HIGH-YIELD LENDING:
IT’S GOOD UNTIL IT’S NOT

Joseph L. Pagliari, Jr.†

June 13, 2017

{A revised version of this paper has been tentatively accepted in the PREA-sponsored, special real estate-issue of the Journal of Portfolio Management}

† University of Chicago Booth School of Business; joseph.pagliari@chicagobooth.edu

The author would like to thank Bassam Barazi, Bill Bennet, Michael Eglit, Jacques Gordon, Drew Ierardi, Tom Mattinson, Paul Mouchakkaa, Randy Mundt and Al Nickerson for their helpful comments. However, any errors or omissions are the responsibility of the author.
HIGH-YIELD LENDING: IT’S GOOD UNTIL IT’S NOT

Of late, one of the most popular opportunistic investment strategies has been high-yield lending (often filling the void left by commercial banks, which are dealing with larger capital requirements imposed by Basel III, and securitized lenders, which are dealing with the implementation of Dodd-Frank’s capital-retention requirements). While the activity is often referred to in a number of different ways (e.g., “B-piece” investing, “distressed” debt, levered loans, mezzanine financing, preferred (or “pref”) equity, etc.), the general principle is the same: the investor advances (debt or equity) capital which is subordinated to a first-mortgage loan (but is senior to the project’s equity). Despite the popularity of such investment strategies, surprisingly little has been written exploring the highly bifurcated nature of these investments: in their most-risky incarnations, they pay off in full in an “up” market but are wiped out in a “down” market. The purpose of this article is to focus on the aspects of pricing these instruments, exploring the potential risks involved and illuminating some of their complexities and nuances.

The balance of this paper addresses these concerns from a theoretical approach. Unfortunately, there is little empirical data to provide context for these theoretical assertions. While a number of potential vendors are exploring the possibilities of creating an index of high-yield debt, such initiatives are bedeviled by these very complexities and nuances of these high-yield instruments – which contribute to the difficulties of deeply understanding the empirical performance of these securities.

I. A GENERAL THEORY OF PRICING RISKY DEBT

In both practice and theory, it is well understood that the cost of indebtedness \( k_d \) increases geometrically as the project’s leverage increases. Conceptually, the cost of indebtedness can be described as:

\[
k_d = r_f + \gamma + \delta \frac{LTV}{1-LTV}
\]

1 As of the beginning of 2017, there were nearly 70 high-yield debt funds in the market, looking to raise nearly $65 billion of equity capital. The median targeted net return was 12.5%, with a weighted average of approximately 13.8% – at a time when the yield on the 5-year U.S. Treasury bond was approximately 1.9%. Sources: Commercial Mortgage Alert, St. Louis Federal Reserve Bank and the author’s calculations.

2 In their literature review of commercial mortgages, Jones and Sirmans (2016) scarcely mention high-yield lending.

3 For now, things will be kept simple by assuming that all such high-yield securities represent debt – as opposed to equity – investments. But, as subsequently discussed, some such securities are, instead, equity investments with debt-like features.

4 In practice, it is more typically the case that the cost of indebtedness has discrete jumps. For example, there may be one interest rate for leverage ratios of, say, 65-70% and another for 70-75%. For our purposes, the continuous version shown in Equation (1) better suits the analyses which follow.
Mortgage lenders require a spread over the risk-free rate ($r_f$) as compensation for the additional costs and structural differences ($\gamma$) between the Treasury bond market and the (commercial) mortgage loan market. In addition, the default-risk premium ($\delta$) can be viewed as reflecting the put option available to borrowers of non-recourse loans. The value of this put option increases as the loan-to-value ($LTV$) ratio increases. Equation (1) is a much-simplified version of more rigorous option-pricing models – as utilized, for example, in Merton (1974), Titman and Torous (1989) and McDonald (2006). These relationships are illustrated in Exhibit 1:

---

5 These structural differences generally fall into three categories: a) differences in duration for securities with identical maturities (while Treasury obligations represent semi-annual interest-only payments (with a “balloon” payment upon the maturity date), commercial mortgage loans often represent monthly self-amortizing payments), b) there are servicing costs (e.g., monitoring of loan covenants and escrow accounts, collection (and disbursement) of escrow payments, asset inspections, etc.) associated with commercial mortgage loans which are not associated with holding Treasury bonds, and c) the illiquidity of the private-market, very-nuanced high-yield loans as compared to the liquidity of the Treasury bonds.

6 The borrower’s ability to relinquish its rights to the collateral without incurring further liability (and, in some cases, to adversely alter the terms of the mortgage loan as the result of a debt restructuring mandated by a bankruptcy court) can be viewed as an option granted to the borrower by the lender in return for a higher interest rate than would otherwise be the case.

7 For convenience, it is assumed that the loan-to-value ratio and the debt-coverage ratio are mathematical inverses of one another. Among other matters, it obviates the need to make a distinction, upon a monetary default stemming from: a) the loan’s debt service exceeds the property’s cash flow (and the owner is unwilling/unable to fund the difference) and b) the loan balance exceeds the property’s fair market value (less transaction costs); both conditions are necessary in order to motivate the borrower to “hand back the keys” (i.e., default) – see, for example, Vandell (1984).

8 Culp, Nozawa and Veronesi (2016) suggest that credit spreads are wider than that suggested by such models – specifically the Merton (1974) model – due particularly to “…tail and idiosyncratic asset risks.”
The graph above is clearly a stylized illustration, holding all other variables constant for a given moment in time. For example, the default-risk premium ($\delta$) varies by the perceived riskiness (or credit quality) of the underlying property type (e.g., spreads are generally narrower for industrial properties than those found with hotels), geography (e.g., spreads are generally narrower in primary markets than in secondary or tertiary markets), sponsor quality (e.g., spreads are generally narrower for high-quality, experienced borrowers), etc. Exhibit 2 illustrates the importance of credit quality (which, for our purposes, is proxied by asset-level volatility, $\sigma$) in loan underwriting; consider the pricing (i.e., the mortgage interest rate) of two commercial mortgage loans where the collateral differs in their riskiness (i.e., where: $0 < \sigma_1 < \sigma_2$):

---

9 While the typical application Modigliani and Miller (1958) (“M&M”) application assumes a constant cost of debt, it should be pointed out that M&M also recognized that the cost of indebtedness rises with the degree of leverage. For example, see footnote 17 of M&M: “We can also develop a theory of bond valuation along the lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be nonlinear…”

10 As an empirical example of the curvature of this credit spread, consider the average spread to swap rates for various investment-grade tranches of 2016 CMBS offerings: “senior” AAA at 124 basis points (bps), “junior” AAA at 153 bps, AA at 314 bps and BBB at 626 bps. Source: Commercial Mortgage Alert and the author’s calculations.

11 In addition to varying collateral quality, spreads are time-varying (i.e., they generally narrow in “up” markets and widen in “down” markets), spreads may widen and/or narrow with varying maturity dates, and spreads may differ as between fixed- and floating-rate instruments.
While interest rates are easily observed for loan-to-value ratios below, say, 70-75%, they are much less so for leverage ratios above this threshold (i.e. the province of high-yield lending). Accordingly, two points are in order: First, this lack of transparency for high-yield (or high-leverage) loans should make us cautious about any strong assertion(s) concerning the risk-adjusted performance of such loans. (As a case in point, the difference in interest rates can be substantial as between loans of varying collateral quality – see above – for a given high loan-to-value ratio.) Second, this imprecision will be to the advantage of some lenders and to the disadvantage of others. In other words, this is an issue of market efficiency in this largely private and unregulated high-yield debt market, also known as the “shadow banking” system. It is an empirical question (unanswerable with current data) as to whether the (vast) majority of lenders find themselves making loans at interest rates that, on average, equal the risk-adjusted rate or, alternatively, some lenders are persistently advantaged (and,  

Two technical points: a) in the limit, the mortgage interest rate approaches the expected return on the unlevered asset: \( E(k_u) \) [i.e., the boundary condition is: as \( LTV \to \text{maximum} \), \( k_d \to E(k_u) \)], and b) lenders’ willingness to lend debt capital stops well short of 100% of the asset value – reflecting both the exogenous and endogenous costs (e.g., exacerbated conflicts of interest, weakening of competitive position, abandonment by key constituents, legal costs, delays and entanglements, etc.) of financial distress [see, e.g., Titman (1984) and Hortaçsu, et al. (2013)].

