stuff is pretty arbitrary, hard to figure out, and the weak point of the paper. Skim!

Fig 3. 1179 their point: May is $t = 0$ when information arrives. My point: markets saw it all coming

Note there is a lot of confusion about high interest rates towards a collapse. Is this “the central bank keeping interest rates high (monetary policy tight) to defend its currency? (Krugman) Or are the *real* rates low – high nominal rates just include a default premium! (JC). Note a lot of banks are playing the game – you make a lot borrowing dollars and lending bhat if you can get out just before the crash. And after the crash...well the government will bail you out! The existence of bailouts gives a huge option to speculate against your own government.

F 4 only used to calibrate M, seignorage

1181 F5. Peg, attack, float monetize. Cagan mechanism of future M growth gives current inflation.

1189 Thailand: little growth in M, then a lot. “Like model” (Fig 4). (Note however, this all comes from the interest elasticity bit. If you have $MV = PY$ then you need M right away. This paper has ppp all the time. What actually happened in these countries is that domestic inflation only slowly caught up with the exchange rate change. You need

M in proportion to domestic inflation (including traded and nontraded goods) We don't need to rely on hyperinflationary dynamics)

1189 Fig 7 and prices. I don't get it - why do export price indices come down?

1191 fig 8 interest rates mixed. Yet the forward rates did see it – covered interest parity not working? Government controls? Too bad – but high nominal interest rates before a crash, and banks betting against the crash are common features of many of these events.

WHY NOT JAPAN? Japan is not pegged. But why isn't the yen drifting down?

Why did all the countries go down at the same time? Could banks etc. be linked? Banks betting against the collapse?

Sims Fiscal Consequences for Mexico of Adopting the Dollar

Background:

JC 1990. I loved currency boards, dollarization. Money = units, we should all use the same units like the metric system. (I'm dubious about frictions; don't trust central banks offsetting shocks).

FTPL: broke my faith in the first, opened big questions on the second.

Big picture. As in "Money is stock" Nominal debt is like Government Equity. Real (dollar) debt is like debt – you have to pay it off (or explicitly default).

Equity has some important risk-sharing advantages over pure debt. That's why it's useful for corporations. (Debt can default but that usually is costlier than having the stock price decline)

Hence, if "Mexico" completely dollarizes, it must at least have greater tax variation; taxes must adapt to at least the present value of spending. Nominal debt allows "state contingent default" through inflation.

Analogies.

1) The model is just like the PIH. Recall PIH.

a) With no markets, $c = y$.

b) With borrowing and lending: consumption still varies, but much less. Consumption is a random walk and varies with permanent income.

$$\begin{aligned}c_t &= rk_t + r\beta E_t \sum \beta^j y_{t+j}; \\c_t - c_{t-1} &= r\beta (E_t - E_{t-1}) \sum \beta^j y_{t+j}\end{aligned}$$

c) PIH with complete markets (state-contingent): Consumption is totally smooth.

$$\begin{aligned}c_t &= rk_0 + r\beta E_0 \sum \beta^j y_t; \\c_t - c_{t-1} &= 0\end{aligned}$$

2) Optimal taxation (clearer in Schmitt-Grohe and Uribe) Inflation = state-contingent debt repayments. Inflation is just a cleaner mechanism than explicit default. Without state contingent debt, you get ~ random walk in taxes. With state contingent debt, you can have fixed taxes.

(Abstract of the working paper. Remind of corp finance equity vs. debt. Equity: risk sharing for management. MM theorem. Make plot, defaultable debt. Total value unaffected, but value of components is. Debt more expensive as becomes more likely to default.)

p.598

$$\frac{B_t}{P_t} = E \sum_{s=1}^{\infty} m_{t,t+s} \left(\tau_{t+s} - G_{t+s} + \frac{e_{t+s}}{P_{t+s}} (F_{t+s} - R_{t+s-1}^* F_{t+s-1}) \right)$$

value of nominal debt = pv of primary surplus + revenues ("interest costs") of dollar debt)

p. 606 similarly

$$\frac{B_t + e_t F_t}{P_t} = E_t \left[\sum_{s=1}^{\infty} m_{t,t+s} \frac{\tau_{t+s} - G_{t+s}}{P_t} \right]$$

(This is like the choice whether to put money on the left or right.) (The second form imposes the condition that you can't borrow dollars forever. Note that e and P typically move together – changing P has no effect on e/PF .)

p. 599. read last paragraph, first paragraph of 600.

