Strategic Dynamic Sourcing from Competing Suppliers with Transferable Capacity Investment

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We study the supplier relationship choice for a buyer that invests in transferable capacity operated by a supplier. With a long-term relationship, the buyer commits to source from a supplier over a long period of time. With a short-term relationship, the buyer leaves open the option of switching to a new supplier in the future. The buyer has incomplete information about a supplier’s efficiency, and thus uses auctions to select suppliers and determine the contracts. In addition, the buyer faces uncertain demand for the product. A long-term relationship may be beneficial for the buyer because it motivates more aggressive bidding at the beginning, resulting in a lower initial price. A short-term relationship may be advantageous because it allows switching, with the capacity transferred at some costs, to a more efficient supplier in the future. We find that there exists a critical level of switching costs above which a long-term relationship is better for the buyer than a short-term relationship. In addition, this critical switching cost decreases with demand uncertainty, implying a long-term relationship more favorable for a buyer facing volatile demand. Finally, we find that in a long-term relationship, capacity can be either higher or lower than in a short-term relationship.

1 Introduction

Facing increasing competition, manufacturers source from external suppliers products and services with increasing complexity and strategic importance (Gottfredson et al., 2005; Maurer et al., 2004). In a dynamic environment, capabilities of suppliers may change and new suppliers may emerge over time. Hence, supplier relationship management is critical for a manufacturer (buyer) (Liker and Choi, 2004; Beth et al., 2003). A buyer may choose to develop a long-term or a short-term relationship with a supplier. A long-term relationship features the commitment of sourcing from a supplier over a long-period of time, while a short-term relationship usually only binds for a short period of time with the buyer frequently switching to other suppliers (Pyke and Johnson, 2003). A long-term commitment with a supplier hampers a buyer in tapping into more efficient alternative sources in the future. Sako (1992) reported that Japanese car manufacturers lamented the difficulties they had in breaking up existing relationships with suppliers (and switching to new suppliers). Yet, the recent years have witnessed a growing trend of companies consolidating their supply base and forming long-term relationships with their key suppliers in which the manufacturer has limited or no business with competing suppliers (Maurer et al., 2004; Pyke and Johnson, 2003).
For example, Chrysler’s average contract length nearly doubled over the period from 1989 to 1994 (Helper, 1991; Dyer, 1997).

In this paper, we study the optimal relationship choice when a buyer invests in transferable and specific capacity operated by a supplier. Investment of specific capacity may be needed for a supplier to produce parts or systems that are customized to the buying firm’s needs. The selection of the supplier relationship influences the level of upfront specific capacity investment. The specific capacity can take the form of facilities that are located in close proximity to the buyer’s plants, physical assets such as customized machinery, tools, or equipment, or employees that have relationship-specific information or experience that allow effective coordination between the buyer and supplier (Dyer, 2000). These assets are dedicated to the buyer and therefore typically owned by the buyer (although operated by a supplier). For example, Ford owns the buildings in its supply park that sit one-half mile from the Chicago assembly plant, and leases these buildings to eleven suppliers (Peter, 2004). In the automotive industry, the manufacturer usually provides the specific tools or reimburses tooling costs to suppliers (Visteon, 2003). It is also common for a manufacturer to assign dedicated employees to work with a supplier, even on a supplier’s site (Maurer et al., 2004; Dyer, 2000). In case of the buyer switching to a new supplier, the specific capacity is typically transferred, at some costs, from the incumbent to the new supplier. This transfer may not only refer to the physical relocation of machines or tools, but may also mean to adapt the new supplier to the dedicated facility, or training specialized workers for the new supplier. In the automotive industry, a supplier that loses the business to a new supplier can even be asked to ship the tools to the new supplier and help the new supplier to get up and running (Dyer, 1997).

When the buyer faces uncertain market demand for the final product, how much specific capacity to invest is a non-trivial question. The return of the investment depends not only on the realization of the demand, but also on the length of the relationship with a supplier and the supplier efficiency. Thus the investment of specific capacity should take into consideration not only the uncertainty of demand but also the governance structure of supplier relations (Dyer, 1997).

No matter which relationship to use, a buyer wants to contract with the most efficient supplier that can provide the same production at the least cost. The information about the efficiency of a supplier is typically not fully accessible to a buyer. Because suppliers tend to understand the technical processes better than the purchasing company, they are better positioned to identify areas of savings: for example, reducing the number of components by combining separate parts, applying advanced fabrication techniques to reduce subsequent steps, changing part geometries and increasing machining tolerances, alternating materials that can reduce cost of noncritical components,
etc. (Ulrich and Eppinger, 2004). This knowledge allows a supplier to hold private information of production costs. For technologically complex products, there is generally substantial private information about productivity or technologies of a supplier (Lyon, 2006). Without knowing the actual cost of a supplier, the buyer usually selects a supplier and negotiates the contract by a Request-for-Proposal, or a competitive bidding process. In this situation the profit of the buyer depends on how much profit a supplier can retain due to information asymmetry.

We study the tradeoff between long-term and short-term relationships when a buyer sources from competing suppliers and invests in transferable capacity that is operated by a supplier. We take both information asymmetry concerning a supplier’s cost structure and uncertainty concerning the market demand into account. In order to understand the fundamental economic determinants of the supplier relationship decision, we analyze a model with a two-period horizon. With a long-term relationship the buyer commits to source from a single supplier over the entire horizon. With a short-term relationship the buyer does not make such commitment but instead keeps the option of switching to a new supplier open in the second period. Competitive bidding processes are used in each period to select a supplier and determine the contract. We study the key operational decisions that the buyer needs to make: How much capacity to contract for, with which supplier and for how long. The questions that we address in this paper are: What is an optimal strategy of a buyer under each supplier relationship? What is the influence of the supplier relationship on the capacity investment decision? How do characteristics of the environment, in particular the switching cost and demand uncertainty, impact the supplier relationship selection?

2 Literature Review

A body of research in economics and operations management has been developed that provides us with insights in dynamic sourcing strategies (see Elmaghraby (2000) for an overview of the literature). In the following we provide a review of the papers that consider similar issues as in our paper: supplier switching, supplier commitment, and capacity investment in a context of sourcing from competing suppliers with private information.

Dynamic sourcing strategies with supplier switching: In a multi-period setting where supplier capabilities are private information and change over time, the buyer must make switching decisions considering the trade-off between competitive advantage realized by switching, and the benefit of working with the same supplier. Laffont and Tirole (1988), Stole (1994) and Rob (1986) study how a buyer should contract with a supplier and make the switching decision when the first supplier strategically makes investments to reduce the future cost. Anton and Yao (1987), Klotz
and Chatterjee (1995), and Lewis and Yildirim (2002) model the dynamic switching decision for a buyer when the incumbent generally has an efficiency advantage over an entrant due to learning-by-doing. The impact of the switching cost on dynamic selection of suppliers is specifically analyzed by Bac (2000) and Cabral and Greenstein (1990). Generally these papers argue that although switching may introduce system inefficiency, it infuses competition between suppliers and therefore reduces the buyer’s information rent. The tradeoff between the inefficiency and information rent determines the optimal switching decision of a buyer. All these papers assume that the buyer never commits to a single supplier, in other words, they only consider short-term relationships.

**Long-term versus short-term relationships:** The conclusion that the buyer should switch under some conditions with a short-term relationship does not deny the benefit of establishing a long-term relationship. Motivated by procurement in the defense industry with certain demand, Riordan and Sappington (1989) compare sole sourcing and second sourcing, which are analogous to long-term and short-term relationships, considering a supplier’s investment to improve the project’s value in the future. By assuming identical future costs for all suppliers, they find that a short-term relationship is of limited value and discourages investments. In their paper, investment in product enhancement increases the probability of switching because it improves the value of the product, and thus reduces the relative significance of the switching cost. Our insight is different in that the capacity of the incumbent decreases the switching probability, and as a result, a short-term relationship may induce more capacity investment than a long-term relationship. This suggests that to choose a supplier relationship, a buyer has to carefully identify the impact of investment on the switching decision and the motivation of switching.

Considering the effect of learning-by-doing, Elmaghraby and Oh (2004) compare an erosion rate policy against two base cases in which the buyer holds an (independent, short-term) auction in each period, and the buyer procures all her demand in a single bundle auction. Grimm (2004) and Menezes and Monteiro (2003) compare sequential auctions and a bundled auction for two items when synergies of item values exist. All these papers focus on the information structure, assuming deterministic demand (or supply as in Menezes and Monteiro (2003) in a forward auction setting). When both demand uncertainty and supplier cost uncertainty exist, as is the case in our research, the supplier relationship and procurement contracting must take into consideration capacity investment along with supplier selection under asymmetric information.

Swinney and Netessine (2007) study the choice of a buyer between offering long-term and short-term contracts to a supplier considering the supplier default risk. Assuming public information, they show that the possibility of supplier default offers a reason for a buyer to prefer long-term
contracts to short-term contracts. We consider private information about the cost of a supplier. The private information provides a different motivation for the buyer to select a long-term relationship.

In this paper, we consider a situation where the supplier’s specific capacity is owned by the buyer and transferable between suppliers. This is different from Li and Debo (2008) (among some other modelling differences), who assume non-transferable capacity. Thus the competition models between the incumbent and entrant supplier(s) are different in these two papers: With non-transferable capacity, the friction of sourcing from an entrant supplier is attributable to the installed capacity advantage of the incumbent and the new capacity investment cost of the entrant. With transferable capacity, the friction is due to the switching cost for transferring the installed capacity from the incumbent to entrant supplier. As a result, although the driver of a long-term commitment is the same in both papers, how the demand uncertainty influences the buyer’s decision is different. Particularly, when demand uncertainty increases, with non-transferable capacity, Li and Debo (2008) show that leaving the option of alternative sourcing open is more favorable, while with transferable capacity, we show that committing to a single supplier is a more likely choice. We thus complement the results of Li and Debo (2008) with insights based on different features of supplier capacity.

