Equity Premium and the link between Macroeconomics and Finance

John H. Cochrane

University of Chicago
Equity Premium puzzles.

- Goal: Understand $E(R^e)$ patterns, relation to macroeconomy.
- Natural Framework

$$1 = E \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right]$$

 Doesn’t work very well. (Yet.) (*Asset Pricing*).
- “Equity premium puzzle.” Why not?

$$E(dR^e) = \gamma \text{cov}(dR^e_t, \frac{dc}{c})$$

$$E(R^e_{t+1}) \approx \gamma \text{cov} \left( R^e_{t+1}, \Delta c_{t+1} \right)$$

$$\frac{E(R^e_{t+1})}{\sigma(R^e_{t+1})} \approx \gamma \sigma (\Delta c_{t+1}) \rho$$

$\rho$ is sensitive to timing. Even more robust, what if $\|\rho\| = 1$?

$$\frac{\|E(R^e_{t+1})\|}{\sigma(R^e_{t+1})} \leq \gamma \sigma (\Delta c_{t+1})$$
Equity Premium puzzles

- **HJ bound**
  \[
  \frac{\| E(R_{t+1}^e) \|}{\sigma(R_{t+1}^e)} \leq \gamma \sigma(\Delta c_{t+1})
  \]

- **Rough numbers**

<table>
<thead>
<tr>
<th>$E(\Delta c)$</th>
<th>$\sigma(\Delta c)$</th>
<th>$E(R^e)$</th>
<th>$\sigma(R^e)$</th>
<th>$\text{corr}(\Delta c, R^e)$</th>
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<td>8%</td>
<td>16%</td>
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\[
\frac{0.08}{0.16} = 0.5 < \gamma \times 0.02 \Rightarrow \gamma > 25?\]

- **“Correlation puzzle.”**, $\rho < 0.5$

\[
\frac{E(R_{t+1}^e)}{\sigma(R_{t+1}^e)} = \gamma \sigma(\Delta c_{t+1}) \rho
\]

\[
0.5 < \gamma \times 0.02 \times 0.5
\]

\[
\gamma > 50?\]
Equity Premium Puzzles

- “Risk free rate puzzle”

\[ r^f_t = \delta + \gamma E_t \left( \frac{d c_t}{c_t} \right) - \frac{1}{2} \gamma (\gamma + 1) \sigma_t^2 \left( \frac{d c_t}{c_t} \right) \]

\[ 0.02 = \delta + \gamma \times 0.02 - \frac{1}{2} \gamma (\gamma + 1) (0.02)^2 \]

1. First term “intertemporal substitution”

\[ 0.02 = \delta + 50 \times 0.02 \rightarrow \delta = -98\%? \]

2. “Precautionary savings.” (0.02)^2 = 0.0004 = 0.04\% = Small. Not with big \( \gamma \)!

\[ 0.02 = 0.02 + \gamma \times 0.02 - \frac{1}{2} \gamma (\gamma + 1) (0.02)^2 \rightarrow \gamma = 99? \]
Equity Premium Puzzles

“Sensitivity puzzle”

\[ r^f_t = \delta + \gamma E_t \left( \frac{dc_t}{c_t} \right) - \frac{1}{2} \gamma (\gamma + 1) \sigma^2_t \left( \frac{dc_t}{c_t} \right) \]

\[ r^f_t = \delta + 99 \times E_t \left( \frac{dc_t}{c_t} \right) - \frac{1}{2} 99(100) \sigma^2_t \left( \frac{dc_t}{c_t} \right) \]

Time-varying equity premium puzzle (dp forecasts)

\[ \frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} = \gamma_t \sigma_t (\Delta c_{t+1}) \rho_t = \sigma_t(m_{t+1}) \rho_t \]

Time-varying Sharpe ratio needs a conditionally heteroskedastic discount factor. Why does everyone get scared in recessions, “reach for yield” in good times?

A Quantitative puzzle. Signs are all great.

A robust puzzle, quibbling about numbers/data will not easily solve. High Sharps pervasive, \( \sigma(\Delta c) \ll 20\%, \).
Why did Finance not notice?

- Finance

\[
E(R^e) = \text{cov}(R^e, \Delta c) \gamma \\
E(R^e) = \frac{\text{cov}(R^e, \Delta c)}{\text{var}(\Delta c)} \left[ \gamma \text{var}(\Delta c) \right] = \beta \lambda
\]

\( \lambda \) is usually a free parameter. Puzzle is economic basis of \( \lambda \! \)!

