Agency Cost Determinants of Bank Risk-Taking*

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Abstract

Is risk-taking ever a privately optimal response to agency problems within banks? In a model where borrower types only matter for safe projects, I show that the answer to this question depends on the nature of the agency problem – that is, whether loan officers are hired to screen or whether they are hired to both screen and monitor. Incentivizing screening favors no risk-taking but involves a non-monotone relationship between performance and compensation. This non-monotonicity undermines incentives to monitor so, when both screening and monitoring are important, the bank instead prefers a strategy which pushes low types into risky projects. That selected risk-taking emerges under impediments to non-monotone compensation is also illustrated in an environment with rank-order tournaments and no monitoring.

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1 Introduction

Banker pay is often criticized for having encouraged undue risk-taking in the years leading up to the recent financial crisis. The following statement from the Federal Reserve, released in October 2009 and cited by Fahlenbrach and Stulz (2011), encapsulates the view:

“Flaws in incentive compensation practices were one of many factors contributing to the financial crisis. Inappropriate bonus or other compensation practices can incent senior executives or lower level employees, such as traders or mortgage officers, to take imprudent risks that significantly and adversely affect the firm.”

In theory, however, compensation contracts are written to address principal-agent problems so an alternative hypothesis is that banker pay encourages risk-taking precisely because risk minimizes agency costs. Although several types of agency problems can arise in banking, I focus on problems between bank executives and lower-level employees. First, the evidence in Fahlenbrach and Stulz (2011), Kaplan (2012), and Murphy (2012) suggests that agency problems between shareholders and bank executives are insufficient to explain what might be construed as excess risk-taking. Second, as shown in John et al (2010), agency problems between debtholders and bank executives work against high-powered incentives. I thus build a model to study whether encouraging more risk-taking is ever a privately optimal solution to asymmetric information within the banking unit.

The baseline environment is one where loan officers must be incentivized to screen borrowers before choosing loan rates. High rates compel borrowers to undertake risky projects and the trigger rate varies across borrower types. Such variation reflects a positive relationship between type and the ability to operate safer projects. High types are better at safer projects and thus have a higher trigger rate. The bank seeks to maximize expected interest income net of agency costs – that is, net of what it must pay officers to implement a particular set of type-contingent loan rates.

When the success rate of risky projects is type-independent, I show that agency costs are lowest in an equilibrium with no risk-taking. In effect, the bank can mitigate its agency
problem by capitalizing on the relationship between borrower types and success rates for safer projects. Mitigating in this fashion, however, involves a non-monotone mapping from performance to compensation. The intuition is as follows. When the bank observes successful repayment, it believes more strongly that the officer had a good borrower, in which case a higher loan rate could have been charged without triggering the risky project. Success is thus rewarded at high loan rates and penalized at low ones. For some borrowers though, low rates are necessary to curb the risky project so the bank cannot also penalize failure at low loan rates: if it did, the loan officer would never charge such rates.

The non-monotonicity just described is key to achieving low agency costs in the no risk-taking equilibrium. In cases where the loan officer must also be incentivized to monitor, I show that non-monotonicity becomes problematic and selected risk-taking emerges. More precisely, when borrowers must be monitored to exert maximum effort, failures at any loan rate may reflect lack of monitoring by the officer. If the monitoring problem is severe, then the bank is reluctant to ever compensate a failure and the no risk-taking strategy is dominated by one which pushes lower types into risky projects. Bank risk-taking thus depends on the nature of the internal agency problem – that is, whether the bank is concerned about screening or whether it is concerned about both screening and monitoring.

No risk-taking in the absence of monitoring extends to more complicated environments and can only be reversed with restrictions on the contracting space that impede the use of non-monotone compensation. For example, if the bank is imperfectly informed about the distribution of available borrowers, it prefers to hire two loan officers and compensate using tournaments. The optimal tournament still favors no risk-taking and can be implemented via rank-order when side payments between agents are precluded. In the presence of side payments, however, rank-order tournaments are not optimal and restricting attention to them will again prompt selected risk-taking.

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1 Higher loan rates for higher types is technically informational capture in the absence of competition but a looser interpretation is that loan rates are standing in for the value of the lending relationship to the bank.
This paper is broadly related to three literatures. First is experimental work on the use of compensation to mitigate agency problems within banks. Kanz (2010) shows how incentive contracts can reduce defaults while Agarwal and Wang (2009) show how incentive contracts can increase defaults. In my paper, the effect on default rates depends on the nature of the agency problem and, in cases where defaults increase, the mechanism does not rely on origination-based pay. Second is the banking literature on front-loaded compensation. In Acharya et al (2013), Bénabou and Tirole (2013), and Thanassoulis (2012), front-loading arises when banks compete for talent. In Inderst and Pfeil (2012), it arises when banks securitize loans. Risk management decisions are affected either way but, in contrast to my paper, the effects stem from individual banks not having full stake in the activities of their agents. Third is the literature on optimal labor contracts and optimal tournaments (e.g., Lazear and Rosen (1981), Green and Stokey (1983), Krishna and Morgan (1998)). Relative to this literature, I embed the optimality of tournaments into a model of project choice and examine the implications for bank risk-taking. While Jarque and Prescott (2013) also study risk-taking in a multi-agent setting, their model is one where compensation only matters for bank risk-taking if loan officer returns are correlated.

The rest of the paper proceeds as follows: Section 2 describes the borrower environment, Section 3 presents the bank’s core agency problem, Section 4 introduces monitoring, Section 5 introduces tournaments, and Section 6 concludes. Proofs are collected in the appendix.

## 2 Environment

The borrower side builds on the two-type environment in Van Tassel (2002). There are two types of risk neutral firms: $H$ and $L$. The fraction of type $H$ firms is $\lambda \in (0, 1)$. Both types have access to an investment project and a speculative project. If operated by a type $i \in \{L, H\}$ firm, the investment project produces $\theta_1$ units of output with probability $p_i \in (0, 1)$ and zero units with probability $1 - p_i$. In contrast, the speculative project
produces $\theta_2$ units of output with probability $q$ and zero units otherwise regardless of which type operates it. Assume $p_H > p_L$ so that type $H$ firms are the ones most likely to succeed at the investment project. Also assume $p_L > q$ and $\theta_2 > \theta_1$ but $p_L \theta_1 > q \theta_2$ so that the speculative project is riskier in the sense that it is second order stochastically dominated by the investment project. I will refer to the investment project as the “safe” project and the speculative project as the “risky” project. All quantities are denominated in units of output.

A firm needs one unit of bank capital to operate either project. I abstract from the extensive margin and assume one risk neutral bank that is willing to finance both types. The key is at what loan rate. To ease the exposition, assume that capital is not destroyed in the production process so the borrowed unit is always recovered by the bank. Also normalize the bank’s cost of funds to zero so that, if paid, the loan rate $R$ reflects the bank’s profit from the loan. In a world where borrowers pay if and only if their projects are successful, a type $i \in \{L, H\}$ borrower will get $p_i [\theta_1 - R]$ from operating the safe project versus $q [\theta_2 - R]$ from operating the risky one. The rate that makes the borrower indifferent between the two projects (i.e., the borrower’s reservation rate) is the rate that equates these payoffs, namely:

$$R_i \equiv \frac{p_i \theta_1 - q \theta_2}{p_i - q}$$

It is trivial to show $R_L < R_H < \theta_1$. In other words, a type $i$ borrower will opt for the safe project when $R \in [0, R_i]$ and the risky project when $R \in (R_i, \theta_2]$. The bank thus chooses between charging $R_i$ or $\theta_2$. Notice that charging below $R_i$ is not optimal: the bank could extract more profit from the borrower without increasing the default probability (i.e., without changing the borrower’s project choice) by just charging $R_i$. Similarly, charging strictly between $R_i$ and $\theta_2$ is not optimal: the bank could extract more profit without increasing the default probability by just charging $\theta_2$. Since the bank does not need to compete for borrowers and any $R \leq \theta_2$ satisfies the borrower participation constraint, there are no other incentives to undercut.
Before proceeding, let me return to the assumption that borrowers repay their loans if and only if their projects succeed. Given the strategies derived so far, a sufficient condition for this assumption is that banks can detect the presence of positive output. Recall that positive output in this model is either $\theta_1$ or $\theta_2$. Since the bank does not charge above $\theta_2$, a borrower can repay if his realization is $\theta_2$. On the other hand, a realization of $\theta_1$ means the safe project was undertaken which itself means the bank did not charge above the borrower’s reservation rate. Both reservation rates are below $\theta_1$ so the borrower can again repay. Therefore, regardless of whether $\theta_1$ or $\theta_2$ materializes, a borrower with positive output can always repay. Allowing the bank to detect the presence of positive output (but not necessarily the exact level) is thus sufficient to induce repayment among successful projects.

**Proposition 1** Suppose $q \leq \left(1 - \sqrt{1 - \frac{\theta_1}{\theta_2}}\right) p_L \equiv q^*$. The bank achieves the socially efficient outcome when borrower types are public information.

The proof of Proposition 1 is straightforward. With $p_i \theta_1 > q \theta_2$ for both types, a social planner would want everyone to undertake the safe project. The bank only replicates this if its expected profit from charging $R_i$ is greater than or equal to its expected profit from charging $\theta_2$. In other words, we need $p_i R_i \geq q \theta_2$ for both types. With $p_H > p_L$ and $R_H > R_L$, it will suffice to have $p_L R_L \geq q \theta_2$ which reduces to $q \leq q^*$ as defined in Proposition 1. Going forward, I will restrict attention to $q = q^*$ so that the bank is indifferent between doing what is socially optimal and triggering risky projects among low types. The question of interest then becomes how agency costs break this indifference.

### 3 Core Agency Problem

To introduce an agency problem into the above environment, suppose borrower types are private information but can be discovered by a loan officer who screens at cost (or disutility) $c > 0$. The screening technology is prohibitively expensive for the bank to operate without an officer but whether the officer actually operates it cannot be observed by the bank. The
bank must thus incentivize the officer to screen and charge type-contingent loan rates. If the bank does not want type-contingent rates (i.e., if it wants the same rate for all borrowers) then there is no reason to hire a loan officer and incur the agency costs of incentivizing him. We can thus restrict attention to three cases: (1) incentivizing $R_i$ for each $i$; (2) incentivizing $R_i$ if and only if $i = H$; and (3) incentivizing $R_i$ if and only if $i = L$.

Notice that case (3) cannot be optimal. Safe projects are more lucrative when operated by type $H$ borrowers so a lender who finds it profitable to induce the safe project among type $L$s must also find it profitable to do so among type $H$s. Therefore, we can further restrict attention to cases (1) and (2). Case (1) induces both borrower types to undertake safe projects so I will label it the safe strategy. Case (2) will then be labeled the risky strategy since it induces type $L$ borrowers to undertake risky projects. Notice that the risky strategy involves only selected risk-taking since type $H$ borrowers still undertake safe projects.

### 3.1 Incentivizing the Safe Strategy

Begin with a bank that wants to incentivize $R_i$ for each $i$, expressed more compactly as $\{R_L, R_H\}$. The bank can ultimately observe what loan rate the officer charged and whether or not the borrower repaid so officer compensation can be conditioned accordingly. In particular, the bank can offer a wage contract of the form $\{w_{SH}, w_{FH}, w_{SL}, w_{FL}\}$. An officer who charges $R_H$ gets $w_{SH}$ if his borrower succeeds (repays) and $w_{FH}$ if his borrower fails (defaults). Similarly, an officer who charges $R_L$ gets $w_{SL}$ if his borrower succeeds and $w_{FL}$ if his borrower fails. Assume limited liability so that wages have to be non-negative. Also assume an outside option of zero for the loan officer.

