

# Equity Premium and the link between Macroeconomics and Finance

John H. Cochrane

University of Chicago

## Equity Premium puzzles.

- ▶ Goal: Understand  $E(R^e)$  patterns, relation to macroeconomy.
- ▶ Natural Framework

$$1 = E \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right]$$

Doesn't work very well. (Yet.) (*Asset Pricing*).

- ▶ “Equity premium puzzle.” Why not?

$$E(dR^e) = \gamma \text{cov} \left( dR_t^e, \frac{dc}{c} \right)$$

$$E(R_{t+1}^e) \approx \gamma \text{cov} (R_{t+1}^e, \Delta c_{t+1})$$

$$\frac{E(R_{t+1}^e)}{\sigma(R_{t+1}^e)} \approx \gamma \sigma(\Delta c_{t+1}) \rho$$

$\rho$  is sensitive to timing. Even more robust, what if  $\|\rho\| = 1$ ?

$$\frac{\|E(R_{t+1}^e)\|}{\sigma(R_{t+1}^e)} \leq \gamma \sigma(\Delta c_{t+1})$$

# Equity Premium puzzles

- ▶ HJ bound

$$\frac{\|E(R_{t+1}^e)\|}{\sigma(R_{t+1}^e)} \leq \gamma \sigma(\Delta c_{t+1})$$

- ▶ Rough numbers

$E(\Delta c)$	$\sigma(\Delta c)$	$E(R^e)$	$\sigma(R^e)$	$\text{corr}(\Delta c, R^e)$
2	2	8%	16%	0.4

$$\frac{0.08}{0.16} = 0.5 < \gamma \times 0.02 \Rightarrow \gamma > 25?$$

- ▶ “Correlation puzzle.”,  $\rho < 0.5$

$$\frac{E(R_{t+1}^e)}{\sigma(R_{t+1}^e)} = \gamma \sigma(\Delta c_{t+1}) \rho$$
$$0.5 < \gamma \times 0.02 \times 0.5$$

$$\gamma > 50?$$

# Equity Premium Puzzles

- ▶ “Risk free rate puzzle”

$$r_t^f = \delta + \gamma E_t \left( \frac{dc_t}{c_t} \right) - \frac{1}{2} \gamma (\gamma + 1) \sigma_t^2 \left( \frac{dc_t}{c_t} \right)$$
$$0.02 = \delta + \gamma \times 0.02 - \frac{1}{2} \gamma (\gamma + 1) (0.02)^2$$

1. First term “intertemporal substitution”

$$0.02 = \delta + 50 \times 0.02 \rightarrow \delta = -98\%$$

2. “Precautionary savings.”  $(0.02)^2 = 0.0004 = 0.04\% =$  Small. Not with big  $\gamma$ !

$$0.02 = 0.02 + \gamma \times 0.02 - \frac{1}{2} \gamma (\gamma + 1) (0.02)^2 \rightarrow \gamma = 99?$$

# Equity Premium Puzzles

- ▶ “Sensitivity puzzle”

$$r_t^f = \delta + \gamma E_t \left( \frac{dc_t}{c_t} \right) - \frac{1}{2} \gamma (\gamma + 1) \sigma_t^2 \left( \frac{dc_t}{c_t} \right)$$

$$r_t^f = \delta + 99 \times E_t \left( \frac{dc_t}{c_t} \right) - \frac{1}{2} 99(100) \sigma_t^2 \left( \frac{dc_t}{c_t} \right)$$

- ▶ Time-varying equity premium puzzle (dp forecasts)

$$\frac{E_t(R_{t+1}^e)}{\sigma_t(R_{t+1}^e)} = \gamma_t \sigma_t (\Delta c_{t+1}) \rho_t = \sigma_t (m_{t+1}) \rho_t$$

Time-varying Sharpe ratio needs a *conditionally heteroskedastic* discount factor. *Why does everyone get scared in recessions, “reach for yield” in good times?*

- ▶ A *Quantitative* puzzle. Signs are all great.
- ▶ A robust puzzle, quibbling about numbers/data will not easily solve. High Sharpes pervasive,  $\sigma(\Delta c) \ll 20\%$ .

# Why did Finance not notice?

- ▶ Finance

$$E(R^e) = \text{cov}(R^e, \Delta c) \gamma$$

$$E(R^e) = \frac{\text{cov}(R^e, \Delta c)}{\text{var}(\Delta c)} [\gamma \text{var}(\Delta c)] = \beta \lambda$$

$\lambda$  is usually a free parameter. Puzzle is economic basis of  $\lambda$ !