- CAPM

\[\Delta c = R^{market}; \ E(R^e) = \beta_m \lambda_m\]

No problem if \( \sigma_{\Delta c} = 20\% \). Must see \( \Delta c \) for puzzle.

\[0.5 = \gamma \times 0.20 \rightarrow 2.5 = \gamma\]

- Portfolio calculations

\[
w = \frac{1}{\gamma \sigma^2(R^e)} \rightarrow 0.6 = \frac{1}{3} \frac{0.06}{(0.18)^2}
\]

But the same theory says \( \Delta c = R^{portfolio} \), ignored.

- The puzzle is that the market price of risk is so high, given that our economy is, in fact so “safe” \( \sigma(\Delta c) \approx 1 - 2\% \), \( \sigma(R) = 20\% \)
Hope for the power utility model

- **How high is $E(R)$ really?**

  1. **Data**: is the observed premium luck/selection bias?
     - 50 years: $\sigma / \sqrt{T} = 16 / \sqrt{49} = 16 / 7 = 2.5!$
     - 20 years: $\sigma / \sqrt{20} = 16 / 4.5 \approx 3.5$. $40 / \sqrt{20} > 10!$
     - US is highest premium!
     - Will we see 6-8% $E(R^e)$? Did our grandparents expect 8%?

  2. **Long run returns depend on economic growth.**

     $$\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k p d_{t+k} - p d_t$$

     Valuation risk is temporary. Knew about growth? Will it last?
     - “Rare disasters.” $\sigma(\Delta c_t)$ a lot bigger? Criticism: “Dark matter.”
Hope for the power utility model

- Long horizons, higher $\rho$, better measurement. Example: Jagannathan and Wang 2005
Hope for the power utility model

But.. Doesn’t fit $R^f$, (excess returns here), high $\gamma$. (Yet).
Utility functions – Habits

- Objective: Match dp regressions, volatility, correlation with business cycle.
- Risk aversion, thus expected return, rises in recession, drive p/d down.

\[ \frac{E_t \left( R^e_{t+1} \right)}{\sigma_t \left( R^e_{t+1} \right)} = \gamma_t \sigma_t \left( \Delta c_{t+1} \right) \]

- A habit in the utility function (Problem set)

Rising risk aversion
Habits

\[ U_t = \frac{1}{1 - \gamma} E \sum \beta^t (C_t - X_t)^{1-\gamma} \]

\[ \Lambda_t = \frac{\partial U}{\partial C_t} = \beta^t (C_t - X_t)^{-\gamma} = \beta^t C_t^{-\gamma} \left( \frac{C_t - X_t}{C_t} \right)^{-\gamma} = \beta^t C_t^{-\gamma} S_t^{-\gamma} \]

\[ M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} \]

- \( S = \) “fear of recession”
- Risk aversion

\[ u_{cc} = -\gamma (C_t - X_t)^{-\gamma-1} \]
\[ -\frac{C u_{cc}}{u_c} = -\gamma \frac{C (C_t - X_t)^{-\gamma-1}}{(C_t - X_t)^{-\gamma}} = -\gamma \frac{C}{C - X} = -\gamma \frac{C}{S_t} \]

As \( C \downarrow X \), curvature rises!
Habits

- Slow-moving habit. Not \((C_t - \theta C_{t-1})^{1-\gamma}\). Idea:

\[
X_t = \sum \phi^j C_{t-j}; \quad X_t = \phi X_{t-1} + C_t
\]

Instead, AR(1) for \(s_t = \log S_t\)

\[
\Delta s_{t+1} = -(1 - \phi) (s_t - \bar{s}) + \lambda(s_t) (\Delta c_{t+1} - g)
\]

\[
ds_t = \phi (\bar{s} - s_t) dt + \lambda(s_t) \left[ \frac{dc_t}{c_t} - g dt \right]; \quad dx_t = f(x_t, c_t) dt + g(x_t, c_t) dc_t
\]

- Really simple, random walk consumption ("endowment")

\[
\Delta c_t = g + \nu_t
\]

