1 Pictures

Sargent’s website:
real primary surplus / GDP
Lucas: Money demand in the United States

\[ \log \left( \frac{M_0}{P_T} \right) \]

Short-Term Interest Rate

* denotes observations from 1900 to 1957.
0 denotes observations from 1958 to 1985.

or Lucas’ Nobel lecture:

\[ \text{Inflation rate (percent)} \]

\[ \text{Money growth (percent)} \]

\[ \text{Fig. 1} \]
Interest rates. Moody’s BAA and AAA; 10 year Treasury constant maturity and 3 month Treasury bill; 3 month nonfinancial and financial commercial paper
Three possible reactions to a 10% expected surplus shock. Calibrated to an estimate of the US Federal debt maturity structure.

Bond yields and inflation, in the delayed inflation scenario.
Output and real and nominal rates in a “delayed inflation” scenario.
2 Overview: Theories of the price level

The price level is the most basic issue. We can do output and business cycles later.

2.1 Fiscal theory: A Simple one period model

Overnight debt and money come due at time t: \( B_{t-1}(t) + M_{t-1} \).

AM: People redeem \( B_{t-1}(t) \) for intraday money (Bonds say “promise to pay $1”). People can use that money to transact. But they can also use credit, lous, private money, maturing debt, barter etc.

PM: Government taxes \( s_t = t_t - g_t = \tau y_t - g_t \) and taxes must be paid in government money.

Night: The world ends. M, B are worthless

The flow constraint: money added to the economy must be soaked up by taxes.

\[
B_{t-1}(t) + M_t = P_t s_t
\]

The equilibrium condition. People hold no money ex post. So,

\[
\frac{B_{t-1}(t) + M_t}{P_t} = s_t
\]

As the day ends, people who have more money than they need to pay their taxes try to buy stuff instead. (One more capuccino..) But equilibrium (C=Y) means then only end up driving up prices instead.

We’re done! Money has value because the government accepts it for taxes.

“A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.” Adam Smith.

“This note is legal tender for all debts public and private” Defines the dollar. “public” is the key!

2.2 Theories of the price level.

That’s where we’re going. Now, let’s motivate it. Let’s look at all the other theories of the price level. For each one, I’ll give a quick overview of how and whether it works. Then we need to ask, can this possibly apply to us, now?

Why are these piece of paper valuable – why do people give you valuable stuff in exchange? The great puzzle!
1) Commodity money?

E.g. gold coins, or wheat, rice.

The valuation question seems simple if the commodity is valuable. Rice is valuable because you can eat it. The story is not really so clear for gold coins – what is their real value independent of money? But let’s come back to that issue. The “commodity money” theory says gold coins/rice/etc. get their value from supply/demand, then serve as numeraire.

Nice theory, but it obviously doesn’t apply to us. We don’t use commodities.

2) Commodity standard. Notes redeemable on demand.

A simple view of the gold standard. The government says “one dollar = 1 oz. of gold, come and get it.” This nails the price level. (Or one bushel of wheat, etc.)

That’s also not so clear on second thought. Again, why is gold valued – is its value independent of its use as reserves?

Also, the gold standard is full of interesting historical problems: periodic crises, runs, devaluations, etc. Using notes instead of gold promises “efficiency” since we don’t need to dig so much, but seems to involve a loss of “stability” in periodic runs since it usually has less than 100% reserves.

We’ll come back to study the gold standard. I view it actually as an instance of fiscal theory, but we’ll get there.

For now, note our currency has no explicit redemption promise. So this doesn’t apply. It’s worth studying because lots of people want to return to the gold standard.

A modern version is an exchange rate peg: one dollar = 1 euro. Begging the question of why Euros are valued, that seems to nail the value of the dollar. It’s also prone to crises and devaluations – even the Argentine currency board failed with 100% reserves.

3) $MV = PY$. The Quantity Theory

This does work for money with no redemption promise. “All debts private” is the key.

Ingredients:

a) Money is intrinsically worthless (or worth more than its intrinsic value). We choose by law or custom one object as money.

b) Money demand as an inventory for making transactions. You hold money despite interest loss, as you hold a peanut butter inventory.

   c) Money supply is limited. This is important, since money’s value $>$ its intrinsic (paper) value!
d) Money is “special.” Law or custom prevent the use of substitutes – Ious, banknotes, small denomination government debt, foreign currency.

Again, does this apply to us?

a) What’s M? We really don’t have a clear separation between money and bonds.

b) Is the Fed limiting it? No. The Fed is explicitly pegging interest rates! Other central banks “provide liquidity to meet the needs of trade,” i.e. set MV = PY, a horizontal money supply curve. Then MV = PY determines M, not P.

Conclusion. Neither money demand nor money supply conform to the needed ingredients for this theory to work.

Note 1: Is a gold standard really a commodity standard, or is it just MV = PY? Should we regard gold as a commodity with set real value, independent of its use as money? Or should we regard it as a worthless commodity in limited supply, so that the gold standard is just a convention useful for limiting M in MV = PY? I suspect the latter. If so, the common prediction that a gold standard means volatile inflation is not true.

Note 2: If \( MV(i) = PY \), velocity is interest elastic, then it’s not even obvious that this can determine the price level. \( i_t = E_t(P_{t+1}/P_t) \) so \( MV(P_{t+1}/P_t) = P_tY \) is now a difference equation. It turns out to have many solutions. We’ll come back to “indeterminacy.” Put a placeholder here though: the quantity theory does not determine the price level uniquely if velocity is interest-elastic. This is only a complete theory of the price level if \( V \) is constant.

