Money and Inflation  Spring 2012 John Cochrane

Assignment 1

1. In this question, we will analyze a joint monetary/fiscal regime. One point is to stress that usually it’s not so simple as “one side is in charge of P and the other is passive.” The other is to stress how fiscal constraints and conditions – real debt vs. nominal debt – affect very conventional monetary policy. This problem also gives the government a tradeoff of “inflation now or more inflation later,” though the mechanism is quite different than the long-term debt mechanism.

The economy has a traditional money demand function,

\[ M_t v = P_t y \]

We will consider the case of real or foreign (euro, if you’re Greece) debt

\[ b_t = \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) \]  \hspace{1cm} (1)

and nominal debt

\[ \frac{B_{t-1}(t)}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) \]  \hspace{1cm} (2)

The economy starts at steady state \( R = 1.05, b = 0.5, y = 1 \) (50% debt/GDP), \( M = 1/6, v = 6, b = R^{1-1/s} \) hence \( s = 0.0238 \). Start by verifying that this is indeed a steady state. Next, the economy suffers an unexpected permanent 10% decline in primary surpluses \( s_t \).

(a) Sargent and Wallace pointed out that (1) limits monetary policy. Sooner or later the Fed will have to inflate, but the Fed does have a choice of when to inflate. To make an easy problem set, consider a one-time increase in the money stock,

1. Immediately following the shock
2. 10 years after the shock

Plot the paths of real debt, money, and price level in each case. Choose the size of the M increase to get the economy back to a steady state. (I picked options i and ii for convenience, and because that’s what my solutions will consider. Obviously, the Fed has great flexibility in how and when it’s going to raise the required seigniorage. If we were writing a paper, we’d try more realistic scenarios, like a constant money growth rate.)

You should reproduce Sargent and Wallace’s famous result: In the face of intractable deficits, the Fed will have to monetize sooner or later – the increase in seigniorage must make up for the decline in \( s \). The longer it puts off the inflation, (and the greater debt accumulates in the meantime) the worse the eventual inflation will be. This was their “unpleasant monetarist arithmetic.” Their point
was aimed at the early 1980s policy mix of large deficits and attempted monetary stringency. They pointed out these two are inconsistent. Interestingly, they turn out to be wrong; we did not get inflation, instead the economy grew so much (perhaps from the effects of lower tax rates) that by the late 1990s the government was running substantial surpluses. Maybe people knew this. Still, the theoretical point is right, and it is one of the founding articles in the resurgence of understanding that monetary policy faces fiscal constraints.

(b) Repeat the experiment with nominal debt rather than real debt, using (2). How does the presence of nominal debt change the result if the Fed chooses “inflation now?” How does the presence of nominal debt change the result if the Fed chooses “inflation later?” Is one or the other option more attractive in the presence of nominal debt? Explain the difference. (Answer: “inflation now”, changing $\Phi \tau$ can devalue the outstanding debt. This option was unavailable with (1). “Inflation later” does not have this property, so the seignorage must completely make up the loss in surplus. Even waiting one period loses the option of devaluing the outstanding debt. Don’t let them roll over the debt after the bad news has come out! Of course you still have to match these words with equations.)

2. Given our fiscal equation

$$b_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

a natural response is to fit an $s$ process, take present values, and then compare the model’s “predictions” to the actual $b_t$. Then we think about inflation as $b_t = B_{t-1}(t)/P_t$, given a process for nominal debt $B_{t-1}(t)$.

This problem is a prelude to that question – we’ll ask what a reasonable surplus process is to match real debt $b_t$, or equivalently one in which the price level could be a constant. In the data, deficits are strongly pro-cyclical – deficits in recessions, and less deficit or even primary surpluses in booms. With this in mind, the natural place to start is an AR(1),

$$s_{t+1} = \rho s_t + \epsilon_{t+1}$$

(Leave off constants and don’t worry about negative debt. Use $s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \epsilon_t$ if this is bugging you.)