- Find

\[
\frac{P_t}{C_t}(S_t) = E_t \left[ m_{t,t+1} \left( \left[ \frac{P_{t+1}}{C_{t+1}}(S_{t+1}) + 1 \right] \frac{C_{t+1}}{C_t} \right) \right].
\]
Fig. 3.—Price/dividend ratios as functions of the surplus consumption ratio
Fig. 4.—Expected returns and risk-free rate as functions of the surplus consumption ratio.
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<tr>
<th>Horizon (Years)</th>
<th>Consumption Claim</th>
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TABLE 5
LONG-HORIZON RETURN REGRESSIONS

Habits
Here, $X_t = k \sum_{j=0}^{\infty} \phi^j C_{t-j}$
Habits, factors and the long-run equity premium

\[ M_{t,t+k} = \delta^k \left( \frac{S_{t+k}}{S_t} \frac{C_{t+k}}{C_t} \right)^{-\gamma}. \]

- In one period \( S \) moves one for one with \( C \), and “amplifies”. 
  \( \Delta s_{t+1} = \ldots + \lambda(s_t) (\Delta c_{t+1} - g) \)

- Longer horizons, \( S, C \) (fear of consumption decline) become uncorrelated. “Fear of recession” is stronger (\( \gamma = 2 \)).

- But \( S \) is stationary. \( C \) is a random walk, so \( \sigma(C_{t+k} / C_t) \) grows with \( k \), while \( \sigma(S_{t+k} / S_t) \Rightarrow \) constant. Long run equity premium?

- Answer \( S^{-\gamma} \) is not stationary! (S fat tails).
General point. Most models below are of the form

$$M_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\gamma} f \left( \frac{x_{t+k}}{x_t} \right)$$

in continuous time

$$\frac{d\Lambda}{\Lambda} = -\delta dt - \gamma \frac{dc_t}{c_t} - f' dx_t$$

$$E_t(dR) = -\gamma \text{cov}(dR, \frac{dc}{c}) - f' \text{cov}(dR, dx).$$

1. $\text{cov}(r, dx)$ helps to explain premiums
2. But with stationary $x$ consumption takes over for long run returns?
Habits – new directions

- Two shocks! Data $\varepsilon^d, \varepsilon^{dp}$ uncorrelated. $\Delta c$ is both a cashflow and a discount rate shock.
- More state variables (?) $Y^{(l)} - y^{(s)}$, etc. all move together. Reality? “single factor model for expected returns”
- Test; Other assets, $1 = E(mR^{ei})$
- Leverage, stock of durable goods to produce habit like behavior?
- In general equilibrium.
Recursive utility-main results

- Nonseparable across states – Epstein Zin, Long run risk

\[ U_t = \left( (1 - \beta) c_t^{1-\rho} + \beta \left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{1-\gamma} \right)^{1-\rho} \]

\( \gamma = \) risk aversion \( \rho = 1/eis \). Power utility for \( \rho = \gamma \).

\[ m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \left( \frac{U_{t+1}}{E_t \left( U_{t+1}^{1-\gamma} \right) \left( \frac{1}{R_c^{t+1}} \right)^{1-\theta}} \right)^{\rho-\gamma} \]

- Using \( R^c = \) claim to consumption to proxy for \( E_t U_{t+1} \)

\[ m_{t+1} = \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\rho} \right]^{\theta} \left( \frac{1}{R_{t+1}^c} \right)^{1-\theta} \]

\( \theta = \frac{1 - \gamma}{1 - \rho} \).
Recursive utility

- U from news of future consumption! \((\rho \approx 1)\).

\[
\Delta E_{t+1} (\ln m_{t+1}) \approx -\gamma \Delta E_{t+1} (\Delta c_{t+1}) + (1 - \gamma) \left[ \sum_{j=1}^{\infty} \beta^j \Delta E_{t+1} (\Delta c_{t+1}) \right]
\]

News about *future* long-horizon consumption growth enters the *current* period \(m_t\), “extra factor.”

- **Features/thoughts**
  1. iid \(\Delta c\), reduces to power utility.
  2. \(\sigma \left[ E_t (R_{t+1}^e) / \sigma_t (R_{t+1}^e) \right]\), \(\sigma_t (m_{t+1})\) must come from \(\sigma_t\) of consumption process.
  3. Is there really a lot of news about long run future \(\Delta c\)? Is that really the fear? or “Dark Matter?”
  4. “Preference for early resolution of uncertainty.” Feature or bug?
  5. “Separates eis from risk aversion.” Yes, but so does habit.
  6. The index is total consumption, no \(u(c) + v(d)\)
Attractive! But puzzle: how can idiosyncratic shocks matter?

\[ E(mR) = E([proj(m|X) + \varepsilon] R) = E([proj(m|X)] R) \]

Answer: idiosyncratic \( m \) isn’t idiosyncratic \( c \)! Utility is nonlinear!