4) Monetarism

There is much more to “monetarism” that MV = PY. Among other propositions (doctrines):

a) The Fed should pay attention to and control money supply. Period. Friedman’s 4% rule. It may peek at inflation but really ignore output and interest rates.

b) The Fed not be “activist” and try to offset shocks. (4% was because Friedman didn’t trust the Fed, not because he thought a stochastic optimal policy exercise would yield 4%. There is a whiff of commitment and rules vs. discretion here. Friedman is searching for a “rule” or “institution” like gold standard to take the place of “discretion” which he did not trust.)

c) Most output and inflation variability is due to Fed mistakes in controlling the money supply, not to external shocks. 4% would have been better.

d) “Operating procedures” The Fed should focus on money supply, not interest rates as instrument, signal, etc. (You might say, controlling i is the same as controlling M. Monetarists say no, it’s better to control M.)

Problems:
a) The theory, like the quantity theory, relies on a rigorous distinction between “money” and “non-money” assets. It has always foundered the question, *What is money anyway?* Cash, reserves, checking accounts? (M1). Money market – can write checks but few do (and pays interest)? Unused credit card balances? Sweep accounts? The Marie Antoinette theory applies – there are a lot of liquid assets that *could* be used as money though their velocities are typically low.

Monetarists quickly abandoned the “transactions” view which motivates the theory in favor of a more nebulous “asset” view, to justify using M2 and larger aggregates rather than narrow transactions assets like M1, because M2 was more correlated with GDP. Correlation is not causation.

A nagging issue: The theory requires a friction, a “special” liquidity service of M. Yet financial innovation really means there is no such thing. A martian would think that actual institutions look much more like unlimited private banknotes or electronic accounting system of exchange than like the textbook base + checking accounts limited by reserve requirements vs non-transactions “bonds.”

It is also uncomfortable to think there is *no* frictionless benchmark – no determinate price level without this friction. Usually in economics we start with a clear frictionless benchmark, then add the effects of frictions.

b) The theory requires government control of the monetary assets. For example, if people can issue circulating IOUs (banknotes) the price level is indeterminate. *How should the Fed control “money”? Should it eliminate financial innovation?* This also requires the government to rigorously control “money substitutes” such as privately-circulating IOUs (checks, banknotes), and to pass laws forbidding the use of other objects (foreign currency). If these are free, demand for government money can go to zero and the price level to infinity. This has put otherwise free-market monetarists in the uncomfortable position of advocating stringent restrictions on financial innovation.

c) Theory requires that the Fed *does* control the money supply.

i) “supply liquidity to meet the needs of trade” MV=PY is now a recipe for M.

ii) But in fact it pegs interest rates (for good reasons, “unstable money demand”) The theory no more applies to the current institutional setting than the gold standard theory applies to a fiat money setting.

Fact central banks set interest rates not monetary aggregates. *If we want a theory that describes our current economy, we can’t use MV=PY.*

6) *Indeterminacy and instability* results (Sargent and Wallace; Friedman, Bernanke)

So, our central bank follows an interest rate target in a fiat-money regime.

There are two classical criticisms of such targets: *indeterminacy and instability.*

a) *Indeterminacy* (Sargent and Wallace): Imagine a simple frictionless model with
constant real rates. Now, $i_t = r + E_t \pi_{t+1}$. So if the Fed sets $i_t = \bar{i}$, this determines $E_t \pi_{t+1}$, but does not determine $p_t$, or even $\pi_{t+1} - E_t \pi_{t+1}$. There are many possible equilibria, “sunspots,” etc. Later I will display some “new-Keynesian” models that display this behavior.

b) **Instability:** $i = \bar{i}$ leads to inflationary or deflationary spiral. (Friedman, Krugman, Bernanke): If $i$ is just a bit too low, it will lead to money expansion (Friedman) or extra “demand” through credit channels. Extra “demand” means more inflation, but then the real rate is even lower, so the whole thing spirals out of control. This is Friedman’s 1968 description of the problem with interest rate targets and looming inflation. The opposite mechanism leads to exploding deflation if $i$ is a bit too high, or if we start getting deflation and $i$ hits the zero bound. This is Bernanke or Krugman’s view of deflation danger of $i = 0$.

This mechanism needs $i_t = r_t + E_t \pi_{t+1}$, some sort of adaptive expectations so $i_t = r_t + \alpha \pi_t$ and thus the Fed affects real interest rates. We also need some sort of price-stickiness and a Phillips curve so that “aggregate demand” exists and too much of it pushes $\pi_{t+1}$ up. Note the view of this model is that inflation is **determinate** – the model predicts one value only – but **unstable** – the time path of that value is explosive. Later I will exhibit an old-Keynesian model that displays this behavior.

Indeterminacy and instability are easy to confuse. They are in fact totally very different mechanisms that apply in very different models of price formation.

Interest rate targets are an instance of “passive money” policy regimes for the central bank. The instability/indeterminacy problem is a criticism of all passive money regimes.

Example 1: the “real bills doctrine” which in one flavor directs the central bank to passively “rediscount” any of a large class of fairly creditworthy debt. In modern parlance, I translate it as an interest rate target for unconventional policy: lend freely against any collateral in a given set at a given rate, accept any repurchase agreements, etc.

Example 2: “Passive money” policy, “provide liquidity to satisfy the needs of trade,” which basically means set $M$ to equal $PY/V$.

7) **“Fed view.”** “Old-Keynesian” ISLM models with expectations-adjusted Phillips curves.

This is the view implicit in every statement by the Fed, newspapers, Op-ed, and most applied macroeconomists writing for non-professional audiences.

- The Fed controls the **Federal funds rate**. This is the overnight rate for reserve requirements (in US) or the overnight rate on reserves (in other countries).

- The causal change to inflation: $\pi \rightarrow$ short term rates $\rightarrow$ long term rates $\rightarrow$ “demand” $\rightarrow$ “gaps” “unused capacity” $\rightarrow$ “price pressure” $\rightarrow$ inflation.