Under an incentive compatible contract, the bank’s expected profit and the loan officer’s expected payoff are given by equations (1) and (2) respectively:

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2 All loans are characterized by one unit of capital, there is no collateral, and the one-shot nature of the game precludes intertemporal incentives. Loans are thus defined by one interest rate so the bank does not have enough instruments to design separating contracts in lieu of hiring officers to screen.
\[ \lambda [p_H(\overline{R}_H - w_{SH}) - (1 - p_H)w_{FH}] + (1 - \lambda) [p_L(\overline{R}_L - w_{SL}) - (1 - p_L)w_{FL}] \] (1)

\[ \lambda [p_Hw_{SH} + (1 - p_H)w_{FH}] + (1 - \lambda) [p_Lw_{SL} + (1 - p_L)w_{FL}] - c \] (2)

Incentive compatibility means the officer incurs the screening cost \( c \) and charges each type their reservation rate. With probability \( \lambda \), the officer’s borrower will turn out to be a type \( H \). He will thus be charged \( \overline{R}_H \) and succeed with probability \( p_H \). With probability \( 1 - \lambda \), the officer’s borrower will turn out to be a type \( L \). He will thus be charged \( \overline{R}_L \) and succeed with probability \( p_L \). For screening to be incentive compatible, (2) must be at least as large as what the officer could get by not screening and always charging the same loan rate. By not screening and always charging \( \overline{R}_L \), the officer would get:

\[ \lambda [p_Hw_{SL} + (1 - p_H)w_{FL}] + (1 - \lambda) [p_Lw_{SL} + (1 - p_L)w_{FL}] \]

By not screening and always charging \( \overline{R}_H \), the officer would get:

\[ \lambda [p_Hw_{SH} + (1 - p_H)w_{FH}] + (1 - \lambda) [qw_{SH} + (1 - q)w_{FH}] \]

Since \( \overline{R}_H > \overline{R}_L \), a type \( L \) will undertake the risky project when charged \( \overline{R}_H \) so the probability of success falls from \( p_L \) to \( q \). The incentive compatibility constraints for screening are thus:

\[ [p_Hw_{SH} + (1 - p_H)w_{FH} - p_Hw_{SL} - (1 - p_H)w_{FL}] \lambda \geq c \] (3)

\[ [p_Lw_{SL} + (1 - p_L)w_{FL} - qw_{SH} - (1 - q)w_{FH}] (1 - \lambda) \geq c \] (4)

Turn next to the loan rate choices conditional on screening. An officer with a type \( H \) borrower will charge \( \overline{R}_H \) instead of \( \overline{R}_L \) provided:
\[ p_H w_{SH} + (1 - p_H) w_{FH} \geq p_H w_{SL} + (1 - p_H) w_{FL} \]

By the same token, an officer with a type \( L \) borrower will charge \( R_L \) instead of \( R_H \) provided:

\[ p_L w_{SL} + (1 - p_L) w_{FL} \geq q w_{SH} + (1 - q) w_{FH} \]

With \( c > 0 \) and \( \lambda \in (0, 1) \), the incentive compatibility constraints for screening also guarantee incentive compatibility of loan rate choices. The bank thus chooses wages to maximize (1) subject to (3), (4), and the non-negativity constraints. This yields:

**Proposition 2** The optimal wage contract for \( \{ R_L, R_H \} \) involves \( w_{SL}^* = w_{FH}^* = 0 \) and unique values \( w_{SH}^* > 0 \) and \( w_{FL}^* > 0 \). The bank’s optimized expected profit is then:

\[
\lambda p_H R_H + (1 - \lambda) p_L R_L - \left( 1 + \frac{[\lambda p_H + (1-\lambda)q][1-\lambda p_H - (1-\lambda)p_L]}{\lambda(1-\lambda)[p_H(1-p_L) - q(1-p_H)]} \right) c
\]

According to Proposition 2, a loan officer who charges \( R_H \) is only compensated if his borrower succeeds. In contrast, one who charges \( R_L \) is only compensated if his borrower fails. To see why, recall that the officer can deviate from screening in one of two ways: he can either charge everyone \( R_L \) or he can charge everyone \( R_H \). The bank wants to eliminate both deviations as cheaply as possible. Given \( p_H > q \) and \( R_H > R_L \), a loan officer who charges \( R_H \) has a higher probability of succeeding with a type \( H \) borrower than he does with a type \( L \) borrower. Since the bank wants \( R_H \) if and only if the borrower is type \( H \), it puts all the compensation for charging \( R_H \) into \( w_{SH} \). A similar argument applies for \( R_L \). In particular, given \( p_H > p_L \), a loan officer who charges \( R_L \) has a higher probability of failing with a type \( L \) borrower than he does with a type \( H \) borrower. Since the bank wants \( R_L \) if and only if the borrower is type \( L \), it puts all the compensation for charging \( R_L \) into \( w_{FL} \). The bank then sets \( w_{FL} \) and \( w_{SH} \) high enough to make the loan officer indifferent between deviating in either direction.
Notice that Proposition 2 can also be interpreted as Bayesian updating by the bank. The probability of a borrower being type $H$ conditional on him succeeding exceeds the unconditional probability of drawing a type $H$. Therefore, when the bank observes a success, it believes more strongly that the loan officer had a type $H$, in which case $\overline{R}_H$ should have been charged. The bank thus penalizes success at $\overline{R}_L$ because, off equilibrium, there is a good chance the officer did not screen. The bank must then compensate failure at $\overline{R}_L$, otherwise the officer has no incentive to ever charge $\overline{R}_L$ and thus no incentive to screen. The result is $\max\{w^*_S, w^*_F\} \geq \min\{w^*_S, w^*_F\} > w^*_S = w^*_F$ so there is a non-monotonic relationship between the profit generated by a loan officer and the wage he receives.

### 3.2 Incentivizing the Risky Strategy

Consider now a bank that wants to incentivize $\overline{R}_i$ if and only if $i = H$, expressed more compactly as $\{\theta_2, \overline{R}_H\}$. The bank can again condition on the loan rate charged and the repayment outcome so the wage contract takes the form $\{\tilde{w}_S, \tilde{w}_F, \tilde{w}_S\theta, \tilde{w}_F\theta\}$. The interpretation of $\tilde{w}_S$ and $\tilde{w}_F$ is as before but now there is no compensation for charging $\overline{R}_L$ since the bank does not want that rate to be charged. Instead, an officer who charges $\theta_2$ gets $\tilde{w}_S\theta$ if his borrower succeeds and $\tilde{w}_F\theta$ if his borrower fails. The analysis proceeds as in the previous subsection. In particular, the bank’s expected profit and the loan officer’s expected payoff under an incentive compatible contract are now:

$$\lambda [p_H (\overline{R}_H - \tilde{w}_S) - (1 - p_H) \tilde{w}_F] + (1 - \lambda) [q (\theta_2 - \tilde{w}_S\theta) - (1 - q) \tilde{w}_F\theta]$$

(5)

$$\lambda [p_H \tilde{w}_S + (1 - p_H) \tilde{w}_F] + (1 - \lambda) [q \tilde{w}_S\theta + (1 - q) \tilde{w}_F\theta] - c$$

(6)

respectively. Incentive compatibility again requires the officer to incur screening costs but now he only charges type $H$s their reservation rate: type $L$s are instead charged the highest possible loan rate $\theta_2$ and thus pushed into risky projects. By not screening and always charging $\overline{R}_H$, the officer would get:
\[
\lambda [p_H \tilde{w}_{SH} + (1 - p_H) \tilde{w}_{FH}] + (1 - \lambda) [q \tilde{w}_{SH} + (1 - q) \tilde{w}_{FH}]
\]

By not screening and always charging \(\theta_2\), the officer would get:

\[q \tilde{w}_{S\theta} + (1 - q) \tilde{w}_{F\theta}\]

The incentive compatibility constraints for screening are thus:

\[
[q \tilde{w}_{S\theta} + (1 - q) \tilde{w}_{F\theta} - q \tilde{w}_{SH} - (1 - q) \tilde{w}_{FH}] (1 - \lambda) \geq c \tag{7}
\]

\[
[p_H \tilde{w}_{SH} + (1 - p_H) \tilde{w}_{FH} - q \tilde{w}_{S\theta} - (1 - q) \tilde{w}_{F\theta}] \lambda \geq c \tag{8}
\]

The screening constraints will once again guarantee incentive compatibility of loan rate choices so the bank just chooses wages to maximize (5) subject to (7), (8), and the non-negativity constraints. This yields:

**Proposition 3** The optimal wage contract for \(\{\theta_2, R_H\}\) involves \(\tilde{w}_{FH}^* = 0\) and unique values \(\tilde{w}_{SH}^* > 0\) and \(q \tilde{w}_{S\theta}^* + (1 - q) \tilde{w}_{F\theta}^* > 0\). Individually though, \(\tilde{w}_{S\theta}^*\) and \(\tilde{w}_{F\theta}^*\) are not uniquely defined. The bank’s optimized expected profit is then:

\[
\lambda p_H R_H + (1 - \lambda) q \theta_2 - \left(1 + \frac{\lambda p_H + (1 - \lambda)q}{\lambda(1 - \lambda)(p_H - q)}\right) c
\]

The intuition for Proposition 3 builds on that for Proposition 2. Once again, the bank puts all the compensation for charging \(R_H\) into \(\tilde{w}_{SH}\) because an officer who charges \(R_H\) has a higher probability of succeeding with a type \(H\) borrower. In contrast, charging \(\theta_2\) prompts any borrower to undertake the risky project so the probability of success is \(q\) regardless of type. Therefore, the bank cannot conserve on agency costs by putting all the compensation for charging \(\theta_2\) into an event that is more likely when the officer has a type \(L\). The implications are two-fold. First, the division between \(\tilde{w}_{S\theta}\) and \(\tilde{w}_{F\theta}\) is irrelevant: it only matters that
the expected compensation \( q\tilde{w}_{S\theta} + (1 - q)\tilde{w}_{F\theta} \) is large enough to dissuade the officer from charging everyone \( \bar{R}_H \) but small enough to also dissuade him from charging everyone \( \theta_2 \). Second, incentivizing the risky strategy does not require a non-monotone wage function: the optimal contract in Proposition 2 is flexible enough to admit \( \tilde{w}^*_{S\theta} > \tilde{w}^*_{S\theta} > \tilde{w}^*_{F\theta} = \tilde{w}^*_{F\theta} \).

3.3 Bank’s Choice of Risk

I will now characterize the bank’s optimal strategy. Recall that the safe strategy of \( \{R_L, R_H\} \) yields bank profits as per Proposition 2 while the risky strategy of \( \{\theta_2, \bar{R}_H\} \) yields bank profits as per Proposition 3. Comparing the coefficients on \( c \) in these profit expressions reveals that incentivizing \( \{\theta_2, \bar{R}_H\} \) is more costly than incentivizing \( \{R_L, \bar{R}_H\} \). This reflects the fact that all borrowers are the same (i.e., everyone selects the type-independent project) when charged \( \theta_2 \). All else constant then, the agency problem imparted by \( c > 0 \) works in favor of the more conservative strategy. With \( q = q^* \), the choice between \( \{R_L, \bar{R}_H\} \) and \( \{\theta_2, \bar{R}_H\} \) is determined solely by agency costs so \( \{R_L, \bar{R}_H\} \) prevails and, by extension, there exists an \( \varepsilon > 0 \) such that \( \{R_L, \bar{R}_H\} \) prevails even if \( q = q^* + \varepsilon \).

The question that remains is whether \( \{R_L, \bar{R}_H\} \) prevails over type-incontingent strategies. In particular, if agency costs are high enough, the bank may prefer to eliminate the loan officer and just charge the same rate on all loans. As Proposition 4 shows, the bank’s assessment of whether agency costs are too high depends on the quality of the borrower pool:

**Proposition 4** Suppose \( q = q^* \) so that \( p_LR_L = q\theta_2 \). There exist scalars \( \bar{c}, \lambda_1, \) and \( \lambda_2 \) such that, if \( c \in (0, \bar{c}) \), the bank chooses a flat rate of \( \bar{R}_L \) for \( \lambda \in (0, \lambda_1) \), the safe strategy of \( \{R_L, \bar{R}_H\} \) for \( \lambda \in [\lambda_1, \lambda_2] \), and a flat rate of \( \bar{R}_L \) for \( \lambda \in (\lambda_2, 1) \).

The cost of always offering \( \bar{R}_H \) is lost profit on type \( L \) borrowers \( (q\bar{R}_H < p_LR_L) \) while the cost of always offering \( \bar{R}_L \) is lost profit on type \( H \) borrowers \( (p_HR_L < p_H\bar{R}_H) \). When \( \lambda \) is very low, there are few type \( H \) borrowers so the bank conserves on agency costs by always charging \( \bar{R}_L \). When \( \lambda \) is very high, there are few type \( L \) borrowers so the bank conserves on agency costs
by always charging $\bar{R}_H$. For moderate values of $\lambda$, both types are common enough that the bank is willing to incur agency costs in order to price discriminate. A corollary of Proposition 4 is that average default rates do not decrease monotonically with average borrower quality. In particular, as the borrower pool improves past $\lambda_2$, the bank’s switch from $\{\bar{R}_L, \bar{R}_H\}$ to a flat rate of $\bar{R}_H$ induces type $L$ borrowers to undertake risky projects and default more often. It is not until improvements in the pool reach $\lambda = \frac{p_L - q + (p_H - p_L)\lambda_2}{p_H - q} \in (\lambda_2, 1)$ that the average default rate returns to its $\lambda_2$ level.

In sum, agency costs have two effects on risk-taking. Within the space of type-contingent strategies, agency costs make the bank more conservative. However, if the costs are high enough relative to dispersion in the borrower pool, type-contingent strategies are no longer desirable and a better pool may be accompanied by an increase in average defaults.