Bottom line:

\[ m_{t+1} = \beta \left( e^{\frac{\gamma (\gamma + 1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \]

\( y_{t+1} = \text{cross-sectional variance of consumption growth.} \)

\[ \Delta c_{t+1}^i = \Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2; \sigma^2(\eta_{i,t+1}) = 1 \]

so \( \text{cov}(R, y) \) can generate premiums.
\[ \Delta c_{t+1}^i = \Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2; \quad \sigma^2 (\eta_{i,t+1}) = 1 \Rightarrow \]

\[ m_{t+1} = \beta \left( e^{\gamma (\gamma + 1) y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \]

- Derivation.

\[ 1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] \]

\[ = E_t \left[ \beta E_{t+1} \left[ e^{-\gamma (\Delta c_{t+1} + \eta_{i,t+1} y_{t+1} - \frac{1}{2} y_{t+1}^2)} \right] R_{t+1} \right] \]

\[ = E_t \left[ \beta e^{-\gamma \Delta c_{t+1} + \gamma \frac{1}{2} y_{t+1}^2 + \frac{1}{2} \gamma^2 y_{t+1}^2} R_{t+1} \right] \]

\[ = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} e^{\frac{1}{2} \gamma (\gamma + 1) y_{t+1}^2} R_{t+1} \right] \]

- Brilliant existence / reverse engineering theorem!
Quantitatively true? is $y_{t+1}$ what we need? (Remember consumption)

$$m_{t+1} = \beta \left( e^{\frac{\gamma(\gamma+1)}{2} y_{t+1}^2} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

$$\sigma(m) = \sigma \left( e^{\frac{1}{2} \gamma(\gamma+1) y_{t+1}^2} \right) \approx \sigma \left( \frac{1}{2} \gamma(\gamma + 1) y_{t+1}^2 \right)$$

$$\gamma = 1 \quad \sigma(y_{t+1}^2) = 0.5.$$  

$y_{t+1}^2 = 0.5$ means $y_{t+1} = \sigma(\Delta c_{it+1}) = 0.71$ cross sectional standard deviation of consumption growth. Need this variation, not the level. Avoid huge $\gamma$?

- New work in data (Schmidt). Maybe individual rare “disasters” in recessions?
Garleanu-Panageas heterogenous risk aversion

- Idea: Less risk averse hold more stocks. Lose more in a recession. The “average investor” gets more risk averse.

- 

\[
\max E \int e^{-\delta t} \frac{c_A^t}{1 - \gamma_A} dt + \lambda \int e^{-\delta t} \frac{c_B^t}{1 - \gamma_B} dt \quad \text{s.t.} \quad c_A^t + c_B^t = c_t 
\]

\[ FOC : \quad c_A^{-\gamma_A} = \lambda c_B^{-\gamma_B} \]

- Sharing rule result:

\[ c_A^t = f(c_t) : \lambda \frac{1}{\gamma_B} c_A^t \gamma^B_A + c_A^t = c_t \]

\[ c_B^t = g(c_t) : \lambda \frac{1}{\gamma_A} c_B^t \gamma^A_B + c_B^t = c_t \]
Sharing rule, $\gamma_A / \gamma_B = 2,$

\[
\frac{1}{2} c_B^t + c_B^t = c_t; \quad c_A^t + c_A^t = c_t
\]
Garleanu-Panageas heterogenous risk aversion

1. Risk premiums:

\[
\frac{dc}{c} = \mu dt + \sigma dz
\]

\[
\lambda \frac{1}{\gamma_B} c_A^\gamma_B + c_At = c_t
\]

\[
d \left( \lambda \frac{1}{\gamma_B} c_A^\gamma_B + c_At \right) = dc_t
\]

\[
\rightarrow \sigma \left( \frac{dc_A}{c_A} \right) = \frac{1}{\gamma_A} \frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A} \frac{c_{At}}{c_t} \sigma \left( \frac{dc}{c} \right)
\]

2. 

\[
\frac{E_t (dR) - rdt}{\sigma_t (dR)} \leq \gamma_A \sigma_t \left( \frac{dc_A}{c_A} \right) = \left( \frac{1}{\gamma_B} \frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A} \frac{c_{At}}{c_t} \right)^{-1} \sigma
\]

Risk aversion is the consumption-weighted risk aversion of the two agents. In bad times, aggregate risk aversion rises!
Production / Q theory

- Tie asset prices to macroeconomics through *producer* FOC.
- Q theory

\[ V_t(k_t, \cdot) = \max_{\{i_t\}} E_t \int_{s=0}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} \pi_{t+s} ds \quad s.t. \, dk_t = (-\delta k_t + i_t) \, dt \]

\[ \pi_t = \theta_t k_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] i_t \]

Envelope: cost of profit \( \pi_t dt \) due to \( idt = \) value of increase in \( k \).