- What about price level determination? $p_{t-1}$ is given; this determines $p_t$.

- 1980-now: this view has changed a little bit, there is more lip service to “expecta-
tions” for better or worse. For example, the Fed now places great faith that announce-
ments of the path of Federal funds have big influences on long rates.

- Financial crisis to now. The Fed is changing a lot, both its ideas and its procedures. Now it tries to affect more rates directly – long bonds, mortgages, commercial paper. It lends directly to institutions it wants to support. It links rates to “demand” by “credit constraints” rather than the usual cost of borrowing.

- There is not much evidence or theory for any step of the chain.

- In the Fed view, the instability of rate targets is solved by the Taylor rule. Rather than \( i_t = \bar{i} \), or even \( \bar{i} \) (a time-varying peg), it’s \( i_t = \bar{i} + \phi \pi_t \) with \( \phi > 1 \). By moving interest rates aggressively with inflation, the instability is reversed. Inflation rises, interest rates rise more, real rates rise, “demand” declines, by Phillips curves future inflation declines. We will see this in an explicit old-Keynesian model.

What’s wrong? There really is no model – no utility function, budget constraint, production function, equilibrium, etc. At best this is a mechanical description of inflation. That’s even worse than my description of monetarism as needing a big friction at the short end of the yield curve. Is there really no simple supply and demand benchmark economic model of inflation?

8) Old-old Keynesian (1930-1970) economics was very weak about inflation. There wasn’t even a Phillips curve until the 1960s, and it was not shifted by expected inflation until the late 1970s.

If anything, Keynesians described a “wage price spiral,” and thought about unions and bargaining power. This is a part of the NRA’s catastrophic confusion of relative prices and the price level; cartelizing industries to cause inflation. Here, inflation is divorced from monetary affairs, or actions of the central bank. Inflation was the first great failure of Keynesian economics, in the early postwar (late 1940s-early 1950s), around the world, and especially in the late 1970s, and early 1980s.

I mention this to emphasize that the recent consensus that inflation has to do with the central bank is actually quite novel. For hundreds of years, inflation was about the gold standard; the central bank just managed the liquidity of government debt, and then started to handle financial crises. The idea that the central bank is centrally responsible for the price level really is due to Milton Friedman in the 1960s, and only became accepted by economists in the 1970s.

I also mention this because we’re going to end up giving the central bank a lot less power than the post-Friedman view suggests.

Needless to say, this view has even less economics.

9) New-Keynesian models. We’ll study these in depth. They posit price stickiness,
but really solve a model with utility functions, budget constraints, and optimization, unlike old-Keynesian models.

These models operate in the world of indeterminacy, not instability, of interest rate pegs. For them, the Taylor rule solves indeterminacy of interest rate targets. If inflation goes up, the Fed raises nominal rates, but in these models that makes future inflation go up even more. Unless, inflation jumps to the unique saddle path value for which there is no explosion. Thus, in these models the Fed deliberately introduces instability in order to solve (apparently) the indeterminacy problem. That seems rather strained, given that our Fed loudly announces precisely the opposite intention. Bernanke says “if inflation rises, we’ll raise nominal rates to bring it back again.” He does not say “if inflation rises, we’re going to hyperinflate the economy back to the stone age. So you guys better not raise prices in the first place.” And, as it turns out, the instability doesn’t really even cure indeterminacy. We’ll study these models in details in “Determinacy and Identification.”

10) Overlapping Generations models are used by many economists to study “money” issues. The young work, hold money to sell to the old when they retire. “Money” repairs a dynamic inefficiency. This is to my mind basically a theory of Social Security, not of medium of exchange. Furthermore, it does not work if the real interest rate exceeds the population growth rate, or if the present value of the government’s taxing ability is finite. Nice model, it just doesn’t have anything to do with the price level in our monetary system.

11) Fiscal theory of the price level. So I conclude this tour deeply unsatisfied with the alternatives, before we even begin empirical work. As I see it, there isn’t even a coherent alternative. The fiscal theory is the only coherent economic model able to describe our world: fiat money, rampant financial innovation heading to electronic barter, trillions of highly liquid assets around, a Federal reserve running the most passive of monetary policies, interest rate pegs and liquidity provision.

3 Fiscal theory

3.0.1 1. Simple one period model:

(Reminder from before.)

Overnight: $B_{t-1}(t) + M_{t-1}$.

AM: Redeem $B_{t-1}(t)$ for $M$ (B say “promise to pay $1”). UseMt to transact if you’d like, but can also use credit, private, barter etc.

PM: Government taxes $s_t = T_t - G_t = \tau Y_t - G_t$ and taxes must be paid in government money.
The world ends. M, B worthless overnight

\[ B_{t-1}(t) + M_{t-1} = P_ts_t \]

Equilibrium condition. Hold no money ex post.

\[ \frac{B_{t-1}(t) + M_{t-1}}{P_t} = s_t \]

We’re done! Money has value because the government accepts it for taxes.

3.0.2 2. Dynamic model

(Danger, it’s easy to get lost in transversality condition technicalities.)

Look at a cashless model, meaning no cash is held overnight. As in current institutions, money is only counted and interest only paid overnight.

Now cash in = cash out, but bond sales can soak up cash as well as taxes.

\[ B_{t-1}(t) = Q_tB_t(t + 1) + P_ts_t \]
\[ Q_t = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right) \]

Iterate forward

\[ \frac{B_{t-1}(t)}{P_t} = E_t \left( \frac{\Lambda_{t+1}B_t(t + 1)}{\Lambda_t} \frac{P_t}{P_{t+1}} \right) = s_t \]
\[ \frac{B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j} \]

The real value of government debt = the present value of the primary surpluses that retire that debt. (primary!)

