Bank Promotions and Credit Quality*

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Abstract

This paper studies the lending decisions of loan officers who compete for a profit-based promotion. Two main results emerge from the theoretical model. First, higher average borrower quality can actually increase default rates in the presence of promotion incentives, especially if some lenders are uninformed about the quality of their own borrowers. Second, longer promotion horizons can also lead to higher default rates.

1 Introduction

Theories of incentive provision abound in economics yet there have been few systematic analyses of the effect of loan officer promotions on borrower default rates. This paper takes a step in that direction, building a model to study the lending decisions of loan officers who compete for a profit-based promotion. I examine the parameters for which promotions induce more versus less risk-taking. I also examine how risk-taking changes with the length of the promotion horizon.

I begin by constructing a model where loan officers essentially choose how much risk to induce among their borrowers. Safe loans generate a small profit with high probability while risky loans generate a large profit with low probability. Safe loans are assumed to dominate

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in terms of expected output and are socially optimal. When a big promotion value is on the line, I find that lenders are less willing to push good borrowers towards very risky projects. However, if the fraction of good borrowers is above some threshold, lenders are more willing to push weaker borrowers in that direction. The intuition is as follows. With a high fraction of good borrowers, a lender expects the quality of his colleague’s borrower to be high and thus expects the colleague to generate a sizeable profit. A safe loan to a weak borrower is very unlikely to outperform such a profit so lenders with weak borrowers attempt to compete against their colleagues by making risky loans. Higher average borrower quality can thus generate higher default rates in the presence of promotion incentives. The result is even more pronounced when some lenders are uncertain about the quality of their own borrowers. In this case, they also expect the quality of their own borrowers to be good and thus make loans that default with high probability if the expectation is not realized. Abstracting from the intertemporal accumulation of skills, I also find that longer promotion horizons can lead to higher default rates. By eliminating the scope for catch-up, a short horizon eliminates risk-taking by laggars and also makes lenders more conservative in the risks they are willing to take early on.

Since a profit-based promotion is effectively a rank-order tournament, my paper is broadly related to the literature on such tournaments (e.g., Lazear and Rosen (1981), Green and Stokey (1983), and Krishna and Morgan (1998)). However, instead of analyzing optimal labor contracts and/or optimal tournaments, I focus on how winner-take-all tournaments within banks affect the quality of credit. Also related are finance papers on risk-taking by mutual fund managers in the presence of tournaments. Different papers reach different conclusions though, with some arguing that laggars take more risk (e.g., Goriaev et al. (2001)) and others arguing that leaders take more risk (e.g., Taylor (2003) and Basak and Makarov (2012)). As we will see, my model nests each of these predictions (as they pertain to loan officers) in different areas of the parameter space. While the pursuit of more risk by laggars arises for likelier parameter combinations, if safe loans are actually risky in absolute terms and
risky loans are only moderately riskier in relative terms, then I find that lenders with small leads become the bigger risk-takers. Finally, my paper is related to micro-banking studies (e.g., Kanz (2010)) that investigate how compensation schemes can be used to mitigate agency problems within banks and reduce defaults. In such papers, however, the focus is on principal-agent problems rather than multi-agent tournaments.

The paper proceeds as follows: Section 2 presents the results when each lender knows his borrower’s type and promotions occur after one period, Section 3 relaxes the information assumption, Section 4 extends the promotion horizon, and Section 5 concludes.

2 Basic Model

The economy is populated by two types of firms: $H$ and $L$. The fraction of type $H$ firms is $\lambda \in (0, 1)$. All firms have access to an investment project and a speculative project. If operated by a type $i \in \{L, H\}$, the investment project produces $\theta_1$ units of output with probability $p_i \in (0, 1)$ and zero units with probability $1 - p_i$. In contrast, the speculative project produces $\theta_2$ units of output with probability $q$ and zero units otherwise regardless of which type operates it. Assume $p_H > p_L$ so that type $H$ firms are the ones most likely to succeed at the investment project. Also assume $p_L > q$ and $\theta_2 > \theta_1$ but $p_L \theta_1 > q \theta_2$ so that the speculative project is riskier in the sense that it is second order stochastically dominated by the investment project. In what follows, I will refer to the investment project as the safe project and the speculative project as the risky one. All quantities are denominated in units of output.

A firm needs one unit of bank capital to operate either project. There are two loan officers per bank and each officer has one unit of capital to intermediate. I abstract from the extensive margin and assume all borrowers are financed. The key margin is thus the loan rate at which each borrower is financed and the project choices that these rates induce. To ease the exposition, assume that capital is not destroyed in the production process so the
borrowed unit is always recovered by the loan officer. Also "normalize" the bank’s cost of funds to zero so that the loan rate, if paid, reflects the bank’s profit from the loan. The rate that makes a type $i$ borrower indifferent between the safe and risky project is $\bar{R}_i = \frac{p_i \theta_1 - q \theta_2}{p_i - q}$, where $\bar{R}_L < \bar{R}_H < \theta_2$. I assume that borrowers repay their loans if and only if their projects are successful so a borrower charged $R$ would get $p_i [\theta_1 - R]$ from operating the safe project and $q [\theta_2 - R]$ from operating the risky one, making $\bar{R}_i$ the loan rate that equates these two payoffs. A loan officer (lender) with a type $i$ borrower must decide whether to charge $\bar{R}_i$ or $\theta_2$. Notice that charging below $\bar{R}_i$ is not optimal: the lender could extract more profit from the borrower without increasing the default probability (i.e., without changing the borrower’s project choice) by just charging $\bar{R}_i$. Similarly, charging strictly between $\bar{R}_i$ and $\theta_2$ is not optimal: the lender could extract more profit without increasing the default probability by just charging $\theta_2$. Since matches are one-to-one and any $R \leq \theta_2$ satisfies the borrower’s participation constraint, lenders do not need to compete for borrowers and thus have no other incentives to undercut.

However, within each bank, loan officers compete for a prize valued at $X$. The prize cannot be split between officers so I interpret it as a promotion, although one could also interpret it as a winner-take-all bonus. For now, the promotion horizon is one period and each lender knows his borrower’s type. These assumptions will be relaxed in subsequent sections. A lender does not know the type of borrower financed by his colleague and thus assigns probability $\lambda$ to the event that the colleague finances an $H$. The lender who generates the most profit at the end of the period wins the promotion. If the two lenders are tied with positive profit (i.e., if both generate $\bar{R}_i$ or both generate $\theta_2$), then one is promoted at random. If the two lenders are tied with zero profit (i.e., if both of their borrowers default), then neither lender gets the promotion. I focus on symmetric pure strategy nash equilibria (PSNE) of the form $(r_L, r_H)$, where $r_i$ represents the loan rate charged to a type $i$ borrower. The results will be compared to a benchmark with no promotion so, to give lenders a decision problem in the benchmark, assume that each gets to keep a share (set to 1 for simplicity) of
the profit he generates regardless of relative performance.