As a non-real estate aside: For highly levered firms which succumbed to financial distress, Andrade and Kaplan (1998) estimate the costs of that financial distress to be 10-20% of firm value.
symmetrically, others persistently disadvantaged). As part of the shadow-banking system, certain financial institutions (often affiliates of Wall Street firms) are able to offer borrowers a “one-stop shop” (i.e., a single source) with regard to their capital needs. Such institutions provide borrowers all of their debt capital needs and, in some cases, most all of their initial equity needs. These institutions utilize their lines of credit to facilitate a swift real estate closing; thereafter, such institutions often look to repay a portion of their corporate borrowing by securitizing a first-mortgage loan in the commercial mortgage-backed securities (CMBS) market (or selling a first-mortgage loan to a portfolio lender) and to repay the remaining portion of their corporate borrowing by selling (or syndicating) their remaining debt interest to a high-yield lender. In those cases in which such institutions also supply the preponderance of the borrower’s initial equity needs, these institutions typically look to the borrower to repay the bridge-equity loan with the proceeds of a subsequent (third-party, institutional) capital raise. Whether these one-stop shops have successfully used their scale and financial dexterity to their advantage is another example of an unexamined empirical question.

For investors contributing capital to high-yield debt funds, the question of being able to identify ex ante which of these funds/lenders (if any) are persistently advantaged by this imprecision is particularly daunting. (Of course, those investors who invest in multiple high-yield debt funds across multiple sponsors (or general partners) will eventually earn the market-clearing return.) For our purposes, we will assume that there is sufficient market activity that any persistent advantage is ultimately competed away for the (vast) majority of high-yield lenders.

II. THE YIELD CURVE PRICES EACH TRANCHE

A given yield curve also implies the pricing of each tranche. That is, the weighted average of each tranche’s interest rate must equal the market-clearing interest rate for the aggregate leverage ratio. This point is particularly apt to high-yield lending where relatively small increments in the leverage ratio can lead to large increases in the interest rate (and, accordingly, the required rate of return); in turn, these increases in the rate of return often imply large increases in the probability of default.

13 This is another instance of an instrument generally being described as a loan, when it is really an equity instrument – with debt-like features.

14 Notable instances of institutions providing one-stop financings, incorporating a bridge-equity loan, include: a) Wachovia and Merrill Lynch facilitating the ≈$7 billion acquisition of the apartment complex known as Peter Cooper Village/Stuyvesant Town in New York City, where the borrowing entity was a joint venture formed between BlackRock and Tishman-Speyer, and b) Lehman Brothers facilitating the ≈$24 billion acquisition of the publicly traded apartment REIT then known as Archstone-Smith, where the borrowing entity was a joint venture formed between Lehman Brothers and Tishman-Speyer. Both transactions took place in 2007. In the first case, the raise of third-party equity capital repaid the bridge-equity loan (of ≈$1 billion); in the second case, third-party equity capital was not raised – thereby failing to repay the bridge-equity loan (of ≈$2 billion).

15 The organizational documents of many of these high-yield funds permit the fund to invest in both mezzanine loans as well as bridge-equity loans. As noted above, the latter may be a particular risky form of subordinated lending.
Perhaps a numerical example best illustrates these points. For convenience’s sake, let’s assume the following regarding mortgage-loan pricing: the risk-free rate \((r_f)\) equals 4% per annum, structural differences \((\gamma)\) equal 0.4% per annum, and the default-risk premium \((\delta)\) equals 0.4% per annum.\(^{16}\) These are the parameters used to produce Exhibit 1. We can then expand Exhibit 1 to include the appropriate interest rates for given leverage ratios, beginning at 60%, as well as the appropriate interest rates for the potential tranches – stated in increments of five percentage points beginning after 60%. This is tantamount to assuming a first-mortgage loan underwritten at 60% leverage, with a mezzanine loan providing an additional 25% in debt financing, then subdivided into five equal-sized tranches of increasing leverage – as illustrated in Exhibit 3:

\(^{16}\) As earlier noted, the mortgage interest rate approaches the expected return on the unlevered asset, \(E(k_a)\), in the limit. However, the mortgage lender falls well short of advancing 100% of the capital structure – due to the adverse effects of financial distress. Assuming that limit (“Max LTV”) is 90% leverage, then, the boundary condition [which is: as \(LTV \rightarrow \text{maximum}, k_j \rightarrow E(k_a)\)] implies that the default-risk premium \((\delta)\) equals 0.4% per annum – based upon inverting Equation (1) and solving for \(\delta\) given the assumptions described herein – including that expected asset-level returns, \(E(k_a)\), equal 8% (see the next section). More generally, \[\delta = \left[ E(k_a) - r_f - \gamma \right] \frac{1 - \text{Max LTV}}{\text{Max LTV}}.\]
Given our assumptions, the 60% first-mortgage loan bears an interest rate of 5.00% and the five mezzanine tranches bear interest rates ranging from 6.86% to 17.33%. An alternative interpretation of Exhibit 3 would be a first-mortgage loan at higher leverage combined with a smaller mezzanine piece. As one example: consider a 75% first-mortgage loan with the mezzanine loan (still tranched in increments of five percentage points) supplying an additional 10% of the capital stack. The 75% first mortgage loan would carry an interest rate of 5.60% and the two mezzanine tranches would carry interest rates of 12.00% and 17.33%, respectively. This discussion is meant to illustrate the point that, in equilibrium, high-yield debt warrants significantly higher stated interest rates (e.g., consider the rates of 12.00% and 17.33%, relative to a 5.60% first-mortgage interest rate – in the example above) merely as compensation for the higher probability of default and its associated costs.

III. CONSIDERING ASSET-LEVEL VOLATILITY

While Exhibit 3 illustrates the contractual interest rate (as between the lenders and the borrower) for various tranches of the capital stack of mortgage indebtedness, the expected return to each tranche is, however, lower than the contractual rate once asset-level volatility is considered. To illustrate the point, let’s further assume the following regarding property returns: the normal distribution of unlevered asset-level returns ($k_a$) has an expected value $E(k_a)$ which equals 8% per annum and the volatility of that return ($\sigma_a$) equals 12%. Importantly, if the high-yield lender is to make a loan against a less (more) risky asset than the one presumed in this illustration, then the default-risk premium ($\delta$) would decrease (increase) in a manner consistent with preserving the boundary condition.

Exhibit 4 displays, via the bell-shaped curve, the assumed asset-level return distribution. Exhibit 4 also displays the payoffs to all pieces of the capital stack (including the equity position) across the illustrated range of likely returns. When asset-level returns are sufficiently disappointing (i.e., instances of negative returns), losses follow the absolute-priority rule, with the equity position absorbing the initial losses. (Of course, the equity position also enjoys the “upside” associated with

---

17 The contrast represents the difference between the average interest rate (as shown on the vertical axis of Exhibit 3 and as identified in Equation (1)) and the marginal interest rate (as identified in the text above the horizontal axis of Exhibit 3), as given by the derivative of Equation (1) with respect to the leverage ratio: $\frac{dk_a}{dLTV} = \frac{\delta}{(1-LTV)^2}$.

18 For high-yield lenders making loans on “transitional” (or non-core) assets, it may be even more difficult to determine “value” and, therefore, the appropriate debt cost vis-à-vis the loan-to-value ratio.

19 As a frame of reference, the average return on the NCREIF Property Index has been approximately 9.3% per annum, with a standard deviation of approximately 7.7% and is generally well described by the normal distribution. (The average real – or inflation-adjusted – return has been approximately 5.6% per annum.) However, the volatility of the Index differs from, and is lower than, the volatility of individual property’s return. In some concurrent work, it is estimated that the volatility of individual property’s return is approximately 12%. Of course, the distribution of returns for a given property can vary substantially from these parameters (including the assumption of normality).