Note the inverse return is also ok as a discount factor.

\[ r_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} \]
\[ Q_t = \frac{1}{r_{t+1}} \frac{P_t}{P_{t+1}} \]
\[ \frac{B_{t-1}(t)}{P_t} = \frac{1}{r_{t+1}} \frac{B_t(t + 1)}{P_{t+1}} + s_t \]
\[ \frac{B_{t-1}(t)}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=0}^{j} \frac{1}{r_{t+k}} \right) s_{t+j} \]
This is an identity, *ex post*. It is the definition of rate of return.

\[
\frac{B_{t-1}(t)}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{k=0}^{j} \frac{1}{r_{t+k}} \right) s_{t+j} \right]
\]

This is a nice form to remember. It lets you think about the discount rate as the return on government debt. And it reminds you we’re looking at an identity really.

With money that does not pay interest (Appendix to “Understanding policy”) there is seignorage revenue. This is tiny for the US. I regard it as an annoyance, but you can get somewhere with fiscal consequences of monetary policy. I regard it as no more or less interesting than the on the run/off the run liquidity spread, or the treasury/agency spread. All of these spreads have small fiscal consequences. Small compared to $1.5 trillion primary deficits. Reserves that pay interest are not.

\[
\frac{B_{t-1}(t) + M_{t-1}}{P_t} = Q_t B_t(t+1) + M_t + P_t s_t
\]

\[
Q_t = \frac{1}{1 + i_t}
\]

\[
\frac{M_{t-1} + B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ s_{t+j} + \frac{M_{t+j}}{P_{t+j}} \frac{i_{t+j}}{1 + i_{t+j}} \right]
\]

\[
\frac{B_{t-1}(t)}{P_t} = E_t \sum_{t=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right].
\]

### 3.0.3 What have we done

1. Intuition: Inflation comes from “Aggregate demand” “wealth effect of government bonds” “Pigou effect.” If the right hand side is too small, people try to get rid of government debt. They try to buy private assets instead, but this sends asset prices up. With asset expected returns low, they try to buy goods and services. If you’re trained in Cambridge or Berkeley, you see “aggregate demand,” if you’re trained in Chicago you see “too much money chasing too few goods.” It’s not a weird or unusual inflation.

2. Intuition: Money/bonds are like stock in the government. (See “money as stock.”) Imagine a world in which we use microsoft shares as medium of exchange, unit of account, etc. This is how we’d start thinking about the value question. Lots of people view exchange rates, inflation as reflecting “faith in the government.” This gives some teeth to the view.

3. It looks like \(MV = PY\) but it’s not. What’s \(M\)? Unlike \(MV=PY\) only government \(M\) (base, debt) are on the left hand side, *not* \(M1\). (Inside money, Ious, banknotes.). As options and futures do not dilute Microsoft stock and have no effect on its value.
4. “What’s B?” Ideally, all net government nominal liabilities! Ideally, we’d line up real and nominal and P is the divisor. Government salaries, defined benefit pensions, nominal depreciation allowances, all are really a bit of B. In the Korean devaluation, Eichenbaum et al show the government benefited by devaluing nominal salaries of government workers. (Or you could model this sort of thing by $s(P)$.) Alas, doing fiscal-theory accounting will be hard, and I encourage people better at accounting than I am to try it.

5. Warning: We are not “assuming s is exogenous”! It’s easy and fun to think of “S exogenous.” But really it isn’t, and we don’t have to think of it that way. Really surpluses are just part of an equilibrium, and we’re thinking conditionall on that choice. Just as dividends are not economically exogenous, but we think about stock prices given the dividend stream. We really should think about s together with other government choices, i.e. model distorting taxes, political imperatives to spending, how the government convinces people of expectations. This is just one equation of an equilibrium.

6. Implications:

   (a) Modigliani Miller theorem, open market operations – more M, less B – are irrelevant to the price level (holding s, r fixed, except very small seignorage terms.) (Or, if the Fed does control the price level it does so indirectly, by setting in motion some chain of events that means changes in future surpluses, or by affecting somehow the real discount rate for government debt. The “Ricardian regime” is the former.)

   (b) There is no need to control M to obtain a determinate or stable price level.

   (c) There is no need to control inside money (reserve requirements, bank notes), No need to control the M1 in $(Mb + M1)V = PY$

   (d) There is no need to control financial innovation. We can allow arbitrary private “moneys” (IOUs, banknotes, etc.) or foreign money to circulate. We can allow arbitrary financial and transactions-technology innovation. If we all want to use debit cards on stock funds, no problem.

   (e) The price level can be determined with NO monetary frictions whatever! The $M = 0$ limit (and limit point) work fine!
      i. It is also a natural “Frictionless benchmark” on which to build more complex theories.
      ii. It allows us to start with a Chicago "free market" approach to money, and add frictions only if we need them. Even Friedman needed one big friction!

   (f) Money can pay market interest! Then $M = B$. "Interest on reserves" used to imply "you’ll lose control of P." Not any more. A good thing!

   (g) The real bills doctrine is ok, does not lose price level control. The Fed can buy arbitrary real assets and issue more $B$, $M$ in exchange. The real assets
show up on the right hand side, so there is no (or even a positive, since there is a liquidity spread) fiscal effect.