2.1 Equilibrium with Only Safe Projects

I begin by analyzing the parameters for which \((\bar{R}_L, \bar{R}_H)\) emerges as a symmetric PSNE. Consider the best response of lender B when his colleague, lender A, plays \((\bar{R}_L, \bar{R}_H)\). Suppose B charges type \(L\) borrowers \(\bar{R}_L\). If B’s borrower repays, then B gets \(\bar{R}_L\). Furthermore, if B’s borrower repays while A’s borrower defaults, then B also gets \(X\). However, if both borrowers repay, then B gets \(X\) with probability \(\frac{1}{2}\) if and only if A’s borrower is also a type \(L\). If instead B charges his type \(L\) borrower \(\theta_2\), then B will always outperform A as long as B’s borrower repays. So, for B to charge \(\bar{R}_L\), we need:

\[
p_L \left[ \bar{R}_L + \lambda (1 - p_H) X + (1 - \lambda) (1 - p_L) X + \left( \frac{1 - \lambda}{2} p_L \right) X \right] > q [\theta_2 + X] \tag{1}
\]

Now suppose B charges type \(H\) borrowers \(\bar{R}_H\). If B’s borrower repays, then B gets \(\bar{R}_H\). Furthermore, if B’s borrower repays while A’s borrower defaults, then B also gets \(X\). If both borrowers repay, then B gets \(X\) with probability 1 if A’s borrower is type \(L\) and \(X\) with probability \(\frac{1}{2}\) if A’s borrower is type \(H\). If instead B charges his type \(H\) borrower \(\theta_2\), then B will always outperform A as long as B’s borrower repays. So, for B to charge \(\bar{R}_H\), we need:

\[
p_H \left[ \bar{R}_H + \lambda (1 - p_H) X + (1 - \lambda) X + \frac{\lambda p_H}{2} X \right] > q [\theta_2 + X] \tag{2}
\]

It is straightforward to show that equation (1) is more restrictive than (2). Substituting for \(\bar{R}_L\) then yields that \((\bar{R}_L, \bar{R}_H)\) is a symmetric PSNE if and only if:

\[
q < \left[ 1 - \frac{|p_L + \lambda(2p_H - p_L)| X}{4(\theta_2 + X)} \right] \sqrt{\frac{\theta_2 - \theta_1}{\theta_2 + X}} + \left( \frac{|p_L + \lambda(2p_H - p_L)| X}{4(\theta_2 + X)} \right)^2 \] p_L \equiv Q \left( p_L \right) \tag{3}
\]

We can also show that \(\frac{dQ(p_L)}{dX} > 0\) amounts to \(\lambda < \frac{2\sqrt{\theta_2 - \theta_1} - p_L}{2p_H - p_L}\). Since \(\lambda \in (0, 1)\), this condition
is only relevant if \( p_H > \sqrt{\frac{\theta_2 - \theta_1}{\theta_2}} \). Proposition 1 summarizes the results so far.

Proposition 1  Fix \( \theta_1 \) and \( \theta_2 \).

1. \((\overline{R}_L, \overline{R}_H)\) is a symmetric PSNE if and only if \( q < Q(p_L) \).

2. If \( p_H \in \left(\sqrt{\frac{\theta_2 - \theta_1}{\theta_2}}, 1\right) \) and \( \lambda \in \left(\frac{2\sqrt{\frac{\theta_2 - \theta_1}{\theta_2} - p_L}}{2p_H - p_L}, 1\right) \), then \( \frac{dQ(p_L)}{dX} < 0 \). Otherwise, \( \frac{dQ(p_L)}{dX} > 0 \).

In words, Proposition 1 says that higher values of \( X \) decrease the set of \( q \)'s for which \((\overline{R}_L, \overline{R}_H)\) is an equilibrium if \( p_H \) and \( \lambda \) are sufficiently large. In particular, unless \( q \) is very low, lenders will prefer to charge type L borrowers \( \theta_2 \), thus inducing the risky project. The intuition is as follows. With high \( \lambda \), it is very likely that B’s colleague has a type \( H \) borrower. Moreover, with high \( p_H \), it is very likely that the colleague’s borrower will be able to repay \( \overline{R}_H \). Therefore, if B charges his type L borrower \( \overline{R}_L \), it is very unlikely that he will outperform the colleague. The higher the promotion value \( X \), the higher the desire to outperform and, therefore, the higher the incentive to charge \( \theta_2 \) instead of \( \overline{R}_L \).

2.2 Equilibrium with Only Risky Projects

I now analyze the parameters for which \((\theta_2, \theta_2)\) emerges as a symmetric PSNE. In particular, consider B’s best response when A plays \((\theta_2, \theta_2)\). Suppose B charges type L borrowers \( \overline{R}_L \). If B’s borrower repays, then B gets \( \overline{R}_L \). Since \( \overline{R}_L < \theta_2 \), the only way B can also get \( X \) is if his borrower repays while A’s borrower defaults. If instead B charges his type L borrower \( \theta_2 \) and the borrower repays, then B gets \( X \) with probability \( 1 \) if A’s borrower defaults and \( X \) with probability \( \frac{1}{2} \) if A’s borrower repays. So, for B to charge L \( \theta_2 \), we need:

\[
p_L \left[ \overline{R}_L + (1 - q) X \right] < q \left[ \theta_2 + (1 - q) X + \frac{q}{2} X \right] \tag{4}
\]

The analysis proceeds similarly if B has a type H so, for B to charge H \( \theta_2 \), we need:

\[
p_H \left[ \overline{R}_H + (1 - q) X \right] < q \left[ \theta_2 + (1 - q) X + \frac{q}{2} X \right] \tag{5}
\]
With \( p_H > p_L \) and \( \overline{R}_H > \overline{R}_L \), equation (5) is more restrictive than (4) so \((\theta_2, \theta_2)\) is a symmetric PSNE if and only if:

\[
f(q) = \frac{X}{2} q^3 - \left[ \theta_2 + (1 + \frac{3p_H}{2}) X \right] q^2 + p_H [2\theta_2 + (2 + p_H) X] q - p_H^2 [\theta_1 + X] > 0 \quad (6)
\]

The next proposition, proven in Appendix A.1, summarizes a key implication of (6):

**Proposition 2** Suppose \( \theta_2 > \left( \frac{\frac{p_H}{1+p_H-\sqrt{1+p_H^2}}}{\theta_1} \right) \) with \( p_H \geq \frac{1}{2} \).

1. There exists a unique \( Q \) such that \( f(Q) = 0 \). Moreover \( \frac{df}{dX} > 0 \).

2. \((\theta_2, \theta_2)\) is a symmetric PSNE if and only if \( X < X \) and \( q \in (Q, \frac{p_H \theta_1}{\theta_2}) \).

For \( \theta_2 \) sufficiently higher than \( \theta_1 \), Proposition 2 says that \((\theta_2, \theta_2)\) will only be an equilibrium if \( q \) is sufficiently high and \( X \) is sufficiently low. Indeed, as \( X \) increases, the set of \( q \)'s for which \((\theta_2, \theta_2)\) is an equilibrium shrinks until empty. With the parameter space restricted by \( p_L \theta_1 > q \theta_2 \), a very high value of \( \theta_2 \) means that only very low values of \( q \) are feasible. Charging a borrower \( \theta_2 \) thus induces a very high probability of default and, by implication, a very high probability of losing the promotion. Proposition 2 basically says that lenders are less willing to push type \( H \) borrowers towards very risky projects when a big promotion value is on the line.