3.4 Discussion

The key insight so far is that the safe strategy $\{\bar{R}_L, \bar{R}_H\}$ dominates the risky strategy $\{\theta_2, \bar{R}_H\}$. This is consistent with Holmström and Milgrom (1987): when output is a less noisy signal of effort, incentives are higher-powered and agency costs are lower. Here, however, agents are risk neutral and noisiness of the risky project comes from type-independence rather than inherent variance. Independence limits how well the bank can condition on output – recall that $\tilde{w}_{S\theta}^*$ and $\tilde{w}_{F\theta}^*$ are not uniquely pinned down – and this is the sense in which incentives for the safe strategy are higher-powered. Moreover, the safe strategy involves a non-monotonicity not usually found in studies of high-powered incentives. The remainder of the paper will show that, absent such non-monotonicity, the risky strategy can in fact dominate the safe one. Monitoring and rank-order tournaments are two forces that work against non-monotonicity so I consider them in turn. Monitoring creates an endogenous reluctance to compensate failures while rank-order tournaments exogenously restrict the contracting space. In both cases though, we will see that the risky strategy emerges.
4 Extension: Monitoring

Along with screening borrowers ex ante, suppose loan officers must also monitor projects ex post. Absent monitoring, a safe project now succeeds with probability $\alpha p$, while a risky project succeeds with probability $\alpha q$, where $\alpha \in (0, 1)$. Setting $\alpha = 1$ returns the environment of Section 2. Assume that an officer’s cost of monitoring is much greater than $c$ if he does not screen and equal to zero otherwise. He then monitors if and only if he screens. This assumption simplifies the exposition but it is not necessary for the main insights.

Proposition 5 If types are public information and the officer must only be incentivized to monitor (at cost $c$), the bank remains indifferent between the safe and risky strategies.

We have now seen that monitoring problems alone do not break the indifference between $\{R_L, R_H\}$ and $\{\theta_2, R_H\}$ while screening problems alone break in favor of the safe strategy. The following proposition shows that having to incentivize both screening and monitoring can actually break the indifference in favor of the risk strategy:

Proposition 6 If the officer must be incentivized to screen and monitor, then there exists an $\alpha \in (0, 1)$ such that the risky strategy dominates the safe one if and only if $\alpha < \alpha^*$. We know from Proposition 2 that compensating failure rather than success at $R_L$ is what allows the safe strategy to better mitigate screening problems. However, when borrowers must be monitored in order to exert maximum effort, failures at any loan rate may reflect a lack of monitoring by the loan officer. If the monitoring problem is severe – that is, if $\alpha$ is sufficiently low – then the bank is reluctant to ever compensate a failure. It thus sets $w_{FL} = w_{FH} = 0$ and incents screening using $w_{SH} > w_{SL} > 0$. Stated otherwise, the bank can no longer conserve on agency costs by putting all the compensation for charging $R_L$ into an event that is more likely when the officer has a type L borrower. This is the same issue encountered with the risky strategy in Subsection 3.2 so the risky strategy is no longer unambiguously dominated. In fact, it becomes dominant because the ratchet effect of
$w_{SL} > 0$ on $w_{SH}$ is stronger than the ratchet effect of $\tilde{w}_{S\theta} > 0$ on $\tilde{w}_{SH}$. To see why, recall that a loan officer who charges $\bar{R}_L$ has a higher probability of succeeding with a type $H$ borrower than he does with a type $L$ whereas the success probability of an officer who charges $\theta_2$ is type-independent. Therefore, the value of $w_{SH}$ needed to eliminate all borrowers being charged $\bar{R}_L$ exceeds the value of $\tilde{w}_{SH}$ needed to eliminate all borrowers being charged $\theta_2$.

5 Extension: Tournaments

Return to the core model where loan officers must only be incentivized to screen. In other words, $\alpha = 1$ and there is no need for monitoring. The analysis in Section 3 assumed one loan officer who was compensated based on his absolute performance. The following proposition establishes that this was without loss of generality so additional ingredients will be necessary to motivate the use of tournaments:

**Proposition 7** Hiring two loan officers and allowing compensation to depend on relative performance cannot improve on the independent wage contracts in Propositions 2 and 3.

For any strategy, the bank minimizes agency costs by making all incentive compatibility constraints hold with equality – that is, by making the officer just indifferent between screening and deviating to either loan rate. The crux of Proposition 7 is that wage contracts involve enough contingencies to implement such indifference. To warrant the additional contingencies introduced by relative performance (tournaments), suppose $\lambda$ is unknown and uncontractible when wages are set but eventually revealed to the loan officer before he decides whether to screen. Realizations of $\lambda$ can take one of two values, $\underline{\lambda}$ or $\overline{\lambda}$, where $0 < \underline{\lambda} < \overline{\lambda} < 1$ and $\Pr(\lambda = \underline{\lambda}) = \pi$. Also assume $\underline{\lambda} + \overline{\lambda} = 1$. This amounts to a symmetry assumption: the probability of drawing a high type from a good distribution, $\overline{\lambda}$, equals the probability of drawing a low type from a bad distribution, $1 - \underline{\lambda}$.
5.1 Incentive Compatible Contracts

Independent wage contracts can no longer implement the desired amount of indifference. Consider \( \{ \overline{R}_L, \overline{R}_H \} \). Conditions (3) and (4) induce screening for a given value of \( \lambda \) so, with \( \lambda \) uncontractible, an incentive compatible wage contract must satisfy these conditions regardless of which \( \lambda \) materializes. Since \( \overline{\lambda} > \lambda \), the best the bank can do is make (3) hold with equality at \( \lambda \) and (4) hold with equality at \( \overline{\lambda} \). One incentive compatibility constraint will thus end up being slack. A similar issue arises for \( \{ \theta_2, \overline{R}_H \} \). The conditions for an incentive compatible wage contract are (7) and (8) and they must also hold regardless of which \( \lambda \) materializes. The best the bank can do is make (7) hold with equality at \( \lambda \) and (8) hold with equality at \( \overline{\lambda} \) so, once again, one of the constraints will end up being slack.

Proposition 8 below establishes the benefit of tournaments in the current environment. To summarize the nature of the contract, define \( \Psi \equiv \{ SL, SH, S\theta, FL, FH, F\theta \} \). The reward of an officer who achieves \( j \in \Psi \) when the other officer achieves \( k \in \Psi \) is now denoted by \( X_{j \mid k} \). If the bank wants \( \{ \overline{R}_L, \overline{R}_H \} \), then it sets \( X_{S\theta \mid k} = X_{F\theta \mid k} = 0 \). If it wants \( \{ \theta_2, \overline{R}_H \} \), then it sets \( X_{SL \mid k} = X_{FL \mid k} = 0 \). An optimal tournament is one that places no additional restrictions on the \( X \)'s. Along with unrestricted tournaments, I will also consider restrictions that impose varying degrees of monotonicity on loan officer pay. In what I will call rank-order tournaments, generating at least as much as the other officer is a necessary condition for non-zero compensation. In what I will call winner-take-all tournaments, generating at least as much is both a necessary and sufficient condition. The winner-take-all tournament randomizes fairly in the event of a tie but may randomize over zeros if both officers fail.

Proposition 8 When \( \lambda \) is not contractible, optimal tournaments dominate independent wage contracts. For \( \{ \overline{R}_L, \overline{R}_H \} \), the optimal tournament can only be implemented via rank-order if \( p_H < 1 \) and can never be implemented via winner-take-all. Moreover, with \( p_H = 1 \), the optimal rank-order tournament only dominates wages if \( \pi < \Pi_0 \in (0, 1) \). For \( \{ \theta_2, \overline{R}_H \} \), the optimal tournament can always be implemented via rank-order or via winner-take-all.
The implications of Proposition 8 are three-fold. First, and in contrast to Proposition 7, relative performance evaluations improve on independent wage contracts. As alluded to above, the bank now needs additional instruments to make loan officers exactly indifferent between screening and deviating in all states of the world. Second, a winner-take-all scheme is optimal for implementing the risky strategy. This underscores the fact that non-monotone payoffs are not necessary to incentivize the risky strategy. Third, it is not always optimal to use a rank-order tournament to incentivize the safe strategy. In particular, a rank-order tournament under $p_H = 1$ will not provide enough instruments to make all incentive compatibility constraints bind for any realization of $\lambda$.

Why is $p_H = 1$ problematic for rank-order tournaments under the safe strategy? The intuition reflects the fact that the bank is dealing with two information problems: agency and uncertainty about $\lambda$. The agency problem is addressed as in Proposition 2. More precisely, a loan officer who charges $R_H$ is only compensated if his borrower succeeds while a loan officer who charges $R_L$ is only compensated if his borrower fails. The optimality of never compensating $R_L$ when it succeeds, even if all else has failed, is enough to rule out winner-take-all schemes. Turn now to uncertainty about $\lambda$ which can be addressed by tilting compensation towards events that have $\lambda$-insensitive probabilities. In my model, the probability of one officer getting a type $L$ borrower and the other getting a type $H$ is $\phi(\lambda) \equiv \lambda (1-\lambda)$. With $\lambda + \bar{\lambda} = 1$, we have $\phi(\lambda) = \phi(\bar{\lambda})$ so the bank deals with uncertainty about $\lambda$ by compensating most when one officer charges $R_L$ and the other charges $R_H$. Putting everything together, an officer who charges $R_L$ is only compensated if his borrower fails and he receives more such compensation if his colleague charged $R_H$. Under $p_H = 1$, type $H$ borrowers always succeed when charged $R_H$ so the bank would be compensating a failure at $R_L$ despite a success at $R_H$. This violates the conditions for a rank-order tournament so

\[^{3}\text{Relative performance contracts also arise in } \text{Gromb and Martimort (2007). Their agents make accept/reject decisions so the principal has no ex post outcomes to condition on in the event of rejection. Hiring two agents to evaluate the same project is thus useful for cross-checking rejections. In my model, all borrowers are accepted so ex post outcomes are always available. Moreover, the two loan officers do not screen the same borrower. Instead, they serve as two independent draws from the same distribution, allowing the bank to overcome its imperfect information about that distribution.} \]
the optimal scheme for incentivizing \{R_L, R_H\} cannot be implemented via rank-order.

5.2 Bank’s Choice of Risk

Recall from Subsection 3.3 that agency costs made \{\theta_2, R_H\} more costly to incentivize than \{R_L, R_H\} when \lambda was known, thus rendering the bank more conservative within the space of type-contingent strategies. Proposition 9 shows that this is still the case when \lambda is non-contractible as long as optimal tournaments are used. If, however, \(p_H = 1\) and the bank only considers rank-order tournaments, then there are parameters for which the risky strategy becomes less costly to incentivize than the safe strategy.

**Proposition 9** With optimal tournaments, \{R_L, R_H\} is less costly than \{\theta_2, R_H\} to incentivize. With a rank-order restriction on tournaments, cutoffs \(Q \in (0, 1)\) and \(\Pi \in [\Pi_0, 1)\) exist such that the reverse is true if (and only if) \(p_H = 1, \lambda < \frac{3-\sqrt{5}}{2}, q < Q,\) and \(\pi \geq \Pi\).

Figure 1 builds on the second part of Proposition 9, showing how the bank’s strategy depends on the first and second moments of \lambda. Given \(\text{Pr}(\lambda = \lambda) = \pi\) and \(\text{Pr}(\lambda = 1 - \lambda) = 1 - \pi,\) the expected value and standard deviation of \lambda are respectively:

\[
E(\lambda) = \pi \lambda + (1 - \pi)(1 - \lambda)
\]

\[
\sigma(\lambda) = (1 - 2\lambda) \sqrt{\pi(1 - \pi)}
\]

Notice that type-contingent strategies only emerge in the interior of Figure 1. When \(\sigma(\lambda)\) is high, the bank adopts a flat rate of \(R_H\) if \(E(\lambda)\) is also high and a flat rate of \(R_L\) otherwise. As uncertainty decreases, expectations about \lambda must be either very high or very low for the bank to prefer a flat rate. With moderate expectations, a bank that restricts itself to rank-order tournaments chooses to incentivize the risky strategy if \(E(\lambda)\) is moderately low and the safe strategy if \(E(\lambda)\) is moderately high. In contrast, willingness to compensate an officer who fails at \(R_L\) precisely when another officer succeeds at \(R_H\) allows the bank to implement
5.2.1 Implications for Economic Activity

Along with comparing optimal and rank-order tournaments when \( p_H = 1 \), Figure 1 can be interpreted as comparing optimal rank-order tournaments when \( p_H = 1 \) and \( p_H = 0.99 \). This alternative interpretation reveals that rank-order tournaments for loan officers introduce a more pronounced relationship between otherwise independent borrowers: when high types are riskless (\( p_H = 1 \)), low types are more likely to end up in risky projects. At an aggregate level then, dispersion in project output will tend to be higher when good types are riskless. Figure 2 provides some numerical examples. Notice that the problem is most severe in the area where rank-order restrictions prompt banks to choose the risky strategy: by the time a

\(^4\)The left panel is the optimal rank-order tournament when \( p_H = 1 \) so it will suffice to explain why the right panel – the optimal tournament when \( p_H = 1 \) – approximates the optimal rank-order tournament when \( p_H = 0.99 \). The argument proceeds in two steps. First, continuity of payoffs within optimal tournaments means that the right panel is roughly equal to the optimal tournament when \( p_H = 0.99 \). Second, the ability to implement any optimal tournament via rank-order when \( p_H < 1 \) means that the right panel is also roughly equal to the optimal rank-order tournament when \( p_H = 0.99 \).
Notes: Parameters as in Figure 1. Dispersion is measured as the coefficient of variation.

flat rate of $R_H$ is optimal, the expected value of $\lambda$ is usually high enough that risky projects by low types are only a small fraction of economic activity.