(h) Interest rate targets – even fixed, non Taylor targets – produce neither instability nor indeterminacy. Lets see that next

3.0.4 Debt sales and interest rate targets

1. What if the government sells more debt at time t-1?

\[
\frac{B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j S_{t+j}
\]

\[
\frac{B_{t-1}(t)}{P_{t-1}} E_{t-1} \left( \frac{\beta P_{t-1}}{P_t} \right) = E_{t-1} \sum_{j=0}^{\infty} \beta^{j+1} S_{t+j}
\]

real revenue = \[
\frac{B_{t-1}(t)}{P_{t-1}} Q_{t-1}^{(1)} = E_{t-1} \sum_{j=0}^{\infty} \beta^{j+1} S_{t+j}
\]

(a) \( \frac{\partial}{\partial B} \vert_{S} \) Increasing \( B_{t-1}(t) \) with no change in \( \{S_{t+j}\} \) gives rise to a proportionate change in expected inflation; rise in nominal interest rate, decline in bond price (unit-elastic) and no extra revenue. The real value of debt fixed. The nominal value = number of shares sold.

i. Application 1: Currency reform. We change \( B, P \) changes proportionally, this is set up to communicate no change in \( s \). It is the same as a share split.

ii. Application 2: Interest rate targets with no \( s \) control are fine. (The Fed). The Fed could say “We offer to buy/sell Treasury debt at \( Q^{(1)} = 1/r_f \)” and thus nail the nominal interest rate and expected inflation rate. (B now becomes endogenous). The existence of a Fed, interest rate targets, and the Fed’s power to affect expected inflation even with no control of surpluses does not invalidate what we’re doing so far. (This Fed can’t affect real rates in this model of course) The choice of \( B \) means any expected inflation/nominal rate the government wants.

(b) \( \frac{\partial}{\partial B} \) with \( \Delta S \) For any bond sale to raise additional revenue, bond sales must come with promises to increase future \( \{S_{t+j}\} \)

i. Danger of comparative statics. What we see (war): B rises, with no change in \( P, R \). Then \( S \) rises. This is not “B caused \( S \),” but the fulfillment of promises.

ii. There is a widespread econometric danger – time series from equilibrium are not partial derivatives! “Tests” find that \( S \) “responds” to \( B \) but that’s exactly what we should see!
iii. (Note: As above, most debt sales come with implicit future surpluses. Implementing this means we need debt sales that convey different expectations. A currency reform vs. debt sale is like split vs. share issue. Open market operations as b without S? Can implement with Q “come and get it” Rather than sell B quantity, just roll over debt at fixed prices. This was how it was done for a long time! A reason for interest rate targets!)

(c) Similarly, decompose the present value relation to expected/unexpected

\[ B_{t-1}(t)E_{t-1}\left(\frac{1}{P_t}\right) = E_{t-1}\sum_{j=0}^{\infty} \beta^j s_{t+j} \]

\[ B_{t-1}(t)(E_t - E_{t-1})\left(\frac{1}{P_t}\right) = (E_t - E_{t-1})\sum_{j=0}^{\infty} \beta^j s_{t+j} \]

Even with “exogenous S” (fixing S), we can freely set expected inflation with B, then ex post inflation soaks up surplus shocks.

(d) The unexpected part

i. This is a first answer to “how will deficits affect the price level?” **Shocks to the PV of future surpluses will result in a price level jump today.** It feels just like stock prices; there is little warning, little momentum to inflation. Note, inflation is in fact very unpredictable! (Thesis Topic) It is contrary to usual fiscal doctrine: deficits may cause inflation when they are monetized, but not now. It is contrary to the usual monetary doctrine: plenty of warning, pitts curves, time to adjust.

ii. There is no “safe” level, debt/gdp, etc. Those may give solace about S, but not necessarily

iii. We have determined the ex-post inflation. So **We get a determinate price level with fixed nominal rate policies – we overturn Sargent and Wallace. Similarly, we overturn all the criticism of passive money, real bills, etc. as giving indeterminacies.** And a good thing too, since central banks do it all the time.

(e) **Whether followed by more inflation or a one time jump is up to the government.**

(f) Note: The dynamics will change a lot when we think about long term debt. Our government rolls over half it’s debt every 2 years, so it’s not a bad approximation.

### 3.1 Surpluses

What surpluses, you say? In fact primary surpluses (net of interest expense) have been positive on average. Note the secular trends that line up with past inflation and deflation well. The main influence on the surplus is the business cycle. Note that “Reagan deficits”
were not particularly bad given the state of the business cycle. Note the huge boom and increase in surpluses in the late 80s and 90s. The fiscal theory tracks the end of inflation well. Note our current state of affairs, and compare to the productivity slowdown of the 1970s and the sharp deficit of 1975.
3.2 Regimes and constraints

I What could go wrong?

\[ \frac{B_{t-1}(t) + M_t}{P_t} = s_t \]

1. What if \( s_t = s(P_t) = \bar{s}/P_t \)? What if, in response to a lower (off-equilibrium) price level, the government raises surpluses just enough to pay off higher debt?

2. Isn’t this policy plausible? Isn’t this what good governments do after all – raise taxes to pay off debts?

3. More deeply, is this a choice or a “constraint?” Doesn’t the government have to raise \( s \) to pay off debts?

Answers:

1. Yes. If \( s_t = s(P_t) = \bar{s}/P_t \), then either any \( P \) is a solution if \( B_{t-1}(t) + M_t = \bar{s} \), or no \( P \) is a solution.

2. If we just read the equation as directions to the Treasury on how to set \( s \) after the fact, then any \( P \) is a solution. (i.e. \( P \) must be set elsewhere, e.g. by \( MV = PY \)). This is called a “passive fiscal” regime.

3. Note this is not true of proportional taxes:

\[ P_t s_t = \tau P_t y_t \rightarrow s_t = \tau y_t \]

If we have proportional taxes, then indeed \( s \) is fixed independently of \( P \). So the “baseline case” is not “passive fiscal.”