20 Exhibit 4 shows a range of returns equal to $E(k_a) \pm 3(\sigma_a)$. 

---
its investment – as indicated by the green triangular area.) After the equity position has been exhausted, the losses travel down the mezzanine capital stack. As indicated by the red-hatched area of Exhibit 4, there is first a diminution of the expected return and ultimately a loss of principal for the various mezzanine holders. (If asset-level returns are excessively disappointing (i.e., losses that exceed those illustrated in Exhibit 4), there is first a diminution of expected return and ultimately a loss of principal on the least-risky mezzanine tranches and ultimately the first-mortgage loan.) The red-hatched area of Exhibit 4 also indicates the expected rates of return on the most-risky mezzanine tranches will, on average, fall short of the contractual interest rate.

When quantifying the expected return (and/or the volatility) of a particular tranche, this paper assumes a simple end-of-period model (e.g., there are no interim cash-flow distributions and/or debt-service payments); those returns – as more fully described in the Appendix – are determined by the assumed asset-level returns vis-à-vis the coupon payment due each tranche in which borrowers
“ruthlessly” exercise their default option when asset-level returns are insufficient to repay the most-junior tranche (and possibly more-senior tranches), while ignoring the costs of financial distress (including, generally, ignoring the “deadweight” costs of foreclosure/bankruptcy). While some might argue that an option-pricing model would more elegantly identify the pricing (and risk/return) of each tranche, such models tend to rely on no-arbitrage assumptions (e.g., the availability of a perfect hedge, which is costlessly and continuously adjustable, and the returns on the underlying asset are independent of the option-holder’s decisions) which are ill-suited to the opaque environment of high-yield lending.

Based on this paper’s model, Exhibit 5 identifies the shortfall between the contractual interest rate and the expected rate of return at various leverage ratios. The return shortfall is based on the marginal rate (as opposed to the average rate) – such that the expected shortfall is most clearly identifiable for the riskiest tranches. (While the divergence between the two rates is most noticeable for leverage ratios above 70%, there is also an imperceptible divergence for the lower leverage ratios as well.)

---

21 The decision to ruthlessly default can be contrasted with the decision to strategically do so. For a discussion and analysis of both the borrower’s strategic considerations and the lender’s response(s), see Brown, Ciochetti and Riddiough (2006).

22 Exhibit 4 illustrates mezzanine tranches as high as 90%, whereas Exhibits 1, 2, 3 and 5 confine the leverage ratio to 85%. The confinement to 85% is done merely to improve the readability of those graphs (because the vertical scale of those graphs would otherwise increase dramatically and, thereby, making it difficult to see the relationships highlighted therein).

23 To improve the readability of the shortfall as between the contract and expected rates, the tranche sizes were decreased to 2.5% of the borrower’s capital stack – whereas most of the rest of this paper assumes tranche sizes of 5%.
A point to which we will return is the notion that the risky mezzanine tranches increasingly face the probability of a loss of principal and forsaken interest income. Naturally, the probability depends on a variety of factors, including: the shape of asset-level distributions, the level of subordination, contractual interest rates on the more-senior piece(s), etc. For now, we will assume that a partial recovery of a mezzanine tranche (i.e., the high-yield lender experiences something less than a complete loss of principal upon a monetary default of the borrower) is possible.

Given our assumptions, Exhibit 6 illustrates the “hurdles” (i.e., the principal plus accrued interest for various tranches) needed for each mezzanine tranche to receive at least a partial payoff upon the borrower’s default. A liquidation of the collateral resulting in net proceeds which is less than one of these hurdles means that the more-junior tranches suffer a complete loss.
These analyses result in a variety of illustrative performance statistics, as shown in Exhibit 7. It should come as no surprise that the expected returns increase with the more-subordinated mezzanine tranches and so do the standard deviations of those returns. The implication is, of course, that such tranches are rationally priced. (In fact and as earlier noted, this paper is largely predicated on the rational pricing of these high-yield instruments.24)

But because the standard deviation of a particular tranche’s return is not a fully informative statistic – given the asymmetric nature (i.e., the “capped” upside) of returns to lending – the probability of a complete repayment (of principal and interest) is shown by tranche, as are the probabilities of: a complete repayment of principal but not interest, a partial repayment of principal (but no interest) and a complete loss of principal.

---

24 For example, the cost of indebtedness is – in part – a function of the asset’s expected return, which is – in turn – a function of the riskiness (or volatility) of that asset’s return.
The probabilities shown in Exhibit 7 – particularly those probabilities associated with partial or complete losses – underscore the asymmetric nature of lending returns, especially so for the riskiest mezzanine tranches. That is, “it’s good until it’s not.” To oversimplify, the returns to the 85% mezzanine tranche (i.e., in this example, assumed to supply the 80% to 85% portion of the capital stack) equal the coupon rate in approximately nine of ten such mezzanine loans; but in one of these ten loans, the 85% mezzanine tranche is (nearly or completely) wiped out. Similarly, the returns to the 90% mezzanine tranche (i.e., in this example, assumed to supply 85% to 90% of the capital stack) equal the coupon rate in approximately four loans in five; but in one of these five loans, the 90% mezzanine tranche is (nearly or completely) wiped out. As discussed elsewhere (and to reiterate an earlier point), the asymmetric nature of these returns also underscores the put option held by the borrower (when the property is financed with a non-recourse mortgage loan).

In essence, the lender’s put premium (i.e., the spread over the risk-free rate \( \eta \) plus structural differences \( \gamma \)) is earned when the loan is paid in full, but the lender incurs losses (first, a diminution of the expected return and, ultimately, a loss of principal) in all other outcomes – as illustrated in Exhibit 8:

---

Notes on Probability of Repayment:

\( (a) \) \( \text{Prob}(k_a \geq \text{Payoff}) \)

\( (b) \) \( \text{Prob}(\text{Payoff} > k_d \geq 0) \)

\( (c) \) \( \text{Prob}(-1 < k_d < 0) \)

\( (d) \) \( \text{Prob}(k_d = -1) \)

---

25 The returns to a given tranche can be viewed in the context of a truncated distribution. For more details, please see the Appendix.
The near-binary payoffs to mezzanine financing are highlighted in Exhibit 8. When the asset produces a return equal to or greater than the “hurdle” (i.e., principal plus accrued interest) for that particular tranche (85% leverage in this case), the high-yield security is paid in full (and receives the coupon yield). When the asset returns an amount less than the “hurdle” for next-most senior tranche (80% leverage in this case), the high-yield security suffers a complete loss. In between these two outcomes, the high-yield security suffers a partial loss (ranging from earning something slightly less than anticipated to nearly a complete loss of principal).

IV. A NOTE ON LEVERED LOANS
As part of the recent activity in high-yield lending, both private funds and public REITs are pursuing a “levered-loans” strategy. That is, the high-yield lender makes a subordinated loan to a borrower and, in turn, finances a portion of that lending activity by borrowing at the fund- or corporate level. Ignoring asset-level volatility gives the mistaken impression that these levered-loan strategies significantly increase the high-yield returns from what they would otherwise be.
However, these levered-loan strategies are tantamount to the high-yield lender selling the less-risky tranche(s). In equilibrium, the high-yield lender should earn exactly what would have been the case had the lender made a loan equal to the most junior tranche of those under consideration.