The figures also suggest that rigid compensation structures in the banking sector can have real economic effects. Suppose the economy starts with $p_H < 1$. Rank-order tournaments are optimal for loan officers and, with a configuration like $\sigma(\lambda) = 0.35$ and $E(\lambda) = 0.5$ in Figure 1, only safe projects will be undertaken. As the economy’s access to riskless assets increases, or as its ability to produce such assets via diversification grows, rank-order tournaments become suboptimal. However, if compensation structures are ingrained and the fixed cost of changing them is high, then the shift from $p_H < 1$ to $p_H = 1$ will result in a risky equilibrium.

5.2.2 Side Payments Between Agents

Up to this point, rank-order tournaments have only interfered with risk-taking when $p_H = 1$. I now show that introducing side payments between agents introduces interference when
\( p_H < 1 \). In particular, suppose loan officers can agree to redistribute their compensation so that each gets \( \frac{X_k + X_j}{2} \) when the pair \(( j, k) \in \Psi \times \Psi \) is realized. These arrangements are unobserved by the bank and impose extra constraints on the contracts in Proposition 8.

**Proposition 10** Suppose \( p_H < 1 \). In the model with side payments, the optimal tournament for \( \{ \theta_2, \overline{R}_H \} \) can still be implemented via rank-order whereas that for \( \{ \overline{R}_L, \overline{R}_H \} \) cannot. Absent a rank-order restriction, \( \{ \overline{R}_L, \overline{R}_H \} \) is less costly than \( \{ \theta_2, \overline{R}_H \} \). With such a restriction, however, the reverse is true for \( q \) and \( \Delta \) sufficiently low and \( \pi \) sufficiently high.

To see why side payments complicate rank-order tournaments for the safe strategy, recall how the bank incentivized the safe strategy in Subsection 5.1. First, it addressed the core agency problem by setting \( X_{SL}^k = X_{FH}^k = 0 \) for all \( k \in \Psi \). Second, it addressed uncertainty about the borrower pool by compensating more when officers charged different loan rates. With side payments, however, the second feature may actually encourage officers to collude on different rates without screening. To eliminate such collusion, the optimal tournament would extend the first feature by penalizing both officers if either one achieves \( j \in \{ SL, FH \} \) after different rates have been charged. This yields \( X_{FH}^F = 0 \) so implementing the second feature involves \( X_{FL}^{SH} > 0 \) which is not rank-order. Once again then, the optimal scheme for incentivizing the safe strategy cannot be implemented via rank-order. The conditions on \( q \), \( \Delta \), and \( \pi \) in Proposition 10 are qualitatively similar to those in Proposition 9 but are now sufficient conditions rather than sufficient and necessary.

6 Conclusion

This paper has studied whether risk-taking is ever a privately optimal response to agency problems within banks. In a model where borrower types only matter for safe projects, I showed that the answer to this question depends on the nature of the agency problem – that is, whether loan officers are hired to screen or whether they are hired to both screen and monitor. Incentivizing screening favors no risk-taking but involves a non-monotone
relationship between performance and compensation. This non-monotonicity undermines incentives to monitor so, when both screening and monitoring are important, the bank instead prefers a strategy which pushes low types into risky projects. That selected risk-taking emerges under impediments to non-monotone compensation was also illustrated in an environment with rank-order tournaments and no monitoring.

References


Appendix

Proof of Proposition 2

Notice that the optimal contract will make both screening constraints bind: slackness of (3) means the bank can decrease \( w_{SH} \) or \( w_{FH} \) without violating (4) while slackness of (4) means it can decrease \( w_{SL} \) or \( w_{FL} \) without violating (3). Given equation (1), decreasing any of the four wages increases bank profits and is thus desirable. Having established that (3) and (4) hold with equality, rearrange them to get \( w_{SH} \) and \( w_{SL} \) as functions of \( w_{FH} \) and \( w_{FL} \) then substitute into equation (1) to write the bank’s expected profit as:

\[
\lambda p_H R_H + (1 - \lambda) p_L R_L - \left( 1 + \frac{\lambda p_H + (1 - \lambda) p_L}{\lambda (1 - \lambda) p_H (p_L - q)} \right) c - \frac{\lambda p_H + (1 - \lambda) q}{\lambda (1 - \lambda) p_H (p_L - q)} w_{FH} \]

A profit-maximizing bank thus sets \( w_{FH} = 0 \) and \( w_{FL} \) as high as possible. The non-negativity constraint on \( w_{SL} \) imposes an upperbound on \( w_{FL} \) and completes the proof. ■

Proof of Proposition 3

By the same logic as the Proposition 2 proof, the optimal contract will again make both screening constraints bind. Imposing equality on (7) and (8), rearrange to get \( \tilde{w}_{SH} \) and \( \tilde{w}_{S\theta} \) as functions of \( \tilde{w}_{FH} \) and \( \tilde{w}_{F\theta} \) then substitute into equation (5) to write profits as:

\[
\lambda p_H \bar{R}_H + (1 - \lambda) q \theta_2 - \left( 1 + \frac{\lambda p_H + (1 - \lambda) q}{\lambda (1 - \lambda) p_H (p_L - q)} \right) c - \tilde{w}_{FH}
\]

Profit maximization thus prompts the bank to set \( \tilde{w}_{FH} = 0 \). Substituting into the expressions for \( \tilde{w}_{SH} \) and \( \tilde{w}_{S\theta} \) completes the proof. ■
Proof of Proposition 4

Let $\Delta_{LH}$ denote expected profits under $\{\overline{R}_L, \overline{R}_H\}$ and define:

$$\phi(\lambda) \equiv 1 + \frac{[\lambda p_H (1-\lambda)q][1-\lambda p_H - (1-\lambda)p_L]}{\lambda(1-\lambda)[p_H (1-p_L) - q(1-p_H)]}$$

Notice $\phi'(\lambda) > 0$ if and only if $\lambda > \left(1 + \sqrt{\frac{p_H (1-p_H)}{q(1-p_L)}}\right)^{-1}$. Also notice $\phi''(\lambda) > 0$. Imposing $p_L \overline{R}_L = q\theta_2$ on Proposition 2 yields:

$$\Delta_{LH}(\lambda) = \lambda p_H \overline{R}_H + (1-\lambda) q\theta_2 - c \phi(\lambda)$$

Recall that $\{\overline{R}_L, \overline{R}_H\}$ dominates $\{\theta_2, \overline{R}_H\}$ when $p_L \overline{R}_L = q\theta_2$ so we need not consider expected profits under $\{\theta_2, \overline{R}_H\}$. Instead, just let $\Delta_i(\lambda)$ denote expected profits under a flat rate of $\overline{R}_i$. We can then write:

$$\Delta_L(\lambda) = \lambda p_H \overline{R}_L + (1-\lambda) q\theta_2$$

$$\Delta_H(\lambda) = \lambda p_H \overline{R}_H + (1-\lambda) q\overline{R}_H$$

Notice $\Delta_L(\lambda) > \lambda p_L \overline{R}_L + (1-\lambda) q\theta_2 = q\theta_2$, where $q\theta_2$ is the bank’s expected profit from charging a flat rate of $\theta_2$. It will thus suffice to compare $\{\overline{R}_L, \overline{R}_H\}$ with a flat rate of either $\overline{R}_L$ or $\overline{R}_H$. The conditions for $\Delta_{LH}(\lambda) > \Delta_L(\lambda)$ and $\Delta_{LH}(\lambda) > \Delta_H(\lambda)$ are respectively:

$$p_H (\overline{R}_H - \overline{R}_L) \lambda > c \phi(\lambda)$$

$$q (\theta_2 - \overline{R}_H) (1-\lambda) > c \phi(\lambda)$$

Tangency between $c \phi(\lambda)$ and $p_H (\overline{R}_H - \overline{R}_L) \lambda$ requires $c = c_l \equiv \frac{p_H (\overline{R}_H - \overline{R}_L)}{\phi(\lambda_l)}$ with $\lambda_l$ implicitly defined by $\phi(\lambda_l) \equiv \phi'(\lambda_l) \lambda_l$. If $c < c_l$, then $\Delta_{LH}(\lambda) \geq \Delta_L(\lambda)$ for $\lambda \in [\lambda_l - \varepsilon_l(c), \lambda_l + \hat{\varepsilon}_l(c)]$ where $\varepsilon_l(\cdot) > 0$, $\varepsilon'_l(\cdot) < 0$, $\hat{\varepsilon}_l(\cdot) > 0$, and $\hat{\varepsilon}'_l(\cdot) < 0$. Similarly, tangency between $c \phi(\lambda)$
and \( q \left( \theta_2 - R_H \right) (1 - \lambda) \) requires \( c = c_h \equiv -\frac{q(\theta_2 - R_H)}{\phi(\lambda_h)} \) with \( \phi(\lambda_h) \equiv -\phi'(\lambda_h)(1 - \lambda_h) \).

Since \( \phi'(\lambda_h) < 0 < \phi'(\lambda_l) \), we have \( \lambda_h < \lambda_l \). If \( c < c_h \), then \( \Delta_{LI}(\lambda) \geq \Delta_H(\lambda) \) for \( \lambda \in [\lambda_h - \varepsilon_h(c), \lambda_h + \tilde{\varepsilon}_h(c)] \) where \( \varepsilon_h(\cdot) > 0, \varepsilon'_h(\cdot) < 0, \tilde{\varepsilon}_h(\cdot) > 0, \) and \( \tilde{\varepsilon}'_h(\cdot) < 0 \). Necessary conditions for optimality of \( \{R_L, R_H\} \) are thus \( c < \min \{c_l, c_h\} \) and \( \lambda_l - \varepsilon_l(c) < \lambda_h + \tilde{\varepsilon}_h(c) \).

The latter reduces to \( \varepsilon_l(c) + \tilde{\varepsilon}_h(c) < \lambda_l - \lambda_h \) which is just another upperbound on \( c \), denoted by \( c < c_k \). Finally, the intersection between \( p_H(R_H - R_L) \lambda \) and \( q \left( \theta_2 - R_H \right) (1 - \lambda) \) is defined by \( \lambda_0 \equiv \frac{q(\theta_2 - R_H)}{q(\theta_2 - R_H) + p_H(R_H - R_L)} \) and it lies on \( c\phi(\lambda) \) when \( c = c_0 \equiv p_H(R_H - R_L)\lambda_0 \). If \( c < c_0 \), then \( \lambda_h - \varepsilon_h(c) < \lambda_l - \varepsilon_l(c) < \lambda_0 < \lambda_h + \tilde{\varepsilon}_h(c) < \lambda_l + \tilde{\varepsilon}_l(c) \). Notice \( \Delta_L(\lambda) \geq \Delta_H(\lambda) \) if and only if \( \lambda \leq \lambda_0 \) so defining \( \tau \equiv \min \{c_l, c_h, c_k, c_0\} \), \( \lambda_1 \equiv \lambda_l - \varepsilon_l(c) \), and \( \lambda_2 \equiv \lambda_h + \tilde{\varepsilon}_h(c) \) completes the proof. \( \blacksquare \)

**Proof of Proposition 5**

Begin with the safe strategy. The bank seeks to minimize:

\[
\lambda p_H (w_{SH} - w_{FH}) + (1 - \lambda) p_L (w_{SL} - w_{FL}) + \lambda w_{FH} + (1 - \lambda) w_{FL} \tag{9}
\]

The officer’s payoff under the incentive compatible contract is still given by (2) while his pay-off from not monitoring is \( \lambda \left( \alpha p_H w_{SH} + (1 - \alpha p_H) w_{FH} \right) + (1 - \lambda) \left[ \alpha p_L w_{SL} + (1 - \alpha p_L) w_{FL} \right] \).