4. And to the extent that \( s(P) \), inflation helps the government; \( s'(P) > 0 \). (Inflation moves people to higher tax brackets, makes sticky government salaries / pensions worth less)

5. Laffer curve: At some point \( s \) is bounded; If the government responds to deflation by raising \( \tau \), \( \tau Y \) stops increasing. Deflation cannot lead to infinite tax revenue!

6. Must the government do this, as a “budget constraint?” No. If people want to wallpaper their coffins with money, it’s ok for the government to leave money outstanding. The existence of the top of the Laffer curve also proves the point. If there is a limit, it can’t be a constraint.

7. In summary, the Government may announce \( \{B, s\} \), then \( P \) is an equilibrium condition. It does not need to announce \( B, s \) that make this work for every \( P \). It may choose to follow a “passive fiscal” policy, but that is a choice.
Don’t call it a government budget constraint, call it an equilibrium condition or a valuation equation.

II. More on Equilibrium condition vs. budget constraint.

\[ B_{t-1}(t) + M_t = P_t s_t \]

First approach: Let’s write the actual Government budget constraint (watch cash flow, the change in liabilities outstanding)

\[
\frac{B_{t-1}(t) + M_t}{P_t} = s_t + \frac{M_{t+1}}{P_t} - d
\]

The government physically can leave money outstanding at the end of the world. We need \( M_{t+1} = 0 \) to conclude the fiscal theory. That’s a specification of preferences, and equilibrium \( (M^s = M^d) \) not a constraint.

Second approach: Let’s look at the private budget constraint (sources and uses of money). Let \( y_t \) be a real endowment.

\[
B_{t-1}(t) + M_t + P_t y_t = P_t c_t + P_t tax_t - P_t transfer_t + M_{t+1}
\]

\[
c_t = \frac{B_{t-1}(t) + M_t - M_{t+1}}{P_t} + (y_t - tax_t + transfer_t)
\]

The equilibrium condition (market clearing) is:

\[
\text{supply} = \text{demand}
\]

\[
y_t = c_t + g_t
\]

\[
\text{demand-supply} = c_t + g_t - y_t
\]

\[
c_t + g_t - y_t = \frac{B_{t-1}(t) + M_t - M_{t+1}}{P_t} + (-tax_t + transfer_t + g_t)
\]

These are demands. Add, as above the specification of preferences \( u(M_{t+1}) = 0, M_{t+1}^d = 0 \). (This becomes the “transversality condition” in intertemporal models)

\[
\text{demand-supply} = c_t + g_t - y_t = \frac{B_{t-1}(t) + M_t - M_{t+1}}{P_t} - s_t
\]

and we’re done.

Note, to get there, we need a) \( y = c + g \) b) \( u(M_{t+1}) = 0 \). The budget constraints alone are not enough.
We also see that violations of our condition are equivalent to violations of the equilibrium, market clearing, condition \( y = c + g \). (no \( i \) in a one-period economy). Inflation comes when “aggregate demand” \( c + g \) is greater than “aggregate supply” \( y \).

**III The same ideas in dynamic models** involve transversality conditions and confuse people:

\[
\frac{B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{k} \frac{1}{R^j} s_{t+j} + E_t \frac{1}{R^k} \frac{B_{t+k-1}(t+k)}{P_{t+k}}
\]

1. The issue is the “transversality condition,” the last term

2. If \( P \), lower does not mean higher \( s \) then \( B/P \) explodes. This violates the transversality condition. That condition is a consumer first order condition. If they see it exploding, they can consume more now. (bonds are “wealth.”) Trying to consume more now raises \( P \). It is not a constraint on the government. If people want to hold ever exploding debt, as the central bank of china seems to want to do for the US, there is no constraint forcing the US not to print up the stuff.

3. Conversely, \( s \) that responds to \( B/P \) (off equilibrium) is enough to guarantee the transversality condition is satisfied for any \( P \). If \( s_t = s(B_{t-1}/P_t) \), and \( s' > 0 \), you can show the constraint holds. This makes “passive fiscal” seem plausible. But note, this requires the ability to raise taxes to arbitrary levels. Taxes 1000% of GDP?

4. **You can’t test, at least using time series from within one equilibrium.** The equation holds in equilibrium, whether \( P \) adjusts to \( s \) or whether \( s \) adjusts to \( P \). The time series in equilibrium will not see explosions, no matter how they are averted. This is part of a General Theorem: you can’t test for threatened explosions. You can’t test off-equilibrium threats. We’ll see this in analyzing the Taylor rule. It applies here too. (See “a frictionless model of US inflation” and “Determinacy and Identification with Taylor Rules”)

5. Some think “The Government can’t commit to let debt explode,” but this is the same as letting \( M \) sit at the end of the world. Microsoft can commit to letting a “bubble” explode its stock price, and no “constraint” forces it to validate prices with more earnings. “Off equilibrium” paths can be weird.

6. **Will** the government let debt explode, or does it responsibly raise taxes to pay off debts? Distinguish “on equilibrium” from “off equilibrium.” If the government borrowed, say to fight a war, then raising surpluses to pay off debts is just fulfilling its promises. The issue is, suppose a self-fulfilling off equilibrium deflation breaks out and halves the price level. Does the government respond by raising taxes to pay off an unexpected bounty to “Wall Street fat-cats and Chinese central bankers?” (Quote from summer 2011 debt ceiling debate). Of course, thanks to the Laffer curve, there is a point at which it must stop raising those taxes!
Summary: In a “passive fiscal regime” the government chooses $s$ ex post to soak up any cash, if so, this no longer determines the price level. It chooses to respond to “off equilibrium” price fluctuations. However, nothing forces the government to do it, and at least in the deflation direction there are limits to its ability to be “passive.”

3.3 Money and coordination

Let’s put $M$ back in. I” be a bit anachronistic, this is $M$ that doesn’t pay interest. ($B_{t-1}(t)$ is the same thing as interest-paying reserves!)