Summary: Dollarization

- 1) removes the temptation to inflate ex-post – solves a time-consistency problem
- 2) removes the "equity cushion" – only (distorting) taxes or (costly, not mentioned) explicit default are now possible.

Model (JC notation)

$$\begin{aligned} \min E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \tau_t^2 \\ \frac{B_{t-1}}{p_t} &= \tau_t - G_t + \beta E_t \left(\frac{1}{p_{t+1}} \right) B_t \\ \frac{B_{t-1}}{p_t} &= E_t \sum \beta^j (\tau_{t+j} - G_{t+j}) \end{aligned}$$

"Barro" $p_t = 1$ (dollars; more generally fixed). "Sims" gets to choose p_t as well

Major claims:

- 1) Barro gives $\tau_t = E_t \tau_{t+1}$
- 2) Sims gives $\tau_t = \tau_{t+1}$. 604. pp1. Surprise inflation is useful!
- 3) If $B_{t-1} E_{t-1} (1/p_t)$ the real value of debt is large. There is a limit – more outstanding debt is good!

$$\begin{aligned} \frac{B_{t-1}}{p_t} &= E_t \sum \beta^j (\tau_{t+j} - G_{t+j}); \tau = \text{constant} \\ B_{t-1} \left[\frac{1}{p_t} - E_{t-1} \left(\frac{1}{p_t} \right) \right] &= -(E_t - E_{t-1}) \sum \beta^j G_{t+j} \end{aligned}$$

The most we can do is $p_t = \infty$. Thus, the mechanism runs out when

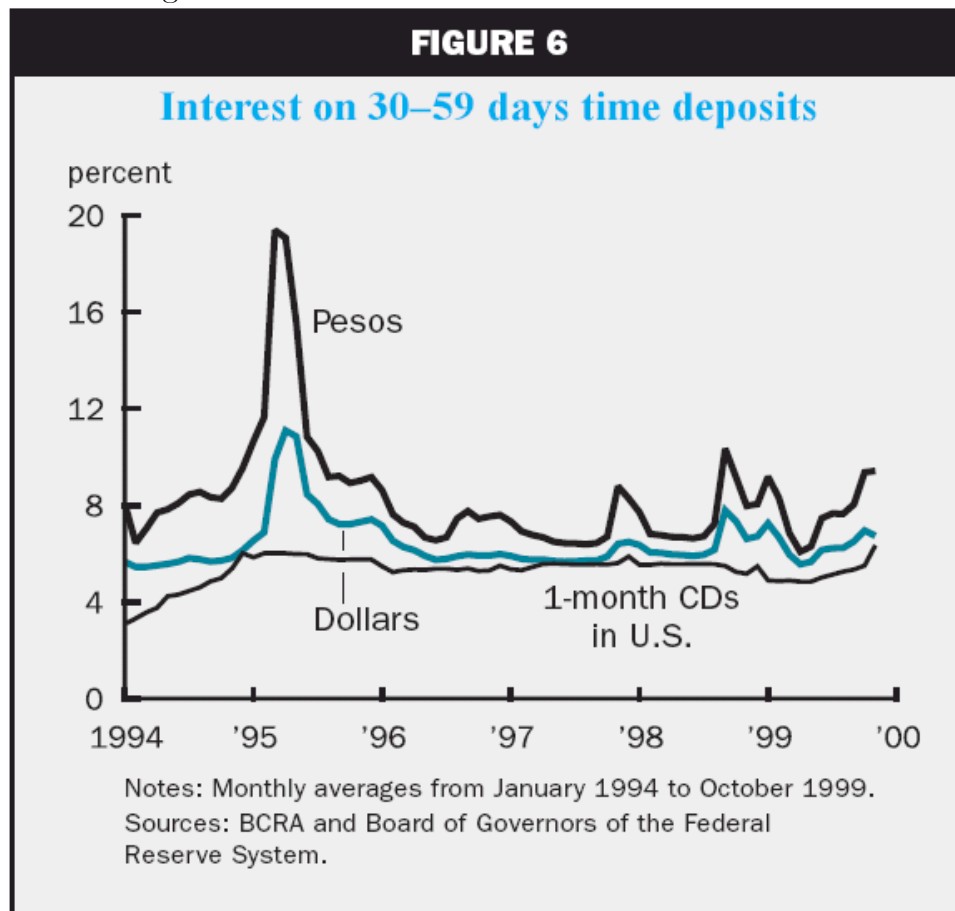
$$B_{t-1}E_{t-1} \left(\frac{1}{p_t} \right) = \text{real value of debt at beginning of } t = - (E_t - E_{t-1}) \sum \beta^j G_{t+j}$$