Here too, an example probably best illustrates the point. Let’s reconsider Exhibit 3 which is renumbered here as Exhibit 9; however, this time let’s assume that a lender wishes to make a high-yield loan that occupies 70 to 80% of the borrower’s capital stack; let’s further assume that the high-yield lender wishes to finance half of this capital advance with debt (making this a levered-loan strategy). Given our assumptions, the equilibrium interest rate on this (70-80%) piece of the capital stack is 10.67% (i.e., the average marginal return on the sum of the 75% and 80% mezzanine tranches); the fund (or corporate) lender should require an interest rate equal to 9.33% (i.e., the marginal return on the 75% mezzanine tranche); and the high-yield lender is left with a rate of return equal to 12.00% (i.e., the marginal return on the 80% mezzanine tranche) – as illustrated below:

Exhibit 9: Illustration of the Cost of Indebtedness as a Function of Loan-to-Value Ratio for a Given Maturity Date

At 60% leverage, the interest rate equals 6.00%.
At 65% leverage, the interest rate equals 6.54%.
At 70% leverage, the interest rate equals 7.00%.
At 75% leverage, the interest rate equals 7.55%.
At 80% leverage, the interest rate equals 8.10%.
At 85% leverage, the interest rate equals 8.65%.

This 5% tranche bears an interest rate of 6.66%.
This 5% tranche bears an interest rate of 7.28%.
This 5% tranche bears an interest rate of 7.90%.
This 5% tranche bears an interest rate of 8.55%.
This 5% tranche bears an interest rate of 9.20%.
This 5% tranche bears an interest rate of 9.85%.

An example of the “levered loan” strategy: The high-yield lender makes a loan occupying 70-80% of the borrower’s capital stack; however, half of the loan is funded with corporate debt.
In other words, the high-yield lender is “long” both the 75% and 80% mezzanine tranches, while simultaneously “short” the 75% mezzanine tranche; the net result is, of course, that the high-yield lender is “long only” the 75% mezzanine tranche. Here, variations of two earlier points are in order: 
a) the lack of transparency in the high-yield lending market should make us cautious about how strongly the equilibrium condition is realized in practice, and 
b) this imprecision will be to the advantage of some high-yield lenders and the disadvantage of others. Again, this is an issue of market efficiency; it is an empirical question (also unanswerable with current data) as to whether the (vast) majority of such high-yield lenders find themselves borrowing at, above or below interest rates equal to the “short” tranche.

However, a common complication of this strategy is often found in practice: Some high-yield lenders utilizing fund- or corporate-level debt do so by cross-collateralizing and -defaulting such loans. These “crossed” borrowings have the impact of changing the nature of the payoff function and, consequently, the self-contained example illustrated in Exhibit 9 no longer applies. In essence, the high-yield lender forsakes some “optionality” (i.e., the high-yield lender no longer enjoys some of the downside limitations with regard to a particular mezz loan had the borrowing not been cross-collateralized and -defaulted). While these “crossed” fund-level loans carry a lower interest rate (i.e., the institution making the fund- or corporate-level debt recognizes that its risk is reduced by cross-collateralizing and -defaulting such loans and, therefore, lowers the interest rate from what it would otherwise have been), the impact on the investors’ returns can be devastating with even moderate defaults on the high-yield portfolio.

Exhibit 10 continues the example illustrated in Exhibit 9, with the exception of assuming that the fund-level borrowings are crossed. The high-yield lender can either: i) make a series of high-yield loans that occupy 70 to 80% of each borrower’s capital stack and finance half of this capital advance with cross-collateralized debt, or ii) make a series of high-yield loans that occupy 75 to 80% of each borrower’s capital stack, which requires no fund-level debt. The reduction in the interest rate due to cross-collateralizing the loan portfolio has been arbitrarily assumed to be worth two percentage points. Both strategies require that same amount of investor equity. Given our earlier assumptions, the equilibrium interest rate on the first strategy is 10.67% (i.e., the average of 9.33% and 12.00%), with the fund-level debt requiring an interest rate equal to 7.33% (i.e., the equilibrium rate of 9.33% less 2.00% due to cross-collateralization), leaving the high-yield lender with an equilibrium interest rate equal to 14.00%. Alternatively, the equilibrium interest rate on the second strategy is 12.00% (which would have been the interest rate on the first strategy if not for the cross-collateralized fund-level loan bearing an assumed interest rate two percentage points below the equilibrium rate).

As shown in Exhibit 10, the benefit of the reduction in the interest rate due to the crossed borrowings has a *de minimus* impact on the analysis once losses are considered. Assume a range of

---

26 More broadly, any high-yield/mezzanine lender can be viewed as effectively having made a loan equal to the entire debt portion of the capital stack less the more-senior tranche(s).
loss rates from 0% to 50% (where the loss rates are measured from a complete payoff of principal plus accrued interest).

Given our assumptions, losses on most mezz tranches are a fairly rare event; moreover, increases in these loss rates clearly occur – on average – with declining probability. Nevertheless, the point here is, conditional on realizing the “down” state (i.e., when asset returns are insufficient to repay the current tranche(s)), “crossing” the borrowings on high-yield assets increasingly worsens the high-yield lender’s losses. For example, a loss rate of approximately 6.3% leaves the “crossed” high-yield lender (i.e., the first strategy) with merely a return of its initial principal advance (i.e., no accrued interest is realized); whereas the comparable loss rate is approximately 10.7% for the “uncrossed” high-yield lender (i.e., the second strategy). At a 25% loss rate, the crossed high-yield lender experiences a return of approximately –45%, whereas the uncrossed high-yield lender experiences a return of approximately –15%. And, at a 50% loss rate, the crossed high-yield lender experiences a near complete annihilation (i.e., a return of approximately –100%), whereas the uncrossed high-yield lender experiences a return of approximately –45%. 
V. SOME ADDITIONAL COMPLEXITIES AND NUANCES

As mentioned earlier, there are a number of complexities and nuances in high-yield lending that hinder our understanding – either on an ex-ante or ex-post basis – the risk-adjusted performance of such securities. These intricacies represent loan characteristics that ought to be priced (but it is notoriously difficult to do so); let’s review some of the more prominent ones:

- Small changes in the amount of debt which is senior to the mezzanine tranche(s) and in the amount which is subordinated to the mezzanine tranche(s) can have a substantial impact on the risk/return characteristics of the mezzanine tranche(s) in question – particularly so for the riskiest mezzanine tranches. As an illustration of these sensitivities, consider the chart presented in Exhibit 11 which indicates the difference between the contractual interest rate and the expected rate of return for various tranches:

<table>
<thead>
<tr>
<th>Most-Junior Tranche</th>
<th>65%</th>
<th>70%</th>
<th>75%</th>
<th>80%</th>
<th>85%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>85%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.69%</td>
</tr>
<tr>
<td>80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.73%</td>
<td>6.71%</td>
</tr>
<tr>
<td>75%</td>
<td></td>
<td></td>
<td></td>
<td>1.76%</td>
<td>2.24%</td>
<td>5.06%</td>
</tr>
<tr>
<td>70%</td>
<td></td>
<td>0.49%</td>
<td>1.13%</td>
<td>1.66%</td>
<td>3.92%</td>
<td></td>
</tr>
<tr>
<td>65%</td>
<td>0.12%</td>
<td>0.30%</td>
<td>0.79%</td>
<td>1.27%</td>
<td>3.16%</td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td>0.02%</td>
<td>0.07%</td>
<td>0.21%</td>
<td>0.60%</td>
<td>1.02%</td>
<td>2.63%</td>
</tr>
</tbody>
</table>

The interpretation of the exhibit is as follows: The most-senior tranche (shown on the vertical axis of Exhibit 10) represents the percentage of the project’s capital stack which is senior to the mezzanine tranche, while the most-junior tranche (shown on the horizontal axis) represents the percentage of the project's capital stack which is subordinate to the mezzanine tranche. As one example, the expected shortfall of a high-yield security which occupies a portion of the capital stack from 70% to 80% is 1.13% per annum.

While this shortfall estimate is just one measure of risk, it nicely serves to illustrate the point that all tranches are not created equally. For example, let’s compare two mezzanine tranches each occupying 15% of the borrower’s capital stack; in so doing, let’s choose the least- and most-risky of such tranches. That is, the expected shortfall of a security which occupies a portion of the capital stack from 60% to 75% is 0.21% per annum; meanwhile, the expected shortfall of a security which occupies a portion of the capital stack from 75% to 90% is 5.06% per annum. The latter is more than 24 times the former.