### 2.3 Equilibrium with Both Safe and Risky Projects

Turn now to the existence of \((\overline{R}_L, \theta_2)\) and/or \((\theta_2, \overline{R}_H)\) as symmetric PSNEs - that is, the existence of pure strategy equilibria where project choice differs across borrower types. The following proposition, proven in Appendix A.2, narrows the focus:

**Proposition 3** \((\overline{R}_L, \theta_2)\) is not a symmetric PSNE.

The intuition behind Proposition 3 is straightforward: safe projects are more lucrative when operated by type \( H \) borrowers so a lender who finds it profitable to induce the safe project
among type $L$s must also find it profitable to induce the safe project among type $H$s. The remainder of this subsection thus formalizes B’s best response when A plays $(\theta_2, \overline{R}_H)$.

If B charges a type $L$ borrower $\overline{R}_L$ and the borrower repays, then B gets $\overline{R}_L$. However, since both $\overline{R}_H$ and $\theta_2$ exceed $\overline{R}_L$, the only way B can also get $X$ is if his borrower repays while A’s borrower defaults. If instead B charges his type $L$ borrower $\theta_2$ and the borrower repays, then B gets $X$ with probability 1 if A’s borrower is a type $H$, $X$ with probability 1 if A’s borrower is a defaulting type $L$, and $X$ with probability $\frac{1}{2}$ if A’s borrower is a repaying type $L$. So, B charges $L \theta_2$ if:

$$p_L \left[ \overline{R}_L + \left[ \lambda (1 - p_H) + (1 - \lambda) (1 - q) \right] X \right] < q \left[ \theta_2 + \left[ \lambda + (1 - \lambda) (1 - q) + \frac{(1-\lambda)q}{2} \right] X \right] \quad (7)$$

Now suppose B charges type $H$ borrowers $\overline{R}_H$. If B’s borrower repays, then B gets $\overline{R}_H$. Furthermore, if B’s borrower repays while A’s borrower defaults, then B also gets $X$. If both borrowers repay, then B gets $X$ with probability $\frac{1}{2}$ if and only if A’s borrower is type $H$. If instead B charges his type $H$ borrower $\theta_2$ and the borrower repays, then B again gets the right-hand side of equation (7). So, for B to charge $\overline{R}_H$, we need:

$$p_H \left[ \overline{R}_H + \left[ \lambda (1 - p_H) + (1 - \lambda) (1 - q) + \frac{\lambda p_H u}{2} \right] X \right] > q \left[ \theta_2 + \left[ \lambda + (1 - \lambda) (1 - q) + \frac{(1-\lambda)q}{2} \right] X \right] \quad (8)$$

In sum, $(\theta_2, \overline{R}_H)$ is a symmetric PSNE if and only if equations (7) and (8) hold.

### 2.4 Numerical Results

In what follows, I normalize $\theta_1 = 1$. What matters is $\theta_2$ and $X$ relative to $\theta_1$ so the normalization is without loss of generality. I also fix $p_H = 0.99$ so that a safe project is very safe when operated by a type $H$ borrower.

Figure 1 illustrates how the incidence of $(\theta_2, \overline{R}_H)$ as a symmetric PSNE varies with the
remaining parameters when $\theta_2$ is high relative to $\theta_1$. Panel (a) shows what happens absent profit-based promotions (i.e., $X = 0$). Panels (b) and (c) then show what happens with such promotions (i.e., $X > 0$) when the economy is characterized by low versus high proportions of type $H$ borrowers. With $p_L \theta_1 > q \theta_2$, only combinations of $p_L$ and $q$ below the solid black lines are feasible. In each figure, the blue triangle represents the parameter combinations for which both lenders choose $(\overline{R}_L, \overline{R}_H)$, thus inducing the safe project among all borrowers. The pink triangle represents the combinations for which both lenders choose $(\theta_2, \overline{R}_H)$, thus inducing the risky project among type $L$ borrowers and the safe project among type $H$ borrowers. Finally, the yellow triangle represents the combinations for which both lenders choose $(\theta_2, \theta_2)$, thus inducing the risky project among all borrowers. Blank spaces below the solid black lines mean that no symmetric PSNEs exist for those combinations of $p_L$ and $q$. For comparison, the dashed lines in panels (b) and (c) trace out the division of the parameter space that prevails in panel (a).

As predicted by Proposition 2, an increase in $X$ (from $X = 0$ to $X = 2$ in Figure 1) decreases the set of parameters for which $(\theta_2, \theta_2)$ is an equilibrium when $\theta_2$ is high. Moreover, as predicted by Proposition 1, an increase in $X$ increases the set of parameters for which $(\overline{R}_L, \overline{R}_H)$ is an equilibrium when $\lambda$ is low but decreases it when $\lambda$ is high. As we might expect, $(\theta_2, \overline{R}_H)$ arises as an equilibrium in the intermediate parameter space - that is, when $q$ is high enough relative to $p_L$ to justify pushing type $L$ borrowers towards the risky project but not high enough relative to $p_H$ to justify pushing the type $H$s. Notice, however, that $(\theta_2, \overline{R}_H)$ does not fill the entire intermediate region when $X > 0$. The blank spaces below the solid black lines are thus associated with asymmetric PSNEs and/or mixed strategy Nash equilibria. Either way then, parameters in the blank area between the pink and yellow triangles would prompt strategies that do not induce all borrowers to choose the risky project.

Since the safe project is socially optimal (recall the expected outputs $p_i \theta_1 > q \theta_2$), this is an improvement over the $(\theta_2, \theta_2)$ equilibrium that would arise under $X = 0$. In contrast, parameters in the blank area between the blue and pink triangles would prompt strategies
that do not induce all borrowers to choose the safe project. With a high proportion of type 
H borrowers as in panel (c), this is a worse outcome than the $(\bar{R}_L, \bar{R}_H)$ equilibrium that would 
arise under $X = 0$. However, with a low proportion as in panel (b), it is a better outcome if 
the parameters are also above the horizontal dashed line but could be either better or worse otherwise.\footnote{A note on uniqueness under $X = 0$: It is straightforward to show that there are no asymmetric PSNEs. What about mixed strategy nash equilibria? With $X = 0$, a lender would only mix if $p_i R_i = q \theta_2$ or, equivalently, $q = \left(1 - \sqrt{1 - \frac{p_i}{\theta_2}}\right) p_i$. These equations trace out the dashed black lines in Figure 1. Since the 
dashes do not intersect the blank areas in panel (c), the symmetric PSNEs under $X = 0$ would be the unique 
NEs in these areas. Same applies to panel (b) except for a small segment along the horizontal dashes.}