The issue is “Regimes.” “Monetary - fiscal coordination.”

All monetary models have two equations

\[
\frac{M_{t-1} + B_{t-1}(t)}{P_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left[ s_{t+j} + \left( \frac{M_{t+j}}{P_{t+j}} \frac{i_{t+j}}{1 + i_{t+j}} \right) \right]
\]

\[
\frac{B_{t-1}(t)}{P_t} = E_t \sum_{t=0}^{\infty} m_{t+j} \left[ s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right]
\]

\[M, V_t(i_t, \cdot) = P_t Y_t\]

(The second equation is a short hand stand in for what is often a very complex monetary model, it may involve Phillips curves, sticky prices, Fed interest rate policies, Taylor rules, etc. In many cases I will argue that the second equation can’t even set $P$ on its own, so there is no coherent “money dominant” regime. But leave aside that doubt for now.)

Issues:

I Regimes

1. Two equations can determine $P$. The government sets $\{B, s, M\}$ (or $i$). It must do so in a way that both equations work

2. Classic. Fed sets $M, P$ from the second equation. But Monetary policy needs fiscal backing. The Treasury must agree (and be able) to change $s$ ex post. (Friedman 1946),

(a) There is a footnote in all monetary papers: “lump sum taxes ex post” to fulfill the “government budget constraint.”

(b) Feel it: Suppose the Fed wants deflation or disinflation? Then the Treasury must raise taxes.
(c) Fiscal theory is not a new discovery. In that sense, the fiscal equation is always and everywhere, the issue is just how achieved.

3. The Fiscal regime: The treasury sets P from the first equation. Then the Fed is passive.

4. Lots of analysis of these two equations. The “Game of chicken.” “Central bank independence” means “resist financing our deficits.”

5. Non-identification results. *In equilibrium, both equations hold.* Many have tried, and failed. Example: can we test Granger causality from B/P to s? B/P rises, then s rises. Does this prove we’re in a passive fiscal regime? A: no because *in equilibrium both equations hold always.* You can’t see response to off-equilibrium prices! (See the problem set)

6. I do not think this “regime” business is helpful. From a fiscal point of view, I now see it as, *is there really any other regime? Does the whole regime business make any sense at all, especially since it’s not testable? I think not.* I can read the same equations as: the Fed sets M. This sets in motion expectations of different $s_{t+j}$, and it is those expectations that drive the price level. Monetary policy is just a signaling device for fiscal policy, a way to help markets distinguish currency reforms (no future $s$) from debt sales (future $s$ will pay off debts). Current talk of monetary policy as a signal rather than doing anything direct (announcements about future interest rate targets) fall in this category of thinking. The Gold standard is (below) just a fiscal signal.

II What about helicopter drops? Doesn’t that prove $MV = PY$?

The helicopter drop works in the top equation too! Helicopter drops are a *fiscal* operation, a transfer payment. Accounting would say this is “spending” financed by borrowing, and the Fed buys the bonds issuing money. The Fed is not allowed to do them!

What if you drop money, then announce tomorrow taxes will rise? There would be no effect! What if you drop bonds and announce credibly no new taxes? You get the same effect!

Helicopters are a brilliant psychological device to say there will be no following surpluses.

*Helicopter drops vs currency reforms vs. bond sales.*

a) They all look the same in the equation: $B$ rises. What’s the difference? Answer: they are only different in that each conveys a different set of expectations/commitments/historical memories about what the subsequent surpluses will be.

$$\frac{B_{t-1}(t)}{P_{t-1}} Q_{t-1}^{(1)} = \text{revenue} = E_{t-1} \sum_{j=0}^{\infty} \beta^{j+1} s_{t+j}$$
b) You can’t even inflate unless you convince people there will not be future s! Joint fiscal/monetary policy is always and everywhere. Did Japan just never convince people they wouldn’t pay back their debts?

c) Helicopter drop vs. open market operations. Monetarists say: all that matters is \( M \). So to them a helicopter drop which does not involve taking back debt is equal to an open market operation in which we buy debt. They deny the “wealth effect.” Well, yes if that’s the fiscal regime! If the fiscal authority responds to \( M \) and \( M \) only, and to inside MA: needs fiscal cooperation.

What about hyperinflations and all the money they print?

Hyperinflations are always fiscal! Would a hyperinflation be any different if the government printed one week debt to pay its bills and refused to print more money? Is there any case of a hyperinflation without a fiscal disaster – a “self fulfilling” inflation in a fiscally sound government?

What about the supposed “Stability of Money Demand” For example, Lucas’ Nobel lecture

![Graph](image)

or From Lucas “Money demand in the united states: a quantitative review” (though note here we get an interest elasticity which means \( MV(i) = PY \) which does not uniquely determine the price level!)
Answer: Both conditions hold in both theories. Causality can run $M -> MV = PY -> P -> FT -> surplus$. Or it can run $surplus -> FT -> P -> MV = PY -> M$. Money demand stability does not mean that money causes inflation. Rich guys smoke cigars; smoking cigars will not necessarily make you rich.

Ball bearing demand vs. income is steady. Ask the US airforce.

(A deeper problem: M here is usually M2, including savings accounts, not transactions accounts. That’s ever more suspect of reverse causality.)

Again: Non obserability theorem 2: You cannot test regimes. at least from time series from within a regime. Both equations hold. No “causality test” works.