- It is not merely a legal technicality that sometimes “mezz debt” really is a mortgage obligation (secured by a mortgage lien) and in other instances it is equity (secured by an interest in the
ownership entity (e.g., partnership, limited liability company, trust, etc.) with debt-like features. Whether such securities are debt or equity instruments depends on the joint considerations of the borrower and the lender (and, as is often the case, the first-mortgage lender as well); those considerations include the permissibility of secondary (mortgage loan) financing, the ease of foreclosing, rights of redemption, income-tax considerations, etc. Nevertheless, the financial issue is that many of these features carry with them economic consequences. For example, the foreclosure of a mortgage loan is generally a slow-moving (state-court-administered) legal process, while the perfection of a security interest in an ownership entity is, alternatively, a fairly swift-moving legal process – often facilitated by simply replacing one managing partner with another. The speed with which the “mezz lender” removes/replaces a borrower in default may help mitigate the costs of financial distress upon the underlying collateral.

- Another technical area is that of loan covenants. While these restrictions (e.g., authority and/or notice provisions regarding debt-coverage ratios, loan-to-value ratios, junior financing, major capital improvements, change of use, change of ownership, etc.) may be decried as simply legal “boilerplate,” their economic ramifications may be substantive. A particularly apt example may be that of reserves (i.e., lender-mandated deposits) for a variety of matters (e.g., property taxes and insurance, future replacements, certain working-capital reserves, etc.); an increase in such reserves improves the collateral position of the lender (and, therefore, reduces the likelihood of default) while concurrently reducing the current cash flow to the borrower. Whether a mezz loan is covenant-light or - heavy can have significant economic consequences. Similarly, another contractual provision that is common to high-yield lending involves reinvestment-rate risk. Whereas the vast preponderance of first-mortgage indebtedness is restricted with regard to loan

---

27 The interest payments on “mezz debt” secured by a mortgage lien are tax-deductible, while preference payments on “mezz debt” secured by an ownership interest are not tax-deductible. The importance of their tax-deductibility depends on the income-tax status of the borrower.

28 To put an even finer point on the matter, those states which follow a judicial-foreclosure process tend to be more slow-moving than those states which follow a non-judicial-foreclosure process. The difference in speed and resources as between the two processes represents a distinction that ought to be priced – see, e.g., Mian, Sufi and Trebbi (2015).

29 Notwithstanding whether the borrowing entity contains a “bankruptcy-remote” entity (see, e.g., Lynch (2010)), the borrower – upon an uncured monetary default – will often attempt to seek protection under Chapter 11 of the U.S. Bankruptcy Code, whereby the debtor’s liabilities may be reorganized (including the “cram down” of certain lenders). Because bankruptcy proceedings take place in federal court (and because federal court supersedes state court), the initiation of a bankruptcy proceedings may substantially slow the foreclosure process – as well as alter the settlements that (secured and unsecured) lenders may receive.

30 See, for example, Leland (1994).

31 There can be significant variation in the degree of restrictive covenants in these private-market mezz loans. The lack of standardized loan documents for mezz loans stands in stark contrast with most first-mortgage loans. Because most first-mortgage loans are prepared as if the loans will be sold to the CMBS market – where standardization is highly prized – there is little variation in these loan documents. (So much so that one running joke among first-mortgage-loan borrowers is: “Press hard, the second copy is yours!” Translation: The loan documents are standardized and non-negotiable.)
prepayment, most high-yield loans are freely pre-payable one year after origination. As such, high-yield lenders are more susceptible to facing reinvestment-rate risk.

- The (possible) existence of loan origination fees (or discount points) would, all else being equal, serve to enhance the expected return of a given mezzanine tranche.

- A point made earlier (see §III) concerned partial settlements (i.e., some payoff between complete repayment and complete loss) upon a default and liquidation. The ability of the high-yield lender to receive a partial settlement in a foreclosure auction\(^\text{32}\) depends, in large part, on its ability to fund its expenses in connection with the dead-weight costs of bankruptcy/foreclosure, repay senior securities, fund necessary items (e.g., capital improvements, leasing commissions and tenant improvements) needed to preserve/enhance the collateral value, etc.; this is particularly true of the holder of the “fulcrum” security.\(^\text{33}\) In turn, this implies that the high-yield lender either has sufficient reserves on hand and/or access to a line of credit which enables the high-yield lender to protects its interest(s) upon a monetary default of the borrower (assuming that the high-yield lender believes that the collateral is worth more than the most-senior hurdle(s)).\(^\text{34}\)

If these forms of liquidity are not available, the expected returns worsen. To be more emphatic, Exhibit 12 assumes that the high-yield lender either has neither sufficient reserves on hand nor access to a line of credit:\(^\text{35}\)

\(^{32}\) If, instead of a mortgage loan, the “mezz debt” is secured by an ownership interest, then a UCC (Uniform Commercial Code) filing must be made. Economically speaking, UCC liquidations involve settlement procedures similar to foreclosure processes but are faster-moving. Such structures may also facilitate the replacement of the borrower, while keeping the senior loan current, upon the borrower’s default (e.g., see Mello and Quintin (2017)).

\(^{33}\) The “fulcrum” security is the most-junior mezzanine tranche for which the collateral value exceeds the balances of the more-senior obligations plus the current tranche. (There may, however, be some further stipulations (e.g., some 3% of the appraised value is used to determine the realizable collateral value) as contained in the inter-creditor agreements; see, e.g., Fileti (2012).) The fulcrum security is typically provided with certain rights in determining whether the defaulted borrower should be foreclosed upon or should be offered a forbearance (or “work-out”) agreement. Most mezz lenders would like to have “cure” provisions which permit the mezz lender to keep the senior mortgage loan current (and/or to purchase the senior loan) and to maintain control over the forbearance agreement (and its enforcement). Clearly, there can be differing financial motivations as between the holders of the most-senior loan instruments and those of the most-junior loan instruments (which are still in-the-money); generally, the former wants to foreclose while the latter (in the absence of a loan-to-own strategy) wants to forebear.

\(^{34}\) Carrying reserves and/or having access to a line of credit are not without their costs. Moreover, finding a “take-out” lender in the midst of a foreclosure process and in a “down” market can be quite challenging.

\(^{35}\) In order to avoid (perhaps) unfairly enriching the immediately senior tranche upon a foreclosure/liquidation event in which the subject tranche has neither sufficient reserves on hand nor access to a line of credit to protect its interest, it is assumed that the deadweight costs of bankruptcy/foreclosure equal whatever windfall the immediately senior tranche would have otherwise received.
As a consequence (and compared to Exhibit 7), the expected returns decline and the volatilities increase; this is particularly true of the riskiest tranches – due to the worsening instances (and, therefore, increasing probabilities) of a complete loss. The difference in expected return and volatility for each tranche is highlighted, with and without (lender) liquidity, in Exhibit 13.