**Capacity investment in competitive sourcing:** Cachon and Zhang (2006) consider simultaneously sourcing and capacity investment strategies in the presence of demand uncertainty. They study single sourcing mechanisms of a buyer with information asymmetry about a supplier’s cost. The buyer and suppliers interact only once to award the contract. The supply capacity and contract are driven by the competition only between simultaneously existing suppliers. However, in a dynamic environment where supplier costs and pools are subject to change, a supplier not only competes with other suppliers in the same period but also with suppliers that emerge (with different costs) in the future.

In summary, while the existing literature provides useful insights in dynamic supplier competition and competitive capacity investment, to the best of our knowledge, there has been no research providing insights that consider both. We develop and analyze a model that allows an understanding of optimal relationship selection concerning capacity investment in the presence of both demand and supplier cost uncertainty.

The remainder of the paper is structured as follows: The dynamic sourcing model with long-term and short-term relationships is described in Section 3. The optimal mechanisms with long-term and short-term relationships are presented in Section 4. Section 5 compares the profits and capacity in the two relationships. We discuss the system efficiency of the optimal mechanisms in Section...
6. Some extended analysis is provided in Section 7. We summarize the main insights and discuss future research in Section 8.

3 The model

3.1 Model description

In order to analyze the tradeoff in the buyer’s supplier relationship selection, we build a model with two periods ($\tau = 1, 2$) and one monopoly buyer that sources the production of a critical component from external suppliers.

**Capacity:** We focus on specific capacity that is owned by the buyer but operated by a supplier. The capacity limits the production quantity of a supplier in each period. We denote by $k$ the unit cost of the capacity. If the buyer switches suppliers, the capacity is transferred from the incumbent to the new supplier at unit cost $s$. Without loss of generality we assume the buyer incurs the switching cost; otherwise the supplier who incurs this cost can be equivalently compensated with the buyer’s contractual payments. Since the capacity has features that are customized to the buyer’s needs, it has no salvage value outside the buyer’s business. We assume that the lead-time of capacity investment is so long that the capacity can only be invested at the beginning of the first period but not in the middle of the horizon. Thus the capacity cannot be expanded in the second period. Since the capacity has zero salvage value, a supplier will never reduce existing capacity dedicated to the buyer. We also assume that the buyer’s revenue is large enough so that in case of switching to a new supplier, all capacity of the incumbent will be transferred to the new supplier. Therefore, the capacity will not be shrunk either in the second period. (In Section 7, we relax the assumption of fixed capacity in the second period, and extend the analysis to the situation when the capacity can be expanded or shrunk in the second period.)

**The supplier pool:** In a dynamic environment, both the supplier pool and the costs of a supplier may change. Denote by $n_1$ the number of available suppliers in period 1, and $n_2$ the number of entrant suppliers, not including the incumbent, in period 2. We assume that the suppliers who lose the first period competition will stay and participate in the second-period auction (in a short-term relationship). Thus the $n_2$ entrant suppliers in the second period include the $n_1$ suppliers except the incumbent in the first period. In each period a supplier’s unit production cost is determined by independent, privately observed shocks. A supplier does not know his cost until he enters that period. All other players (the buyer and other suppliers) only know that the unit production cost of a supplier in each period is drawn from an identical distribution $F(c)$ (density $f(c)$) on the
support $[\underline{c}, \overline{c}]$, with mean $\mu_c$ and standard deviation $\sigma_c$. This assumption of independent costs across periods is reasonable when the duration of a period is long, or the cost shocks are tied to changes in, e.g., labor and subcontractor management, scrap rates, materials management, inbound and outbound logistics, etc., that can happen quickly and frequently. Similar assumptions are also carried in Menezes and Monteiro (2003). In line with auction models (Krishna, 2002; Laffont and Tirole, 1993) we assume that $F(c)$ is log-concave, i.e., $F (c) / f (c)$ is an increasing function of $c$. This condition is satisfied by a wide variety of commonly used distributions, such as the normal and Beta distributions (Rosling, 2002; Bagnoli and Bergstrom, 2005). We denote by $F_n(c) = 1 - (1 - F(c))^n$ ($f_n(c) = n f (c) (1 - F(c))^{n-1}$) the cumulative probability (probability density) of the lowest value realized amongst $n$ variables that are identically and independently distributed according to $F(\cdot)$. The complementary distribution is $F_n(c) = 1 - F_n(c)$.

**Market demand:** The buyer sells the final product at unit price $r$. The demand $D$ for this product in each period is uncertain and independently drawn from a cumulative probability distribution $G(d)$ (density $g(d)$ and $\overline{G(d)} = 1 - G(d)$) on the support $[\underline{d}, \overline{d}]$. Let $X$ be a random variable with a symmetric density function, mean 0 and standard deviation 1. We consider $D$ of the form $\mu_d + \sigma_d X$, which has $\mu_d$ as expected demand and $\sigma_d$ as standard deviation. $G(d)$ is known by the buyer and all the suppliers. When the supplier capacity is $Q$, the demand to be satisfied by the supplier in one period is $\min\{Q, D\}$, and its expectation is $S(Q) = \mathbb{E}[\min\{Q, D\}]$. It is easily proven that for $Q \geq \mu_d$, $\frac{\partial}{\partial \sigma_d} S(Q) \leq 0$ increases with $Q$, i.e., when the demand uncertainty increases, the expected satisfied demand decreases, but at a lower rate when the capacity is higher.

**The contracts:** Both the buyer and supplier follow a make-to-order policy, i.e., the buyer places an order to the supplier after the demand is known, and given the order the supplier produces subject to the capacity constraint. The contract in the first period is specified by $(Q, w)$, where $Q$ is the size of the capacity to be installed, and $w$ is the unit price to be paid for each unit delivered. In the second period, the supply contract only specifies the unit price, with the capacity given in the first period.

**Long-term and short-term relationships:** At the beginning of the first period, the buyer decides whether to establish a long-term or a short-term relationship with a supplier. With the former selection, the buyer commits to source from the first-period winner in both periods. With the latter selection, the buyer is allowed to switch suppliers in the second period. We indicate long-term and short-term relationships by subscripts $\ell$ and $s$, respectively. Due to the transaction cost of sourcing from multiple suppliers, for example, the costs of monitoring suppliers’ performances, communicating and coordinating with suppliers, achieving quality consistency across suppliers, etc. (Richardson
and Roumasset, 1995), we assume that the buyer sources from only one supplier at a time, i.e., even with multiple suppliers available in the second period of a short-term relationship, the buyer will source exclusively from the incumbent or an entrant instead of splitting the production over multiple suppliers.

The auction protocol: As suppliers’ production cost is private information, in each period, the buyer selects one supplier from the available pool and decides the supply contract via an auction. In the second period of a long-term relationship, since the buyer can only source from the incumbent, the auction has only one bidder—the incumbent—and can be regarded as a renegotiation between the buyer and the incumbent. To eliminate the dependence of results on the specific auction forms used, we analyze the optimal auction mechanism for the buyer. Based on the revelation principle (Myerson, 1981), the consideration of an optimal auction mechanism can be restricted, without loss of generality, to direct mechanisms that are incentive compatible and individually rational. In a direct mechanism, the buyer offers each supplier a menu of contracts, with each contract corresponding to a profile of suppliers’ costs. Each supplier then bids their costs, which may differ from the true costs. Given the bids, the contracts that correspond to the profile of bids are awarded to suppliers. A direct mechanism is incentive compatible (IC) if bidding the true cost constitutes a Bayes-Nash equilibrium. A mechanism is individually rational (IR) if the expected profit is non-negative for any supplier with any cost. Other papers built on direct auction mechanisms include Cachon and Zhang (2006), Dasgupta and Spulber (1989/90) and Laffont and Tirole (1988).

Sequence of events: The sequences of events in long-term and short-term relationships are illustrated in Figure 1.

3.2 Formulation

We shall first formulate the second period decision problems of the buyer in long-term and short-term relationships, and then the first period decision problem with each relationship.