The analysis so far has focused on high values of $\theta_2$ relative to $\theta_1$. What happens if $\theta_2$ 
is only slightly higher than $\theta_1$? Figure 2 presents those results. The most notable difference 
relative to Figure 1 is that the area covered by $(\theta_2, \theta_2)$ now actually increases with $X$.\footnote{At first glance, another difference may appear to be the shrinking of the area covered by $(\bar{R}_L, \bar{R}_H)$ in panel (b). However, with $\theta_2 = 1.1$, even $\lambda = 0.25$ satisfies $\lambda > \frac{\sqrt{\frac{p_i}{\theta_2}} - p_L}{2 \frac{p_H}{p_H} - p_L}$ for most values of $p_L$. A lower 
value of $\lambda$ would thus be needed to illustrate the demarcation in Proposition 1 when $\theta_2$ is low.} At 
a lower value of $\theta_2$, higher values of $q$ satisfy $p_L \theta_1 > q \theta_2$ and are thus admitted into the 
parameter space. Since $q$ is also the repayment probability of borrowers who undertake the 
risky project, the set of $q$’s for which lenders find it optimal to pursue risk in an attempt 
to outperform their colleagues increases. Taken together with Proposition 2, this suggests 
that promotions induce more risk-taking when the risks to be taken are moderate but less 
risk-taking when the risks to be taken are high.

3 Extension One: Some Uninformed Lenders

I now relax the assumption that each lender knows his borrower’s type. Let $\pi \in (0, 1)$ denote 
the probability that any given lender is uninformed about his borrower. As in Section 2, a 
lender who knows that his borrower is type $i$ would not find it optimal to charge anything 
other than $\bar{R}_i$ or $\theta_2$. What about a lender who does not know $i$? Any $R < \bar{R}_L$ would 
induce both types to operate the safe project, an outcome that the lender could achieve with
more profit by just charging $R_L$. Similarly, any $R \in (\bar{R}_L, \bar{R}_H)$ would induce only type $H$ borrowers to operate the safe project, an outcome that could be achieved with more profit by just charging $R_H$. Finally, any $R \in (\bar{R}_H, \theta_2)$ would induce both types to operate the risky project, an outcome that could be achieved with more profit by just charging $\theta_2$. Therefore, uninformed lenders choose between $\bar{R}_L, \bar{R}_H,$ and $\theta_2$. Proposition 4, proven in Appendix A.3, reduces the number of relevant equilibria:

**Proposition 4** Only $(\bar{R}_L, \bar{R}_H, \bar{R}_L)$, $(\bar{R}_L, \bar{R}_H, \bar{R}_H)$, $(\theta_2, \bar{R}_H, \cdot)$, and $(\theta_2, \theta_2, \theta_2)$ satisfy the conditions for a symmetric PSNE.

In words, this proposition says that any symmetric PSNE has three features. First, if it is optimal for an informed lender with a type $L$ borrower to charge $\bar{R}_L$, then it must be optimal for an informed lender with a type $H$ to charge $\bar{R}_H$. Second, if it is optimal for an informed lender with a type $L$ to charge $\bar{R}_L$, then it cannot be optimal for an uninformed lender to charge $\theta_2$. Third, if it is optimal for an informed lender with a type $H$ to charge $\theta_2$, then it must be optimal for an uninformed lender to charge $\theta_2$. To analyze when each equilibrium arises, we need to specify the expected payoffs. As in the basic model, these payoffs are the product of two parts: (1) the probability that B’s borrower repays and (2) B’s earnings conditional on repayment. Since (2) depends only on the loan rate charged and how it compares to the performance of B’s colleague, the expected payoffs that support each relevant equilibrium can be summarized as follows. First, B’s expected payoff from charging $\bar{R}_L$ is $p_L J_L$ when informed with a type $L$ and $[\lambda p_H + (1 - \lambda) p_L] J_L$ when uninformed, where:

$$J_L = \begin{cases} 
\bar{R}_L + \left(1 - \frac{\lambda(1+\pi)p_H+(1-\lambda)p_L}{2}\right) X & \text{for } (\bar{R}_L, \bar{R}_H, \bar{R}_L) \\
\bar{R}_L + [1 - (1 - \lambda) (1 - \pi) q] X - \left[\lambda p_H + \frac{(1-\lambda)p_L}{2}\right] X & \text{for } (\bar{R}_L, \bar{R}_H, \bar{R}_H) \\
\bar{R}_L + [1 - (1 - \lambda) \pi q] X - \frac{\lambda(1+\pi)p_H+(1-\lambda)(1-\pi)p_L}{2} X & \text{for } (\theta_2, \bar{R}_H, \bar{R}_L) \\
\bar{R}_L + (1 - q) X - \lambda (p_H - q) X & \text{for } (\theta_2, \bar{R}_H, \bar{R}_H) \\
\bar{R}_L + (1 - q) X - \pi \lambda (p_H - q) X & \text{for } (\theta_2, \bar{R}_L, \theta_2) \\
\bar{R}_L + (1 - q) X & \text{for } (\theta_2, \theta_2, \theta_2)
\end{cases}$$
Next, B’s expected payoff from charging $\bar{R}_H$ is $p_H J_H$ when informed with a type $H$ and $[\lambda p_H + (1 - \lambda) q] J_H$ when uninformed, where:

$$J_H = \begin{cases} \bar{R}_H + (1 - \frac{\lambda p_H}{2}) X & \text{for } (\bar{R}_L, \bar{R}_H, \bar{R}_L) \\ \bar{R}_H + X - \frac{\lambda p_H + (1 - \lambda)(1 - \pi) q}{2} X & \text{for } (\bar{R}_L, \bar{R}_H, \bar{R}_H) \\ \bar{R}_H + [1 - (1 - \lambda) \pi q] X - \frac{\lambda p_H}{2} X & \text{for } (\theta_2, \bar{R}_H, \bar{R}_L) \\ \bar{R}_H + X - \frac{\lambda p_H + (1 - \lambda)(1 + \pi) q}{2} X & \text{for } (\theta_2, \bar{R}_H, \bar{R}_H) \\ \bar{R}_H + (1 - q) X - \pi \lambda \left(\frac{\mu_H}{2} - q\right) X & \text{for } (\theta_2, \bar{R}_H, \theta_2) \\ \bar{R}_H + (1 - q) X & \text{for } (\theta_2, \theta_2, \theta_2) \end{cases}$$

Finally, regardless of his information set or borrower’s type, B’s expected payoff from charging $\theta_2$ is $qK$, where:

$$K = \begin{cases} \theta_2 + X & \text{for } (\bar{R}_L, \bar{R}_H, \bar{R}_L) \text{ and } (\bar{R}_L, \bar{R}_H, \bar{R}_H) \\ \theta_2 + \left(1 - \frac{(1 - \lambda) \pi q}{2}\right) X & \text{for } (\theta_2, \bar{R}_H, \bar{R}_L) \text{ and } (\theta_2, \bar{R}_H, \bar{R}_H) \\ \theta_2 + \left(1 - \frac{(1 - \pi \lambda) q}{2}\right) X & \text{for } (\theta_2, \bar{R}_H, \theta_2) \\ \theta_2 + \left(1 - \frac{q}{2}\right) X & \text{for } (\theta_2, \theta_2, \theta_2) \end{cases}$$