Constant returns, so \( V(k_t, \cdot) = k_t V(1, \cdot) \)

\[ - \frac{\partial \pi_t}{\partial i_t} = \frac{\partial V_t}{\partial k_t} \]

\[ 1 + \alpha \left( \frac{i_t}{k_t} \right) = \frac{\partial V_t}{\partial k_t} = \frac{V_t}{k_t} = Q_t \]

Investment = function of M/B = Q (no error!)
Returns – “first-differenced q theory”

\[ dR_t = \frac{dV_t + \pi_t dt}{V_t} ; \quad \frac{V_t}{k_t} = 1 + \alpha \left( \frac{i_t}{k_t} \right) \]

(algebra)

\[ dR_t = \frac{\left[ \theta_t - \delta - \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right)^2 \right] dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \left( \frac{i_t}{k_t} \right)} = dR_t^l \]

Discrete time

\[ R_{t+1} = (1 - \delta) \frac{1 + \theta_{t+1} + \frac{\alpha}{2} \left( \frac{i_{t+1}}{k_{t+1}} \right)^2 + \alpha \left( \frac{i_{t+1}}{k_{t+1}} \right)}{1 + \alpha \left( \frac{i_t}{k_t} \right)} = R_{t+1}^l \]

\[ R_{t+1} = R_{t+1}^l \approx a + b \Delta i_{t+1} , \text{ ex post.} \]

\[ \text{Intuition: } R \text{ high when you go from low investment - log adj cost - low price to high investment - high adj cost - high price.} \]

\[ \text{Hence } E_t R_{t+1} = E_t R_{t+1}^l \]
Production algebra

\[ dR_t = \frac{dV_t + \pi_t dt}{V_t}; 1 + \alpha \left( \frac{i_t}{k_t} \right) = \frac{V_t}{k_t} \]

\[ V_t = k_t + \alpha i_t \]

\[ dV_t = dk_t + \alpha di_t = (i_t - \delta k_t) dt + \alpha di_t \]

\[ \frac{dV_t}{V_t} = \frac{(i_t - \delta k_t) dt + \alpha di_t}{k_t + \alpha i_t} = \frac{(\frac{i_t}{k_t} - \delta) dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \frac{i_t}{k_t}} \]

\[ \frac{\pi_t}{V_t} dt = \frac{\theta_t k_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] i_t}{k_t + \alpha i_t} dt = \frac{\theta_t - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] \frac{i_t}{k_t} dt}{1 + \alpha \left( \frac{i_t}{k_t} \right)} \]

\[ dR_t = \frac{(\frac{i_t}{k_t} - \delta) dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t} + \theta_t dt - \left[ 1 + \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right) \right] \frac{i_t}{k_t} dt}{1 + \alpha \frac{i_t}{k_t}} \]

\[ dR_t = \left[ \theta_t - \delta - \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right)^2 \right] dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t} \]

\[ dR_t = \frac{\left[ \theta_t - \delta - \frac{\alpha}{2} \left( \frac{i_t}{k_t} \right)^2 \right] dt + \alpha \left( \frac{i_t}{k_t} \right) \frac{di_t}{i_t}}{1 + \alpha \left( \frac{i_t}{k_t} \right)} \]
The forecasting variables are jointly significant predictors of stock returns: the $x^2$ test for the joint significance has a probability value of 0.03% for $R_{t+1} = R_{I_t+1}$, from "Production-Based Asset Pricing."
Panel A. Single Regression

1. Quarterly Returns

\[ \text{Return (t - 1 \rightarrow t) = \alpha + \beta X(t - 2) + \varepsilon(t) } \]