- ▶ CAPM

$$\Delta c = R^{\text{market}}; E(R^e) = \beta_m \lambda_m$$

No problem if  $\sigma_{\Delta c} = 20\%$ . Must see  $\Delta c$  for puzzle.

$$0.5 = \gamma \times 0.20 \rightarrow 2.5 = \gamma$$

- ▶ Portfolio calculations

$$w = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e)} \rightarrow 0.6 = \frac{1}{3} \frac{0.06}{(0.18)^2}$$

But the same theory says  $\Delta c = R^{\text{portfolio}}$ , ignored.

- ▶ The puzzle is that the market price of risk is so high, given that our economy is, in fact so “safe”  $\sigma(\Delta c) \approx 1 - 2\%$ ,  $\sigma(R) = 20\%$

# Hope for the power utility model

- ▶ How high is  $E(R)$  really?
  1. Data: is the observed premium luck/selection bias?
    - ▶ 50 years:  $\sigma/\sqrt{T} = 16/\sqrt{49} = 16/7 = 2.5!$
    - ▶ 20 years:  $\sigma/\sqrt{20} = 16/4.5 \approx 3.5$ .  $40/\sqrt{20} > 10!$
    - ▶ US is highest premium!
    - ▶ Will we see 6-8%  $E(R^e)$ ? Did our grandparents expect 8%?
  2. Long run returns depend on economic growth.

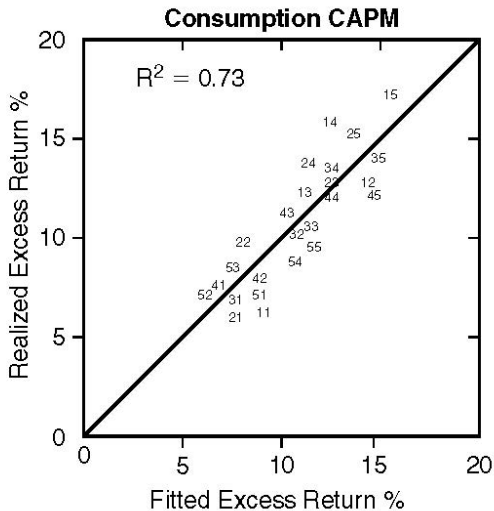
$$\sum_{j=1}^k \rho^{j-1} r_{t+j} = \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} + \rho^k p d_{t+k} - p d_t$$

Valuation risk is temporary. Knew about growth? Will it last?

- ▶ “Rare disasters.”  $\sigma(\Delta c_t)$  a lot bigger? Criticism: “Dark matter.”

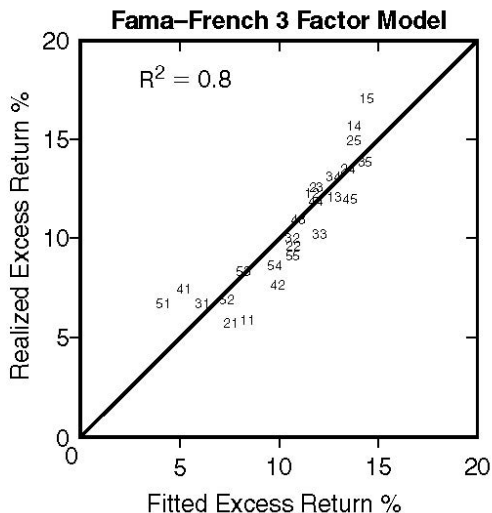
## Hope for the power utility model

- ▶ Long horizons, higher  $\rho$ , better measurement. Example: Jagannathan and Wang 2005





## Hope for the power utility model



But.. Doesn't fit  $R^f$ , (excess returns here), high  $\gamma$ . (Yet)

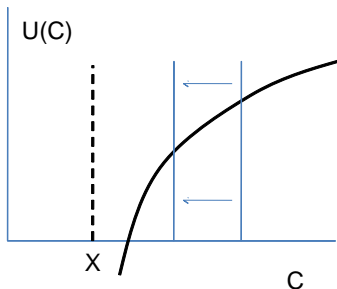
# Utility functions – Habits

- ▶ Objective: Match dp regressions, volatility, correlation with business cycle.
- ▶ Risk aversion, thus expected return, rises in recession, drive p/d down.

$$\frac{E_t (R_{t+1}^e)}{\sigma_t (R_{t+1}^e)} = \gamma_t \sigma_t (\Delta c_{t+1})$$

- ▶ A habit in the utility function (Problem set)

Rising risk aversion



# Habits



$$U_t = \frac{1}{1-\gamma} E \sum \beta^t (C_t - X_t)^{1-\gamma}$$

$$\Lambda_t = \frac{\partial U}{\partial C_t} = \beta^t (C_t - X_t)^{-\gamma} = \beta^t C_t^{-\gamma} \left( \frac{C_t - X_t}{C_t} \right)^{-\gamma} = \beta^t C_t^{-\gamma} S_t^{-\gamma}$$

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma}$$

- ▶ S = “fear of recession”
- ▶ Risk aversion

$$u_{cc} = -\gamma (C_t - X_t)^{-\gamma-1}$$
$$-\frac{C u_{cc}}{u_c} = \frac{-\gamma C (C_t - X_t)^{-\gamma-1}}{(C_t - X_t)^{-\gamma}} = \frac{-\gamma C}{C - X} = \frac{-\gamma}{S_t}$$

As  $C \searrow X$ , curvature rises!