So, $(\bar{R}_L, \bar{R}_H, \bar{R}_L)$ is a symmetric PSNE if and only if $p_L J_L > qK$, $p_H J_H > qK$, and $[\lambda p_H + (1 - \lambda) p_L] J_L > \max \{qK, [\lambda p_H + (1 - \lambda) q] J_H\}$. Since $p_H > p_L$ and $J_H \geq J_L$, these conditions reduce to $J_L > \max \left\{ \frac{qK}{p_L}, \frac{[\lambda p_H + (1 - \lambda) q] J_H}{\lambda p_H + (1 - \lambda) p_L} \right\}$ and imply that all borrowers face a loan rate less than or equal to their reservation rate. Therefore, when $(\bar{R}_L, \bar{R}_H, \bar{R}_L)$ arises, the safe project is always undertaken. To take another example, $(\theta_2, \theta_2, \theta_2)$ is an equilibrium if and only if $qK > \max \{p_L J_L, \lambda p_H J_L, [\lambda p_H + (1 - \lambda) p_L] J_L, [\lambda p_H + (1 - \lambda) q] J_H\}$. This condition reduces to $qK > p_H J_H$ and implies that all borrowers face a loan rate above their reservation rate. Therefore, when $(\theta_2, \theta_2, \theta_2)$ arises, the risky project is always undertaken.

Figure 3 shows the results when $\theta_2$ is high while Figure 4 shows the results when $\theta_2$ is low. The key insight relative to the basic model is that, when $\lambda$ is high and we account for the possibility that some lenders are uninformed about their borrowers’ types, large
values of $X$ eliminate the equilibrium where all borrowers undertake the safe project. At best, $(R_L, R_H, R_L)$ is replaced by $(R_L, R_H, R_H)$ which induces more defaults because type $L$ borrowers with uninformed lenders choose the risky project (recall that $R_H$ is above their reservation rate $R_L$). Therefore, even when the risky project is extremely risky (i.e., $q$ is close to zero), a higher fraction of good borrowers actually results in higher default rates because of promotion incentives.

4 Extension Two: Longer Promotion Horizon

Now suppose lenders are judged on the profit they accumulate over two periods. To ease exposition, I consider a model with only one borrower type whose probability of success in the safe project is denoted by $p$ and whose reservation loan rate is $R = \frac{p^{\theta_1-q\theta_2}}{p-q}$. With only one type, lenders are necessarily informed about their borrowers. In the first period, lenders choose whether to charge $R$ or $\theta_2$. The gap that results from the realization of their borrowers’ projects then determines the state for the second period. In particular, the gap between the leading lender and the lagging lender can take on one of four values: $0$, $R$, $\theta_2$, or $\theta_2 - R$. Notice that a gap of 0 can reflect two situations: both lenders generated zero profit in the first period or both lenders generated the same amount of positive profit. I refer to the first situation as $Fire = 1$ and the second as $Fire = 0$. These situations have different implications for the second period because a lender who accumulates zero profit over two periods has no chance of being promoted, even if his colleague also accumulates zero profit.

In the second period, lenders again choose whether to charge $R$ or $\theta_2$ given their state. To solve the model, I determine the second period choices of leading and lagging lenders in each state then solve for the first period choices given the second period value functions.

\footnote{This mirrors the assumption in Sections 2 and 3 that lenders who are tied with zero profit at the end of the tournament do not get promoted.}
4.1 Second Period with a Leader

Define \( \xi(y) \equiv \mathcal{I}(y > 0) + \frac{1}{2} \mathcal{I}(y = 0) \), where \( \mathcal{I}(\cdot) \) is an indicator function that equals 1 if its argument is true and 0 otherwise. Consider some arbitrary state \( G \), where \( G \) denotes the gap in first period profits from lender B’s perspective. Since this subsection assumes a leader, \( |G| \in \{ \overline{R}, \theta_2, \theta_2 - \overline{R} \} \).

Suppose lender A plays \( \overline{R} \). B’s expected payoffs from \( \overline{R} \) and \( \theta_2 \) are respectively:

\[
\Pi_{\overline{R}|\overline{R}}(G) = p \overline{R} + [p^2 + (1 - p)^2] \xi(G) X + p (1 - p) [\xi(G + \overline{R}) + \xi(G - \overline{R})] X
\]

\[
\Pi_{\theta_2|\overline{R}}(G) = q \left[ \theta_2 + p \xi(G + \theta_2 - \overline{R}) \right] X + (1 - p) \xi(G + \theta_2) X
\]

\[
+ (1 - q) \left[ p \xi(G - \overline{R}) + (1 - p) \xi(G) \right] X
\]

Now suppose A plays \( \theta_2 \). B’s expected payoffs are:

\[
\Pi_{\overline{R}|\theta_2}(G) = p \left[ \overline{R} + q \xi(G + \theta_2 - \overline{R}) \right] X + (1 - q) \xi(G + \overline{R}) X
\]

\[
+ (1 - p) \left[ q \xi(G - \theta_2) + (1 - q) \xi(G) \right] X
\]

\[
\Pi_{\theta_2|\theta_2}(G) = q \theta_2 + [q^2 + (1 - q)^2] \xi(G) X + q (1 - q) [\xi(G + \theta_2) + \xi(G - \theta_2)] X
\]

There are four possible PSNEs:

1. Both lenders play \( \overline{R} \). This occurs for parameters where \( \Pi_{\overline{R}|\overline{R}}(|G|) > \Pi_{\theta_2|\overline{R}}(|G|) \) and \( \Pi_{\overline{R}|\overline{R}}(-|G|) > \Pi_{\theta_2|\overline{R}}(-|G|) \). The value of the leader is \( V(|G|) = \Pi_{\overline{R}|\overline{R}}(|G|) \) and the value of the lagger is \( V(-|G|) = \Pi_{\overline{R}|\overline{R}}(-|G|) \).

2. Both lenders play \( \theta_2 \). This occurs for parameters where \( \Pi_{\theta_2|\theta_2}(|G|) > \Pi_{\overline{R}|\theta_2}(|G|) \) and \( \Pi_{\theta_2|\theta_2}(-|G|) > \Pi_{\overline{R}|\theta_2}(-|G|) \). The value of the leader is \( V(|G|) = \Pi_{\theta_2|\theta_2}(|G|) \) and the value of the lagger is \( V(-|G|) = \Pi_{\theta_2|\theta_2}(-|G|) \).
3. Leader plays $R$, lagger plays $\theta_2$. Occurs for parameters where $\Pi_{R|\theta_2}(|G|) > \Pi_{\theta_2|R}(|G|)$ and $\Pi_{\theta_2|\theta_2}(-|G|) > \Pi_{R|\theta_2}(-|G|)$. The value of the leader is $V(|G|) = \Pi_{R|\theta_2}(|G|)$ and the value of the lagger is $V(-|G|) = \Pi_{\theta_2|R}(-|G|)$.

