# Facts 1: Equity Premium and Risk

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<th>Stocks Real</th>
<th>Bonds Real</th>
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<th>GDP</th>
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**BIG EQUITY PREMIUM** $ECR^{Stock} - R^{Bond}$ = 7%

**BIG VOLATILITY OF STOCK RETURNS** $\sigma(R^{Stock} - R^{Bond}) = 18.4$

Stocks correlated with economy

Why so risky?
FACTS 2: TIME VARYING RISK PREMIUM

\[ R_{t+1} = a + bX_t + \varepsilon_{t+1} \]

\[ E_t(R_{t+1}) = a + bX_t \]

\[ b \rightarrow 0 \Rightarrow E(R_{t+1}) = \text{const.} \]

\[ P_t \] forecasts returns

\[ \frac{P_t}{D_t} \]

\[ E_t(R_{t+1}) \text{ is big; varies a lot over time} \]

\[ \text{Big in recessions} \]

\[ P_t \] does not forecast \( D_t \)

\[ P_t = E \left[ \text{discounted div.} \right] \]
FACTS 3: THE CROSS SECTION OF STOCK RETURNS

\[ E(R^i) = \alpha_i + \beta_i E(R^m) \]

\[ E(R^i) = \alpha_i + b_i E(r_{mt}) + h_i E(hml) < S_i E(smb) \]

WHY? WHY \( E(R^m) \)? WHY \( E(hml) \)?

"MODEL OF RETURNS" - FACTORS
**FACTS SUMMARY**

**RISK PREMIUM**

**EXPECTED RETURN**

1) $E(R_e) \approx 7\%$ → Buy? Risk?

$\sigma(R_e)$ is big, 18%.

$R_e$ is correlated with $\Delta S$, $\Delta Y$ → Stocks fall in bad times

2) $E_t(\Delta R_{t+m})$ varies over time $\sigma[E_t(\Delta R_{t+m})] \approx 6\%$

→ Buy! Time varying risk premium?

3) $E_i(R_{t+m})$ varies across assets a lot!

**Factor models**

Why are "factors priced" premium for Value - Growth?
Theory Overview. Preview

- What’s it worth? Time and risk
- $\frac{dV}{dx}$ Risk Management
- Investment vs Equilibrium
- What does the market look like after investment?

Goal

$P_t = E_t \left( \beta \frac{V(t+1)}{V(t)} X_{t+1} \right)$

$P = E(mX)$

All classic issues of finance.
Theory Overview: \( X \) and \( U \) to \( P = E(M_X) \)

Payoff:
- Stock: \( P_t \rightarrow x_k = P_t + d_t \)
- Bond: \( P_t \rightarrow 1 \)

BET: \( P_t = 0 \)

+1 win

-1 lose

\( X_{1, i} \) (Random)

\( \mathbb{E} \)

Utility: \( X \rightarrow P? \) Value to who?

\( U(C_0, C_1, \ldots) = u(C_0) + \beta E_u [U(C_1, \ldots)] \)

Discount factor \( \rightarrow \) time
\[ u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \quad u'(c) = c^{-\gamma} \]

\[ \gamma = 1 \quad u(c) = \log(c) \]

\[ \max \quad u(c_t + \beta E_t u(c_{t+1} + \xi_{t+1})) \]

\[ p_t = \mathbb{E}_t \left[ \frac{u'(c_{t+h})}{u'(c_t)} \xi_{t+h} \right] = \mathbb{E}_t \left[ \beta \left( \frac{c_{t+h}}{c_t} \right) \xi_{t+h} \right] \]

- After investment
  - Your $A$ adjusts
  - Marginal

\[ p_t = \mathbb{E}_t (c_{t+1} | X_{t+1}) ? \]

\[ p_t = \mathbb{E}_t \left( \frac{1}{R} x_{t+1} \right) ? \]

\[ p_t = \mathbb{E}_t (M_{t+1} | X_{t+1}) \]

Random, Stochastic