Second period, long-term relationship: Assume the capacity invested in the first period is \( Q \). In the second period of a long-term relationship, the buyer offers the incumbent supplier a menu \( \{ w_{I,2}(\cdot), p_{I,2}(\cdot) \} \), where \( p_{I,2}(\hat{c}_I^2) \) is the probability that the incumbent stays as the supplier and \( w_{I,2}^I(c_I^2) \) is the unit price if his reported cost is \( \hat{c}_I^2 \). The supplier’s profit is \( u_{I,2}(\hat{c}_I^2, c_I^2, Q) = \left( w_{I,2}^I(\hat{c}_I^2) - c_I^2 \right) S(Q) p_{I,2}(\hat{c}_I^2) \) if he reports \( \hat{c}_I^2 \) while his true cost is \( c_I^2 \). Recall that the buyer’s unit revenue is \( r \). Therefore, the buyer’s decision problem in the second period of a long-term relationship is
Figure 1: Sequence of events in long-term vs. short-term relationships. The two relationships are different in the second period events. While with a long-term relationship the buyer commits to work with the same supplier in the second period, with a short-term relationship the buyer is allowed to switch. The capacity is transferred from the incumbent to the new supplier if switching happens. The supplier and contract in each period are determined via an auction (with the incumbent as the only bidder in the second period of a long-term relationship).

\[
\begin{align*}
\pi_{\ell,2}(Q) &= \max_{\{w_{s,2}, p_{s,2}\}} \mathbb{E}_{c_2^I} \left[ (r - w_{s,2}(c_2^I)) p_{s,2}(c_2^I) \right] S(Q) \\
\text{s.t. } \forall c_2^I \in [\underline{c}, \bar{c}] : \left\{ u_{\ell,2}(c_2^I, c_2^I, Q) = \max_{c_2^E \in [\underline{c}, \bar{c}]} u_{\ell,2}(c_2^E, c_2^I, Q) \right\} \text{ (IC)} \\
&\quad \left\{ u_{\ell,2}(c_2^I, c_2^I, Q) \geq 0 \right\} \text{ (IR)}
\end{align*}
\]

The first constraint ensures that the mechanism is incentive compatible (IC), i.e., the supplier will report his cost truthfully. The second constraint guarantees individual rationality (IR), i.e., the supplier will achieve non-negative profit from participation. Define \( \pi_{\ell,2}(Q) = \mathbb{E}_{c_2} [u_{\ell,2}(c_2^E, c_2^I, Q)] \) as the \textit{ex ante} profit of the winner in the second period of a long-term relationship.

**Second period, short-term relationship:** Let \( c_2^E = (c_{2,1}^E, \ldots, c_{2,n_2}^E) \) be the cost profile of all entrant suppliers in the second period. Define \( c_{2,-j}^E = (c_{2,1}^E, \ldots, c_{2,j-1}^E, c_{2,j+1}^E, \ldots, c_{2,n_2}^E) \) as the profile of all suppliers except supplier \( j \). The incumbent’s cost in the second period is \( c_2^I \). Depending on the capacity \( Q \), the buyer offers a menu \( \{w_{s,2}(\cdot), p_{s,2}(\cdot)\} \) to the incumbent and \( \{w_{s,2}^E(\cdot), p_{s,2}^E(\cdot)\} \), \( j = 1, \ldots, n_2, \) to each entrant. Given the incumbent’s bid \( c_2^I \) and entrants’ bids \( c_2^E = (c_{2,1}^E, \ldots, c_{2,n_2}^E) \), the probability of the incumbent selected as the winner is \( p_{s,2}(c_2^I, c_2^E) \) with unit price \( w_{s,2}(c_2^I, c_2^E) \), and the probability of the entrant \( j \) selected as the winner is \( p_{s,2}^E(c_2^I, c_2^E) \) with unit price \( w_{s,2}^E(c_2^I, c_2^E) \).

Let

\[
\begin{align*}
u_{s,2}(c_2^I, c_2^I, Q) &= \mathbb{E}_{c_2^E} \left[ (w_{s,2}(c_2^E, c_2^I) - c_2^I) p_{s,2}(c_2^I, c_2^E) \right] S(Q)
\end{align*}
\]
be the expected second period profit of the incumbent with cost $c^1_i$ if he bids $\hat{c}^2_j$ and all entrants bid truthfully. Similarly, let
\[
u_{s,2}^{E,j}(\hat{c}^2_j, c^2_j, Q) = \mathbb{E}_{c^1_i, c^2_j} \left[\left(u_{s,2}^{E,j}(c^1_i, \hat{c}^2_j, c^2_j, Q) - c^2_j\right) p_{s,2}^{E,j}(c^1_i, \hat{c}^2_j, c^2_j, Q)\right] S(J)
\]
be the expected second period profit of entrant $j$ with cost $c^2_j$ if he bids $\hat{c}^2_j$ and all other suppliers bid truthfully. Recall that the unit capacity transferring cost is $s$. The buyer’s decision problem in the second period is
\[
\pi_{s,2}(Q) = \max_{\{u_{s,2}^{E,j}(\cdot), p_{s,2}^{E,j}(\cdot)\}} \mathbb{E}_{c^1_i, c^2_j} \left[\left(r - w_{s,2}^{I,j}(c^2_j, Q)\right) p_{s,2}^{I,j}(c^2_j, Q)\right] + \sum_{j=1}^{n_2} \left(\left(r - w_{s,2}^{E,j}(c^1_i, c^2_j)\right) S(J) - s Q\right) p_{s,2}^{E,j}(c^1_i, c^2_j, Q)
\]
s.t. $\forall j = 1, ..., n_2$, $c^1_i, c^2_j, \hat{c}^2_j \in \mathbb{R}^+$
\[
\left\{\begin{array}{l}
\hat{c}^2_j \in [c, P] \\
u_{s,2}^{I,j}(c^2_j, Q) = \max_{\hat{c}^2_j \in [c, P]} \nu_{s,2}^{I,j}(c^2_j, \hat{c}^2_j, Q) \\
u_{s,2}^{E,j}(c^1_i, c^2_j, Q) = \max_{\hat{c}^2_j \in [c, P]} \nu_{s,2}^{E,j}(c^1_i, c^2_j, \hat{c}^2_j, Q) \\
u_{s,2}^{I,j}(c^2_j, Q) \geq 0, \nu_{s,2}^{E,j}(c^1_i, c^2_j, Q) \geq 0
\end{array}\right\} \quad \text{(IC)}
\]
\[
\text{Let } \bar{\nu}_{s,2}^{I,j}(Q) = \mathbb{E}_{c^2_j} \left[\nu_{s,2}^{I,j}(c^1_i, c^2_j, Q)\right] \text{ and } \bar{\nu}_{s,2}^{E,j}(Q) = \mathbb{E}_{c^1_i} \left[\nu_{s,2}^{E,j}(c^1_i, c^2_j, Q)\right] \text{ be the ex ante profits of the incumbent and an entrant supplier in the second period of a short-term relationship (for given capacity investment all entrants have the same expected second-period profits because they are ex ante identical).}
\]

**Second period summary:** For both the short- and long-term relationships we have obtained expressions for the incumbent and entrant supplier’s second-period expected profits $\bar{\nu}_{s,2}^{I,j}(Q)$, $\bar{\nu}_{s,2}^{I,j}(Q)$ and $\bar{\nu}_{s,2}^{E,j}(Q)$, and expressions for the buyer’s second period profits, $\pi_{s,2}(Q)$ and $\pi_{s,2}(Q)$. We will use these expressions when formulating the first period problem.

**First period:** The buyer’s first period decisions in long-term and short-term relationships are similar: They both maximize the buyer’s profits over the entire horizon, taking the second period optimization as an embedded problem. Therefore, in the formulation we do not specify the relationship but replace the relationship indication by subscript “$\ast$”.

Let $c_1 = (c_1^1, ..., c_1^{n_1})$ be the cost profile of all suppliers in the first period. Define $c^1_i \overset{\ast}{=} (c_1^1, ..., c_1^{i-1}, c_1^{i+1}, ..., c_1^{n_1})$ as the profile of all suppliers excluding supplier $i$. In the first period, the buyer offers a menu $\{Q^i(\cdot), w^i(\cdot), p^i(\cdot)\}$, $i = 1, ..., n_1$, where $p^i(\cdot)$ is the probability that supplier $i$ is selected as the winner, $w^i(\cdot)$ is the unit price of supplier $i$, and $Q^i(\cdot)$ is the capacity to be invested for supplier $i$.
bids are $\hat{c}_1 = (c_1^1, ..., c_1^n)$. Define

$$u^t_t(\hat{c}_1, c_1) \doteq \mathbb{E}_{c_1^{-t}} \left[ \left( \left( w^t_{t,1}(c_1^1, c_1^{-i}) - c_1^i \right) S \left( Q^t_t(\hat{c}_1^i, c_1^{-i}) \right) - kQ^t_t(\hat{c}_1^i, c_1^{-i}) \right) \right]$$

and

$$u^s_s(\hat{c}_1, c_1) \doteq \mathbb{E}_{c_1^{-s}} \left[ \left( \left( w^s_s(\hat{c}_1^i, c_1^{-i}) - c_1^i \right) S \left( Q^s_s(\hat{c}_1^i, c_1^{-i}) \right) - kQ^s_s(\hat{c}_1^i, c_1^{-i}) \right) \right]$$

as the total expected profit of supplier $i$ over two periods in long-term and short-term relationships, if he bids $\hat{c}_1^i$ with true cost $c_1^i$ and all other suppliers bid truthfully. Then the buyer’s optimal decision in the first period is:

$$\max_{\{Q^t(.), w^t_{s,1}(.), p^t_{s,1}(.), w^s_s(.), p^s_{s,1}(.), \alpha, \beta, \gamma, \delta, \epsilon\}} \mathbb{E}_{c_1} \left[ \sum_{i=1}^{n_1} \left( \left( r - w^t_{s,1}(c_1) \right) S \left( Q^t_t(c_1) \right) + \pi_{s,2} \left( Q^t_t(c_1) \right) \right) \right]$$

s.t. $\forall i = 1, ..., n_1$, $c_1^i \in [s, \bar{s}] : \left\{ \begin{array}{l} u^t_t(c_1^i, c_1^i) = \max_{\hat{c}_1^i \in [s, \bar{s}]} u^t_t(\hat{c}_1^i, c_1^i) \quad \text{(IC)} \ \ u^t_t(c_1^i, c_1^i) \geq 0 \quad \text{(IR)} \end{array} \right.$

4 The Optimal Mechanisms of Long-term and Short-term Relationships

We first analyze the optimal mechanisms in the second period (Section 4.1) for each relationship, and then characterize the optimal solution in the first period (Section 4.2).