The incentive compatibility constraint thus amounts to:

\[
\lambda p_H (w_{SH} - w_{FH}) + (1 - \lambda) p_L (w_{SL} - w_{FL}) \geq \frac{c}{\alpha - 1}
\]

Driving this down to equality and substituting into the bank’s problem yields agency costs of \( \frac{c}{\alpha - 1} + \lambda w_{FH} + (1 - \lambda) w_{FL} \). The bank thus sets \( w_{FH} = w_{FL} = 0 \), making its optimized cost \( c/(1 - \alpha) \). Turn now to the risky strategy. The bank seeks to minimize:

\[
\lambda p_H (\bar{w}_{SH} - \bar{w}_{FH}) + (1 - \lambda) q (\tilde{w}_{S0} - \tilde{w}_{F0}) + \lambda \bar{w}_{FH} + (1 - \lambda) \tilde{w}_{F0} \tag{10}
\]
The officer’s payoff under the incentive compatible contract is still given by (6) while his payoff from not monitoring is $\lambda [\alpha p_H \tilde{w}_{SH} + (1 - \alpha p_H) \tilde{w}_{FH}] + (1 - \lambda) [\alpha q \tilde{w}_{S\theta} + (1 - \alpha q) \tilde{w}_{F\theta}]$. The incentive compatibility constraint thus amounts to:

$$\lambda p_H (\tilde{w}_{SH} - \tilde{w}_{FH}) + (1 - \lambda) q (\tilde{w}_{S\theta} - \tilde{w}_{F\theta}) \geq \frac{c}{1 - \alpha}$$

Imposing equality and subbing into the bank’s problem yields $\frac{c}{1 - \alpha} + \lambda \tilde{w}_{FH} + (1 - \lambda) \tilde{w}_{F\theta}$. The bank thus sets $\tilde{w}_{FH} = \tilde{w}_{F\theta} = 0$, making its optimized cost $c/(1 - \alpha)$ once again. ■

**Proof of Proposition 6**

As modeled, $\alpha$ does not affect the trigger rates $\bar{R}_i$. To implement the safe strategy, (2) must now exceed both of the following:

$$\lambda [\alpha p_H w_{SL} + (1 - \alpha p_H) w_{FL}] + (1 - \lambda) [\alpha p_L w_{SL} + (1 - \alpha p_L) w_{FL}]$$

$$\lambda [\alpha p_H w_{SH} + (1 - \alpha p_H) w_{FH}] + (1 - \lambda) [\alpha q w_{SH} + (1 - \alpha q) w_{FH}]$$

Substituting the resulting constraints (with equality) into (9) makes the bank’s agency costs:

$$\alpha [\lambda p_H + (1 - \lambda) q] \frac{(1 - \lambda)(\lambda p_H + (1 - \lambda) p_L) - p_L}{\lambda p_H - \alpha(\lambda p_H + (1 - \lambda) q)} (w_{SL} - w_{FL}) + \left( \frac{\lambda p_H + (1 - \lambda)(\lambda p_H + (1 - \lambda) q)}{\lambda p_H - \alpha(\lambda p_H + (1 - \lambda) q)} \right) c + w_{FH}$$

(11)

The bank thus sets $w_{FH} = 0$. If $\alpha > \frac{p_L}{\lambda p_H + (1 - \lambda) p_L} \equiv \overline{\alpha}_0$, then it also sets $w_{SL} = 0$ and $w_{FL}$ as high as possible which yields:

$$w_{FL} = \frac{\alpha [\lambda p_H + (1 - \lambda) q] c}{\lambda p_H [1 - \alpha p_L - \alpha(\lambda p_H - p_L)] - \alpha[\lambda p_H + (1 - \lambda) q][\lambda (1 - \alpha p_H) + (1 - \alpha)(1 - \lambda) p_L]}$$

(12)

If $\alpha < \overline{\alpha}_0$, then it instead sets $w_{FL} = 0$ and $w_{SL}$ as high as possible which yields:

$$w_{SL} = \frac{[\lambda p_H + (1 - \lambda) q] c}{\lambda (1 - \lambda)p_H (p_L - q) + (1 - \alpha)[\lambda p_H + (1 - \lambda) q][\lambda p_H + (1 - \lambda) p_L]}$$

(13)
Consider now the risky strategy. The payoff (6) must now exceed both of the following:

$$\lambda [\alpha p_H \tilde{w}_{SH} + (1 - \alpha p_H) \tilde{w}_{FH}] + (1 - \lambda) [\alpha q \tilde{w}_{SH} + (1 - \alpha q) \tilde{w}_{FH}]$$

$$\alpha q \tilde{w}_{S\theta} + (1 - \alpha q) \tilde{w}_{F\theta}$$

Substituting the resulting constraints (with equality) into (10) makes the agency costs:

$$\frac{\lambda [1 + \alpha (1 - \lambda)] p_H + \alpha (1 - \lambda)^2 q_c}{\lambda [(1 - \alpha) (p_H - q) + (1 - \alpha) q]} \left( \tilde{w}_{S\theta} - \tilde{w}_{F\theta} \right) + \tilde{w}_{FH}$$

(14)

The bank thus sets $$\tilde{w}_{FH} = \tilde{w}_{F\theta} = 0$$ and $$\tilde{w}_{S\theta}$$ as high as possible, yielding:

$$\tilde{w}_{S\theta} = \frac{\frac{\lambda p_H + (1 - \lambda) q}{q [1 - \lambda (p_H - q) + (1 - \alpha) q]} \tilde{w}_{S\theta}}{(1 - \alpha) q}$$

(15)

For the safe strategy to dominate the risky one, we need (11) less than (14). There are two cases. If $$\alpha < \overline{\alpha}_0$$, substitute $$w_{FL} = w_{FH} = 0$$ into (11) and $$\tilde{w}_{FH} = \tilde{w}_{F\theta} = 0$$ into (14). The desired condition then simplifies to $$w_{SL} > \frac{(1 - \alpha) q}{p_L - \alpha [\lambda p_H + (1 - \lambda) p_L]} \tilde{w}_{S\theta}$$. With $$w_{SL}$$ and $$\tilde{w}_{S\theta}$$ given by (13) and (15) respectively, the condition further reduces to $$\frac{\alpha}{1 - \alpha} < -\frac{p_H}{(1 - \lambda) (p_H - q)}$$ which is false. If $$\alpha > \overline{\alpha}_0$$, substitute $$w_{SL} = w_{FH} = 0$$ into (11) and $$\tilde{w}_{FH} = \tilde{w}_{F\theta} = 0$$ into (14). The desired condition then simplifies to $$w_{FL} > \frac{(1 - \alpha) q}{\alpha [\lambda p_H + (1 - \lambda) p_L] - p_L} \tilde{w}_{S\theta}$$. With $$w_{FL}$$ and $$\tilde{w}_{S\theta}$$ given by (12) and (15) respectively, the condition further reduces to:

$$\alpha^2 - \left( \frac{1}{1 + \lambda (p_H - p_L)} + \frac{p_H}{\lambda p_H + (1 - \lambda) q} \right) \alpha + \frac{p_H}{1 + \lambda (p_H - p_L) [\lambda p_H + (1 - \lambda) q]} < 0$$

We thus need $$\alpha > \frac{1}{1 + \lambda (p_H - p_L)} > \overline{\alpha}_0$$ so defining $$\overline{\alpha} \equiv \frac{1}{1 + \lambda (p_H - p_L)}$$ completes the proof. ■

Proof of Proposition 7

Part 1 Begin with $$\{ \overline{R}_L, \overline{R}_H \}$$. Define the space $$\Omega \equiv \{ SL, SH, FL, FH \}$$, where $$SL$$ and $$SH$$ denote success under $$\overline{R}_L$$ and $$\overline{R}_H$$ respectively while $$FL$$ and $$FH$$ denote failure. I will
denote by $X^j_k$ the payment to an officer who achieves $j \in \Omega$ when the other officer achieves achieves $k \in \Omega$. To simplify the exposition, also define:

$$N_1 \equiv \lambda \left[ p_H X_{SH}^H + (1 - p_H) X_{FH}^H \right] + (1 - \lambda) \left[ p_L X_{SL}^H + (1 - p_L) X_{FH}^H \right]$$
$$N_2 \equiv \lambda \left[ p_H X_{SH}^H + (1 - p_H) X_{FH}^H \right] + (1 - \lambda) \left[ p_L X_{SL}^H + (1 - p_L) X_{FH}^H \right]$$
$$N_3 \equiv \lambda \left[ p_H X_{SL}^H + (1 - p_H) X_{FH}^H \right] + (1 - \lambda) \left[ p_L X_{SL}^H + (1 - p_L) X_{FH}^H \right]$$
$$N_4 \equiv \lambda \left[ p_H X_{FL}^H + (1 - p_H) X_{FH}^H \right] + (1 - \lambda) \left[ p_L X_{SL}^H + (1 - p_L) X_{FH}^H \right]$$

Under an incentive compatible contract, the loan officer’s expected payoff is:

$$\lambda [p_H N_1 + (1 - p_H) N_2] + (1 - \lambda) [p_L N_3 + (1 - p_L) N_4] - c$$ (16)

Always charging $R_L$ would give him $[\lambda p_H + (1 - \lambda) p_L] N_3 + [1 - \lambda p_H - (1 - \lambda) p_L] N_4$ while always charging $R_H$ would give him $[\lambda p_H + (1 - \lambda) q] N_1 + [1 - \lambda p_H - (1 - \lambda) q] N_2$. The incentive compatibility constraints are thus:

$$p_H N_1 + (1 - p_H) N_2 \geq p_H N_3 + (1 - p_H) N_4 + \frac{c}{\lambda}$$ (17)
$$q N_1 + (1 - q) N_2 \leq p_L N_3 + (1 - p_L) N_4 - \frac{c}{1 - \lambda}$$ (18)

The bank will make both constraints bind so we can combine to get:

$$N_1 - N_2 = \frac{[\lambda p_H + (1 - \lambda) p_L] c}{\lambda (1 - \lambda) p_H (p_L - q)} - \frac{(p_H - p_L) (N_4 - N_2)}{p_H (p_L - q)}$$ (19)
$$N_3 - N_4 = \frac{[\lambda p_H + (1 - \lambda) q] c}{\lambda (1 - \lambda) p_H (p_L - q)} - \frac{(p_H - q) (N_4 - N_2)}{p_H (p_L - q)}$$ (20)

To complete the argument, note that the bank’s expected profit per loan officer is just:
\[
\lambda p_H \bar{R}_H + (1 - \lambda) p_L \bar{R}_L - \lambda [p_H N_1 + (1 - p_H) N_2] - (1 - \lambda) [p_L N_3 + (1 - p_L) N_4]
\] (21)

Substituting equations (19) and (20) into the expression for expected profit yields:

\[
\lambda p_H \bar{R}_H + (1 - \lambda) p_L \bar{R}_L - \left( \frac{2p_L}{p_L - q} + \frac{\lambda^2 p_H^2 + (1 - \lambda)^2 p_L q}{\lambda (1 - \lambda)p_H(p_L - q)} \right) c - \left( \frac{(p_H - q)[\lambda p_H + (1 - \lambda) p_L] N_2}{p_H(p_L - q)} + \frac{(p_H - p_L)[\lambda p_H + (1 - \lambda) q] N_4}{p_H(p_L - q)} \right)
\]

The bank thus sets \( N_2 = 0 \) and \( N_4 \) as high as possible. Given equations (19) and (20), non-negativity of \( N_3 \) yields \( N_4 = \frac{[\lambda p_H + (1 - \lambda) q c]}{\lambda (1 - \lambda)p_H(1 - p_L - q - q(p_H - p_L))} \). Substituting back into expected profit then yields the same expression as Proposition 2.

**Part 2**

Consider now \( \{ \bar{R}_H, \theta_2 \} \). Define \( \tilde{\Omega} \equiv \{ SH, S\theta, FH, F\theta \} \), where \( S\theta \) and \( F\theta \) denote success and failure respectively under \( \theta_2 \). Denote by \( \tilde{X}_j^k \) the payment to an officer who achieves \( j \in \tilde{\Omega} \) when the other officer achieves \( k \in \tilde{\Omega} \). It will also help to define:

\[
\tilde{N}_1 \equiv \lambda \left[ p_H \tilde{X}_{SH}^S + (1 - p_H) \tilde{X}_{SH}^F \right] + (1 - \lambda) \left[ q \tilde{X}_{SH}^S + (1 - q) \tilde{X}_{SH}^F \right]
\]

\[
\tilde{N}_2 \equiv \lambda \left[ p_H \tilde{X}_{FH}^S + (1 - p_H) \tilde{X}_{FH}^F \right] + (1 - \lambda) \left[ q \tilde{X}_{FH}^S + (1 - q) \tilde{X}_{FH}^F \right]
\]

\[
\tilde{N}_3 \equiv \lambda \left[ p_H \tilde{X}_{S\theta}^S + (1 - p_H) \tilde{X}_{S\theta}^F \right] + (1 - \lambda) \left[ q \tilde{X}_{S\theta}^S + (1 - q) \tilde{X}_{S\theta}^F \right]
\]

\[
\tilde{N}_4 \equiv \lambda \left[ p_H \tilde{X}_{F\theta}^S + (1 - p_H) \tilde{X}_{F\theta}^F \right] + (1 - \lambda) \left[ q \tilde{X}_{F\theta}^S + (1 - q) \tilde{X}_{F\theta}^F \right]
\]

The incentive compatibility constraints are:

\[
p_H \tilde{N}_1 + (1 - p_H) \tilde{N}_2 \geq q \tilde{N}_3 + (1 - q) \tilde{N}_4 + \frac{c}{\lambda}
\] (22)

\[
q \tilde{N}_3 + (1 - q) \tilde{N}_4 \geq q \tilde{N}_1 + (1 - q) \tilde{N}_2 + \frac{c}{1-\lambda}
\] (23)
Both constraints will bind so we can combine them to get:

\[ \tilde{N}_1 = \tilde{N}_2 + \frac{c}{\lambda(1-\lambda)(p_H-q)} \]  \hspace{1cm} (24)

\[ q\tilde{N}_3 + (1-q)\tilde{N}_4 = \tilde{N}_2 + \frac{[\lambda p_H+(1-\lambda)q]c}{\lambda(1-\lambda)(p_H-q)} \]  \hspace{1cm} (25)

Turn next to the bank’s expected profit per loan officer, namely:

\[ \lambda p_H \bar{R}_H + (1-\lambda) q\theta_2 - \lambda \left[p_H \tilde{N}_1 + (1-p_H) \tilde{N}_2\right] - (1-\lambda) \left[q \tilde{N}_3 + (1-q) \tilde{N}_4\right] \]  \hspace{1cm} (26)