# Habits

- ▶ Slow-moving habit. Not  $(C_t - \theta C_{t-1})^{1-\gamma}$ . Idea:

$$X_t = \sum \phi^j C_{t-j}; \quad X_t = \phi X_{t-1} + C_t$$

Instead, AR(1) for  $s_t = \log S_t$

$$\Delta s_{t+1} = -(1 - \phi)(s_t - \bar{s}) + \lambda(s_t)(\Delta c_{t+1} - g)$$

$$ds_t = \phi(\bar{s} - s_t)dt + \lambda(s_t) \left[ \frac{dc_t}{c_t} - gdt \right]; \quad dx_t = f(x_t, c_t)dt + g(x_t, c_t)dc_t$$

- ▶ Really simple, random walk consumption (“endowment”)

$$\Delta c_t = g + v_t$$

- ▶ Find

$$\frac{P_t}{C_t}(S_t) = E_t \left[ m_{t,t+1} \left( \left[ \frac{P_{t+1}}{C_{t+1}}(S_{t+1}) + 1 \right] \frac{C_{t+1}}{C_t} \right) \right].$$

# Habits

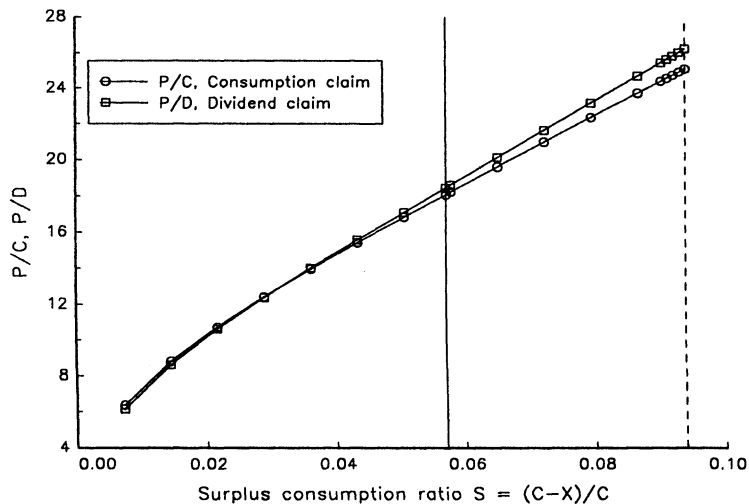


FIG. 3.—Price/dividend ratios as functions of the surplus consumption ratio

# Habits

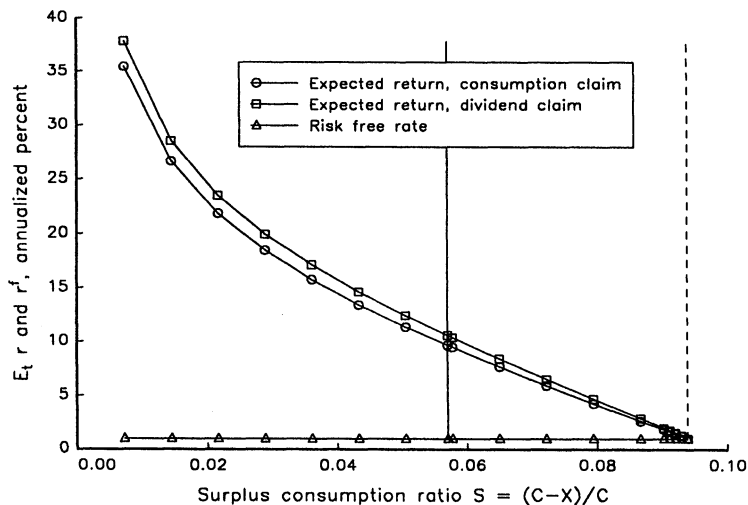
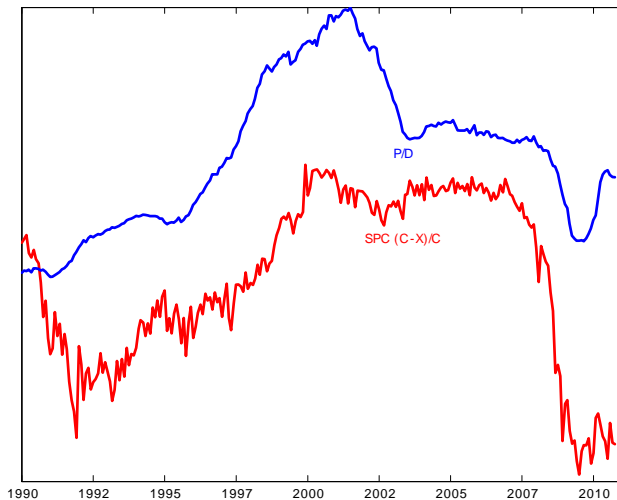


