1. Consider the frictionless New Keynesian model from class, with a monetary policy shock

\[ \begin{align*}
    i_t &= E_t \pi_{t+1} \\
    i_t &= \phi_\pi \pi_t + x_t \\
    x_t &= \rho x_{t-1} + \varepsilon_t
\end{align*} \]

(a) Compute the standard “New Keynesian” solution to this model, assuming \( \phi_\pi > 1 \) and solving the unstable root forward. You’re looking for \( \pi_t \) and \( i_t \) (observables) in terms of current and past shocks.

(b) Plot the impulse-response function \( \pi_t, \pi_t \) and \( x_t \) to a tightening — a positive unit shock to \( x_t \). Choose sensible numbers for the parameters. Does tighter monetary policy produce inflation or deflation?

(c) In b, you may find that a positive \( x \) shock produces a lower \( i \) as well. How can a positive interest rate shock produce lower interest rates?

(d) This is an entirely frictionless model! Interest rates certainly do not lower “demand” and thus lower inflation by a Phillips curve. Interest rates equal expected inflation, the first equation. How can raising nominal interest rates lower future inflation in a frictionless model?

(e) The fiscal equation \( B_{t-1}(t)/P_t = E_t \sum \beta^j S_{t+j} \) holds in this model by assumption — any change in \( P_t \) is assumed to trigger a corresponding change in \( E_t \sum \beta^j S_{t+j} \). Thus, in computing this impulse-response function you are really changing two things at once, \( x_t \) and \( E_t \sum \beta^j S_{t+j} \). In general, we should think of this model as having two shocks, a shock to monetary policy \( x_t \) and a shock to fiscal policy \( E_t \sum \beta^j S_{t+j} \). Rather than assume these are perfectly correlated, we should assume they are less than perfectly correlated. In the extreme, it is interesting to plot the response to a monetary policy shock that has no fiscal effect, and to a fiscal shock that has no monetary \( (x) \) effect. Though typical events in the data will combine the two shocks, this might give us better intuition about how the model works.

We will do only the first of these: Plot the response of \( x, i, \pi \) to a shock to \( x_t \) that does not change \( E_t \sum \beta^j S_{t+j} \). You will have to assume \( \phi_\pi < 1 \) — as noted in class, “active fiscal” policy needs to come with “passive monetary” policy. As in class, if we hold \( E_t \sum \beta^j S_{t+j} \) constant, then \( B_{t-1}(t)/P_t = E_t \sum \beta^j S_{t+j} \) means that there can be no shock \((E_t - E_{t-1}) \pi_t = \delta_t = 0 \) — inflation must be predetermined by one period. This does not mean that \((E_t - E_{t-1}) \pi_{t+j} = 0 \), however, so there is an interesting response function. (As in class, the government can change \( B_t \) and hence \( E_t/P_{t+1} \); this is how it follows an interest rate target in the first place.)

(f) Explain the difference between the response function in this case and above. Which strikes you as a more reasonable characterization of the data, or at least of common stories of how the Fed thinks it affects inflation? Which response strikes you as a more reasonable characterization of the Fed’s power to affect inflation in
2. Now, we’ll compute the impulse-response function for the three-equation New Keynesian model.

\[
\begin{align*}
y_t &= E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + x_{dt} \\
\pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + x_{\pi t} \\
i_t &= \phi_{\pi} \pi_t + x_{it}
\end{align*}
\]

(You may omit the shocks in the first two equations. My solutions will keep them for later use, so we can plot responses to all shocks. Here you will only plot responses to the \(x_i\) shock. But keeping the first two shocks around will give you a better sense of overall model solution technique.) Let the shock again follow \(AR(1)\),

\[
\begin{align*}
x_{dt} &= \rho_d x_{dt-1} + \varepsilon_{dt} \\
x_{\pi t} &= \rho_{\pi} x_{\pi t-1} + \varepsilon_{\pi t} \\
x_{it} &= \rho_{i} x_{it-1} + \varepsilon_{it}
\end{align*}
\]

(a) Compute the response of \(y, \pi, i\), to the \(x_{it}\) shock.

(b) Give the intuition, or at least a verbal description of how this shock affects output, inflation, and interest rates. Compare this response to the response in the frictionless model. Does the frictionless model capture much of the intuition here? Or is the mechanism in Fed statements – a shock raises interest rates, that raises real rates, output declines, and the Phillips curve pushes inflation down – important to understanding this response function?

The solutions will follow the method described on p. 10 of the online appendix to “Determinacy and Identification.” Other methods are fine. I’d rather you actually calculate it rather than just plug it in a canned computer program. The point is to learn how the solution methods work. Feel free to jump to numerical techniques at any point. I use \(\sigma = 1, \gamma = 1, \beta = 0.95\) and \(\phi_{\pi} = 1.2\), but other reasonable numbers are ok. I used Scientific Workplace’s analytic calculations of eigenvalues and eigenvectors to produce analytic solutions, but this is a classic point at which you may want to jump to numbers. I will spare you calculating the corresponding “fiscal” solution, but I’ll put them in the answers if you’re curious.