Substituting equations (24) and (25) into the expression for expected profit gives:

\[ \lambda p_H \bar{R}_H + (1-\lambda) q\theta_2 - \left(1 + \frac{\lambda p_H+(1-\lambda)q}{\lambda(1-\lambda)(p_H-q)}\right) c - \tilde{N}_2 \]

The bank thus sets \( \tilde{N}_2 = 0 \), yielding the same expected profit as in Proposition 3. ■

**Proof of Proposition 8 – Part 1: Risky Strategy**

**Wage Scheme** To satisfy (7) and (8) for both \( \lambda \)s, the bank makes (7) hold with equality at \( \bar{\lambda} \) and (8) hold with equality at \( \lambda \). The resulting expressions can be combined to get:

\[ \tilde{w}_{SH} - \tilde{w}_{FH} = \frac{1-\bar{\lambda}+\lambda}{\lambda(1-\lambda)(p_H-q)}c \]  \hspace{1cm} (27)

\[ \tilde{w}_{S\theta} - \tilde{w}_{F\theta} = \frac{1}{q} (\tilde{w}_{FH} - \tilde{w}_{F\theta}) + \frac{\lambda p_H+(1-\bar{\lambda})q}{\lambda(1-\lambda)q(p_H-q)}c \]  \hspace{1cm} (28)

Consider now the bank’s expected profit per loan officer, namely:

\[ \left[\pi\lambda + (1-\pi) \bar{\lambda}\right] \left[p_H \left(\bar{R}_H - \tilde{w}_{SH}\right) - (1-p_H) \tilde{w}_{FH}\right] \]

\[ + \left[1-\pi\lambda - (1-\pi) \bar{\lambda}\right] \left[q \left(\theta_2 - \tilde{w}_{S\theta}\right) - (1-q) \tilde{w}_{F\theta}\right] \]
Substituting equations (27) and (28) into the expression for expected profit yields:

\[
\pi \lambda + (1 - \pi) \lambda \] p_H R_H + [1 - \pi \lambda - (1 - \pi) \lambda] q_\theta_2 - \left( \pi + \frac{(1 - \pi) \lambda}{\Delta} + \frac{\lambda p_H + (1 - \lambda) q}{\Delta (\pi - \lambda)} \right) c - \tilde{w}_{FH} \]

The bank thus sets \( \tilde{w}_{FH} = 0 \). Invoking \( \lambda + \bar{\lambda} = 1 \), we can then write expected profit as:

\[
[1 - \lambda - (1 - 2\lambda) \pi] p_H R_H + [\lambda + (1 - 2\lambda) \pi] q_\theta_2 - \left( \frac{2p_H - (\lambda + (1 - 2\lambda) \pi)(\pi - q)}{\lambda (\pi - q)} \right) c \quad (29)
\]

**Tournament Scheme** Define \( \bar{\Omega} \) and \( \bar{X}_j^k \) as in the proof of Proposition 7. Also define:

\[
\bar{A} \equiv p_H \left[ p_H \bar{X}_{S_H} - q \bar{X}_{S_\theta} - (1 - q) \bar{X}_{F_\theta} \right]
\]

\[
+ (1 - p_H) \left[ p_H \left( \bar{X}_{F_H} + \bar{X}_{S_H} \right) + (1 - p_H) \bar{X}_{F_H} - q \bar{X}_{S_H} - (1 - q) \bar{X}_{F_H} \right]
\]

\[
\bar{B} \equiv q \left( p_H \bar{X}_{S_H} - q \bar{X}_{S_\theta} \right) + (1 - p_H) \left[ q \bar{X}_{F_H} + (1 - q) \bar{X}_{F_\theta} \right]
\]

\[
+ (1 - q) \left[ p_H \bar{X}_{S_H} - q \left( \bar{X}_{S_\theta} + \bar{X}_{S_\theta} \right) - (1 - q) \bar{X}_{F_\theta} \right]
\]

\[
\bar{Y} \equiv p_H \left[ q \left( \bar{X}_{S_H} - \bar{X}_{S_\theta} \right) + (1 - q) \left( \bar{X}_{F_\theta} - \bar{X}_{F_H} \right) \right]
\]

\[
+ (1 - p_H) \left[ q \left( \bar{X}_{S_\theta} - \bar{X}_{S_H} \right) + (1 - q) \left( \bar{X}_{F_\theta} - \bar{X}_{F_H} \right) \right]
\]

\[
\bar{Z} \equiv q^2 \left( \bar{X}_{S_\theta} - \bar{X}_{S_H} \right) + q (1 - q) \left( \bar{X}_{S_\theta} + \bar{X}_{S_\theta} - \bar{X}_{SH} - \bar{X}_{SH} \right) + (1 - q)^2 \left( \bar{X}_{F_\theta} - \bar{X}_{F_\theta} \right)
\]

The incentive compatibility constraints (22) and (23) can now be written as:

\[
\tilde{f} (\lambda) \equiv \lambda^2 \bar{A} + \lambda (1 - \lambda) \bar{B} \geq c \quad (30)
\]

\[
\tilde{g} (\lambda) \equiv \lambda (1 - \lambda) \bar{Y} + (1 - \lambda)^2 \bar{Z} \geq c \quad (31)
\]
Notice that $\lambda + \bar{\lambda} = 1$ implies $\tilde{f}(\lambda) = \tilde{f}(\bar{\lambda}) + (1 - 2\lambda) \tilde{A}$ and $\tilde{g}(\lambda) = \tilde{g}(\bar{\lambda}) - (1 - 2\lambda) \tilde{Z}$.

Therefore, setting $\tilde{A} = 0$ and $\tilde{B} = \frac{c}{\Lambda(1 - \Lambda)}$ will make (30) hold with equality for both $\lambda$ and $\bar{\lambda}$ while setting $\tilde{Z} = 0$ and $\tilde{Y} = \frac{c}{\Lambda(1 - \Lambda)}$ will make (31) hold with equality for both $\lambda$ and $\bar{\lambda}$.

Combining these restrictions with the definitions of $\tilde{A}$, $\tilde{B}$, $\tilde{Y}$, and $\tilde{Z}$ then yields:

\[
p_H \tilde{X}^{SH}_{SH} + (1 - p_H) \tilde{X}^{FH}_{SH} = p_H \tilde{X}^{SH}_{FH} + (1 - p_H) \tilde{X}^{FH}_{FH} + \frac{c}{\Lambda(1 - \Lambda)(p_H - q)} \tag{32}
\]

\[
q \tilde{X}^{s0}_{SH} + (1 - q) \tilde{X}^{F0}_{SH} = q \tilde{X}^{s0}_{FH} + (1 - q) \tilde{X}^{F0}_{FH} + \frac{c}{\Lambda(1 - \Lambda)(p_H - q)} \tag{33}
\]

\[
q \tilde{X}^{s0}_{SH} + (1 - q) \tilde{X}^{F0}_{SH} = \tilde{X}^{s0}_{SH} + (1 - q) \tilde{X}^{F0}_{SH} - \frac{(1 - q)[q \tilde{X}^{s0}_{FH} + (1 - q) \tilde{X}^{F0}_{FH}]}{q} + \frac{c}{\Lambda(1 - \Lambda)(p_H - q)} \tag{34}
\]

\[
p_H \tilde{X}^{SH}_{s0} + (1 - p_H) \tilde{X}^{FH}_{s0} = \frac{v_H \tilde{X}^{FH}_{FH} + (1 - v_H) \tilde{X}^{FH}_{s0}}{q} - \frac{(1 - q)[p_H \tilde{X}^{s0}_{FH} + (1 - p_H) \tilde{X}^{F0}_{FH}]}{q} + \frac{p_H c}{(1 - \Lambda)(p_H - q)} \tag{35}
\]

Conditional on $\lambda$, the bank’s expected profit per officer is (26) which we can rewrite as:

\[
\tilde{H}(\lambda) \equiv \lambda p_H \tilde{R}_H + (1 - \lambda) q \theta_2
\]

Taking into account the possible realizations of $\lambda$, the full expression for expected profit is just $\pi \tilde{H}(\lambda) + (1 - \pi) \tilde{H}(1 - \lambda)$. Combining with equations (32) to (35) then yields:

\[
[1 - \Delta - (1 - 2\Delta) \pi] p_H \tilde{R}_H + [\Delta + (1 - 2\Lambda) \pi] q \theta_2 - \left( \frac{\Lambda^2 + (1 - 2\Lambda) \pi}{\Lambda(1 - \Lambda)(p_H - q)} \right) c
\]

\[- [1 - \Delta - (1 - 2\Delta) \pi] \left( p_H \tilde{X}^{SH}_{FH} + (1 - p_H) \tilde{X}^{FH}_{FH} \right) + [\Delta + (1 - 2\Lambda) \pi] \left( q \tilde{X}^{s0}_{FH} + (1 - q) \tilde{X}^{F0}_{FH} \right)
\]

The bank thus sets $\tilde{X}^{s0}_{FH} = \tilde{X}^{F0}_{FH} = \tilde{X}^{s0}_{FH} = \tilde{X}^{F0}_{FH} = 0$, making its expected profit:
\[(1 - \lambda - (1 - 2\lambda)\pi) p_H \bar{R}_H + [\lambda + (1 - 2\lambda)\pi] q_\theta - \left(\frac{p_H - [\lambda^2 + (1 - 2\lambda)\pi] (p_H - q)}{\Delta (1 - \lambda) (p_H - q)}\right) c \quad (36)\]

For the optimal tournament to dominate independent wage contracts, \((36)\) must exceed \((29)\). This condition reduces to \(\pi > 1 - \frac{p_H}{\Delta (p_H - q)}\) which is true given \(\pi \in (0, 1)\). To see if the optimal tournament can be rank-order, impose \(\tilde{X}_{SH} = \tilde{X}_{FH} = \tilde{X}_{F\theta} = 0\) and the additional rank-order restrictions \((\tilde{X}_{S\theta}^S = \tilde{X}_{F\theta}^S = \tilde{X}_{FH}^S = 0)\) on \((32)\) to \((35)\). The result is:

\[
p_H \tilde{X}_{S\theta}^S + (1 - p_H) \tilde{X}_{F\theta}^S + \frac{(1-q) (1-p_H)}{q} \tilde{X}_{F\theta}^F = \frac{p_H c}{\Delta (1 - \lambda) (p_H - q) q} \quad (37)
\]

\[
p_H \tilde{X}_{SH} + (1 - p_H) \tilde{X}_{FH} = (1 - q) \tilde{X}_{S\theta}^S + (1 - q) \tilde{X}_{F\theta}^S + \frac{(1-q)^2}{q} \tilde{X}_{F\theta}^F = \frac{c}{\Delta (1 - \lambda) (p_H - q)} \quad (38)
\]

which is feasible. The optimal tournament can thus be implemented via rank-order. Since none of the restrictions involve not compensating a straight win, the optimal tournament can also be implemented via winner-take-all. □

**Proof of Proposition 8 – Part 2: Safe Strategy**

**Wage Scheme** Equations \((3)\) and \((4)\) are the conditions that must be satisfied to induce screening for a given value of \(\lambda\). To ensure that both conditions hold regardless of which \(\lambda\) is realized, the bank makes \((3)\) hold with equality at \(\lambda\) and \((4)\) hold with equality at \(\bar{\lambda}\). The resulting expressions can be combined to get:

\[
w_{SL} - w_{FL} = \frac{p_H - q}{p_H (p_L - q)} (w_{FH} - w_{FL}) + \frac{\lambda p_H + (1-\lambda) q} {\lambda (1-\lambda) p_H (p_L - q)} c \quad (37)
\]

\[
w_{SH} - w_{FH} = \frac{p_H - p_L}{p_H (p_L - q)} (w_{FH} - w_{FL}) + \frac{\lambda p_H + (1-\lambda) p_L} {\lambda (1-\lambda) p_H (p_L - q)} c \quad (38)
\]

Consider now the bank’s expected profit per loan officer, namely:
\[
\left[\pi \lambda + (1 - \pi) \bar{X}\right] \left[p_H \left(R_H - w_{SH}\right) - (1 - p_H) w_{FH}\right]
+ \left[1 - \pi \lambda - (1 - \pi) \bar{X}\right] \left[p_L \left(R_L - w_{SL}\right) - (1 - p_L) w_{FL}\right]
\]

Substituting equations (37) and (38) into the expression for expected profit yields:

\[
\left[\pi \lambda + (1 - \pi) \bar{X}\right] p_H R_H + \left[1 - \pi \lambda - (1 - \pi) \bar{X}\right] p_L R_L - \frac{p_L \left[\pi \lambda + (1 - \pi) \bar{X}\right]}{p_H} \left[p_H w_{FH} - p_L w_{FL}\right] - \frac{q \left[\pi \lambda + (1 - \pi) \bar{X}\right]}{p_H} (p_H - q) w_{FH}
+ \frac{q \left[\pi \lambda + (1 - \pi) \bar{X}\right]}{p_H} (p_H - q) w_{FL} - \frac{\left[\pi \lambda + (1 - \pi) \bar{X}\right]}{\Delta(p_H - q)} w_{FH} = 0 \quad \text{as high as possible. Given (37) and (38), non-negativity of } w_{SL} \text{ yields } w_{FL} = \frac{\left[\Delta p_H + (1 - \pi) \Delta q\right]}{\Delta(p_H - q)} \frac{c}{\Delta(1 - \pi)}. \text{ Using } \lambda + \bar{X} = 1, \text{ we can then write profit as:}
\]

\[
\left[1 - \lambda - (1 - 2\lambda) \pi\right] p_H R_H + \left[\lambda + (1 - 2\lambda) \pi\right] p_L R_L - \frac{\left[\lambda + (1 - 2\lambda) \pi\right]}{\Delta(1 - \pi)} \left[p_H \left(1 - p_H\right) - q(1 - p_H)\right] c
\]