<table>
<thead>
<tr>
<th>Portion of Capital Stack</th>
<th>Incremental Interest Rate</th>
<th>Incremental Payoffs ($2.68)</th>
<th>Incremental Cumulative Payoffs ($68.34)</th>
<th>Expected Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Complete Payoff (%)</th>
<th>Complete Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60% -- 65%</td>
<td>6.86%</td>
<td>$2.68</td>
<td>$68.34</td>
<td>6.81%</td>
<td>2.33%</td>
<td>99.95%</td>
<td>0.05%</td>
</tr>
<tr>
<td>65% -- 70%</td>
<td>7.81%</td>
<td>$2.70</td>
<td>$73.73</td>
<td>7.58%</td>
<td>4.99%</td>
<td>99.79%</td>
<td>0.21%</td>
</tr>
<tr>
<td>70% -- 75%</td>
<td>9.33%</td>
<td>$2.75</td>
<td>$79.20</td>
<td>8.44%</td>
<td>9.86%</td>
<td>99.18%</td>
<td>0.82%</td>
</tr>
<tr>
<td>75% -- 80%</td>
<td>12.00%</td>
<td>$2.82</td>
<td>$84.80</td>
<td>9.02%</td>
<td>18.02%</td>
<td>97.34%</td>
<td>2.66%</td>
</tr>
<tr>
<td>80% -- 85%</td>
<td>17.33%</td>
<td>$2.98</td>
<td>$90.67</td>
<td>8.61%</td>
<td>30.77%</td>
<td>92.57%</td>
<td>7.43%</td>
</tr>
<tr>
<td>85% -- 90%</td>
<td>30.67%</td>
<td>$3.40</td>
<td>$97.20</td>
<td>6.62%</td>
<td>50.64%</td>
<td>81.59%</td>
<td>18.41%</td>
</tr>
</tbody>
</table>

Notes on Probability of Repayment:

(a) \( \text{Prob}(k_a \geq \text{Payoff}) \)

(b) \( \text{Prob}(k_a = -1) \)

Asset Assumptions:

\( k_a = 8.00\% \)

\( \sigma_a = 12.00\% \)
These sorts of losses can be exacerbated by “thin” tranches (there is nothing sacrosanct with regard to the 5% tranches used throughout much of this paper). Consider, as one example, a mezz loan providing funding for 20% of the borrower’s capital stack and structured as one tranche; next, consider, as another example, an otherwise identical mezz loan which is structured with ten (thin) tranches – each sequentially providing 2% of the capital stack. The dead-weight costs associated with the borrower's default make it less likely that these thin tranches just barely “in the money” will find it economically compelling to protect its interest upon default.

On the other hand, the sorts of losses associated with an illiquid mezz lender holding the fulcrum security can be mitigated if the mezz lender can sell its position, albeit at a discount, or a more-junior lender steps in (e.g., pays off the more-senior loans in a foreclosure, cures the default on the most-senior mortgage loan, etc).

- Whereas Exhibit 8 highlights the asymmetric payoffs attributable to the put option sold to the borrower by the lender, one rational response to this asymmetry is to consider a “participating”
loan (i.e., an instrument with contingent interest – tied to the “upside” performance of the asset).
From an option-pricing perspective, the high-yield lender is both selling a put option to the
borrower and, simultaneously, buying a call option from the buyer. The net result is a reduction
in the contractual interest rate (in return for a partial participation in the project’s profits).

Because of the private nature of many of these contractual arrangements (as between high-yield
lender and borrower), most of these intricacies are unobservable to investors. Moreover and as the
foregoing discussions highlight, the vast number of these technical intricacies – even if observable –
makes it extremely difficult for investors to understand, analyze and price the complexities and
nuances of these high-yield instruments.

Finally and in the spirit of keeping matters as simple as possible, the model presented here is a
single-period, single-asset (total-return) model. However, neither a single period nor a single asset is
representative of how such high-yield debt funds operate in practice. (In other words, there can be
“model risk” quite apart from real estate risk.) For example, as the holding period lengthens, the
annualized volatility of private-market real estate returns decreases – even in the presence of auto-
correlation (see, e.g., Pagliari (2017)). However, this long-term volatility is complicated by the path-
dependent nature (including capital-market cycles, differences in cash-flow and appreciation returns,
amortization provisions, possible covenant violations, etc.) of these high-yield instruments; so, it is
often preferable to consider a survival function (see, e.g., Li (2000)) that examines the marginal
default probability given that the security has survived to the current period. Moreover, the model
used herein assumes that asset-level returns are stationary (i.e., time invariant). Given that the high-
yield lender’s return (in the absence of loan-to-own strategies) is essentially capped at the coupon
rate, the lender’s downside exposure should be of particular concern when the real estate market is
late in its cycle.36 Vintages originated in this part of the cycle tend to reflect more-aggressive
underwriting practices as well as heightened real estate risks. While these refinements are beyond the
scope of this article, they are worth keeping in mind as this evolving area of opportunistic real estate
investing matures.

VI. THE SECURITY’S ASYMMETRIC NATURE IS ATTENUATED BY THE “PROMOTE”
As earlier noted, investors contributing capital to these high-yield debt funds (whether private or
public) face daunting challenges when attempting to compare fund strategies and structures on a
risk-adjusted basis. Moreover, the asymmetric nature of these high-yield instruments is attenuated by

36 Among others, see Malpezzi and Wachter (2005), Mueller (2009), and Wheaton (1999) for a discussion of real estate’s
market cycles.
the (base) fees paid to the general partner and the promoted (or carried) interest paid to the general partner if a given minimum return is met.\textsuperscript{37}

Again, an example probably best illustrates the point. Let’s continue with our earlier assumptions and further assume that the general partner is paid base fees of 1.5% per annum (provided that the fund’s profits are non-negative) and a promoted interest of 20% of the fund’s net profits in excess of an 8% preferred return on the limited partners’ contributed capital.\textsuperscript{38} The effects of such a structure are illustrated in Exhibit 14:

The existence of the partnership’s (base and incentive) fees is to dilute the limited partners’ return in the “up” case (i.e., when asset-level returns exceed the (principal and interest) hurdle for the 85% mezzanine tranche) by slightly more than 300 basis points (i.e., 150 basis points for base fees plus

\textsuperscript{37} It is assumed for our purposes that, in the case of public fund (e.g., a publicly traded mortgage REIT), the general and administrative expenses (including incentive-stock awards to senior management) are roughly equal to the base and incentive fees paid in a private fund.

\textsuperscript{38} According to Preqin (2017), the mean base management fee for private debt funds was 1.43% per annum in 2016. According to \textit{Real Estate Alert} (2016), the median preferred return for high-yield debt funds was 8% per annum and the median promoted (or carried) interest was 20% (without a “catch-up” provision).
more than 150 basis points of carried interest). It is clear that the partnership’s (base and incentive) fees can substantively dilute the investor’s return. As argued in Jennings and Payne (2016) and elsewhere, such fees should be viewed in the context of the “alpha” (i.e., the risk-adjusted rate of return) generated by the investment.

VII. RETURNS TO HIGH-YIELD LENDING v. LEVERED EQUITY
An oft-asked question by investors in high-yield debt funds is how those returns compare to levered-equity returns. So, this section attempts to crudely frame the expected return and volatility of mezzanine lending in comparison to levered equity.

Naturally, the earlier-assumed (unlevered) asset’s return \[ \sim N(0.08, 12^2) \] represents the starting point for the analysis of levered equity and, as the borrower’s leverage ratio increases, the borrower’s cost of indebtedness increases as indicated in Equation (1). The results of this analysis, for levered equity up to 75% leverage, are shown in Exhibit 15. There, it is clear that aggressive high-yield lending – assuming the lender has sufficient liquidity\(^{40}\) to protect its interests in the event of the borrower’s default – looks, from a risk/return perspective, to be quite similar to moderate-leverage equity investing (e.g., the 85% mezzanine tranche produces anticipated results similar to a levered equity investor employing 45-55% leverage and the 90% mezzanine tranche produces anticipated results similar to a levered equity investor employing 65-70% leverage). Less-aggressive high-yield lending – again assuming the lender has sufficient liquidity – produces comparable to low-leverage equity investing (e.g., the 75% mezzanine tranche produces anticipated results similar to a levered equity investor employing less than 30% leverage).

It is also apparent from Exhibit 15 that the aggressive high-yield lending strategies, when the lender has insufficient liquidity, quickly represents a situation in which the risk/returns characteristics look less rewarding than levered equity.\(^ {41}\)

\(^{39}\) Once we consider asset-level volatility, the difference between the expected gross return and the expected net return is slightly more than 290 basis points (i.e., 150 basis points for base fees plus more 140 basis points of carried interest).

\(^{40}\) As before, no attempt has been made to quantify the costs of providing such liquidity. These costs can come in the form of the drag on returns caused by carrying cash balances, the costs associated with maintaining a credit facility (such as a line of credit) and/or uncalled capital commitments.