(a) Find the real value of debt $b_t$ if surpluses follow this process.

(b) Part a gives a strange result. Debt should accumulate from surpluses, $b_{t+1} = R(b_t - s_t); R = 1/\beta$ This equation captures the pattern we see in the data, debt is decreasing when surpluses are high, and increasing when surpluses are low. But in part a, the level of debt and surpluses are perfectly correlated – debt is high when surpluses are high, not, apparently, after several years of bad deficits. How can you reconcile your answer from part a with this identity and the correlation in the data? To explore this question, find $b_{t+1}$ from the identity, and see if you can get it to equal $s_{t+1}/(1 - \rho \beta)$. The answer is, you can’t. You will see that something is desperately wrong with the AR(1) surplus process combined with real debt, or equivalently nominal debt and a steady price level. Your job is to
state what the problem is. Then, state what restriction on the surplus process we need to have in order for real (non-defaulting) debt to exist, or equivalently for the price level to be constant. (Hint: if you’re getting confused, look back at our expected - unexpected inflation decomposition from class.)

(c) With the insight of part b, let’s find a surplus process that works. Suppose surpluses consist of a cyclical component and a long-run component,

\[
\begin{align*}
    s^c_t &= \rho s^c_{t-1} + \varepsilon^c_t \\
    s^l_t &= \phi s^l_{t-1} + \varepsilon^l_t \\
    s_t &= s^c_t + s^l_t
\end{align*}
\]

(I think of \(s^c\) as the cyclical component the government can’t do much about, and \(s^l\) as reflecting long term tax and spending policies, plus commitments on how to pay off debts slowly over time. I think of \(\varepsilon^c\)). We want this process to solve the problem you found with the AR(1) in b.

1. Find the restrictions on this process that solves the restriction from part b. (Hint: You find a restriction on the \(\varepsilon^l_t\) given \(\varepsilon^c_t\), which you can interpret as how the government has to change long-run surpluses in order to account for shocks to the cyclical component.)

2. Verify that your process does solve both \(b_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}\) and \(b_t = R(b_{t-1} - s_{t-1})\).

3. Plot artificial data on \(\{s_t\}\) and \(\{b_t\}\) in your model, and verify that it looks reasonable: Debt increases when there are deficits and vice versa. Verify that the AR(1) model does not look reasonable in this way.

(d) Now

1. Find the univariate representation of the total surplus process, in terms of the structural shocks \(\varepsilon\). (It turns out you will have shown \(\varepsilon^c_t\) and \(\varepsilon^l_t\) are perfectly correlated. Hence, you only need \(\varepsilon^c_t\) so you’re looking for the representation \(A(L)s_t = B(L)\varepsilon^c_t\).)

2. Also find the form of the Wold representation – the univariate representation of \(s_t\) in terms of regression errors \(s_t = \sum_j a_j s_{t-j} + v_t\), i.e. find \(A(L)s_t = C(L)v_t\). To do this, recall that the Wold representation is the unique ARMA representation for \(s_t\) that has convergent AR and MA terms, i.e. that the AR and MA polynomials are invertible. You will notice that the representation in part i does not have invertible polynomials. Normally finding the Wold representation is an annoying job of factoring spectral density matrices. But it’s easy in this case. You will have a form \(A(L)s_t = (1 + \theta L)\varepsilon_t\) with \(\theta > 1\) and \(A(L)\) convergent. Convince yourself that the Wold representation is \(A(L)s_t = (1 + \theta^{-1} L)v_t\), by showing that both representations have the same autocorrelation function or spectral density, but the roots of the latter are convergent/the polynomials are invertible.)

“Find the form” means you don’t have to solve for the variance of the univariate shock \(v_t\). Just find the form of the convergent AR and MA lag polynomials.
3. Plot the impulse response function from the structural and Wold representation – how $s_{t+j}$ evolves after a shock. I used $\beta = 0.95; \phi = 0.98$ and $\rho = 0.7$. Does the true response function show that higher deficits today are expected to be paid off by higher surplusses tomorrow? Does the univariate impulse-response (the one you would recover from regressions of surplus on lagged surplus) show this same fact?