**Tournament Scheme**  
Define \( \Omega \) and \( X_j^k \) as in the proof of Proposition 7. Also define:

\[
A \equiv p_H^2 \left(X_{SH}^H - X_{SL}^H\right) + p_H \left(1 - p_H\right) \left(X_{SH}^H + X_{SL}^H - X_{SL}^F - X_{FL}^H\right) + (1 - p_H)^2 \left(X_{FH}^H - X_{FL}^H\right)
\]

\[
B \equiv p_H \left[p_L \left(X_{SH}^H - X_{SL}^H\right) + (1 - p_L) \left(X_{SH}^F - X_{SL}^F\right)\right] + \left(1 - p_H\right) \left[p_L \left(X_{FH}^H - X_{SL}^H\right) + (1 - p_L) \left(X_{FH}^F - X_{FL}^F\right)\right]
\]

\[
Y \equiv p_H \left[p_L X_{SL}^H + (1 - p_L) X_{SL}^H - qX_{SL}^H - (1 - q) X_{FH}^H\right] + \left(1 - p_H\right) \left[p_L X_{SL}^F + (1 - p_L) X_{SL}^F - qX_{SL}^H - (1 - q) X_{FH}^H\right]
\]
\[ Z \equiv p_L [p_L X_{SL}^{FL} - q X_{SH}^{SL} - (1 - q) X_{FH}^{SL}] + (1 - p_L) [p_L (X_{SL}^{FL} + X_{SL}^{FL}) + (1 - p_L) X_{FL}^{FL} - q X_{SH}^{FL} - (1 - q) X_{FH}^{FL}] \]

The incentive compatibility constraints (17) and (18) can now be written as:

\[ f(\lambda) \equiv \lambda^2 A + \lambda (1 - \lambda) B \geq c \quad (40) \]

\[ g(\lambda) \equiv \lambda (1 - \lambda) Y + (1 - \lambda)^2 Z \geq c \quad (41) \]

As in the proof of Proposition 8 – Part 1, setting \( A = 0 \) and \( B = \frac{c}{\Delta(1-\Delta)} \) will make (40) hold with equality for both \( \lambda \) and \( \bar{\lambda} \) while setting \( Z = 0 \) and \( Y = \frac{c}{\Delta(1-\Delta)} \) will make (41) hold with equality for both \( \lambda \) and \( \bar{\lambda} \). Combining the newly-found restrictions on \( A, B, Y, \) and \( Z \) with the definitions of those variables then yields:

\[ p_L X_{SH}^{SL} + (1 - p_L) X_{SH}^{FL} = \frac{(1-p_H)(p_L-q)[p_L X_{FL}^{SL} + (1-p_L) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} + \frac{(1-p_H)c}{\Delta(1-\Delta)(p_H(1-p_L)-q(1-p_H))} \quad (42) \]

\[ p_H X_{SH}^{SL} + (1 - p_H) X_{SH}^{FL} = \frac{(1-p_H)(p_L-q)[p_H X_{FL}^{SL} + (1-p_H) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} + \frac{(1-p_H)c}{\Delta(1-\Delta)(p_H(1-p_L)-q(1-p_H))} \quad (43) \]

\[ p_L X_{FL}^{SL} + (1 - p_L) X_{FL}^{FL} = \frac{(p_L-q)[p_L X_{FL}^{SL} + (1-p_L) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} - \frac{p_H(p_L-q)[p_L X_{FL}^{SL} + (1-p_L) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} + \frac{qc}{\Delta(1-\Delta)(p_H(1-p_L)-q(1-p_H))} \quad (44) \]

\[ p_H X_{FL}^{SL} + (1 - p_H) X_{FL}^{FL} = \frac{(p_L-q)[p_H X_{FL}^{SL} + (1-p_H) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} - \frac{p_H(p_L-q)[p_H X_{FL}^{SL} + (1-p_H) X_{FH}^{FL}]}{p_H(1-p_L)-q(1-p_H)} + \frac{p_H c}{\Delta(1-\Delta)(p_H(1-p_L)-q(1-p_H))} \quad (45) \]
Conditional on $\lambda$, the bank’s expected profit per officer is \[(21)\] which we can rewrite as:

\[
H(\lambda) = \lambda p_H \bar{R}_H + (1 - \lambda) p_L \bar{R}_L
- \lambda^2 \left[ p_H^2 X_{SH}^H + p_H (1 - p_H) \left( X_{SH}^F + X_{SL}^H \right) + (1 - p_H)^2 X_{FH}^F \right]
- \lambda (1 - \lambda) p_H \left[ p_L \left( X_{SL}^H + X_{SH}^H \right) + (1 - p_L) \left( X_{FL}^F + X_{SL}^H \right) \right]
- \lambda (1 - \lambda) (1 - p_H) \left[ p_L \left( X_{SH}^F + X_{SL}^L \right) + (1 - p_L) \left( X_{FL}^L + X_{SL}^F \right) \right]
- (1 - \lambda)^2 \left[ p_L^2 X_{SL}^L + p_L (1 - p_L) \left( X_{SL}^F + X_{SL}^L \right) + (1 - p_L)^2 X_{FL}^L \right]
\]

The full expression for expected profit is then $\pi H(\lambda) + (1 - \pi) H(1 - \lambda)$ which, with equations \[(42)\] to \[(45)\] substituted in, becomes:

\[
\begin{align*}
&\left[1 - \lambda - (1 - 2\lambda) \pi\right] p_H \bar{R}_H + \left[\lambda + (1 - 2\lambda) \pi\right] p_L \bar{R}_L \\
&- \left(2\lambda(1-\lambda)p_H(p_H-p_L)+p_H(1-p_H)+q(1-p_L)-p_H(1-p_H)\right)\left[\lambda^2+(1-2\lambda)\pi\right] c \\
&- \left[(1-p_H)(1-\lambda-(1-2\lambda)\pi)+(p_H-p_L)\lambda(1-\lambda)(p_H-q)\left[p_H X_{SH}^H+(1-p_H) X_{FH}^F\right]\right] \\
&- \left[(1-p_L)(\lambda^2+(1-2\lambda)\pi)+(1-p_H)\lambda(1-\lambda)(p_H-q)\left[p_L X_{SL}^L+(1-p_L) X_{FL}^F\right]\right] \\
&- \left[p_H(\lambda^2+(1-2\lambda)(1-\pi)+q(1-\lambda))\left[p_H-p_L\left[p_H X_{SH}^H+(1-p_H) X_{FH}^F\right]\right] \\
&- \left[q(\lambda^2+(1-2\lambda)\pi)+p_H(1-\lambda)\left[p_H-p_L\left[p_L X_{SL}^L+(1-p_L) X_{FL}^F\right]\right] \\
&\end{align*}
\]

The bank thus sets $X_{FH}^F = X_{FH}^F = X_{SL}^L = X_{SL}^L = 0$ and $X_{SH}^F = X_{SL}^H = X_{SL}^F = X_{SL}^L = 0$, making its expected profit:

\[
\left[1 - \lambda - (1 - 2\lambda) \pi\right] p_H \bar{R}_H + \left[\lambda + (1 - 2\lambda) \pi\right] p_L \bar{R}_L
- \left(2\lambda(1-\lambda)p_H(p_H-p_L)+p_H(1-p_H)+q(1-p_L)-p_H(1-p_H)\right)\left[\lambda^2+(1-2\lambda)\pi\right] c
\]

For the optimal tournament to dominate independent wage contracts, \[(46)\] must exceed \[(39)\].

This condition reduces to:
\[ \Delta [q (1 - p_L) - p_H (1 - p_H)] \pi < [\Delta q + (1 - \Delta) p_H] (1 - p_L) \] (47)

If \( q \leq \frac{p_H (1 - p_H)}{1 - p_L} \), then (47) is true. If \( q > \frac{p_H (1 - p_H)}{1 - p_L} \), write (47) as \( \pi < \frac{\Delta q + (1 - \Delta) p_H}{\Delta q (1 - p_L) - p_H (1 - p_H)} \equiv \bar{\pi} \) and notice that \( \bar{\pi} > 1 \) reduces to \( \Delta p_H + (1 - \Delta) p_L < 1 \) which is also true. Therefore, (47) is always true. To see whether the optimal tournament can be implemented via rank-order, impose the following on equations (42) to (45):

\[
X_{SH} = X_{OH} = X_{SL} = X_{OL} = 0 \quad (\text{the first set of conditions for an optimal tournament}),
\]

\[
X_{SH} = X_{SH} = X_{SL} = X_{SL} = 0 \quad (\text{the second set of conditions for an optimal tournament}), \quad \text{and} \quad X_{FL} = X_{FL} = 0 \quad (\text{the additional conditions for a rank-order tournament}).
\]

The result is:

\[
p_L X_{SL} + (1 - p_L) X_{SH} = \frac{(1 - p_L)c}{\Delta (1 - \Delta)(p_H (1 - p_L) - q(1 - p_H))}
\]

\[
p_H X_{SH} + (1 - p_H) X_{FH} = \frac{(1 - p_H)c}{\Delta (1 - \Delta)(p_H (1 - p_L) - q(1 - p_H))}
\]

\[
(1 - p_L) X_{FL} = \frac{qc}{\Delta (1 - \Delta)(p_H (1 - p_L) - q(1 - p_H))}
\]

\[
(1 - p_H) X_{FL} = \frac{p_H c}{\Delta (1 - \Delta)(p_H (1 - p_L) - q(1 - p_H))}
\]

which is feasible if and only if \( p_H < 1 \). Therefore, it is only possible to implement the optimal tournament via rank-order when \( p_H < 1 \). Notice that it is not possible to implement the optimal tournament via winner-take-all because the former requires \( X_{FL} = 0 \).

**Rank-Order with \( p_H = 1 \)**

Return to equations (40) and (41). The focus on rank-order tournaments introduces the following restrictions:

\[
X_{SL} = X_{OH} = X_{SH} = X_{FH} = X_{FL} = 0
\]

Imposing these restrictions along with \( p_H = 1 \) yields \( A = X_{SH}, Y = -q X_{SH} \), and:
\[ B = p_L \left( X_{SH}^{SL} - X_{SL}^{SL} \right) + (1 - p_L) \left( X_{SH}^{FL} - X_{SL}^{FL} \right) \]

\[ Z = p_L \left( p_L X_{SL}^{SL} - q X_{SH}^{SL} \right) + (1 - p_L) \left[ p_L X_{SL}^{FL} + (1 - p_L) X_{FL}^{FL} - q X_{SH}^{FL} - (1 - q) X_{FH}^{FL} \right] \]

Setting \( A = 0 \) and \( B = \frac{c}{\lambda(1-\lambda)} \) will again make (40) hold with equality for both \( \lambda \) and \( \bar{\lambda} \). Given \( Y = -q X_{SH}^{SL} \leq 0 \), however, the bank must now set \( Z > 0 \). In other words, (41) cannot hold with equality for both \( \lambda \) and \( \bar{\lambda} \). Instead, we will have \( g(\lambda) = c < g(\bar{\lambda}) \) with \( Y = 0 \) and \( Z = \frac{c}{(1-\lambda)^2} \). Combining \( A = Y = 0 \) and \( B = Z = \frac{c}{\lambda(1-\lambda)} \) yields:

\[ X_{SH}^{SL} = 0 \] (48)

\[ p_L X_{SL}^{SL} + (1 - p_L) X_{SL}^{FL} = \frac{(1-\lambda + \lambda p_L)c}{\lambda^2(1-\lambda)(1-p_L-q)} - \frac{(1-p_L)^2}{p_L-q} X_{FL}^{FL} \] (49)

\[ p_L X_{SH}^{SL} + (1 - p_L) X_{SH}^{FL} = \frac{(1-\lambda + \lambda p_L)c}{\lambda^2(1-\lambda)(1-p_L-q)} - \frac{(1-p_L)^2}{p_L-q} X_{FL}^{FL} \] (50)

The bank’s expected profit per loan officer is still \( \pi_H(\lambda) + (1-\pi)H(1-\lambda) \) with \( H(\cdot) \) as defined earlier in this proof. Substituting in \( p_H = 1 \), the rank-order restrictions, and equations (48) to (50) then yields:

\[ \left[ 1 - \lambda - (1-2\lambda) \pi \right] \bar{R}_H + \left[ \lambda + (1-2\lambda) \pi \right] p_L \bar{R}_L - \left( \frac{1-\lambda + \lambda p_L}{\lambda^2(1-\lambda)(1-p_L-q)} + \frac{\lambda^2(1-2\lambda)\pi p_L (1-\lambda + \lambda p_L)}{\lambda^2(1-\lambda)(1-p_L-q)} \right) c + \left[ \lambda(1-\lambda) + \frac{\lambda^2(1-2\lambda)\pi q^2 (1-p_L)}{p_L-q} \right] X_{FL}^{FL} \]