a) p. 604. middle: If you have more B outstanding, smaller changes in p give the same revenue. It's "good" to have been through a period of debt.

JC: the model rules out government saving. The government can issue nominal debt, and buy dollar debt. This idea gets rid of most of the pathologies in Sims model (problem set)

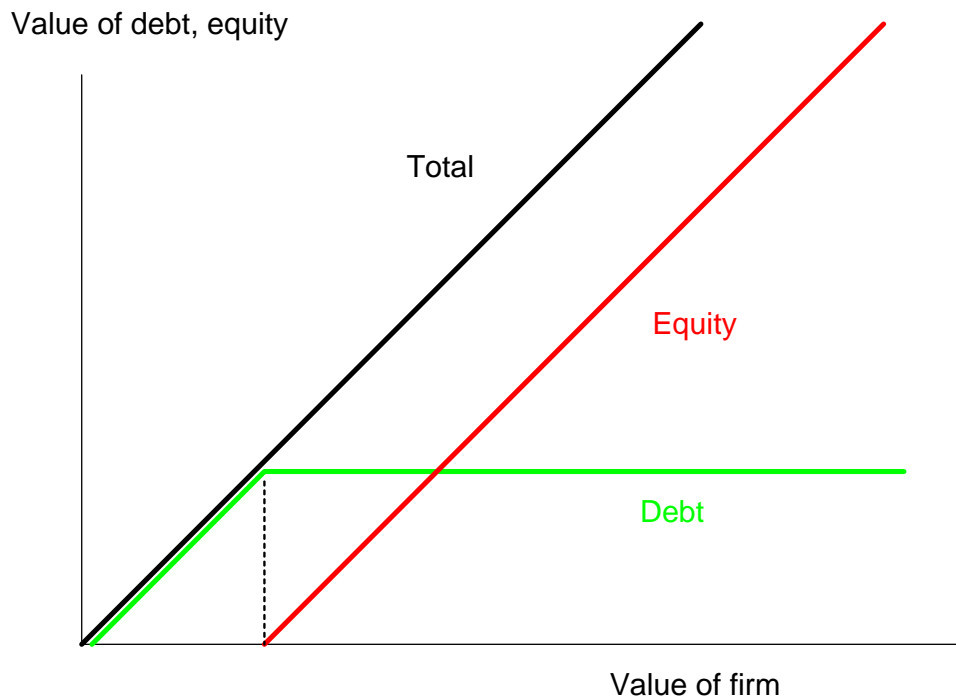
604 pp2. Inflation may also have costs. Schmitt-Grohe and Uribe address this question.