4.1 The second period

Before presenting the outcome of the optimal mechanisms, we define $J(c) = c + F(c)/f(c)$ as the virtual cost of a supplier with the real production cost $c$ (Myerson, 1981). Because $F(c)$ is log-concave, the virtual cost $J(c)$ is an increasing function of the actual cost $c$. We also assume the revenue is large enough so that $r \geq J(\bar{s}) + k + s$. Under this condition, it is always profitable for the buyer to contract with a supplier in each period even if taking the virtual cost as the supplier’s cost. This assumption also implies that the the optimal capacity investment in either relationship will be at least $\mu$, if the demand distribution is symmetric. This is because the marginal return of capacity investment over two periods is at least $2(r - J(\bar{s})) - s$, which is greater than twice of the marginal cost $k$ given this assumption. Define virtual profit as the profit of the supply chain if the virtual cost were considered as the unit cost of the supplier. Then for a given capacity $Q,$
$v_2^l(Q, c_2^l) \doteq (r - J(c_2^l)) S(Q)$ is the virtual profit of sourcing from the incumbent in the second period if the incumbent’s actual production cost is $c_2^l$.

**Proposition 1** In the second period of a long-term relationship, given capacity $Q$, the expected second-period profit of the buyer is

$$\pi_{l,2}(Q) = \mathbb{E}_{c_2^l} \left[ r - J(c_2^l) \right] S(Q), \quad (1)$$

and the expected second-period profit of the incumbent is

$$\overline{\pi}_{l,2}(Q) = \mathbb{E}_{c_2^l} \left[ J(c_2^l) - c_2^l \right] S(Q). \quad (2)$$

Based on Proposition 1, the buyer’s profit is equal to the expectation of the virtual profit generated by the supplier. The virtual profit is lower than the actual profit of the supply chain. Their difference is $\overline{\pi}_{l,2}(Q)$, representing the *information rent* extracted by the supplier.

Next we analyze the optimal results for the second period of a short-term relationship. Let $c_{2}^{E^*}$ be the lowest cost of the entrants.

**Proposition 2** In the second period of a short-term relationship,

(i) the buyer stays with the incumbent if $c_2^{E^*} \geq \tilde{c}(c_2^l, Q)$, otherwise switches to the lowest-cost entrant, where $\tilde{c}(c_2^l, Q) \in [c, \tilde{c}]$ is the solution to

$$(J(c_2^l) - J(\tilde{c})) S(Q) = sQ \quad (3)$$

if such solution exists, otherwise $\tilde{c}(c_2^l, Q) = c$;

(ii) the expected second-period profit of the buyer is

$$\pi_{s,2}(Q) = \mathbb{E}_{c_2^l, c_2^{E^*}} \left[ (r - J(c_2^{E^*})) S(Q) - sQ | c_2^{E^*} \leq \tilde{c}(c_2^l, Q) \right] \cdot \Pr \left( c_2^{E^*} \leq \tilde{c}(c_2^l, Q) \right) \quad (4)$$

$$+ \mathbb{E}_{c_2^l} \left[ (r - J(c_2^l)) S(Q) \cdot F_{n_2}(\tilde{c}(c_2^l, Q)) \right],$$

and the expected second-period profit of the incumbent is

$$\overline{\pi}_{s,2}(Q) = \mathbb{E}_{c_2^l} \left[ (J(c_2^l) - c_2^l) S(Q) \cdot F_{n_2}(\tilde{c}(c_2^l, Q)) \right]. \quad (5)$$

The buyer selects a winning supplier by comparing the virtual profits created by the suppliers. The virtual profit generated an entrant with cost $c_2^E$ is $v_2^E(Q, c_2^E) \doteq (r - J(c_2^E)) S(Q) - sQ$, where $sQ$ is the total switching cost. The log-concavity of $F(\cdot)$ guarantees that the buyer favors the lowest-cost entrant $c_2^{E^*}$ over the other entrants. The buyer will favor the lowest-cost entrant
over the incumbent if and only if \( v^I_2 (Q, c^I_2) \leq v^E_2 (Q, c^E_2) \), i.e., \( c^E_2 \) is below \( \tilde{c} (c^I_2, Q) \). This is stated in Equation (3) of Proposition 2.

Equation (4) states that the second-period expected profit of the buyer is the expectation of the virtual profit from the incumbent or from the lowest cost entrant, whichever is greater. From (5), the incumbent achieves \textit{ex ante} a gain equal to the expected difference between his total virtual cost and actual cost.

4.2 The first period: the buy-in effect

Taking the optimal second period mechanism and the expected outcome as given, the first period mechanism is designed to maximize the buyer’s total expected profit over time. Let \( c^*_1 \) be the lowest cost of suppliers in the first period. The optimal mechanism in the first period is characterized in Proposition 3:

\[ \text{Proposition 3} \]

\textit{In both long-term and short-term relationships, the buyer selects in the first period the supplier with the lowest cost. The expected first-period profit of the buyer is} \( E_{c^*_1} [\pi_{-,1} (c^*_1, Q, (c^1))] \), \textit{where for given capacity} \( Q \) \textit{and first-period supplier cost} \( c_1 \),

\[
\pi_{-,1} (c_1, Q) = (r - J (c_1)) S (Q) - kQ + \pi_{+,2}^I (Q), \tag{6}
\]

\( and \ Q_1 (c_1) = \arg \max_Q (\pi_{-,1} (c_1, Q) + \pi_{+,2} (Q)) \) \textit{is the optimal capacity invested for a supplier with cost} \( c_1 \).

Note from Equation (6) that the incumbent’s expected profit in the second period \( \overline{\pi}_{-,2}^I (Q) \) is included in the buyer’s first period profit. In other words, in the first period the buyer can capture \textit{ex ante} the second period information rent of the incumbent by lowering the first period price. This happens because a supplier will accept any price in the first period that generates him non-negative expected profits over two periods (the IR constraint). Therefore, anticipating the incumbent supplier’s second-period profit, the buyer can accordingly lower by the same amount the payment offered to a supplier in the first period. The more the incumbent’s expected second-period profit, the lower the payment and the higher the buyer’s profit in the first period. Such lowering of the first period payment due to the future profit flow is referred to as the “buy-in effect” (Dhebar and Oren, 1985; Padmanabhan and Bass, 1993; Anton and Yao, 1987; Becker, 1962). The incumbent’s expected second period profit (rent), which is captured by the buyer via the buy-in effect, is called the buyer’s \textit{buy-in profit}. 

13
The buy-in effect can be interpreted as more aggressive bidding by a supplier in the first period, motivated by the future benefit that a winning supplier will obtain. With a long-term relationship, a supplier secures the second period profit if he wins in the first period. With a short-term relationship, winning in the first period allows the supplier to be favorably discriminated against entrants in the second period because of the switching cost.

Given the optimal mechanisms, we are now ready to compare the performance of long-term and short-term relationships and identify the key forces that drive the difference.

5 Relationship comparison

In order to gain an understanding of the driving forces, we first examine the profit difference between long-term and short-term relationships assuming equal capacity investment. Define \( \pi_s(c_1, Q) = \pi_{1}(c_1, Q) + \pi_{.2}(Q) \) as the total profit associated with a first-period supplier type \( c_1 \) given capacity investment \( Q \). Then the optimal total profit is \( \pi = \mathbb{E}_{c_1^*} \left[ \max_Q \pi_s(c_1^*, Q) \right] \). Lemma 1 characterizes the difference between \( \pi_s \) and \( \pi_\ell \).

Lemma 1

\[
\pi_s(c_1, Q) - \pi_\ell(c_1, Q) = \mathbb{E}_{c_2^I, c_2^E^*} \left[ \left( J(c_2^I) - J(c_2^E^*) \right) S(Q) \right] - sQ \\
(\text{(i) Gain from efficiency improvement}) - \left( J(c_2^I) - J(c_2^E^*) \right) S(Q) \left| c_2^{E^*} \leq \tilde{c}(c_2^I, Q) \right\} \cdot \Pr \left( c_2^{E^*} \leq \tilde{c}(c_2^I, Q) \right) . \\
(\text{ii) Switching cost}) \\
(\text{iii) Loss of buy-in profits})
\]

From Lemma 1, the advantage of a short-term relationship over a long-term relationship is influenced by three terms. First, a short-term relationship benefits the buyer by allowing him to switch to a new supplier that has a lower cost than the incumbent. If the buyer were not to switch in the second period, the virtual cost would be \( J(c_2^I) \) instead of \( J(c_2^{E^*}) \), \( J(c_2^I) > J(c_2^{E^*}) \) if \( c_2^I > c_2^{E^*} \). This is the gain from efficiency improvement (term (i)). Second, a short-term relationship costs the buyer the switching cost (term (ii)). Third, by not committing to a supplier, the buyer lowers the incumbent’s expected profits in the second period, which results in less profits captured via the buy-in effect (term (iii)). These three effects jointly determine when a long-term or a short-term relationship is more profitable for the buyer.

While the two relationships may result in different capacity investments, Lemma 1 reveals the driving forces that have the first-order effect on the relationship choice, without considering the
difference of the capacity investment. In the following, we first examine the profit difference between long-term and short-term relationships, focusing on the impact of the switching cost and demand uncertainty. Then we compare the capacity investment in the two relationships.