4. Leader plays $\theta_2$, lagger plays $R$. Occurs for parameters where $\Pi_{\theta_2|R}(|G|) > \Pi_{R|\theta_2}(|G|)$ and $\Pi_{R|\theta_2}(-|G|) > \Pi_{\theta_2|\theta_2}(-|G|)$. The value of the leader is $V(|G|) = \Pi_{\theta_2|R}(|G|)$ and the value of the lagger is $V(-|G|) = \Pi_{R|\theta_2}(-|G|)$.

For each $|G|$, the division of the parameter space into these equilibria is shown in the bottom three panels of Figures 5 and 6. There are two notable features when $p$ is high. First, a gap of $\theta_2 - R$ generates increased risk-taking by the lagging lender at middling values of $q$, where "middling" is judged relative to the maximum feasible $q$. Below the dashed line, both lenders would have charged $R$ if $X$ was zero but, under $X > 0$, some of this area is now occupied by the equilibrium where only the leader plays $R$. Second, a gap of $\theta_2$ generates decreased risk-taking by the leading lender at slightly higher values of $q$. Above the dashed line, both lenders would have charged $\theta_2$ if $X$ was zero but, under $X > 0$, some of this area is now also occupied by the aforementioned equilibrium. Therefore, when the profit gap between lenders is small (i.e., $\theta_2 - R$) and the risky project has a low but non-negligible probability of success, lagging lenders attempt to close the gap by taking on more risk than they would have absent the promotion. For larger gaps (i.e., $\theta_2$) and risky projects with a somewhat higher probability of success, leading lenders take on less risk than they would have absent the promotion in order to solidify their lead with small but almost certain profits.

An interesting phenomenon also occurs at low $p$: leaders sometimes take more risk than laggars (i.e., the equilibrium where only the lagger plays $R$, denoted in green, emerges for some values of $q$ when $X > 0$). This tends to occur at small gaps. When $\theta_2$ is high enough, $R$ is the smallest gap for all parameter combinations and some green emerges in panel (d) of Figure 5. When $\theta_2$ is low, $\theta_2 - R$ is the smallest gap for a large part of the feasible parameter

$^4$If $\theta_2$ is high, then a gap of $R$ is qualitatively similar to a gap of $\theta_2 - R$ for high $p$. If $\theta_2$ is low, then a gap of $R$ is qualitatively similar to a gap of $\theta_2$ for high $p$. 

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space (in particular, $q < \left( \frac{2p}{\theta_2} - 1 \right) p$) and some green emerges in panel (e) of Figure 6.

### 4.2 Second Period with Tied Lenders

Following the notation of the previous subsection, B’s expected payoffs are:

$$\Pi_{\Theta|\overline{R}}(Fire) = p \left( \overline{R} + \left( 1 - \frac{q}{2} \right) X \right) + \frac{(1-p)^2X}{2} \cdot \mathcal{I}(Fire = 0)$$

$$\Pi_{\theta_2|\overline{R}}(Fire) = q \left[ \theta_2 + X \right] + \frac{(1-q)(1-p)X}{2} \cdot \mathcal{I}(Fire = 0)$$

$$\Pi_{\overline{R}|\theta_2}(Fire) = p \left[ \overline{R} + (1 - q)X \right] + \frac{(1-p)(1-q)X}{2} \cdot \mathcal{I}(Fire = 0)$$

$$\Pi_{\theta_2|\theta_2}(Fire) = q \left[ \theta_2 + \left( 1 - \frac{q}{2} \right) X \right] + \frac{(1-q)^2X}{2} \cdot \mathcal{I}(Fire = 0)$$

There are three possible PSNEs:

1. Both lenders play $\overline{R}$. Occurs for parameters where $\Pi_{\Theta|\overline{R}}(Fire) > \Pi_{\theta_2|\overline{R}}(Fire)$. The value of each lender is $U(Fire) = \Pi_{\Theta|\overline{R}}(Fire)$.

2. Both lenders play $\theta_2$. Occurs for parameters where $\Pi_{\theta_2|\theta_2}(Fire) > \Pi_{\overline{R}|\theta_2}(Fire)$. The value of each lender is $U(Fire) = \Pi_{\theta_2|\theta_2}(Fire)$.

3. Asymmetric: one plays $\overline{R}$, one plays $\theta_2$. Occurs where $\Pi_{\theta_2|\Theta}(Fire) > \Pi_{\overline{R}|\overline{R}}(Fire)$ and $\Pi_{\overline{R}|\theta_2}(Fire) > \Pi_{\theta_2|\theta_2}(Fire)$. Ex ante, the value is $U(Fire) = \frac{\Pi_{\theta_2|\Theta}(Fire) + \Pi_{\overline{R}|\overline{R}}(Fire)}{2}$.

The results are shown in panels (b) and (c) of Figures 5 and 6. The highlights are qualitatively similar to the basic model under low $\lambda$: if $\theta_2$ is high so that only low values of $q$ are permissible, $X > 0$ can decrease risk-taking but, if $\theta_2$ is low so that relatively high values of $q$ are permissible, $X > 0$ can increase risk-taking.\(^5\)

\(^5\)Why are the highlights similar to the basic model under low rather than high $\lambda$? With only one type, a parameter combination of $(p, q)$ means that all borrowers would succeed with probability $p$ in the safe project. With two types, however, $(p_L, q)$ means that most borrowers would succeed with probability $p_L$ in the safe project if and only if $\lambda$ is low (i.e., with high $\lambda$, the relevant probability would be $p_H$). Therefore, low $\lambda$ provides a better parallel between the axes in Figures 5 and 6 and the axes in Figures 1 and 2.
4.3 First Period

Suppose A plays \( R \) at \( t = 1 \). B’s expected payoffs from \( R \) and \( \theta_2 \) are respectively:

\[
\tilde{\Pi}_{R|\theta_1} = p \left[ R + pU(0) + (1 - p)V(\theta_1) \right] + (1 - p) \left[ pV(-R) + (1 - p)U(1) \right]
\]
\[
\tilde{\Pi}_{\theta_2|R} = q \left[ \theta_2 + pV(\theta_2 - R) + (1 - p)V(\theta_2) \right] + (1 - q) \left[ qV(\theta_2) + (1 - q)U(1) \right]
\]

Now suppose A plays \( \theta_2 \). B’s expected payoffs are:

\[
\tilde{\Pi}_{\theta_2|R} = p \left[ R + qV(-(\theta_2 - R)) + (1 - q)V(\theta_1) \right] + (1 - p) \left[ qU(0) + (1 - q)V(\theta_2) \right] + (1 - q) \left[ qV(-\theta_2) + (1 - q)U(1) \right]
\]
\[
\tilde{\Pi}_{\theta_2|\theta_2} = q \left[ \theta_2 + qU(0) + (1 - q)V(\theta_2) \right] + (1 - q) \left[ qU(0) + (1 - q)V(\theta_2) \right] + (1 - q) \left[ qV(-\theta_2) + (1 - q)U(1) \right]
\]

There are three possible PSNEs:

1. Both lenders play \( R \). Occurs for parameters where \( \tilde{\Pi}_{R|R} > \tilde{\Pi}_{\theta_2|R} \).

2. Both lenders play \( \theta_2 \). Occurs for parameters where \( \tilde{\Pi}_{\theta_2|\theta_2} > \tilde{\Pi}_{R|\theta_2} \).