### 3.4 Real, Nominal, Debt, and Equity

1. One period (floating rate) nominal debt is equity in the government!

   Microsoft
   
   $\# \text{shares} \times (\$/\text{Share}) = E_t \sum_{j=0}^{\infty} m_{t,t+j}s_{t+j}(\$)$

   Government: B is the “number of shares” into which s will be divided

   $\frac{B_{t-1}(t) (\$)}{P_t(\$/\text{good})} = E_t \sum_{j=0}^{\infty} m_{t,t+j}s_{t+j}(\text{goods})$

   - “Money as stock” We could use microsoft stock as numeraire, unit of account medium of exchange.
- Again, this is not a “budget constraint” In “bubble” price, MS does not have to raise earnings!

2. Indexed, foreign debt is debt – The government must either repay (raise taxes ex post) or default. (P given elsewhere)

\[ b_t = E_t \sum_{j=0}^{\infty} m_{t,j} s_{t+j}(goods) \]

(Our government could also default, I’m just assuming that it would prefer to inflate. The inflate/default question is actually worth thinking about.)

3. Sargent and Wallace, “unpleasant monetarist arithmetic”

There is a real vs. nominal debt difference between this analysis and SW

\[ \frac{B_{t-1}(t)}{P_t} = E_t \sum_{t=0}^{\infty} m_{t+j} \left[ (T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_t} \right] \].

\[ b_{t-1} = E_t \sum_{t=0}^{\infty} m_{t+j} \left[ (T_{t+j} - G_{t+j}) + \frac{M_{t+j} - M_{t+j-1}}{P_t} \right] \].

Sargent and Wallace: inadequate T-G means \( \Delta M \). \( \Delta M \) plus \( MV = PY \) means more \( P_{t+j} \). More \( P_{t+j} \) plus interest elastic money demand can mean more inflation today.

\[ M_t V (P_{t+1}/P_t) = P_t Y \]

\( \rightarrow \) future M means current P. (See the problem set: the more they put it off put off, the worse it is)

With nominal debt, there is a direct mechanism. It works without seignorage. A price level jump can devalue outstanding government debt. Lucas/Stokey state-contingent default.

This one disappears if you put it off at all.

Note: “monetization.” In the frictionless model, the government prints money to pay off debt, doesn’t soak it up by the end of the day. So, yes, it’s still “monetization” and in theory a Fed could refuse. It all happens intra “day” (or instantly) so nobody holds money overnight, and \( \Delta M \) does not help fiscally.

3.5 Accounting for recessions

\[ \frac{B_{t-1}(t)}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{k=0}^{j} \frac{1}{r_{t+k}} \right) s_{t+j} \right] \]
Why do we see procyclical $P$? What are sensible processes?

a) Surplus processes. What will not work: $s$ is an AR(1) $r$ is constant

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \varepsilon_t$$

$$\frac{B_{t-1}(t)}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \beta^j s_{t+j} \right] = \frac{\bar{s}}{1 - \beta} + \frac{1}{1 - \rho \beta} (s_t - \bar{s})$$

$s_t$ is worse in recessions. Also $B$ is not constant, increasing in recessions. This is totally wrong – it says $P$ is higher in recessions when $s$ is lower! How can we fix that?

Start by thinking of a $s$ process that can give rise to a constant price level. By

$$B_{t-1}(t) (E_t - E_{t-1}) \left( \frac{1}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

the right hand side cannot move! Thus a negative shock to surplus today (borrowing) must be followed by a positive shock to future surpluses – paying off the debt. The impulse response function must be negative, but then positive, with the weighted sum of responses equal to zero! An AR(1) doesn’t do that.

b) $r$ declines in recessions! “Flight to quality” and Fed actions. This is deflationary! Yes, lower $r$ means you lower $P$, even with no action on $s$. In “understanding policy” I argued this is why “aggregate demand” fell in 2008. It looks like a good mechanism for understanding why inflation is lower in recessions in general, even though deficits are worse in recessions. The present value of government debt rises in recessions because interest rates decline. Or vice versa – people want to hold government debt, sending up its price and down the price of everything else.

This seems a good way to start to understand the data.

A variance decomposition that showed whether variation in government debt corresponds to returns or to future surpluses would be a great thesis topic!

“Understanding policy.” That also suggests that the Fed can raise “demand” or inflate, by working on the interest rate. Raising real rates would cause inflation, an unusual sign prediction. The most effective would be to raise real rates on Government debt relative to other debt, which is precisely what the Fed has been doing.

It also suggests fiscal stimulus – more $B$, worse current $s$ – can only work if you don’t try to persuade people about future $s$.

A graph to emphasize the “flight to quality”. People are dumping BAA debt and trying desperately to buy treasuries of all maturity.
3.6 Long Term Debt

1. Dynamics change a lot!

2. Example: Perpetuity outstanding, no buying and selling.

\[ \frac{c}{P_t} = s_t. \]  \hspace{1cm} (1)

Inflation only happens when the actual poor surpluses \( s_{t+j} \) are realized, and not in anticipation of those surpluses! The whole present value depended on short term debt!

3. The present-value equation (26) still holds, in the form

\[ \frac{B_t}{P_t} = \int_{j=0}^{\infty} Q_t^{(j)} B_t^{(j)} \frac{dj}{P_t} = E_t \int_{\tau=0}^{\infty} \frac{\Lambda_{t+\tau} s_{t+\tau}}{\Lambda_t} d\tau, \]  \hspace{1cm} (2)

\( B_t = \) market value of nominal government debt, \( B_t^{(j)} \) denotes maturity \( j \) debt and

\[ Q_t^{(j)} = E_t \left( \frac{\Lambda_{t+j} P_t}{\Lambda_t P_{t+j}} \right) \]

the market value of debt – interest rates can absorb expected-surplus shocks. Coupon example. \( s_{t+j} \) raises only \( P_{t+j} \), lowers bond prices \( Q_t^{(j)} \), so bond prices in the numerator of (2) do all the adjusting at \( t \) rather than time-\( t \) prices \( P_t \) in the dominator.