605: The cost of borrowing. The claim: countries pay too high interest on own currency debt because of the chance of inflation. If they don't intend to inflate they are paying too much. Then, dollarizing would lower interest costs. Graph from Velde and Veracierto, "Dollarization in Argentina"



606: NO! Modigliani-Miller theorem. If we issue debt to retire equity (dollarize) then the debt is riskier, and will command a higher risk premium. The overall value of the

firm and its overall cost of capital is unaffected by the debt equity split. As you dollarize, expect your dollar interest rate to rise.



Counterargument? Explicit default is costly, so dollarization forces you to take painful declines in G .

Alternative ways to implement “government equity”? (especially that don’t impose all the frictions of inflation during a “default”?)

p. 607 “Suspension of convertibility” of a dollarized economy, as under the gold standard. (Note this contradicts p. 604 bottom that thought of the 1700s English experience as building up a stock of debt. I think 604 was a bit misleading because under a gold standard ftpl doesn’t really apply. Though a larger stock of debt on which to suspend convertibility means the discount required is smaller.) This has the advantage that not everyone gets hurt

JC: but is that an advantage? Corporate equity comes with control and thus monitoring. Where are the control rights for government equity? In fact, defaultable debt makes bankers the ones who monitor. If lots of voters are hurt by inflation that may be good. Note that fiat money is only successful in democracies. I think this is the last pp on p. 607.

Other options: Argentine “contingent repo facility”

p.610 ”reserve currency” Bonds in pesos, all transactions in dollars. FTPL shows this is possible!

Perpetuities with state-contingent coupons.

Equity comes with control! But where is the control function?

For us: voters. Notice you only have price stability in functioning democracies!

Who should dollarize (work on debt)? Countries with poor political systems.

→ *Invent "government equity" for sovereigns.*

610 Lender of last resort.

Background: What can stop a bank run? A "lender of last resort" that gets you through liquidity crises. This is what the Fed was founded to do.

Problems: Moral hazard (can you tell which banks really should fail? If you bail everyone out there is an incentive to make bad loans)

Credibility. Does the lender have enough? US: In the end the Fed can always print so the lender is credible.

If you dollarize, who is the lender?

A: The government can be the lender if it can borrow dollars in a crisis. But there is a borrowing limit – future taxes. Thus, when the government gets in trouble dollarized bank deposits make the banking system more prone to runs.

IMF.

One answer: more foreign banks. (I think Mexico had foreign banks get letters from their central banks promising LLR function.)

p. 607-608. Was US inflation a state-contingent default? The important task of explaining 70s and 80s.

(19) is terrible notation.

$$B(t) - B(t-1) [1 + r(t-1) + \pi(t) - E_{t-1}\pi(t)] - \tau(t)$$

The τ adjusts the new value of the debt for surplus/deficit, so you're seeing return on initial debt, not increases in value of debt because you're borrowing more.

Result: graph 1. inflation especially of late 70s devalues debt after oil shocks. Then unexpected disinflation of 80s goes the other way, after good shocks.

Important: a story for 70s/80s.

Not terribly convincing: the timing is off. Inflation comes in booms, when surpluses get better. Magnitudes are small. (See Cochrane "A frictionless view of US inflation, Hall and Sargent, and "Long term debt" for better calculations). *Can we account for inflation of the 70s as a "state-contingent default"? Both the trend and cyclical correlations?*

Conclusions 612. Why is dollarization across states good and across countries bad? The usual answer is that labor and other factors can flow, so its an “optimal currency area” – fed offsets common shocks. (But that says open trade!) The usual fiscal theory answer is that currency unions must come with promises of fiscal transfers (Sims, Woodford on Euro). JC: Yeah? Is it a terrible problem if Italy must now explicitly default?