5.1 Comparison of the profit

5.1.1 Impact of switching cost

From Lemma 1, we can see that the profits from the two relationships differ due to the events $c_2E^* \leq \tilde{c}(c^I_2, Q)$, in which case the buyer switches to an entrant in the short-term relationship. Based on the definition of $\tilde{c}(c^I_2, Q)$ in Proposition 2, it can be easily proved that $\tilde{c}(c^I_2, Q)$ decreases with the unit switching cost $s$. Proposition 4 characterizes the impact of switching cost on the relationship choice.

**Proposition 4**

(i) When $s$ is small, $\pi_s \geq \pi_\ell$ and $\pi_s - \pi_\ell$ decreases with $s$.

(ii) There exists $\bar{s} > 0$ such that $\pi_s = \pi_\ell$ for $s > \bar{s}$ and $\pi_s - \pi_\ell$ increases with $s$ when $s$ is less than but close to $\bar{s}$.

(iii) If a supplier’s cost follows a uniform distribution $U(c, c + \Delta)$ and there is no demand uncertainty, then there exists $\hat{s} \in [0, 2\Delta)$ such that $\pi_s - \pi_\ell > 0$ for $s < \hat{s}$, $\pi_s - \pi_\ell < 0$ for $s \in (\hat{s}, 2\Delta)$, and $\pi_s - \pi_\ell = 0$ for $s \geq 2\Delta$.

When switching is inexpensive ($s$ small), a short-term relationship is better than a long-term relationship. This is because with negligible switching cost, the chance of switching is high ($\tilde{c}$ is low). Thus the gain from efficiency improvement (term (i)) is significant and even higher than the loss of buy-in profits (term (iii)). This advantage of a short-term relationship decreases with a higher switching cost.

When switching is expensive ($s$ close to $\bar{s}$), the relationship choice is reversed–a long-term relationship generates more profits than a short-term relationship. This is because the gain from efficiency improvement, after being offset by the switching cost, is dominated by the loss of buy-in profits; i.e., the net gain from switching is dominated by the advantage of a long-term relationship resulting from suppliers’ more aggressive bidding. When $s$ further increases, this advantage of a long-term relationship becomes lower becomes the switching probability in a short-term relationship is smaller. In fact, when the switching cost is very high ($s > \bar{s}$), a short-term relationship reduces to a long-term relationship because the buyer will never switch even with the option of switching open.
Table 1: Parameter values for numerical experiments.

<table>
<thead>
<tr>
<th>Economic parameters</th>
<th>Demand parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>normal dist.</td>
</tr>
<tr>
<td>$r$</td>
<td>$d, d$</td>
</tr>
<tr>
<td>$k$</td>
<td>$[0.2, 0.8], [0, 1]$</td>
</tr>
<tr>
<td>$(n_1, n_2)$</td>
<td>$(1, 1), (1, 3), (1, 6), (3, 3), (3, 5)$</td>
</tr>
<tr>
<td>$[\varepsilon, \bar{\varepsilon}]$</td>
<td>$[0.35, 0.65], [0, 1]$</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.1, 0.3, 0.5, 0.7, 1</td>
</tr>
</tbody>
</table>

Assuming a uniform distribution of a supplier’s cost and deterministic demand, Proposition 4 further shows that the above cases of relationship choice compose the entire pattern of decisions with the full range of switching cost: There exist two critical values of switching cost such that the buyer strictly prefers a short-term to a long-term relationship when the switching cost is less than the lower critical value, the opposite when the switching cost is between the two values, and is indifferent on the relationship choice for switching costs above the higher value. Therefore, we conclude that a long-term relationship is beneficial for the buyer if the switching cost is beyond a threshold, and the benefit is most significant for moderately high switching costs.

We verify the above insights in more general conditions with numerical experiments. The profit difference between the two relationships is compared numerically with (truncated) normal cost distributions and (truncated) normal or gamma demand distributions. The parameter values used in the numerical experiments throughout the paper are summarized in Table 1

We find that the results of Proposition 4 still hold in the numerical experiments. A representative numerical result is illustrated in Figure 2.

Dyer (2000) has pointed out that a firm should use arm’s length (short-term) relationships for non-strategic inputs. Inputs are non-strategic when the outsourced items are (1) commodities or standardized products, (2) stand-alone, or modular with no or few interaction effects with other inputs, and (3) characterized by a low degree of supplier-buyer interdependence. We have shown

---

1Since Proposition 4 and the later results are driven by the switching cost, cost variance and demand variance, we vary these parameters in the experiments but keep $r, k, \mu_c$ and $\mu_d$ fixed. The cost uncertainty has to be significant in order for suppliers to gain information rent and the comparison of relationships not to be trivial. Therefore, the value of $\sigma_c$ is chosen so that the relative variability of costs ranges from very low but still significant (0.2) to relatively high (2). For a normal distribution of the demand, the value of $\sigma_d$ is chosen to cover a range of the relative variability from being very low (close to zero) to high (2). In order to ensure positive values of cost and demand, the normal distributions for the cost and demand are truncated, and for each distribution we test two value supports for robustness. For the gamma distribution of demand, $\sigma_d$ is set to realize different shapes of the distribution (the shape parameter varies from about 0.8 to 4). For convenience of computation, the gamma distribution is truncated on the upper end but still covers a great majority of the mass.
that a short-term relationship is preferable when the switching cost is small. These two insights can be connected by the fact that the switching cost tends to be low when the supplier’s input to the buyer is not strategic: It is easier to shift to a new supplier if the production has a standardized process or low interdependence on other inputs or systems of the buyer.

5.1.2 Impact of demand uncertainty

We compare the sensitivity (derivative) of the profit difference, $\pi_s - \pi_\ell$, with respect to the standard deviation of demand $\sigma_d$. In order not to make the notation too cumbersome, we drop the dependence on $\sigma_d$ in the formulas. We first analyze how relationship choice changes when demand uncertainty is introduced to a certain market. In this case, the two relationships result in the same capacity investment equal to $\mu_d$. Then we numerically examine the impact of demand uncertainty in general situations when the capacities in the two relationships may be different.

Lemma 2 characterizes the derivative of $\pi_s - \pi_\ell$ with respect to $\sigma_d$ when $\sigma_d$ is very small.
Lemma 2  The derivative of $\pi_s - \pi_\ell$ with respect to $\sigma_d$ when $\sigma_d = 0$, can be characterized by:

$$
\frac{d}{d\sigma_d} (\pi_s - \pi_\ell) = E_{c_2, c_2^*} \left[ \left( J(c_2^l) - J(c_2^E) \right) - \left( J(c_2^l) - c_2^l \right) \Big| c_2^E \leq \hat{c} \left( c_2^l, \mu_d \right) \right] \cdot \text{Pr} \left( c_2^E \leq \hat{c} \left( c_2^l, \mu_d \right) \right) \frac{\partial}{\partial \sigma_d} S (\mu_d)
$$

\( (i) \) difference in second period profit if an entrant wins

$$
- E_{c_2^l} \left[ (J(c_2^l) - c_2^l) S (\mu_d) \frac{\partial}{\partial \sigma_d} F_{n_2} \left( \hat{c} \left( c_2^l, \mu_d \right) \right) \right]
$$

\( (ii) \) change of the buy-in profits due to the change of the switching probability

In order to understand in which direction demand uncertainty drives the relationship choice, we focus on the sign of $\frac{d}{d\sigma_d} (\pi_s - \pi_\ell)$ when the two relationships achieve same profits (but with a positive switching probability in the short-term relationship). The derivative being positive (negative) means that an increase of market uncertainty favors the short-term (long-term) relationship.

The term \( (i) \) of Equation (7) characterizes the direct impact of demand uncertainty through its influence on the expected sales, $\frac{\partial}{\partial \sigma_d} S (\mu_d)$. It is the multiplication of two factors. The second factor $\frac{\partial}{\partial \sigma_d} S (\mu_d)$ is negative; i.e., as the market uncertainty increases, the expected sales decrease. The first factor is the difference between the expected gain from efficiency improvement and the loss of buy-in profits generated by sourcing a unit from the winning entrant, which can be interpreted as the marginal profit advantage of a short-term relationship. When the two relationships achieve same profits, this factor is positive because it exactly covers the switching cost. This means that compared to a long-term relationship, the short-term relationship has a higher marginal profit, and hence has the total profit more sensitive to the sales. Given that the expected sales decrease with demand uncertainty, the term \( (i) \) is negative.

The term \( (ii) \) characterizes the indirect impact of demand uncertainty through its influence on the switching probability in a short-term relationship. With higher demand uncertainty, the expected sales are lower. Hence the buyer requires the entrant to be more efficient for switching to happen so that the total gain from switching covers the switching cost: $\frac{\partial c_2}{\partial \sigma_d} < 0$. Therefore, with demand uncertainty increasing, the switching probability $F_{n_2} \left( \hat{c} \left( c_2^l, Q_s \right) \right)$ decreases, generating more buy-in profits. As a result, term \( (ii) \) is positive, which reverses the effect of term \( (i) \).

Based on a uniform cost distribution, Proposition 5 shows that the effect of term \( (i) \) dominates that of term \( (ii) \). A long-term relationship is thus preferred in a more volatile market. Recall from Proposition 4 that $\hat{s}$ is the threshold switching cost at which $\pi_s = \pi_\ell$ and the relationship choice is reversed.
**Proposition 5** With a supplier’s cost following a uniform distribution on $[\underline{c}, \overline{c}]$, \[ \frac{d}{d\sigma_d} (\overline{\pi}_s - \overline{\pi}_\ell) < 0 \] for $\sigma_d = 0$ and $s = \hat{s}$.