\(^{41}\) For purposes of this section, the returns on debt and equity instruments are gross (or before-fee) returns.
However, several caveats are in order with regard to the volatility of returns: As earlier discussed, the standard deviation imperfectly characterizes the riskiness of mezzanine tranches. But, as the percentage of debt financing increases for the levered equity investor, the standard deviation of its return also becomes an increasingly imperfect measure of risk. Exhibit 16 illustrates the differences in the payoff functions, assuming 80-85% of the borrower’s capital stack is the most-junior mezzanine tranche and, accordingly, a levered equity investor utilizing 85% leverage. Whereas the lender’s upside is capped (effectively at its coupon yield, as indicated by the blue dashed line), the equity investor’s upside is unlimited (as indicated by the green dashed line). As earlier noted, the equity investor has purchased a put option from the lender, leaving itself with a call option on future returns. And while both the equity and debt positions have downsides capped at a complete loss, those losses occur with greater frequency for the equity investor (i.e., any asset-level return which is insufficient to repay the most-junior debt tranche results in a complete loss for the equity investor) than they do for the debt investor (i.e., any asset-level return which is insufficient to repay the next-most-junior debt tranche results in a complete loss for the debt investor).
The inadequacies of the standard deviation as an indication of the dispersion in mezzanine returns is illustrated in Exhibit 17, which is identical to Exhibit 16 in all respects but two: 1) the expected (gross) return to the mezzanine tranche is shown by the solid blue line, and 2) the probability density function of that return distribution is shown by the blue-dashed bell-shaped curve. Considering the relatively small difference between the mezzanine loan’s expected return and its coupon yield, the standard deviation of that return substantially overstates the “upside” of mezzanine investing.

42 To be technically correct, the probability density function of the asset-level returns, as graphed in Exhibits 16 and 17, differs from the probability density function of the mezzanine returns, as added to Exhibit 17. More accurately, the probability density function of the mezzanine returns is implicitly represented by a third dimension, not shown in this two-dimensional depiction.

43 Something similar can be said about the pattern of equity returns vis-à-vis its standard deviation; the overstatement occurs however on the “downside” – where the investor’s loss is limited to 100% of the invested capital. However, this misstatement is a far smaller problem for the equity than it is the mezzanine piece.
Considering the relatively small difference in the mezzanine loans expected return and its coupon yield, the standard deviation of that return substantially overstates the upside of mezzanine investing, as indicated by the blue bars within the upper tail of the bell-shaped curve. This inadequacy also leads to thoughts about “downside” risk measures. While an overview of these measures is beyond the scope of this paper, the interested reader might consider measures such as semi-variance, the Sortino ratio (1991), value-at-risk (VaR), as well as other performance measures (e.g., see: Ingersoll, et al. (2007)). Of course, different investors will prefer one payoff structure to another based on their levels of risk aversion as well as concerns about regulatory constraints, liability-matching, current income, etc.

VIII. CONCLUDING REMARKS
This paper has attempted to examine the province of high-yield lending; for which, we have little empirical evidence. In so doing, the paper has used an equilibrium approach in which the cost of indebtedness: a) is tied to the expected return on the asset (which, in turn, is a function of the asset’s risk) and b) increases geometrically with increases in the project’s loan-to-value ratio. The lender has effectively sold a put option to the non-recourse borrower. Therefore, as the contractual interest rate increases with leverage, so too does the expected default probability. Consequently, the expected
rate of return on any given mezzanine tranche is lower than the contractual rate – with the shortfall widening for the most-junior tranches.

While the standard deviation of returns also increases with the leverage ratio, the standard deviation is an imperfect measure of risk – given the asymmetric nature of the payoffs to mezzanine lending. In order to improve our appreciation for the payoff characteristics of these securities, it is also helpful to examine – by tranche – the probabilities associated with underperformance.

While much is often made in practice of levered-loan strategies, they are essentially little more than the high-yield lender going “long” one or more of the mezz tranches and going “short” the senior-mezz tranche(s). The net result, in equilibrium, is identical to high-yield lender simply going “long” one or more of the junior-mezz tranches. The exception occurs when these levered loan strategies are financed with cross-collateralized debt; then, realized losses can quickly dissipate the high-yield lender’s capital position.

In practice, there can be a mind-numbing set of complexities and nuances (e.g., tranches of different sizes and of different positions in the capital stack, “mezz debt” which is really equity with debt-like features, covenant-light v. -heavy loan documents, whether the high-yield has sufficient liquidity in a market downturn to protect its position in a foreclosure auction, etc.). It is a daunting challenge to determine whether these intricacies have been fairly priced. And in turn, whether or not investors are fairly compensated for all the risks to which they have been exposed is an open question. The answer is further hampered by the (base and incentive) fees charged in many of these high-yield funds.

Finally and for simplicity, a single-period and single-property model has been used to illustrate certain key concepts. We have assumed that asset-level returns are stationary and normally distributed. Markets, however, experience cycles; it may be particularly disadvantageous to be a high-yield lender when such loans are originated at the tail end of the real estate cycle. Moreover, the path-dependent nature of these high-yield instruments merits more-advanced models; they ought to incorporate the patterns of auto-correlation found in private-market real estate returns and estimates of survival functions that examine the marginal default probability given that the security has survived to the current period. While these refinements are beyond the scope of this article, they are worth keeping in mind as this evolving area of opportunistic real estate investing matures.
IX. REFERENCES


*Real Estate Alert*, “Fee Scorecard for High-Yield Real Estate Funds,” June 8, 2016, p. 20.


X. **APPENDIX**

**Mezzanine Tranches with Full Lender Liquidity**

For purposes of this section, we assume that the high-yield lender has sufficient liquidity such that it is capable of funding its expenses in connection with the dead-weight costs of bankruptcy/foreclosure, repay senior securities (in the case of a foreclosure auction), fund necessary items (e.g., capital improvements, leasing commissions and tenant improvements) needed to preserve/enhance the collateral value, *etc.*

Let us first consider the contractual payoff (or hurdle) to the immediately senior mezzanine tranche (\(A\)) and the contractual payoff to the current mezzanine tranche (\(B\)), where \(A\) and \(B\) are expressed as a percentage of the total capital structure:

\[
\text{Payoff}_A = A \left( 1 + r_f + \gamma + \delta \frac{LTV_A}{1 - LTV_A} \right)
\]

\[
\text{Payoff}_B = B \left( 1 + r_f + \gamma + \delta \frac{LTV_B}{1 - LTV_B} \right)
\]

(A.1)

Moreover the asset-level returns associated with reaching each of these hurdles are: \(a = \text{Payoff}_A - 1\) and \(b = \text{Payoff}_B - 1\); these asset-level returns can be expressed by their standardized distances (given our assumptions about normally distributed asset-level returns) from the asset’s expected return:

\[a = \frac{a - E(k)}{\sigma_a}\] \(\text{ and }\)

\[b = \frac{b - E(k)}{\sigma_a}\]

respectively. Following Greene (2011), the returns to a given tranche can be viewed in the context of a truncated distribution. Accordingly, the expected return on the B tranche, \(E(k_{d,B})\), equals:

\[
E(k_{d,B}) = -\Phi(\alpha) + \left[ \Phi(\beta) - \Phi(\alpha) \right] \frac{E(k_d) + \sigma_a \frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)} - a}{B - A} + [1 - \Phi(\beta)] \left[ \frac{b - a}{B - A} - 1 \right]
\]

(A.2)

where: \(\Phi(\bullet)\) is the cumulative distribution function, \(\phi(\bullet)\) is the probability density function, and \(\frac{b - a}{B - A} - 1\) equals the contractual interest rate\(^{44}\) for the current mezzanine tranche (\(B\)).