My version of Sims’ model.

$$\begin{aligned} \min E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \tau_t^2 \\ \frac{B_{t-1}}{p_t} &= \tau_t - G_t + \beta E_t \left(\frac{1}{p_{t+1}} \right) B_t \\ b_t &= \tau_t - G_t + \beta E_t (b_{t+1}) \end{aligned}$$

Well understood in the optimal tax literature as a “market clearing condition” not a “budget constraint”

(Where did that come from?)

$$\begin{aligned} \frac{B_{t-1}}{p_t} &= \tau_t - G_t + \frac{Q_t}{p_t} B_t \\ Q_t &= E_t \left(m_{t+1} \frac{p_t}{p_{t+1}} \right) \\ m_{t+1} &= \beta = 1/R \end{aligned}$$

We assume risk neutral or constant c , and private discount rate = government discount rate.)

Note: the PV form of the constraint is

$$b_t = \sum_{j=0}^{\infty} \beta^j (\tau_{t+j} - G_{t+j}).$$

Thus, this is the PIH random walk model with different symbols.

Barro: $p_t = 1$. (More generally exogenous) Thus, b_t is in the time $t - 1$ information set. Let’s be clear about states and information. State

$$s^t = \{s_0, s_1, \dots, s_t\}$$

Problem:

$$\begin{aligned} \min \sum_{t=0}^{\infty} \beta^t \pi(s^t) \frac{1}{2} \tau_t(s^t)^2 \\ b_t(s^{t-1}) &= \tau_t(s^t) - G_t(s^t) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) b_{t+1}(s^t) : \beta^t \pi(s^t) \lambda(s^t) \end{aligned}$$

$$\begin{aligned}
b_t(s^{t-1}) &= \tau_t(s^t) - G_t(s^t) + \beta b_{t+1}(s^t) : \beta^t \pi(s^t) \lambda(s^t) \\
b_{t-1}(s^{t-2}) &= \tau_{t-1}(s^{t-1}) - G_{t-1}(s^{t-1}) + \beta b_t(s^{t-1}) : \beta^{t-1} \pi(s^{t-1}) \lambda(s^{t-1})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial \tau_t(s^t)} &: \beta^t \pi(s^t) \tau_t(s^t) = \beta^t \pi(s^t) \lambda_t(s^t) \\
\tau_t &= \lambda_t
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial b_t(s^{t-1})} &: \sum_{s_{t+1}} \beta^t \pi(s^t) \lambda(s^t) = \beta^{t-1} \pi(s^{t-1}) \lambda(s^{t-1}) \\
\sum_{s_{t+1}} \beta^t \pi(s_t | s^{t-1}) \pi(s^{t-1}) \lambda(s^t) &= \beta^{t-1} \pi(s^{t-1}) \lambda(s^{t-1}) \\
\sum_{s_{t+1}} \beta \pi(s_t | s^{t-1}) \lambda(s^t) &= \lambda(s^{t-1}) \\
E_t(\tau_t) &= \tau_{t-1}
\end{aligned}$$

Thus,

$$\tau_t = E_t(\tau_{t+1})$$

This is the central result. We can fully solve for tax rates as we do in the PIH model. Plug into constraint

$$\begin{aligned}
b_t &= E_t \sum_{j=0}^{\infty} \beta^j (\tau_{t+j} - G_{t+j}) \\
&= \frac{1+r}{r} \tau_t - E_t \sum_{j=0}^{\infty} \beta^j G_{t+j} \\
\left(\frac{1}{1-\beta} = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r} \right) \\
\tau_t &= \frac{r}{1+r} b_t + r \beta E_t \sum_{j=0}^{\infty} \beta^j G_{t+j}
\end{aligned}$$

Taxes pay off the annuity value of the debt plus "permanent spending." As with the pih, we now can express the random walk innovations as innovations in "permanent spending",

$$\tau_t - \tau_{t-1} = r \beta (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j G_{t+j}$$