Next, we examine numerically the impact of demand uncertainty on relationship choice in more general settings based on the parameter values in Table 1. In the experiments we calculate $\overline{\pi}_s - \overline{\pi}_\ell$ over a range of different $\sigma_d$ values while keeping the expected demand $\mu_d$ constant. As an example, Figure 3 draws $\overline{\pi}_s - \overline{\pi}_\ell$ with different values of $\sigma_d$ and a varying switching cost $s$. Consistent with the insights revealed in Proposition 5, the numerical results show that increasing demand uncertainty only shifts the threshold switching cost at which the relationship choice is reversed towards the left. This indicates that a long-term relationship becomes more favorable for the buyer when he faces a more volatile market.

### 5.2 Comparison of the capacity: the lock-in effect

Comparing $\overline{\pi}_s$ and $\overline{\pi}_\ell$ at $\sigma_d = 0$ allows us to understand the first-order effect of the environment parameters without considering the difference of capacity investment in the two relationships. When $\sigma_d > 0$, the two relationships generally result in different capacity investments in the first period. In order to compare the optimal capacity investment, we first compare the marginal return of capacity investment in these two relationships.
Lemma 3 For any \( c_1 \), the derivative of \( \pi_s(Q, c_1) - \pi_\ell(Q, c_1) \) with respect to \( Q \) is:

\[
\frac{\partial}{\partial Q} (\pi_s(Q, c_1) - \pi_\ell(Q, c_1)) = \mathbb{E}_{c^*_2, c^*_1} \left[ (J(c^*_2) - J(c^*_1^e)) \middle| c^*_2 \leq \tilde{c}(c^*_2, Q) \right] \cdot \Pr \left( c^*_2 \leq \tilde{c}(c^*_2, Q) \right) \mathbb{G}(Q)
\]

\[
= \mathbb{E}_{c^*_2, c^*_1} \left[ \left( J(c^*_2) - c^*_2 \right) F_{n_2} \left( \tilde{c}(c^*_2, Q) \right) \right] \mathbb{G}(Q) - \mathbb{E}_{c^*_2} \left[ F_{n_2} \left( \tilde{c}(c^*_2, Q) \right) \right] \mathbb{G}(Q)
\]

\[
\text{(i) Gain from efficiency improvement (+)} - \mathbb{E}_{c^*_2} \left[ J(c^*_2) - c^*_2 \right] S(Q) \frac{\partial}{\partial Q} F_{n_2} \left( \tilde{c}(c^*_2, Q) \right)
\]

\[
\text{(ii) Loss of buy-in profits (-)} - \mathbb{E}_{c^*_2} \left[ F_{n_2} \left( \tilde{c}(c^*_2, Q) \right) \right] \mathbb{G}(Q)
\]

\[
\text{(iii) Switching cost (-)} - \mathbb{E}_{c^*_2} \left[ \left( J(c^*_2) - c^*_2 \right) S(Q) \frac{\partial}{\partial Q} F_{n_2} \left( \tilde{c}(c^*_2, Q) \right) \right]
\]

\[
\text{(iv) Lock-in (+)}
\]

where the sign of each term is indicated in the bracket following the name.

From Proposition 3, we know that with a short-term relationship, the buyer gains from the efficiency improvement (term (i)), but captures less buy-in profits (term (ii)) and pays the switching cost (term (iii)). Lemma 3 shows that the capacity directly influences the magnitudes of these three terms by affecting the volumes: Both the gain from efficiency improvement and the loss of buy-in profits depend on the expected volume \( S(Q) \) provided by the supplier in the second period (with \( S'(Q) = \mathbb{G}(Q) \)), and the total switching cost is proportional to \( Q \).

The capacity also indirectly influences the buyer’s profit by affecting the switching decision in a short-term relationship (term (iv)): The switching probability \( F_{n_2} \left( \tilde{c} \right) \) decreases with the capacity \( Q \). This is because a higher capacity increases the cost of switching, and in turn requires the entrant supplier to be more efficient (\( \tilde{c} \) becomes lower) when switching takes place. We refer to this lowering of the switching probability resulting from a higher capacity as the lock-in effect of the capacity in a short-term relationship. The lower the switching probability, the higher the incumbent’s expected second-period profit, and the more buy-in profits obtained by the buyer. Therefore, the lock-in effect increases the marginal value of capacity investment in a short-term relationship.

Thus based on Lemma 3, compared to a long-term relationship, a short-term relationship may increase the capacity due to the gain from efficiency improvement (term (i)) and the lock-in effect (term (iv)), whereas it may also decrease the capacity due to the loss of buy-in profits (term (ii)) and the cost of switching (term (iii)). The overall effect is characterized in Proposition 6. Recall from Proposition 4 that \( \pi \) is the threshold switching cost beyond which a short-term relationship reduces to a long-term relationship.

**Proposition 6** (i) For any first period supplier cost \( c_1 \), when \( s \) is small, \( Q_S(c_1) > Q_L(c_1) \) with the difference decreasing with \( s \).
(ii) For any first period supplier cost $c_1$, when $s < \bar{s}$ and $s$ is sufficiently large, $Q_S(c_1) > Q_L(c_1)$ with the difference decreasing with $s$.

(iii) If a supplier’s cost follows a uniform distribution $U[c, c + \Delta]$, and the demand uncertainty, $\sigma_d$, is very small, then there exist $s_L$ and $s_H$, $0 < s_L < s_H < 2\Delta$, such that $Q_s(c_1)$ increases with $s$ for $s \in [s_L, s_H]$, and decreases with $s$ for $s \in [0, s_L] \cup [s_H, 2\Delta]$, and there exists $\tilde{s} \in (s_L, 2\Delta)$ such that $Q_s(c_1) > Q_{\ell}(c_1)$ for $s \in (\tilde{s}, 2\Delta)$.

Proposition 6 shows that a short-term relationship results in more capacity investment than a long-term relationship not only when the switching cost is very small but also when it is relatively large (but less than the level above which a short-term relationship reduces to a long-term relationship). The result for the first range is intuitive because that is when the gain from efficiency improvement is significant while the switching cost is negligible. The result for the second range is somewhat counter-intuitive because one would think that higher switching cost reduces the return of capacity investment in a short-term relationship. This result is due to the indirect lock-in effect, which provides an incentive to increase the investment. When switching cost is relatively large, the capacity has a significant impact on the switching probability, and hence the lock-in effect is prominent.

Based on uniform cost distributions and small demand uncertainty, Proposition 6 further characterizes the movement of capacity investment in a short-term relationship over the full range of switching cost. The optimal capacity investment in a short-term relationship changes in three phases: the capacity first decreases, then increases, and finally decreases until the short-term relationship reduces to a long-term relationship. Note the capacity investment in a long-term relationship does not change with the switching cost. Thus this movement also means that the short-term relationship induces a higher capacity level than the long-term relationship for switching costs relatively high.

Again, numerical experiments show that the above insights can be extended to more general situations. Figure 4 shows a representative example. In this figure, the capacity difference between short-term and long-term relationships is drawn as a function of the switching cost with different $n_2$ (left plot) and $\sigma_d$ (right plot).

The difference of capacity investment between the two relationships in the presence of demand uncertainty adds a new element to the impact of demand uncertainty on the profit difference, besides those revealed in Lemma 2. Higher capacity implies that the expected sales are more robust to the change of demand uncertainty. This is because when the capacity is higher, the expected sales
Figure 4: Expected capacity difference $\mathbb{E}[Q_s(c_1^*) - Q_l(c_1^*)]$ between the short-term and long-term relationships as a function of switching cost $s$. $r = 3$, $n_1 = 1$, $k = 0.5$, and supplier production costs are drawn from truncated normal distribution $\mathcal{N}(0.5, 1)$ on the support $[0.35, 0.65]$. For the left plot, $n_2 = \{1, 3, 6\}$, and the demand follows a truncated normal distribution $\mathcal{N}(0.5, 1)$ on the support $[0.2, 0.8]$. For the right plot, $n_2 = 3$ and the demand follows a truncated normal distribution $\mathcal{N}(0.5, \sigma_d)$, $\sigma_d \in \{0.01, 0.1, 0.2, 1\}$ on the support $[0, 1]$.

decrease slower with the increase of demand uncertainty. Numerical evaluations show that when the buyer is indifferent between long-term and short-term relationships, the capacity in a long-term relationship is always higher than in the other. Therefore, the expected sales in a long-term relationship are more robust to demand volatility than in a short-term relationship. This, besides a lower profit margin, further leads the preference shifting to a long-term relationship when demand uncertainty is higher.

6 System efficiency

We have analyzed the optimal mechanisms of long-term and short-term relationships with the objective of maximizing the buyer’s profits. In this section we examine how the buyer’s optimal decision influences the profit of the supply chain. This is done by comparing the buyer’s optimal results to the first-best solution, which maximizes the total expected profit of the supply chain. In a first-best solution, the price is not a concern because it is a transfer within the system. Therefore, we focus on the supplier selection decision in each period and capacity investment in the first period of both relationships. Supplier selection in the second period of a long-term relationship is trivial.