3. Asymmetric: one plays \( R \), one plays \( \theta_2 \). Occurs where \( \tilde{\Pi}_{\theta_2|R} > \tilde{\Pi}_{R|R} \) and \( \tilde{\Pi}_{\theta_2|\theta_2} > \tilde{\Pi}_{R|\theta_2} \).

The results are shown panel (a) of Figures 5 and 6. For high values of \( p \) and middling values of \( q \), we see an increase in risk-taking due to promotions: under \( X = 0 \), both lenders would charge \( R \) but, under \( X > 0 \), the asymmetric equilibrium prevails in the grey area. For slightly higher values of \( q \), however, we see a decrease in risk-taking as long as \( q \) is still low relative to \( p \): in Figure 5, there is a small area above the dashed line where both lenders would charge \( \theta_2 \) if \( X = 0 \) but, under \( X > 0 \), the asymmetric equilibrium prevails again.

4.4 Comparison to a Short Promotion Horizon

When \( Fire = 1 \), the payoffs in Subsection 4.2 are equivalent to those of a one-type, one-period model. This will provide a useful benchmark for understanding the effects of a longer
promotion horizon. Consider first Figure 5. If $p$ is high and $q$ is middling, then a two-period promotion horizon can lead to increased risk-taking in both the first and second periods. To see this, notice that the grey area below the dashed line in panel (a) and the pink areas below the dashed lines in panels (d) and (f) are actually blue in panel (b). Moreover, the grey area above the dashed line in panel (b) is generally yellow in the other panels. Figure 6 tells a similar story: at the marked point, for example, a two-period promotion horizon again increases risk-taking relative to a one-period horizon. Notice that the asymmetric equilibrium prevails in the first period and, with $q$ close to 0.6, a gap of $\theta_2 - \bar{R}$ in the second period occurs with reasonable probability.

An important question following the 2007-2009 financial crisis was why bankers were compensated short-term (see, for example, Cai et al. (2010)). Overall, the results in this section provide one potential rationale: shutting down all other incentive effects, long horizons can increase risk-taking both in the early stages as lenders try to build leads and then in the later stages as laggers try to catch up. With a shorter horizon, there is no scope for catch-up, thus eliminating risk-taking by laggers and also making lenders more conservative in the risks they are willing to take early on.

5 Conclusion

This paper has analyzed the lending decisions of loan officers who compete for a profit-based promotion in three simple models. I began by considering the effect of a one-period promotion horizon when officers are informed about the quality of their own borrowers but not about the quality of their colleagues’ borrowers. In the first extension, I allowed some loan officers to be uninformed about their borrowers and, in the second extension, I studied the effects of a two-period promotion horizon. There were two main findings. First, higher average borrower quality can actually increase default rates in the presence of promotion incentives, especially if some lenders are uninformed about their own borrowers. Second,
longer promotion horizons can also lead to higher default rates. The basic model presented in this paper is ripe for further extensions. Future work will embed it into an overlapping generations model and endogenize the promotion value as the value of managing the bank next period.

References


Figure 1: Equilibrium loan rates under high \( \theta_2 \) in the basic model. All panels use \( p_H = 0.99 \) and \( \theta_2 = 2 \). Dashed black lines are the same in all panels and represent the boundaries between \((R_L, R_H)\), \((\theta_2, R_H)\), and \((\theta_2, \theta_2)\) under \( X = 0 \). The solid black lines trace out \( q = \frac{p_L \theta_1}{\theta_2} \). Blank spaces below these lines indicate a combination of \( p_L \) and \( q \) for which no symmetric PSNE exists.

Figure 2: Equilibrium loan rates under low \( \theta_2 \) in the basic model. All panels use \( p_H = 0.99 \) and \( \theta_2 = 1.1 \).
Figure 3: Equilibrium loan rates under high $\theta_2$ in the model of Section 3 (some uninformed lenders). All panels use $p_H = 0.99$ and $\theta_2 = 2$ and have the same dashed lines. In the feasible area above the flat dashes, $(\theta_2, \theta_2, \theta_2)$ arises when $X = 0$. In the area below the curved dashed line, $(\bar{R}_L, \bar{R}_H, \bar{R}_L)$ arises when $X = 0$.

Figure 4: Equilibrium loan rates under low $\theta_2$ in the model of Section 3. All use $p_H = 0.99$ and $\theta_2 = 1.1$. 

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Figure 5: Equilibrium loan rates under high $\theta_2$ in the model of Section 4 (two periods, one borrower type). All panels use $\theta_2 = 2$ and $X = 2$. For $t = 2$ and $|\text{Gap}| > 0$, $V(\cdot)$ denotes the leading lender and $V(-)$ denotes the lagging lender. Dashed lines are the same in all panels. In the feasible area above the dashed line, both lenders would charge $\theta_2$ if $X = 0$. In the area below it, both would charge $\bar{R}$ if $X = 0$.

Figure 6: Equilibrium loan rates under low $\theta_2$ in the model of Section 4. All use $\theta_2 = 1.1$ and $X = 2$. 
A Appendix

A.1 Proof of Proposition 2

Since the risky project yields less in expected value than the safe one, the parameter space is restricted by \( q \in \left(0, \frac{p_H \theta_1}{\theta_2}\right) \). I start by establishing that \( f(\cdot) \) has no critical points over this interval. The critical points of \( f(\cdot) \) amount to \( q = p_H + \frac{2(\theta_2 + X)}{3X} \pm \frac{2(\theta_2 + X)^2 + \frac{4}{3}(p_H X)^2}{3X} \) so it will be enough to establish that even the smaller point exceeds \( \frac{p_H \theta_1}{\theta_2} \). The desired inequality simplifies to

\[
\left[\frac{2\theta_2 - 6\theta_1 \theta_2 + 3\theta_1^2}{\theta_2^2}\right] \frac{p_H X}{4} + (\theta_2 - \theta_1)(\theta_2 + X) > 0 \text{ so a sufficient condition is} \left[\frac{2\theta_2 - 6\theta_1 \theta_2 + 3\theta_1^2}{\theta_2^2}\right] \frac{1}{4} + (\theta_2 - \theta_1) > 0 \text{ or, equivalently} \theta_2 > \left(\frac{5+\sqrt{7}}{6}\right) \theta_1. \]