(Proof, not in class:

$$\begin{aligned}
b_1 &= \frac{1+r}{r}\tau_1 + E_1 \sum_{j=0}^{\infty} \beta^j G_{1+j} \\
b_2 &= R(b_1 - \tau_1 - G_1) \\
&= R\left(\frac{1+r}{r}\tau_1 + E_1 \sum_{j=0}^{\infty} \beta^j G_{t+j} - \tau_1 - G_1\right) \\
&= \frac{1+r}{r}\tau_1 - E_1 \sum_{j=0}^{\infty} \beta^j G_{t+2+j}
\end{aligned}$$

$$\begin{aligned}
b_2 &= \frac{1+r}{r}\tau_2 + E_2 \sum_{j=0}^{\infty} \beta^j G_{2+j} \\
\frac{1+r}{r}\tau_1 - E_1 \sum_{j=0}^{\infty} \beta^j G_{t+2+j} &= \frac{1+r}{r}\tau_2 + E_2 \sum_{j=0}^{\infty} \beta^j G_{2+j}.
\end{aligned}$$

)

Note no state contingent default. No default at all.

Sims version

$$\begin{aligned}
&\min E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \tau_t^2 \\
\frac{B_{t-1}}{p_t} &= \tau_t - G_t + \beta E_t \left(\frac{1}{p_{t+1}} \right) B_t \\
b_t &= \tau_t - G_t + \beta E_t (b_{t+1})
\end{aligned}$$

Now p_t can be chosen as well. This means that we have exactly the same problem, but b_t is in the time t , not the time $t-1$ information set.

$$\begin{aligned}
&\min \sum_{t=0}^{\infty} \beta^t \pi(s^t) \frac{1}{2} \tau_t(s^t)^2 \\
b_t(s^t) &= \tau_t(s^t) - G_t(s^t) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) b_{t+1}(s^{t+1}) : \beta^t \pi(s^t) \lambda(s^t)
\end{aligned}$$

$$b_t(s^t) = \tau_t(s^t) - G_t(s^t) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) b_{t+1}(s^{t+1}) : \beta^t \pi(s^t) \lambda(s^t)$$

$$b_{t-1}(s^{t-1}) = \tau_{t-1}(s^{t-1}) - G_{t-1}(s^{t-1}) + \beta \sum_{s_t} \pi(s_t|s^{t-1}) b_t(s^t) : \beta^{t-1} \pi(s^{t-1}) \lambda(s^{t-1})$$

$$\frac{\partial}{\partial \tau_t(s^t)} : \beta^t \pi(s^t) \tau_t(s^t) = \beta \sum_{s_t} \pi(s_t | s^{t-1}) b_t(s^t)$$

$$\tau_t = \lambda_t$$

$$\frac{\partial}{\partial b_t(s^t)} : \beta^t \pi(s^t) \lambda(s^t) = \beta^{t-1} \pi(s^{t-1}) \lambda(s^{t-1}) \beta \pi(s_t | s^{t-1})$$

$$\lambda(s^t) = \lambda(s^{t-1})$$

$$\tau_t = \tau_{t-1}$$

Ex-post too!

Here is the big intuition. With real debt, taxes must rise to meet the present value of spending; like PIH consumers. With nominal debt, taxes can be constant; like PIH consumers with state-contingent risk sharing contracts not just borrowing and lending.

Are we done? Not quite. If we read the constraints as giving the government full control of b in every period, then it can achieve $\tau = 0$. That satisfies the first order condition, and just set

$$b_t = 0 - G_t + \beta E_t(b_{t+1})$$

$$b_t = - \sum \beta^j G_{t+j}$$

That's nuts, of course – the value of the debt cannot be negative. Aha! There are some nonnegativity constraints lurking out here, and so the region where $\tau_t \neq \tau_{t+1}$ and the fact that $\tau > 0$ must reflect those constraints.

Constraints:

$$B_t > 0;$$

$$\infty \geq p_t > 0$$

The government can't lend, and it can do no more than devalue all outstanding debt. Now, intuition on how those constraints affect things.

The solution $\tau_t = \tau_{t+1}$ applies when these constraints are not binding. Ergo, the cases in which $\tau_t \neq \tau_{t+1}$ must apply to a binding constraint. These are Kuhn-Tucker constraints, so alas you have to try each case and see what works.