\(^{44}\) Equivalently, the contractual interest rate can be written as: \(\frac{b - a}{B - A} - 1 = r_f + \gamma + \frac{\delta}{B - A} \left( \frac{B^2}{1 - B} - \frac{A^2}{1 - A} \right)\).
That is, the expected return on the current tranche, \( E(k_d B) \), can be viewed – given the option-like payoff of any mezzanine tranche (see Exhibit 8) – as having three components:

- When the asset’s realized return falls below \( a \) (i.e., a complete loss), the return on the current tranche equals \(-1.0\);

- When the asset’s realized return falls between \( a \) and \( b \) (i.e., a partial loss), the return on the current tranche is, on average, equal to the difference between the asset’s expected return, \( E(k_d | a < k_d < b) = E(k_d) + \sigma_a \phi(a) - \phi(b) - \Phi(b) + \Phi(a) \), and the payoff to the immediately senior mezzanine tranche, \( a \), as compared to the initial value \( (B - A) \) of the current tranche; and

- When the asset’s realized return exceeds \( b \) (i.e., complete repayment), the return on the current tranche equals the contractual interest rate, \( b - a \).

The variance of the current mezzanine tranche is similarly derived by recognizing that the payoffs to the three components described above completely and fully partition the entire outcome space (i.e., these events are mutually exclusive and exhaustive). In its most general form, the variance of \( X \) over the entire outcome space, given partitions \( P_1, \ldots, P_n \), equals:

\[
\text{Var}(X) = \sum_{i=1}^{n} \text{Var}(X | P_i) \text{Pr}(P_i) + \sum_{i=1}^{n} E(X | P_i)^2 \left( 1 - \text{Pr}(P_i) \right) \text{Pr}(P_i) - 2 \sum_{i=2}^{n} \sum_{j=1}^{i-1} E(X | P_i) \text{Pr}(P_i) E(X | P_j) \text{Pr}(P_j)
\]

(A.3)

Using the earlier notation and recognizing certain simplifying arguments\(^{45}\), the variance of the return on the B tranche, \( \text{Var}(k_d B) \), equals:

---

\(^{45}\) These simplifying arguments include: \( \text{Var}(X | P_i) = \text{Var}(X | P_j) = 0 \) and \( E(X | P_i) = -1 \).
\[
\text{Var}(k_{d,B}) = \sigma^2 \left[ 1 + \frac{\alpha \phi(\alpha) - \beta \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)} \left( \frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)} \right)^2 \right] \left( \Phi(\beta) - \Phi(\alpha) \right)
\] + \left(1 - \Phi(\alpha)\right)\Phi(\alpha) + \left[ \frac{E(k_d) + \sigma s (\phi(\alpha) - \phi(\beta))}{B - A} - \frac{\Phi(\beta) - \Phi(\alpha) - a}{B - A} \right] \left(1 - \Phi(\beta) + \Phi(\alpha)\right) \left(\Phi(\beta) - \Phi(\alpha)\right)
\] + \left[ \frac{b - a}{B - A} - 1 \right] \left(1 - \Phi(\beta)\right) \left(\Phi(\beta)\right)
\]

\[
\text{Mezzanine Tranches without Lender Liquidity}
\]

For purposes of this section, we assume that the high-yield lender has insufficient liquidity (such that it is incapable of funding the dead-weight costs of bankruptcy/foreclosure, repay senior securities (in the case of a foreclosure auction, fund necessary items (e.g., capital improvements, leasing commissions and tenant improvements) needed to preserve/enhance the collateral value, etc.). Accordingly, any realized asset-level return that falls beneath the payoff \(b\) to the current mezzanine tranche \(B\) results in a complete loss. So when the high-yield lender has no liquidity, the returns to the current tranche, \(E(k^*_{d,B})\), can also be viewed in the context of a truncated distribution.

\[
E(k^*_{d,B}) = -\Phi(\beta) + \left[1 - \Phi(\beta)\right] \left[ \frac{b - a}{B - A} - 1 \right]
\] (A.5)

In a similar fashion, the variance of the return \(\text{Var}(k^*_{d,B})\) on the current tranche – when the high-yield lender has no liquidity – equals:\(^{46}\)

\[
\text{Var}(k^*_{d,B}) = \Phi(\beta) \left(1 - E(k^*_{d,B})\right)^2 + \left(1 - \Phi(\beta)\right) \left[ \frac{b - a}{B - A} - 1 - E(k^*_{d,B})\right]^2.
\]

\(^{46}\) Given the bifurcated nature of the lender’s payoffs in this case (i.e., lender receives either its coupon plus principal or loses all), the variance of the return also equals: \(\text{Var}(k^*_{d,B}) = \Phi(\beta) \left(1 - E(k^*_{d,B})\right)^2 + \left(1 - \Phi(\beta)\right) \left[ \frac{b - a}{B - A} - 1 - E(k^*_{d,B})\right]^2\).
Borrower’s Risk and Expected Return

Irrespective of the lender’s liquidity, the borrower’s return (ignoring strategic defaults) is shaped by the most-junior tranche. As such, the borrower’s expected return and volatility can also be viewed as being generated by a truncated distribution. [In order to maintain the earlier notation, assume the payoff to B represents the payment owed through the most-junior tranche.] Accordingly, the expected return on equity, debt-financed through tranche B, \( E(k_{e,B}) \), equals:

\[
E(k_{e,B}) = -\Phi(\beta) + \left[ 1 - \Phi(\beta) \right] \left[ \frac{E(k_a) + \sigma_a \frac{\phi(\beta)}{1 - \Phi(\beta)} - b}{1 - B} \right] - 1
\]

(A.7)

where: \( E(k_a) + \sigma_a \frac{\phi(\beta)}{1 - \Phi(\beta)} = E(k_a | k_a > b) \) (i.e., the asset’s expected return provided the debt stack is completely repaid). And, the variance of the return on equity, debt-financed through tranche B, \( Var(k_{e,B}) \), equals:

\[
Var(k_{e,B}) = \frac{\sigma_a^2}{(1 - B)^2} \left[ 1 + \beta \phi(\beta) - \left( \frac{\phi(\beta)}{1 - \Phi(\beta)} \right)^2 \right] + \left[ \frac{E(k_a) + \sigma_a \frac{\phi(\beta)}{1 - \Phi(\beta)} - b}{1 - B} \right]^2 (1 - \Phi(\beta)) \Phi(\beta)
\]

(A.8)

Senior Lender’s Risk and Expected Return

While this paper does not specifically address senior lending, it is helpful, from a completeness perspective, to consider the expected value and the volatility of the senior lender’s return distribution. For convenience, assume a simple senior/subordinate (or “A/B”) loan structure where
the senior lender provides a portion of the capital stack equal to $A$. Then, the expected return on the senior loan, $E(k_{d,A})$, equals:

$$E(k_{d,A}) = \Phi(\alpha) \left[ \frac{E(k_s) - \sigma_s \frac{\phi(\alpha)}{\Phi(\alpha)}}{A} - 1 \right] + [1 - \Phi(\alpha)] \left[ r_j + \gamma + \delta \left( \frac{A}{1 - A} \right) \right]$$  \hspace{1cm} (A.9)

In a similar fashion, the variance of the return, $Var(k_{d,A})$, on the senior loan equals:

$$Var(k_{d,A}) = \sigma^2 \left[ 1 - \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} - \left( \frac{\phi(\alpha)}{\Phi(\alpha)} \right)^2 \right] \Phi(\alpha)$$

$$+ \left[ \frac{E(k_s) - \sigma_s \frac{\phi(\alpha)}{\Phi(\alpha)}}{A} - 1 \right] \left( 1 - \Phi(\alpha) \right) \Phi(\alpha)$$

$$+ \left[ r_j + \gamma + \delta \left( \frac{A}{1 - A} \right) \right] \left( 1 - \Phi(\alpha) \right) \Phi(\alpha)$$

$$+ 2 \left[ \frac{E(k_s) - \sigma_s \frac{\phi(\alpha)}{\Phi(\alpha)}}{A} - 1 \right] \Phi(\alpha) \left[ r_j + \gamma + \delta \left( \frac{A}{1 - A} \right) \right] \left( 1 - \Phi(\alpha) \right)$$  \hspace{1cm} (A.10)

---

47 The return distribution for the senior lender resembles the near mirror image of the borrower and is akin to the condition that results for the lender of the more-senior mezz tranche when the lender of the more-junior mezz tranche has no liquidity.