FIG. 4.—Expected returns and risk-free rate as functions of the surplus consumption ratio.

TABLE 5  
LONG-HORIZON RETURN REGRESSIONS

HORIZON (Years)	CONSUMPTION CLAIM		DIVIDEND CLAIM		POSTWAR SAMPLE		LONG SAMPLE	
	10 × Coefficient	R <sup>2</sup>	10 × Coefficient	R <sup>2</sup>	10 × Coefficient	R <sup>2</sup>	10 × Coefficient	R <sup>2</sup>
1	-2.0	.13	-1.9	.08	-2.6	.18	-1.3	.04
2	-3.7	.23	-3.6	.14	-4.3	.27	-2.8	.08
3	-5.1	.32	-5.0	.19	-5.4	.37	-3.5	.09
5	-7.5	.46	-7.3	.26	-9.0	.55	-6.0	.18
7	-9.4	.55	-9.2	.30	-12.1	.65	-7.5	.23

# Habits – and consumption risk



Here,  $X_t = k \sum_{j=0}^{\infty} \phi^j C_{t-j}$



# Habits, factors and the long-run equity premium

$$M_{t,t+k} = \delta^k \left( \frac{S_{t+k}}{S_t} \frac{C_{t+k}}{C_t} \right)^{-\gamma}.$$

- ▶ In one period  $S$  moves one for one with  $C$ , and “amplifies”.  
( $\Delta s_{t+1} = \dots + \lambda(s_t) (\Delta c_{t+1} - g)$ )
- ▶ Longer horizons,  $S$ ,  $C$  (fear of consumption decline) become uncorrelated. “Fear of recession” is stronger ( $\gamma = 2$ ).
- ▶ But  $S$  is stationary.  $C$  is a random walk, so  $\sigma(C_{t+k}/C_t)$  grows with  $k$ , while  $\sigma(S_{t+k}/S_t) \Rightarrow$  constant. Long run equity premium?
- ▶ Answer  $S^{-\gamma}$  is not stationary! ( $S$  fat tails).

# Habits, factors and the long-run equity premium

- ▶ General point. Most models below are of the form

$$M_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\gamma} f \left( \frac{x_{t+k}}{x_t} \right)$$

in continuous time

$$\frac{d\Lambda}{\Lambda} = -\delta dt - \gamma \frac{dc_t}{c_t} - f' dx_t$$
$$E_t(dR) = -\gamma \text{cov}(dR, \frac{dc}{c}) - f' \text{cov}(dR, dx).$$

1.  $\text{cov}(r, dx)$  helps to explain premiums
2. But with stationary  $x$  consumption takes over for long run returns?

## Habits – new directions

- ▶ Two shocks! Data  $\varepsilon^d, \varepsilon^{dp}$  uncorrelated.  $\Delta c$  is both a cashflow and a discount rate shock.
- ▶ More state variables (?)  $Y^{(l)} - y^{(s)}$ , etc. all move together. Reality? “single factor model for expected returns”
- ▶ Test; Other assets,  $1 = E(mR^{ei})$
- ▶ Leverage, stock of durable goods to produce habit like behavior?
- ▶ In general equilibrium.

## Recursive utility-main results

- ▶ Nonseparable across states – Epstein Zin, Long run risk

$$U_t = \left( (1 - \beta)c_t^{1-\rho} + \beta \left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} .$$

$\gamma$  = risk aversion  $\rho = 1/\text{eis}$ . Power utility for  $\rho = \gamma$ .



$$m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \left( \frac{U_{t+1}}{\left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} .$$

- ▶ Using  $R^c$  = claim to consumption to proxy for  $E_t U_{t+1}$

$$m_{t+1} = \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \right]^{\theta} \left( \frac{1}{R_{t+1}^c} \right)^{1-\theta} ,$$

$$\theta = \frac{1-\gamma}{1-\rho} .$$

# Recursive utility

- ▶ U from news of future consumption! ( $\rho \approx 1$ ).

$$\Delta E_{t+1} (\ln m_{t+1}) \approx -\gamma \Delta E_{t+1} (\Delta c_{t+1}) + (1 - \gamma) \left[ \sum_{j=1}^{\infty} \beta^j \Delta E_{t+1} (\Delta c_{t+1j}) \right]$$

News about *future* long-horizon consumption growth enters the *current* period  $m$ , “extra factor.”