Start in period 0.

$$\frac{B_{-1}}{p_0} = E_0 \sum_t \beta^t (\tau_t - G_t)$$

As we raise p_0 we lower the left hand side, so taxes can be lowered on the right hand side. We'd like to go to negative values on the left, but $p_0 = \infty$ is as far as we can go. This is “repudiation of debt in the initial period.” So (assuming of course that $G > 0$ so $\tau = 0$ is not possible) we have

$$p_0 = \infty; \frac{B_{-1}}{p_0} = 0$$

Now, let's think about τ_0 and B_0 . Look both at the flow and PV constraints

$$0 = E_0 \sum_t \beta^t (\tau_t - G_t)$$

$$0 = \tau_0 - G_0 + \beta B_0 E_0 \left(\frac{1}{p_1} \right)$$

Both have to work (603, top) One possibility is that the inequality constraints will never bind. Then $\tau_t = \tau_0$. Does that work?

$$\text{if } \tau_t = \tau_0 : 0 = \frac{\beta}{1-\beta} \tau_0 - E_0 \sum_t \beta^t G_t \rightarrow \tau_0 = \frac{1-\beta}{\beta} E_0 \sum_t \beta^t G_t \quad \tau_0 = \text{permanent } G$$

$$0 = \tau_0 - G_0 + \beta B_0 E_0 \left(\frac{1}{p_1} \right) \rightarrow \beta B_0 E_0 \left(\frac{1}{p_1} \right) = G_0 - \tau_0 \text{ must be } > 0$$

Yes, *if* current G_0 is greater than permanent. Then, some of G_0 is financed by taxes τ_0 and some by borrowing. If G_0 is too small, then we run in to the constraint that the government can't save so $\tau_0 < \tau_1$. Taxes are thus a rising function of G_0 , first rising at $\tau_0 = G_0$, then rising much less steeply as $\tau_0 = \frac{1-\beta}{\beta} E_0 \sum_t \beta^t G_t$. (p. 603)

Now, let's look at a typical period with B_{t-1} outstanding. If the inequality constraints don't bind so the $\tau_t = \tau_{t-1}$ solution is going we have

$$\frac{B_{t-1}}{p_t} = E_t \sum \beta^j (\tau_{t+j} - G_{t+j}) = \frac{\beta}{1-\beta} (\tau_{t-1}) - E_t \sum \beta^j G_{t+j}$$

$$\frac{B_{t-1}}{p_t} - E_{t-1} \frac{B_{t-1}}{p_t} = B_{t-1} \left(\frac{1}{p_t} - E_{t-1} \frac{1}{p_t} \right) = - (E_t - E_{t-1}) \sum \beta^j G_{t+j}$$

The price level acts as the "shock absorber" to match variations in the present value of future spending.

That's fine, if it can do it. But there is a limit of course, once $p_t = \infty$ or $1/p_t = 0$, the entire stock of debt has been "repudiated" *If the shock to the present value of G is large enough, the price level goes to infinity and the government must start raising taxes.* How much? $p_t = \infty$ occurs where

$$B_{t-1} E_{t-1} \left(\frac{1}{p_t} \right) = (E_t - E_{t-1}) \sum \beta^j G_{t+j}$$

Tax smoothing is abandoned when the shock to the present value of government spending exceeds the real value of outstanding debt. Then, we start up as in period zero above, with a higher tax rate.

This leads to Sims' conclusion that *the government is better off (better able to tax smooth) with a larger stock of outstanding debt.*

JC version

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \tau_t^2$$

$$b_{t-1} + \frac{B_{t-1}}{p_t} = \tau_t - G_t + \beta E_t \left(\frac{1}{p_{t+1}} \right) B_t + \beta b_t$$

b_{t-1} may be of either sign – the government may lend as well as borrow internationally. *This removes the force of the constraint $B > 0$.* As long as there is some G_{\max} , we can now build up a large enough stock of B_t so that tax smoothing $\tau_{t+1} = \tau_t$ is globally true. Now the problem is truly isomorphic to the PIH problem with insurance.