<table>
<thead>
<tr>
<th>Forecasting Variable</th>
<th>Stock Return</th>
<th>Investment Return</th>
<th>Stock—Inv.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta )</td>
<td>( % p \text{ value} )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Term</td>
<td>0.16</td>
<td>0.53</td>
<td>0.10</td>
</tr>
<tr>
<td>Corp</td>
<td>0.35</td>
<td>0.94</td>
<td>0.16</td>
</tr>
<tr>
<td>Ret</td>
<td>0.16</td>
<td>2.51</td>
<td>0.15</td>
</tr>
<tr>
<td>( d/p )</td>
<td>1.32</td>
<td>0.26</td>
<td>0.11</td>
</tr>
<tr>
<td>( I/k )</td>
<td>-1.53</td>
<td>2.12</td>
<td>-1.71</td>
</tr>
</tbody>
</table>

\[ E_t (R_{t+1}) = E_t \left( R^I_{t+1} \right), \] From "Production-Based Asset Pricing."
Figure 3. **Forecasts of quarterly stock returns and investment returns.** Forecasts are from linear regressions of returns on the term premium, corporate premium, lagged return and investment to capital ratio.

Longer-term component that forecasts a long-term component in the stock return not found in the investment return. The fact that each of the variables except the dividend-price ratio is significant in single regressions, and individually insignificant but jointly significant in multiple regressions, suggests that these variables are all forecasting the same component of stock returns. Since these variables do not forecast the difference between stock and investment returns, singly or jointly, the forecastable component is the same in stock and investment returns. The fact that the dividend-price ratio is individually significant in multiple regression stock return forecasts suggests that it forecasts a different component of stock returns than the other variables. Since it forecasts the return difference, that component is not found in the investment return. The strong serial correlation of the dividend price ratio, and hence its forecast of returns, suggests the "long horizon" label.

**D. Regressions of Returns on Investment/Capital Ratios**

Figure 4 and Table IV present regressions of stock returns, investment returns, and their difference on investment/capital ratios. The regressions include investment/capital ratios before, contemporaneous, and subsequent to the return dates, so these regressions address all three issues—whether forecasts of the two returns from investment/capital ratios are the same, whether the association of the two returns with subsequent investment/capital ratios is the same, and whether the projections of returns on investment/capital ratios at many dates are the same. I start with the last issue and then consider the first two.

\[ E_t (R_{t+1}) = E_t \left( R^I_{t+1} \right), \text{ From "Production-Based Asset Pricing."} \]
$1 + \alpha \frac{i_t}{k_t} = \frac{\text{market}_t}{\text{book}_t} = Q_t$. From "Discount Rates"
Moral: Q Theory works pretty well! Investment responds to risk premiums, not to interest rates.

Cross section as well: Growth (high B/M) invests a lot. (Zhan, Liu and Whited, JPE, etc.)

Challenge: technologies that allow producers to transfer output across states of nature?

General Equilibrium!
Alternatives overview

- Goal: understand economics of (time-varying) risk premiums, connection to macro. Goal is not smaller alphas than hml, smb! Goal is to explain rmrf, smb, hml premiums.

- New utility functions.
  1. Separable:
     \[ U(c_t, x_t) = u(c_t) + v(x_t); U_c(t) = u_c(c_t) \]
  2. Nonseparable: new “factor”
     \[ U(c_t, x_t); U_c(c_t, x_t). \]
     \[ \frac{d\Lambda}{\Lambda} = \frac{cU_{cc}(c_t, x_t)}{U_c} \frac{dc_t}{c_t} + \frac{U_{cx}(c_t, x_t)}{U_c} dx_t \]
     \[ m_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{X_{t+1}}{X_t} \right)^{-\delta} \]

2.1 Across goods (leisure, houses, etc. influence \( u_c \))
2.2 Across time – habits, durables, \( c_{t-k} \) influences \( u_c(t) \).
2.3 Across states of nature/non expected utility
     \[ E[u(c)] \neq \sum_s \pi_s u[c(s)] \]

3. Psychology in place of utility function? \( \sum \pi u(c) \pi \) wrong?
Overview

- Keep utility, change market structure (full insurance!)
- Heterogeneity matters
  1. Idiosyncratic risk – not perfect risk sharing.
  2. Shifts in wealth change aggregate risk aversion.
- Production side; General Equilibrium
Segmented markets, narrowly held risks, consumption of intermediaries/stockholders, “institutional finance/frictions
Overview

- Trading/information matter for prices?

- Why are people scared to hold stocks in recessions? What’s “bad times? / high $m$?” Much to do!