Recall that in the second period of a short-term relationship, the buyer switches to the lowest-cost entrant if and only if the lowest-cost of entrants, $c_{E2}^*$, is lower than $\tilde{c}(c_{I2}^*, Q)$. $\tilde{c}$ is determined based on the virtual profit (see Proposition 2). In the first-best solution, the buyer will switch to the lowest-cost entrant if and only if the system profit of the supplier chain from continuing with
the incumbent is less than that from switching to the entrant, \((r - c^I_2) S(Q) < (r - c^{E*}_2) S(Q) - sQ\). This results in a different threshold level of the entrant’s cost, \(\hat{c}(c^I_2, Q)\), as characterized in Proposition 7. It can also be shown that \(\hat{c}(c^I_2, Q) \geq \hat{c}(c^E_2, Q)\). Thus asymmetric information makes an entrant, rather than the incumbent, more favorable in the competition (see also Laffont and Tirole (1988)). Such inefficient switching decision benefits the buyer because it lowers the overall information rent.

**Proposition 7**  
(i) In the first-best solution of short-term sourcing, the buyer switches to the lowest-cost entrant if \(c^{E*}_2 < \hat{c}(c^I_2, Q)\) where \(\hat{c}(c^I_2, Q) = \max\{c^I_2 - sQ S(Q), c^I_2\}\), otherwise stays with the incumbent.  
(ii) \(\hat{c}(c^I_2, Q) \geq \hat{c}(c^E_2, Q)\).

In the first period, note that the buyer always selects the lowest-cost supplier even with information asymmetry. Therefore, supplier selection in the first period is system-efficient in the buyer’s optimal solution. However, the situation of the capacity investment decision is more complicated. Information asymmetry leaves rent to the supplier that increases the buyer’s marginal cost of sourcing. As a result, the capacity investment in a long-term relationship is lower compared to the first-best solution. But information asymmetry does not necessarily result in a lower capacity as well in a short-term relationship. This is because the lock-in effect drives up the capacity investment level in the presence of information asymmetry (see Lemma 3).

Next we compare the relationship choice decision. Note that if there is no information asymmetry in the second period, then the incumbent should expect zero second-period profit, and hence the buy-in effect does not exist any more in either short-term or long-term relationships. Therefore, in the first-best solution, a long-term relationship does not have the advantage of generating more buy-in profits as in the case with information asymmetry (see Lemma 1). As a result, a long-term relationship is always inferior to a short-term relationship.

**Proposition 8**  
In the first-best solution, a short-term relationship is always better than a long-term relationship.

Proposition 8 can be understood further by noting that if there is no information asymmetry in the second period, a long-term relationship can be regarded as a constrained case of a short-term relationship in which the switching probability is zero. We conclude that the second period information asymmetry is a key driver for a long-term relationship.
In summary, when there is information asymmetry, the buyer’s optimal decision deviates from
the system-optimal solution in that 1) it favors too much the entrants in the second period of a
short-term relationship, 2) it leads to capacity investment that is too low in a long-term relationship
(but not necessarily in a short-term relationship), and 3) it may choose long-term commitment as
the preferred relationship choice.

7 Extension: Capacity change in the second period

We have assumed that capacity in the second period is fixed at the level invested in the first period.
In this subsection, we study the situation when the capacity is flexible, i.e. in the second period
the capacity may be expanded or shrunk. Recall that the salvage value of capacity is zero as the
capacity is typically not worth much outside the business with the buyer. The buyer may want
to expand the capacity (invest in new capacity) if the supplier (an entrant or the incumbent) in
the second period is so efficient that the marginal profit is larger than the unit cost of capacity
investment. If the buyer switches to an entrant, the capacity of the entrant is either transferred
from the incumbent or is newly invested. It is reasonable to assume that the cost of investing in
new capacity is higher than the cost of transferring capacity, $k > s$; otherwise, no capacity is ever
transferred and the entrant will always invest in new capacity in order to serve the buyer. Therefore,
for the entrant, new capacity will only be invested on top of the existing capacity transferred from
the incumbent. The buyer may also want to shrink the capacity (transferring only a part of the
capacity) when switching to an entrant if the entrant is not efficient enough to justify the switching
cost for transferring all capacity. If the buyer stays with the incumbent, shrinkage is never optimal
since the salvage value is zero.

Recall from Section 4 that the buyer’s second-period virtual profits of sourcing from the incum-
ent and entrant, $v_{2}^{I}(Q, c_{2}^{I})$ and $v_{2}^{E}(Q, c_{2}^{E})$, determine the switching decision, influencing capacity
investment and relationship choice. With flexible second period capacity, these profits change from
the case with fixed capacity. Given capacity $Q$, the second-period virtual profit of sourcing from
the incumbent with type $c_{2}^{I}$ (in both short- and long-term relationships) is:

$$v_{2}^{I}(Q, c_{2}^{I}) = \max_{Q'} \left\{ (r - J(c_{2}^{I})) S(Q') - k (Q' - Q)^{+} \right\}.$$  

The virtual profit of sourcing from an entrant with type $c_{2}^{E}$ is:

$$v_{2}^{E}(Q, c_{2}^{E}) = \max_{Q'} \left\{ (r - J(c_{2}^{E})) S(Q') - k (Q' - Q)^{+} - s \min(Q', Q) \right\}.$$
In the optimal mechanism of a short-term relationship, the buyer will switch to the lowest-cost entrant if and only if $v^E_2 (Q, c^E_2) \geq v^I_2 (Q, c^I_2)$. Similar as in Proposition 2, given the first period capacity $Q$ and incumbent’s second period cost $c^I_2$, let $c^I' (c^I_2, Q)$ be the threshold cost of the entrant such that switching to the entrant occurs if and only if $c^E_2 \leq c^I' (c^I_2, Q)$. Then, the probability of switching is $F_{n_2} (c^I' (c^I_2, Q))$ (i.e. the probability that the lowest entrant cost is less than $c'$). Similar as in the fixed capacity case, this threshold cost decreases, and hence the probability of switching declines, with the existing capacity $Q$:

**Lemma 4** $\frac{\partial}{\partial Q} F_{n_2} (c^I' (c^I_2, Q)) \leq 0$.

Lemma 4 shows that the lock-in effect (see Section 5.2) still exists for the flexible capacity case. As a result, it remains true that a short-term relationship induces more initial capacity investment than a long-term relationship when the switching cost $s$ is large (as in Proposition 6). It can also be shown that the buy-in effect still exists with flexible capacity. Therefore, the qualitative trade-offs between short- and long-term relationships remain the same as in the case with fixed capacity in the second period. It can be proved that when the switching cost is very small or large, the comparison of the profits from the two relationships is similar to the fixed capacity case (as in Proposition 4). These results are summarized in Proposition 9:

**Proposition 9**  
(i) When $s$ is sufficiently large, $Q_S (c_1) \geq Q_L (c_1)$ for any $c_1$.

(ii) When $s$ is small, $\pi_s \geq \pi_\ell$ and $\pi_s - \pi_\ell$ decreases with $s$.

(iii) There exists $s'$ such that $\pi_s = \pi_\ell$ for $s > s'$ and $\pi_s - \pi_\ell$ increases with $s$ when $s$ is less than but close to $s'$.

Proposition 9 demonstrates analytically that for the switching cost very small or very large, the major insights do not change with the capacity becoming flexible in the second period. This is confirmed through numerical experiments (see Figure 5 in the Appendix for example results). Therefore, we may conclude that having flexible capacity in the second period does not change our qualitative insights.

### 8 Conclusion and Discussion

We study when a buyer should select a long-term relationship by committing to source from a single supplier over a long period of time, or choose a short-term relationship by keeping the option open to switch to a new supplier in the future. Facing uncertain demand, the buyer invests in transferable
specific capacity that is operated by a supplier. Considering private information of production costs of suppliers, we model a relationship by means of repeated auctions over two periods: with a short-term relationship, the buyer selects the supplier in each period by repetitively auctioning a supply contract to the supplier pool, while with a long-term relationship, the buyer retains over time the supplier selected in the first period auction. We compare the two relationships by analyzing the buyer’s optimal mechanism for each relationship.

With a long-term relationship, since a supplier expects more second-period profits than with a short-term relationship, suppliers will bid more aggressively and thus result in lower prices in the first period. This is referred to as the ‘buy-in’ effect. This effect may make a long-term relationship more profitable for the buyer than a short-term relationship. In addition, when a short-term relationship is adopted, since more capacity of the incumbent reduces the probability of switching (the lock-in effect) and hence improves his profit in the second period, a higher capacity investment also motivates suppliers to bid more aggressively in the first period. This provides another incentive for capacity investment in a short-term relationship—besides the one of mitigating demand uncertainty.

Our analysis reveals the following insights: (1) The buyer should choose a long-term relationship when the switching cost is above a certain threshold value, (2) when facing more volatile market demand, a long-term relationship tends to be more favorable than a short-term relationship, and (3) a short-term relationship may induce a higher capacity investment level than a long-term relationship when the switching cost is relatively high.

In our model, we assume that the suppliers’ costs are independent across periods. More realistically, a supplier’s costs may be correlated across periods—a supplier that is efficient in the first period may be efficient in the second period as well. Introducing correlated costs in repeated auctions gives rise to the ratchet effect (Laffont and Tirole, 1993, Chapter 9): If a supplier reports truthfully a low cost in the first period, the buyer will learn about his low cost and take advantage of this information by offering a low price to that supplier in the second period. Due to the ratchet effect, a supplier has an additional incentive to exaggerate his cost in the first period, considering the impact of his bid on future profits (besides the impact on the current-period profits). As a result, it becomes more expensive for the buyer to induce truthful bidding of suppliers. If the cost of inducing truth telling dominates the benefit of knowing supplier types in the second period, separating supplier types in the first period (with the IC constraint) does not give the optimal results any more for the buyer. Thus in an optimal mechanism, the suppliers may or may not reveal their costs in the first period. In the presence of the ratchet effect, a complete analysis of the optimal
mechanism is very difficult (Laffont and Tirole, 1993, Chapter 9). A restricted analysis may be done by focusing on some special cases. Separating and pooling mechanisms are two extreme mechanisms in which the buyer acquires complete and no information, respectively, about the suppliers’ first-period costs. Restricting the consideration to separating\(^2\) and pooling equilibria and assuming perfectly correlated costs, it can be shown that a short-term relationship is always better than a long-term relationship. This may suggest that cost correlation makes the short-term relationship more favorable. Nevertheless, a complete analysis of the optimal mechanism with inter-temporally correlated supplier types is a rich (and difficult) problem. Further study of this problem would be interesting work in future research.