- ▶ Features/thoughts
  1. iid  $\Delta c$ , reduces to power utility.
  2.  $\sigma [E_t (R_{t+1}^e) / \sigma_t (R_{t+1}^e)]$ ,  $\sigma_t (m_{t+1})$  must come from  $\sigma_t$  of consumption process.
  3. Is there really a lot of news about long run future  $\Delta c$ ? Is that really the fear? or “Dark Matter?”
  4. “Preference for early resolution of uncertainty.” Feature or bug?
  5. “Separates eis from risk aversion.” Yes, but so does habit.
  6. The index is total consumption, no  $u(c) + v(d)$
  7. News matters? ICAPM? Long run risk vs. ICAPM. ICAPM: news is reflected in current consumption.

## Constantinides and Duffie – idiosyncratic risk

- ▶ Attractive! But puzzle: how can idiosyncratic shocks matter?

$$E(mR) = E([\text{proj}(m|X) + \varepsilon] R) = E([\text{proj}(m|X)] R)$$

Answer: idiosyncratic  $m$  isn't idiosyncratic  $c$ ! Utility is nonlinear!

- ▶ Bottom line:

$$m_{t+1} = \beta \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

$y_{t+1}$  = *cross-sectional variance* of consumption growth.

$$\Delta c_{t+1}^i = \Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2; \sigma^2(\eta_{i,t+1}) = 1$$

so  $\text{cov}(R, y)$  can generate premiums.

## Constantinides and Duffie – idiosyncratic risk

$$\Delta c_{t+1}^i = \Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2; \sigma^2(\eta_{i,t+1}) = 1 \Rightarrow$$

$$m_{t+1} = \beta \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

► Derivation.

$$m_{t+1} = \beta \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

$$\begin{aligned} 1 &= E_t \left[ \beta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} R_{t+1} \right] \\ &= E_t \left[ \beta E_{t+1} \left[ e^{-\gamma(\Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2)} \right] R_{t+1} \right] \\ &= E_t \left[ \beta e^{-\gamma \Delta c_{t+1} + \gamma \frac{1}{2} y_{t+1}^2 + \frac{1}{2} \gamma^2 y_{t+1}^2} R_{t+1} \right] \\ &= E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} e^{\frac{1}{2} \gamma(\gamma+1) y_{t+1}^2} R_{t+1} \right] \end{aligned}$$

► Brilliant existence / reverse engineering theorem!

## Constantinides and Duffie – idiosyncratic risk

- ▶ Quantitatively true? *is*  $y_{t+1}$  what we need? (Remember *consumption*)

$$m_{t+1} = \beta \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

$$\sigma(m) = \sigma \left( e^{\frac{1}{2} \gamma(\gamma+1) y_{t+1}^2} \right) \approx \sigma \left( \frac{1}{2} \gamma(\gamma+1) y_{t+1}^2 \right)$$

$$\gamma = 1 \quad \sigma(y_{t+1}^2) = 0.5.$$

$y_{t+1}^2 = 0.5$  means  $y_{t+1} = \sigma(\Delta c_{it+1}) = 0.71$  cross sectional standard deviation of consumption growth. Need this *variation*, not the *level*. Avoid huge  $\gamma$ ?

- ▶ New work in data (Schmidt). Maybe individual rare “disasters” in recessions?



# Garleanu-Panageas heterogenous risk aversion

- ▶ Idea: Less risk averse hold more stocks. Lose more in a recession. The “average investor” gets more risk averse.



$$\max E \int e^{-\delta t} \frac{c_{At}^{1-\gamma_A}}{1-\gamma_A} dt + \lambda \int e^{-\delta t} \frac{c_{Bt}^{1-\gamma_B}}{1-\gamma_B} \quad \text{s.t.} \quad c_{At} + c_{Bt} = c_t$$

$$\text{FOC: } c_{At}^{-\gamma_A} = \lambda c_{Bt}^{-\gamma_B}$$

- ▶ Sharing rule result:

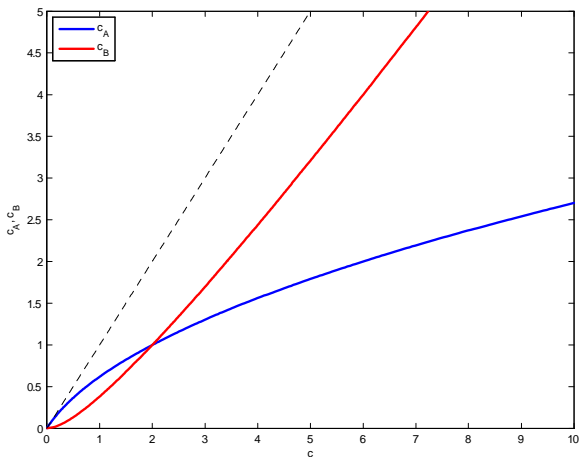
$$c_{At} = f(c_t) : \lambda^{\frac{1}{\gamma_B}} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} = c_t$$

$$c_{Bt} = g(c_t) : \lambda^{-\frac{1}{\gamma_A}} c_{Bt}^{\frac{\gamma_B}{\gamma_A}} + c_{Bt} = c_t$$

# Garleanu/Panageas heterogenous risk aversion

- ▶ Sharing rule,  $\gamma_A/\gamma_B = 2$ ,

$$c_{Bt}^{\frac{1}{2}} + c_{Bt} = c_t; \quad c_{At}^2 + c_{At} = c_t$$



# Garleanu-Panageas heterogenous risk aversion

1. Risk premiums:

$$\frac{dc}{c} = \mu dt + \sigma dz$$

$$\lambda \frac{1}{\gamma_B} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} = c_t$$

$$d \left( \lambda \frac{1}{\gamma_B} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} \right) = dc_t$$

$$\rightarrow \sigma \left( \frac{dc_A}{c_A} \right) = \frac{\frac{1}{\gamma_A}}{\frac{1}{\gamma_B} \frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A} \frac{c_{At}}{c_t}} \sigma \left( \frac{dc}{c} \right)$$

2.

$$\frac{E_t(dR) - rdt}{\sigma_t(dR)} \leq \gamma_A \sigma_t \left( \frac{dc_A}{c_A} \right) = \left( \frac{1}{\gamma_B} \frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A} \frac{c_{At}}{c_t} \right)^{-1} \sigma$$

Risk aversion is the consumption-weighted risk aversion of the two agents. In bad times, aggregate risk aversion rises!

# Production / Q theory

- ▶ Tie asset prices to macroeconomics through *producer* FOC.
- ▶ Q theory

$$V_t(k_t, \cdot) = \max_{\{i_t\}} E_t \int_{s=0}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} \pi_{t+s} ds \quad \text{s.t. } dk_t = (-\delta k_t + i_t) dt$$
$$\pi_t = \theta_t k_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] i_t$$

Envelope: cost of profit  $\pi_t dt$  due to  $idt =$  value of increase in  $k$ .  
Constant returns, so  $V(k_t, \cdot) = k_t V(1, \cdot)$

$$-\frac{\partial \pi_t}{\partial i_t} = \frac{\partial V_t}{\partial k_t}$$
$$1 + \alpha \left( \frac{i_t}{k_t} \right) = \frac{\partial V_t}{\partial k_t} = \frac{V_t}{k_t} = Q_t$$

Investment = function of  $M/B = Q$  (no error!)

# Production Q/ Theory

- ▶ Returns – “first-differenced q theory”

$$dR_t = \frac{dV_t + \pi_t dt}{V_t}; \quad \frac{V_t}{k_t} = 1 + \alpha \left( \frac{i_t}{k_t} \right)$$

(algebra)

$$dR_t = \frac{\left[ \theta_t - \delta - \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right)^2 \right] dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \left( \frac{i_t}{k_t} \right)} = dR_t^I$$

Discrete time

$$R_{t+1} = (1 - \delta) \frac{1 + \theta_{t+1} + \frac{\alpha}{2} \left( \frac{i_{t+1}}{k_{t+1}} \right)^2 + \alpha \left( \frac{i_{t+1}}{k_{t+1}} \right)}{1 + \alpha \left( \frac{i_t}{k_t} \right)} = R_{t+1}^I$$

- ▶  $R_{t+1} = R_{t+1}^I \approx a + b\Delta i_{t+1}$ , ex post.
- ▶ Intuition: R high when you go from low investment - log adj cost - low price to high investment - high adj cost - high price.
- ▶ Hence  $E_t R_{t+1} = E_t R_{t+1}^I$