The bank thus sets \( X_{FL}^{FL} \) as high as possible. Given equations (49) and (50), the non-negativity constraint on \( p_L X_{SL}^{SL} + (1 - p_L) X_{FL}^{FL} \) yields \( X_{FL}^{FL} = \frac{(1-\lambda + \lambda p_L)c}{\lambda^2(1-\lambda)(1-p_L-q)} \). We can now write the bank’s expected profit as:

\[ \left[ 1 - \lambda - (1-2\lambda) \pi \right] \bar{R}_H + \left[ \lambda + (1-2\lambda) \pi \right] p_L \bar{R}_L - \left( \frac{\lambda^2(1-\lambda) + \lambda^2(1-2\lambda)\pi q (1-p_L)}{\lambda^2(1-\lambda)} \right) c \] (51)
For the rank-order tournament to dominate independent wage contracts, (51) must be greater than (39) evaluated at \( p_H = 1 \). This condition reduces to \( \pi < \Pi_0 \equiv \frac{\lambda^2 (1-\lambda q^2)}{1-\lambda q^2} \in (0, 1) \). □

**Proof of Proposition 9**

Begin with the optimal tournament (i.e., not necessarily rank-order) which has been shown to dominate independent wage contracts. The maximized profits under \( \{\theta_2, R_H\} \) and \( \{R_L, R_H\} \) are given by (36) and (46) respectively. For \( \{\theta_2, \overline{R}_H\} \) to be more costly to implement, the coefficient on \( c \) in (36) must exceed the coefficient on \( c \) in (46). In other words:

\[
\frac{p_H - \left[\lambda^2(1-2\lambda)\pi\right](p_H-q)}{\Delta(1-\lambda)(p_H-q)} > \frac{2\lambda(1-\lambda)p_H(p_H-p_L)+p_H(1-p_H)+[q(1-p_L)-p_H(1-p_H)]\left[\lambda^2(1-2\lambda)\pi\right]}{\Delta(1-\lambda)(p_H-1-p_H-q)}
\]

With some algebra, this condition reduces to \( \pi < \frac{[\lambda q + (1-\lambda) p_H]^2}{(1-2\lambda)(p_H^2-q^2)} \). Notice that \( \frac{[\lambda q + (1-\lambda) p_H]^2}{(1-2\lambda)(p_H^2-q^2)} > 1 \) simplifies to \( [\lambda p_H + (1-\lambda) q]^2 > 0 \) which is true. Therefore, \( \{\theta_2, \overline{R}_H\} \) is more costly to implement than \( \{R_L, \overline{R}_H\} \). Turn now to rank-order tournaments when \( p_H = 1 \). The maximized profit under \( \{\theta_2, \overline{R}_H\} \) is still given by (36) which, with \( p_H = 1 \) imposed, becomes:

\[
[1-\lambda - (1-2\lambda)\pi]\overline{R}_H + [\lambda + (1-2\lambda)\pi]q\theta_2 - \left(\frac{1-\left[\lambda^2(1-2\lambda)\pi\right](1-q)}{\Delta(1-\lambda)(1-q)}\right)c
\]

(52)

If \( \pi < \Pi_0 \), then the maximized profit under \( \{\overline{R}_L, \overline{R}_H\} \) is given by (51). For \( \{\theta_2, \overline{R}_H\} \) to be more costly to implement than \( \{\overline{R}_L, \overline{R}_H\} \), the coefficient on \( c \) in (52) must exceed the coefficient on \( c \) in (51). In other words:

\[
\frac{1-\left[\lambda^2(1-2\lambda)\pi\right](1-q)}{\Delta(1-\lambda)(1-q)} > \frac{\lambda^2(1-\lambda q^2)+\left[\lambda^2(1-2\lambda)\pi\right](1-\lambda+\lambda q)}{\Delta^2(1-\lambda)}
\]

This condition simplifies to \( \pi < \frac{\lambda - \lambda^2 (q^2(1-\lambda+\lambda q))}{(1+2\lambda)(1-q)(1-2\lambda)} \equiv \Pi_1 \). Notice that \( \Pi_1 \geq \Pi_0 \) reduces to \( \lambda (1-\lambda)^2 q^2 + (1-\lambda)(1-\lambda+\lambda^2) q + \lambda^2 \geq 0 \) which is true. Consider next \( \pi \geq \Pi_0 \). The

5 Recall that the optimal tournament can always be implemented via rank-order when \( p_H < 1 \) so the cost result on optimal tournaments carries over to rank-order tournaments when \( p_H < 1 \).
maximized profit under $\{R_L, R_H\}$ is now given by (39) evaluated at $p_H = 1$:

$$[1 - \lambda - (1 - 2\lambda) \pi] R_H + [\lambda + (1 - 2\lambda) \pi] p_L R_L - \left(\frac{1 + [\lambda + (1 - 2\lambda) \pi] q}{\lambda}\right) c$$

(53)

For $\{\theta_2, R_H\}$ to be more costly to implement, the coefficient on $c$ in (52) must exceed the coefficient on $c$ in (53). In other words:

$$\frac{1 - [\lambda^2 + (1 - 2\lambda) \pi] (1 - q)}{\lambda (1 - \Delta) (1 - q)} > \frac{1 + [\lambda + (1 - 2\lambda) \pi] q}{\lambda}$$

This condition simplifies to $\pi < \frac{q + \lambda (1 - \lambda) (1 - q)^2}{1 + q (1 - \Delta) (1 - q) (1 - 2\lambda)} \equiv \Pi_2$. Therefore, $\{\theta_2, R_H\}$ is more costly to implement than $\{R_L, R_H\}$ when $\pi < \min \{\Pi_0, \Pi_2\}$. We know $\Pi_0 < 1$ so, for $\{\theta_2, R_H\}$ to always be more costly, we would need $\Pi_2 \geq 1$. Notice $\Pi_2 \geq 1$ if and only if $q \geq \frac{\sqrt{5} - 12\lambda + 4\lambda^2 - 1}{2(1 - \Delta)} \equiv Q$ where $Q < 1$. Moreover, $Q \leq 0$ if and only if $\lambda \in \left[\frac{3 - \sqrt{5}}{2}, \frac{1}{2}\right]$. Therefore, any $\lambda \in \left(0, \frac{3 - \sqrt{5}}{2}\right)$ yields $Q > 0$ and thus $\Pi_2 < 1$ for $q \in (0, Q)$. We can thus conclude that the cost of implementing $\{\theta_2, R_H\}$ is actually lower than the cost of implementing $\{R_L, R_H\}$ when $p_H = 1$, $\lambda \in \left(0, \frac{3 - \sqrt{5}}{2}\right)$, $q \in (0, Q)$, $\pi \in \max \{\Pi_0, \Pi_2\}, 1)$, and tournaments are restricted to be rank-order.

**Proof of Proposition 10**

Begin with the risky strategy. The officer’s payoff under an incentive compatible contract is:

$$\lambda p_H \tilde{N}_1 + \lambda (1 - p_H) \tilde{N}_2 + (1 - \lambda) q \tilde{N}_3 + (1 - \lambda) (1 - q) \tilde{N}_4 - c$$

(54)

with the $\tilde{N}$s as defined in the proof of Proposition 7. There are three new deviations that the bank would like to eliminate. In each one, the officers split rewards equally and do not screen. The first deviation involves both officers charging $R_H$, making the payoff to each:

$$[\lambda p_H + (1 - \lambda) q]^2 \tilde{X}_{SH}^H + \frac{2[\lambda p_H + (1 - \lambda) q][1 - \lambda p_H - (1 - \lambda) q)(\tilde{X}_{SH}^H + \tilde{X}_{FH}^H)]}{2} + [1 - \lambda p_H - (1 - \lambda) q]^2 \tilde{X}_{FH}^H$$
The second deviation involves both officers charging $\theta_2$, making the payoff to each:

$$q^2 \tilde{X}_{S\theta} + \frac{2q(1-q)(\tilde{X}_{S\theta}^F + \tilde{X}_{F\theta}^S)}{2} + (1 - q)^2 \tilde{X}_{F\theta}^F$$

The third deviation involves each officer charging a different rate, making the payoff to each:

$$[\lambda p_H + (1 - \lambda) q] \left[ q \frac{\tilde{X}_{S\theta}^F + \tilde{X}_{F\theta}^S}{2} + (1 - q) \frac{\tilde{X}_{SD}^F + \tilde{X}_{FS}^H}{2} \right] + [1 - \lambda p_H - (1 - \lambda) q] \left[ q \frac{\tilde{X}_{F\theta}^F + \tilde{X}_{FH}^S}{2} + (1 - q) \frac{\tilde{X}_{FSD}^H + \tilde{X}_{FSH}^F}{2} \right]$$

Under the optimal tournament, (54) must exceed all of these payoffs. Recall the optimal tournament from the first part of Proposition 8, namely equations (32) to (35) with $\tilde{X}_{FH}^k = 0$ for all $k \in \tilde{\Omega}$. For this contract to eliminate the first deviation regardless of which $\lambda$ is realized, we need $\tilde{X}_{FH}^F \leq \frac{c}{\Delta(1-\Delta)(p_H-q)}$. Eliminating the second deviation then requires $p_H \geq q$ (trivially true) while eliminating the third requires $\tilde{X}_{S\theta}^F = \tilde{X}_{F\theta}^F = 0$. All of these restrictions can be accommodated by the optimal tournament in Proposition 8 as well as the optimal rank-order tournament. Therefore, agency costs for the risky strategy are still $\frac{p_H - \frac{x^2 + (1-2\lambda)\pi(p_H-q)}{\Delta(1-\Delta)(p_H-q)}}{c}$. Turn now to the safe strategy. The officer’s payoff under an incentive compatible contract is still given by (16) from the proof of Proposition 7. Payoffs from the first deviation – both officers charging $\bar{R}_H$ – are as above but with $X$s instead of $\tilde{X}$s. The second deviation now involves both officers charging $\bar{R}_L$, making the payoff to each:

$$[\lambda p_H + (1 - \lambda) p_L]^2 X_{S\theta}^L + \frac{2[\lambda p_H + (1-\lambda)p_L][1-\lambda p_H - (1-\lambda)p_L](X_{S\theta}^F + X_{F\theta}^S)}{2} + [1 - \lambda p_H - (1 - \lambda) p_L]^2 X_{F\theta}^L$$

The third deviation still involves each officer charging a different loan rate but the rates are now $\bar{R}_L$ and $\bar{R}_H$ so the payoff becomes:

$$[\lambda p_H + (1 - \lambda) p_L] \left[ \frac{\lambda p_H + (1-\lambda)q}{2} \left( X_{S\theta}^F + X_{F\theta}^S \right) + \frac{1-\lambda p_H - (1-\lambda)q}{2} \left( X_{S\theta}^F + X_{F\theta}^S \right) \right] + [1 - \lambda p_H - (1 - \lambda) p_L] \left[ \frac{\lambda p_H + (1-\lambda)q}{2} \left( X_{S\theta}^F + X_{F\theta}^S \right) + \frac{1-\lambda p_H - (1-\lambda)q}{2} \left( X_{S\theta}^F + X_{F\theta}^S \right) \right]$$
Under the optimal tournament, (16) must exceed all of these payoffs. Recall the optimal tournament from the second part of Proposition 8, namely equations (42) to (45) with $X^k_{FH} = X^k_{SL} = 0$ for all $k \in \Omega$. Eliminating the first deviation requires $X^{FH}_{SH} \leq \frac{c}{\Delta(1-\lambda)(p_H-q)}$ while eliminating the second requires $X^{SL}_{FL} \leq \frac{c}{\Delta(1-\lambda)(p_H-p_L)}$. Eliminating the third then requires $X^{SL}_{SH} = X^{FH}_{FL} = 0$. All of these restrictions can be accommodated by the optimal tournament in Proposition 8. However, the third restriction implies $X^{FL}_{SH} = X^{FH}_{FL} > 0$ which violates rank-order. After much algebra (available upon request), the optimal rank-order tournament for the safe strategy now involves a minimum agency cost of:

$$\left\{\frac{\Delta^2 + \pi(1-2\lambda)(1-p_H)+\left[\Delta^2 + \pi(1-2\lambda)(1-p_H)(1-q) + \frac{2\Delta(1-\lambda)p_H(1-p_L)(2-p_L-q)}{p_H(1-p_L)+q(1-p_H)}\right]p_Hc}{\Delta(1-\lambda)(p_H(1-p_L)-q(1-p_H))}\right\}_{\theta_2, R_H}$$

A sufficient condition for $\{\theta_2, R_H\}$ to dominate is then $\Delta^2 + \pi(1-2\lambda) > \frac{p_H(1-p_H)(p_H-p_L)}{(1-p_L)(1-q)(p_H-q)}$, which is true for $q$ and $\Delta$ sufficiently low and $\pi$ sufficiently high.