Notice that the sufficient condition is guaranteed by \( \theta_2 > \left(\frac{1+p_H - \sqrt{1+p_H^2}}{p_H}\right) \theta_1 \) when \( p_H > \frac{1}{2} \). We can now conclude that \( f'(\cdot) \) will not change sign within \( q \in \left(0, \frac{p_H \theta_1}{\theta_2}\right) \). Moreover, with \( f(0) < 0 \), we can also conclude that equation \( (6) \) will not be satisfied by any \( q \in \left(0, \frac{p_H \theta_1}{\theta_2}\right) \) if \( f\left(\frac{p_H \theta_1}{\theta_2}\right) < 0 \). In other words, \( f\left(\frac{p_H \theta_1}{\theta_2}\right) > 0 \) is necessary for \( (\theta_2, \theta_2) \) to be an equilibrium. With some algebra, we can show that \( f\left(\frac{p_H \theta_1}{\theta_2}\right) > 0 \) is equivalent to \( \theta_1 > \left(\frac{\theta_1}{\theta_2} - \frac{1+p_H - \sqrt{1+p_H^2}}{p_H}\right) X \) which, given the assumption on \( \theta_2 \), amounts to \( X < \frac{2p_H \theta_1^2}{2p_H \theta_1^2 - 2(1+p_H) \theta_1 \theta_2 + 2\theta_2^2} \equiv X. \) Furthermore, with \( f\left(\frac{p_H \theta_1}{\theta_2}\right) > 0 \) and \( f(\cdot) \) monotone over \( q \in \left(0, \frac{p_H \theta_1}{\theta_2}\right) \), there exists a unique \( \overline{Q} \) such that \( f(q) < 0 \) for all \( q \in (0, \overline{Q}) \) and \( f(q) > 0 \) for all \( q \in (\overline{Q}, \frac{p_H \theta_1}{\theta_2}) \). In other words, there exists a unique \( \overline{Q} \) such that \( (\theta_2, \theta_2) \) is an equilibrium if and only if \( q \in (\overline{Q}, \frac{p_H \theta_1}{\theta_2}) \).

Now differentiate \( f(\overline{Q}) = 0 \) to get an expression for \( \frac{d\overline{Q}}{dX} \). Notice that the numerator is negative when \( \theta_2 > \left(\frac{p_H}{1+p_H - \sqrt{1+p_H^2}}\right) \theta_1 \). For the denominator to also be negative, we need

\[
h(\overline{Q}, X) = \left(\frac{2\overline{Q} + 3p_H \overline{Q} - \frac{3}{2} \overline{Q}^2 - 2p_H - p_H^2}{p_H - \overline{Q}}\right) X < 2\theta_2. \]

Define \( \tilde{q} \) such that \( h(\tilde{q}, X) = 2\theta_2. \) Since \( h_q(q, X) > 0 \), we will have \( h(\overline{Q}, X) < 2\theta_2 \) if and only if \( \overline{Q} < \tilde{q} \). Therefore, to show \( \frac{d\overline{Q}}{dX} > 0 \), it will be sufficient to show \( \tilde{q} > \frac{p_H \theta_1}{\theta_2} \). After some algebra, we see that the desired condition is guaranteed by \( \theta_2 > \left(\frac{5+\sqrt{7}}{6}\right) \theta_1 \) which, as demonstrated above, is true. ■
A.2 Proof of Proposition 3

Suppose A plays \((\bar{R}_L, \theta_2)\). B will also play \((\bar{R}_L, \theta_2)\) if and only if the following hold:

\[
p_L [\bar{R}_L + \lambda (1 - q) X + (1 - \lambda) \left(1 - \frac{p_L}{2}\right) X] > q \left[\theta_2 + \lambda \left(1 - \frac{q}{2}\right) X + (1 - \lambda) X\right] \quad (A.1)
\]

\[
p_H [\bar{R}_H + \lambda (1 - q) X + (1 - \lambda) X] < q \left[\theta_2 + \lambda \left(1 - \frac{q}{2}\right) X + (1 - \lambda) X\right] \quad (A.2)
\]

With \(p_H > p_L\) and \(\bar{R}_H > \bar{R}_L\), however, the left-hand side of equation (A.2) exceeds the left-hand side of equation (A.1). Therefore, (A.2) is false if (A.1) is true. \(\blacksquare\)

A.3 Proof of Proposition 4

The proof amounts to ruling out the following as symmetric PSNEs: \((\theta_2, \theta_2, \bar{R}_L)\), \((\theta_2, \theta_2, \bar{R}_H)\), \((\bar{R}_L, \bar{R}_H, \theta_2)\), and \((\bar{R}_L, \theta_2, \cdot)\). I go through each in turn.

Suppose lender A plays \((\theta_2, \theta_2, \bar{R}_L)\). Both types undertake the risky project when charged \(\theta_2\) so, regardless of his information set, B’s expected payoff from charging \(\theta_2\) is \(qK \equiv q \left[\theta_2 + \left(1 - \frac{q}{2}\right) X\right]\). If instead B charges \(\bar{R}_L\) and his borrower repays, then B will get \(J_L \equiv \bar{R}_L + (1 - q\pi) X - (1 - \pi) \left(\frac{p_L + \lambda (p_H - p_L)}{2}\right) X\), making his expected payoff \(p_L J_L\) if he is informed with a type \(L\) and \(\left[p_H + (1 - \lambda) p_L\right] J_L\) if he is uninformed. Finally, if B charges \(\bar{R}_H\) and his borrower repays, then B will get \(J_H \equiv \bar{R}_H + (1 - q\pi) X\), making his expected payoff \(p_H J_H\) if he is informed with a type \(H\) and \(\left[p_H + (1 - \lambda) q\right] J_H\) if he is uninformed. So, for \((\theta_2, \theta_2, \bar{R}_L)\) to be an equilibrium, it is necessary to have (i) \(qK > p_H J_H\), and (ii) \(\left[p_H + (1 - \lambda) p_L\right] J_L > qK\). However, with \(p_H > \left[p_H + (1 - \lambda) p_L\right]\) and \(J_H > J_L\), (ii) implicitly requires \(p_H J_H > qK\) which violates (i).

Suppose A plays \((\theta_2, \theta_2, \bar{R}_H)\) and redefine \(K, J_L,\) and \(J_H\) accordingly. For B to charge \(\theta_2\) when informed with a type \(H\), we need \(qK > p_H J_H\). For B to charge \(\bar{R}_H\) when uninformed, we need \(\left[p_H + (1 - \lambda) q\right] J_H > qK\). With \(p_H > \left[p_H + (1 - \lambda) q\right]\), the second condition implicitly requires \(p_H J_H > qK\), thus violating the first.
Suppose A plays \((R_L, R_H, \theta_2)\) with \(K\), \(J_L\), and \(J_H\) appropriately redefined. For B to charge \(R_L\) when informed with a type \(L\), we need \(p_L J_L > qK\). For B to charge \(\theta_2\) when uninformed, we need \(qK > [\lambda p_H + (1 - \lambda) p_L] J_L\). With \([\lambda p_H + (1 - \lambda) p_L] > p_L\), the second condition implicitly requires \(qK > p_L J_L\), thus violating the first.

Suppose lender A plays \((R_L, \theta_2, \cdot)\) and redefine \(K\), \(J_L\), and \(J_H\) accordingly. For B to charge \(R_L\) when informed with a type \(L\), we need \(p_L J_L > qK\). For B to charge \(\theta_2\) when informed with a type \(H\), we need \(qK > p_H J_H\). In all cases (i.e., whether A plays \(R_L\), \(R_H\), or \(\theta_2\) when uninformed), we can show that the corresponding \(J_H\) exceeds the corresponding \(J_L\). Therefore, \(qK > p_H J_H\) implicitly requires \(qK > p_L J_L\), thus violating \(p_L J_L > qK\).