# Production algebra

$$dR_t = \frac{dV_t + \pi_t dt}{V_t}; 1 + \alpha \left( \frac{i_t}{k_t} \right) = \frac{V_t}{k_t}$$

$$V_t = k_t + \alpha i_t$$

$$dV_t = dk_t + \alpha di_t = (i_t - \delta k_t) dt + \alpha di_t$$

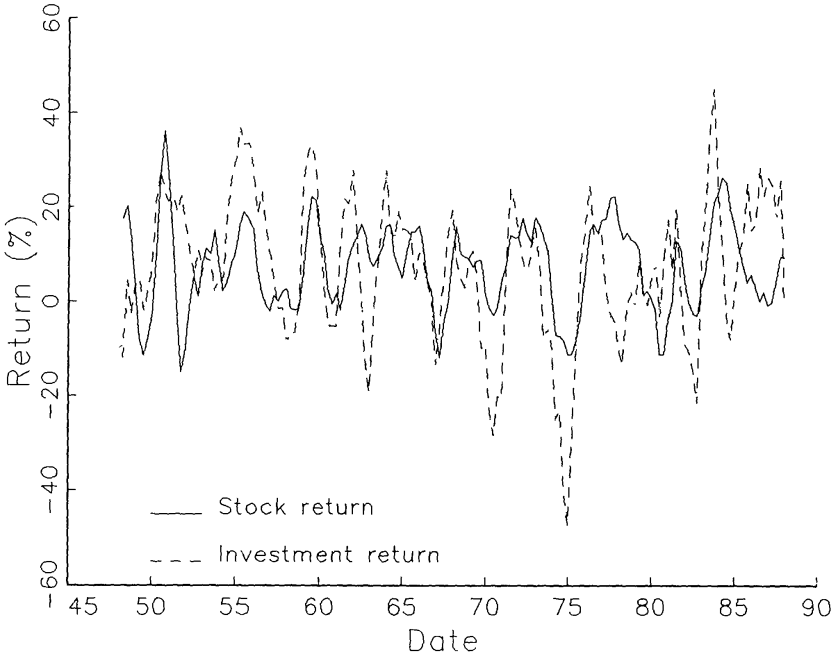
$$\frac{dV_t}{V_t} = \frac{(i_t - \delta k_t) dt + \alpha di_t}{k_t + \alpha i_t} = \frac{\left( \frac{i_t}{k_t} - \delta \right) dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \frac{i_t}{k_t}}$$

$$\frac{\pi_t}{V_t} dt = \frac{\theta_t k_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] i_t}{k_t + \alpha i_t} dt = \frac{\theta_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] \frac{i_t}{k_t}}{1 + \alpha \left( \frac{i_t}{k_t} \right)} dt$$

$$dR_t = \frac{\left( \frac{i_t}{k_t} - \delta \right) dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t} + \theta_t dt - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] \frac{i_t}{k_t} dt}{1 + \alpha \frac{i_t}{k_t}}$$

$$dR_t = \frac{\left[ \theta_t - \delta - \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right)^2 \right] dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \left( \frac{i_t}{k_t} \right)}$$

# Production



# Production

## Panel A. Single Regression

### 1. Quarterly Returns

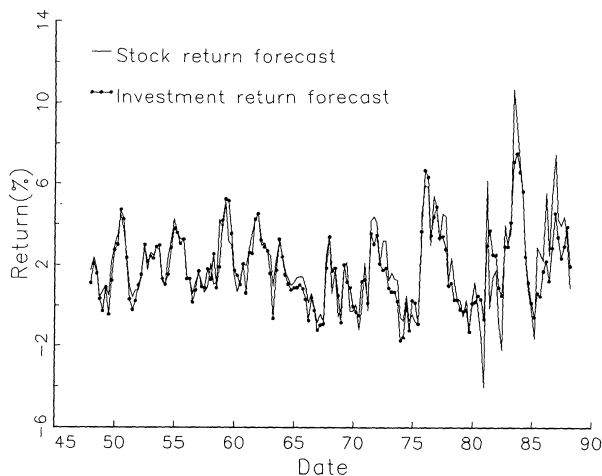
$$\text{Return}(t-1 \rightarrow t) = \alpha + \beta X(t-2) + \varepsilon(t)$$

Forecasting Variable	Stock Return		Investment Return		Stock—Inv.
	$\beta$	% <i>p</i> value	$\beta$	% <i>p</i> value	% <i>p</i> value
Term	0.16	0.53	0.10	0.05	24.10
Corp	0.35	0.94	0.16	0.23	12.44
Ret	0.16	2.51	0.15	0.00	88.56
<i>d/p</i>	1.32	0.26	0.11	70.70	1.22
<i>I/k</i>	-1.53	2.12	-1.71	0.00	79.96

$$E_t(R_{t+1}) = E_t(R_{t+1}^I), \text{ From "Production-Based Asset Pricing."}$$