(e) Calculate the response of the present value $(E_t - E_{t-1}) \sum_{i=0}^{\infty} \beta^i s_{t+i}$ to the time-t shock in the true ($\varepsilon'$) and univariate ($\nu$) case. (You only have to calculate the impact multiplier, how it moves on the day of the shock. You can compute the full response if you like but it’s harder.)

1. Do this first numerically (just sum the discounted response function from your plot).
2. Then do it analytically for each of the impulse response functions from part d
3. Does the “structural” univariate representation obey the restriction on the $s$-process you derived above? Does the Wold representation do so?

(f) Implications. (Really short answers)

1. Now, suppose you fit an ARMA process to $\{s_t\}$, and recovered its Wold representation perfectly. Then, you calculate $E_t \sum \beta^i s_{t+j}$ based on that representation, and compared it to the $b_t$ in the data. Do you recover $b_t$?
2. Suppose this economy really had nominal debt but a constant price level, $B_{t-1}(t)/P_t = E_t \sum \beta^i s_{t+j}$. You fit the univariate (Wold) $s$ process perfectly, and calculate predictions for price level shocks, $B_{t-1}(t) (E_t - E_{t-1}) (1/P_t) = (E_t - E_{t-1}) \sum \beta^i s_{t+j}$ using that process. Would you correctly predict that there aren’t any price level shocks or would you be puzzled?
3. We have nominal debt and the price level can change. Is it possible to have an AR(1) surplus process with nominal debt, $B_{t-1}(t)/P_t = E_t \sum \beta^i s_{t+j}$? Is this a sensible model? (Note, to think about this one, it will help to add a constant, $s_t = \bar{s} + \rho(s_{t-1} - \bar{s}) + \varepsilon_t$ in order to avoid negative prices.)

(g) (Note: You may be wondering, what should you do here? The answer is, that to the econometrician, $b_t$ will help to predict $s_{t+j}$ along with past $s_t$, even though $s_t$ is “really” an exogenous process and $b_t$ is driven entirely by $s_t$. The point of the example is really to drive home that modeling $s_t$, finding $E_t \sum \beta^i s_{t+j}$ and comparing it to $b_t$ is not a good idea, because in the bivariate regression of $s_{t+1}$ on $\{s_{t-j}, b_{t-j}\}$ the bs will help the econometrician to forecast $s$. If you want to, feel free to read the last section of “Long Term Debt” which contains this analysis. My hope is that this guided tour in a simple example is clearer and easier than reading the paper.)

3. Consider a simple economy in which a constant coupon $c$ comes due each period, the government neither buys nor sells additional debt, and there is a stochastic surplus $s_t$. As I argued in “understanding policy,” the equilibrium is $c/P_t = s_t$. Let $s_t = s$
a constant. Assume a constant real interest rate so $Q_t^{(j)} = E_t \left( \beta^j P_t / P_{t+j} \right)$ with $\beta = 1/(1 + r)$

(a) Now suppose that at time $t$, the government suddenly decides to sell additional nominal debt $B$ which comes due at time $t+j$. Find and plot the response to this shock of $\{P_t\}$ and explain why it makes sense. I used $c = 10, s = 10, B = 5, r = 0.05, T = 5$ to make the plot.

(b) Plot at each date from 0 to 6 the term structure of interest rates (yield vs maturity) at that date, and explain why it makes sense.

(c) What effect does this intervention have on the total value of outstanding government debt at each date? Find the value of 1) the perpetuity 2) the bond $B$, and 3) the sum of the two. Also 4) find an analytical expression for the sum. Plot 1-4 and explain. (Evaluate the value of the debt at the beginning of the period, debt “coming due.”)