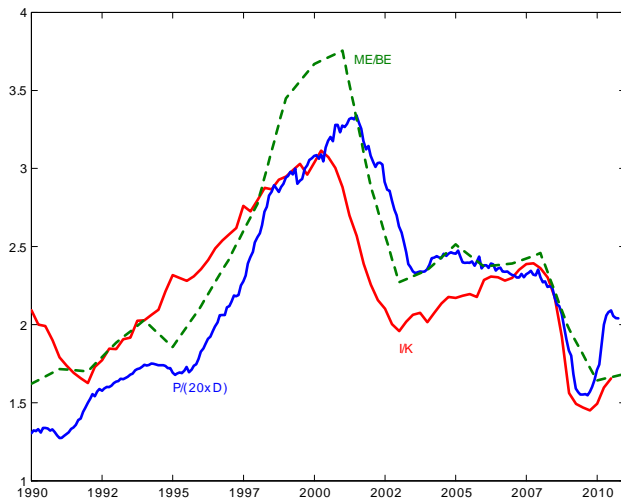
# Production



**Figure 3. Forecasts of quarterly stock returns and investment returns.** Forecasts are from linear regressions of returns on the term premium, corporate premium, lagged return and investment to capital ratio.

$$E_t(R_{t+1}) = E_t(R'_{t+1}), \text{ From "Production-Based Asset Pricing."}$$

# Production



$$1 + \alpha \frac{i_t}{k_t} = \frac{\text{market}_t}{\text{book}_t} = Q_t . \text{ From "Discount Rates"}$$

# Production

- ▶ Moral: Q Theory works pretty well! Investment responds to risk premiums, not to interest rates.
- ▶ Cross section as well: Growth (high B/M) invests a lot. (Zhan, Liu and Whited, JPE, etc.)
- ▶ Challenge: technologies that allow producers to transfer output *across states of nature*?
- ▶ General Equilibrium!

## Alternatives overview

- ▶ Goal: understand economics of (time-varying) risk premiums, connection to macro. Goal is not smaller alphas than hml, smb!  
Goal is to explain rmrf, smb, hml premiums.
- ▶ New utility functions.

1. Separable:

$$U(c_t, x_t) = u(c_t) + v(x_t); U_c(t) = u_c(c_t)$$

2. Nonseparable: new “factor”

$$U(c_t, x_t); U_c(c_t, x_t).$$

$$\frac{d\Lambda}{\Lambda} = \frac{cU_{cc}(c_t, x_t)}{U_c} \frac{dc_t}{c_t} + \frac{U_{cx}(c_t, x_t)}{U_c} dx_t$$

$$m_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right)^{-\delta}$$

- 2.1 Across goods (leisure, houses, etc. influence  $u_c$ )
- 2.2 Across time – habits, durables,  $c_{t-k}$  influences  $u_c(t)$ .
- 2.3 Across states of nature/non expected utility

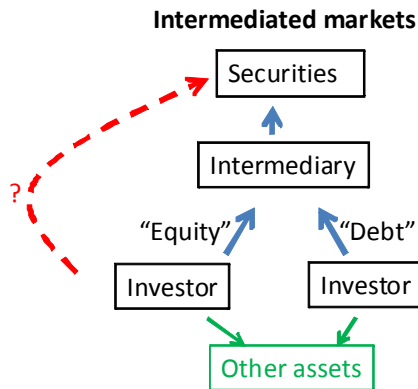
$$E[u(c)] \neq \sum_s \pi_s u[c(s)]$$

3. Psychology in place of utility function?  $\sum \pi u(c)$   $\pi$  wrong?

# Overview

- ▶ Keep utility, change market structure (full insurance!)
- ▶ Heterogeneity matters
  1. Idiosyncratic risk – not perfect risk sharing.
  2. Shifts in wealth change aggregate risk aversion.
- ▶ Production side; General Equilibrium
- ▶ Segmented markets, narrowly held risks, consumption of intermediaries/stockholders, “institutional finance/frictions,” trading/information matter.

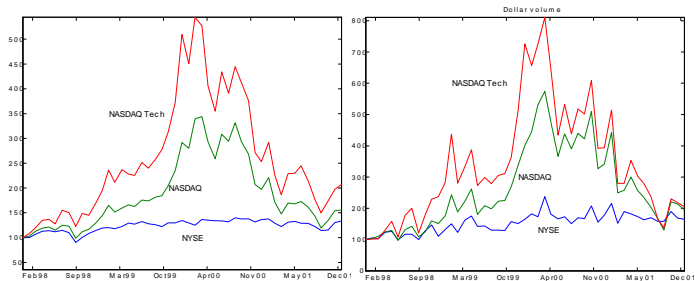
# Overview



- ▶ Segmented markets, narrowly held risks, consumption of intermediaries/stockholders, “institutional finance/frictions

# Overview

- ▶ Trading/information matter for prices?



- ▶ Why *are* people scared to hold stocks in recessions? What's "bad times? / high  $m$ ?" Much to do!