In our model, we also assume identical cost distributions in the two periods for a supplier. In reality, the incumbent may have his second-period efficiency improved over the first period due to the learning effect (Yelle, 1979; Lapre and Wassenhove, 2001). The learning effect can be incorporated in our model by changing the incumbent’s cost distribution in the second period so that it is stochastically dominated by the first-period distribution (but the independence remains). This would not affect the analysis and the qualitative insights on the tradeoffs between the two relationships still hold. But the learning effect will influence the choice of the relationships. With a long-term relationship, the buyer can fully take advantage of the incumbent’s cost reducing achieved from learning. With a short-term relationship, on one hand, the incumbent’s learning benefits the buyer only when the buyer does not switch the supplier, but on the other hand, learning reduces the switching probability, and thus results in more buy-in profits. Based on the analysis of learning effects in Li and Debo (2008), we conjecture that more learning effect should benefit the long-term relationship. The impact of supplier learning on supplier relationship choice would be another topic for future research.

Finally, we also assume that the buyer invests in the supplier’s capacity. This arrangement is appropriate for the capacity that is specific to the buyer’s business and cannot be redeployed for another buyer, referred to as quasi-integration (McWilliams and Gray, 1995). Alternatively, it can be possible that the supplier, instead of the buyer, invests in specific capacity, but a supply contract cannot be fully specified before the investment. In that case, since the capacity investment cost is sunk when the contract is negotiated, the buyer can demand a larger share of the profits. In other words, suppliers face a hold-up problem, which may lead to under-investment of specific capacity and change the value of commitment (Taylor and Plambeck, 2007; Tunca and Zenios, 2006). It

\(^2\)A separating equilibrium exists under the assumption that a supplier will not quit in the second period if he enters in the first period.
would also be interesting to study in future research the value of commitment in such a case when a supplier faces the hold-up problem in capacity investment.

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References


Appendix

We shall first present and prove Lemma A 1, in order to prove Propositions 1 and 2.

**Lemma A 1** Let the capacity of the incumbent be $Q$. Define $\overline{p}_{s,2}^I \left( c_2^I, Q \right) = \mathbb{E}_{c_2^I} \left[ p_{s,2}^I (c_2^I, c_2^E, Q) \right]$ and $\overline{p}_{s,2}^{E,j} \left( c_2^E, Q \right) = \mathbb{E}_{c_2^E} \left[ p_{s,2}^{E,j} \left( c_2^E, c_2^E, Q \right) \right]$ as the expectations, with respect to the other suppliers' costs, of the winning probabilities of the incumbent and an entrant in the second period of a short-term relationship. Define $u_{s,2}^I(c_2^I, Q) = u_{s,2}^I(c_2^I, c_2^I, Q)$ and $u_{s,2}^{E,j}(c_2^E, Q) = u_{s,2}^{E,j}(c_2^E, c_2^E, Q)$ as the second-period profits of the incumbent and an entrant when all suppliers bid truthfully in the second period. The mechanism in the second period of a short-term relationship is incentive compatible if and only if

1. $u_{s,2}^I(c_2^I, Q)$ and $u_{s,2}^{E,j}(c_2^E, Q)$ satisfy
   \[
   u_{s,2}^I(c_2^I, Q) = u_0^I(Q) + S(Q) \int_{c_2^I}^c \overline{p}_{s,2}^I (\rho, Q) \, d\rho \quad (8)
   
   u_{s,2}^{E,j}(c_2^E, Q) = u_0^{E,j}(Q) + S(Q) \int_{c_2^E}^\pi \overline{p}_{s,2}^{E,j} (\rho, Q) \, d\rho \quad (9)
   \]
   where $u_0^I(Q) \geq 0$ and $u_0^{E,j}(Q) \geq 0$ are constants for a given capacity $Q$.

2. $\overline{p}_{s,2}^I \left( c_2^I, Q \right)$ and $\overline{p}_{s,2}^{E,j} \left( c_2^E, Q \right)$ are decreasing functions of $c$.

**Proof** (i) Prove the necessity: With incentive compatibility, $u_{s,2}^I(c, Q) = \max_{c' \in [c, c]} u_{s,2}^I(c', c, Q)$ and $u_{s,2}^{E,j}(c, Q) = \max_{c' \in [c, c]} u_{s,2}^{E,j}(c', c, Q)$. Then based on the envelop theorem,

\[
\begin{align*}
\frac{d}{dc} u_{s,2}^I(c, Q) &= \frac{d}{dc} u_{s,2}^I(c', c, Q)|_{c'} = -S(Q) \overline{p}_{s,2}^I (c, Q), \\
\frac{d}{dc} u_{s,2}^{E,j}(c, Q) &= \frac{d}{dc} u_{s,2}^{E,j}(c', c, Q)|_{c'} = -S(Q) \overline{p}_{s,2}^{E,j} (c, Q).
\end{align*}
\]

Then Equations (8) and (9) follow. Since $u_{s,2}^I(c, Q)$ is the maximum of a family of affine functions of $c$, $u_{s,2}^I(c, Q)$ is a convex function of $c$. Hence $\overline{p}_{s,2}^I (c, Q)$ decreases with $c$. Similarly we can prove that $\overline{p}_{s,2}^{E,j} (c, Q)$ decreases with $c$.

(ii) Prove the sufficiency: Assume $\hat{c} > c$ without loss of generality. From Equation (8),

\[
\begin{align*}
u_{s,2}^I(\hat{c}, Q) - u_{s,2}^I(c, Q) &= \int_{\hat{c}}^c S(Q) \overline{p}_{s,2}^I (\rho, Q) \, d\rho.
\end{align*}
\]

But the right hand side is no greater than $\int_{\hat{c}}^c -S(Q) \overline{p}_{s,2}^I (\hat{c}, Q) \, d\rho = S(Q) \overline{p}_{s,2}^I (\hat{c}, Q) (c - \hat{c})$, as $\overline{p}_{s,2}^I (\rho, Q)$ is a decreasing function of $\rho$. Therefore

\[
u_{s,2}^I(c, Q) \geq u_{s,2}^I(c, Q) - S(Q) \overline{p}_{s,2}^I (\hat{c}, Q) (c - \hat{c}) = u_{s,2}^I(\hat{c}, c, Q).
\]

32
Hence the mechanism is incentive compatible for the incumbent. Following the same reasoning we can show that the mechanism is incentive compatible for an entrant.

**Proof of Proposition 1.** The second period of a long-term relationship can be regarded as a special situation of the second period in short-term sourcing when the number of entrants is zero, and therefore \( p_{I,2}^I(c_2^I) = 1 \). Following Lemma A 1, the ex ante second period profit of the incumbent, with a capacity \( Q \), is

\[
\pi_{I,2}(Q) = u_0^I(Q) + \int_c^{c_2^I} \int_c^{c_2^I} S(Q)p_{I,2}^I(\rho) \, d\rho \, f(c_2^I) \, dc_2^I \\
= u_0^I(Q) + S(Q) \int_c^{c_2^I} F\left(\frac{c_2^I}{f(c_2^I)}\right) p_{I,2}^I(c_2^I) \, f(c_2^I) \, dc_2^I.
\]

To maximize the buyer’s profit, \( u_0^I(Q) = 0 \). The expected profit of the buyer is equal to the expected profit of the supply chain minus the supplier’s profit:

\[
\pi_{I,2}(Q) = \int_c^{c_2^I} (r - c_2^I) S(Q)p_{I,2}^I(c_2^I) \, f(c_2^I) \, dc_2^I - \pi_{I,2}(Q)
\]

\[
= \int_c^{c_2^I} (r - J(c_2^I)) S(Q)p_{I,2}^I(c_2^I) \, f(c_2^I) \, dc_2^I.
\]

Since \( r > J(c_2^I) \), the maximum of the buyer’s profit is achieved at \( p_{I,2}^I(c_2^I) = 1 \) for any \( c_2^I \). It follows \( \pi_{I,2}(Q) = \mathbb{E}_{c_2^I}[(r - J(c_2^I))] S(Q) \) and \( \pi_{I,2}(Q) = \mathbb{E}_{c_2^I} [J(c_2^I) - c_2^I] S(Q) \). In addition, from Lemma A 1, \( u_{I,2}^I(c, Q) = S(Q)(c - \bar{c}) \). But based on the contract, \( u_{I,2}^I(c, Q) = \left(w_{I,2}^I(c) - c\right) S(Q) \). Therefore \( w_{I,2}^I(c) = \bar{c} \).

**Proof of Proposition 2.**

i) From Lemma A 1, the expectation of the second period profit \( \pi_{s,2}^I(Q) \) of the incumbent with respect to its second period cost \( c_2^I \) is:

\[
\pi_{s,2}^I(Q) = u_0^I(Q) + \int_c^{c_2^I} \int_c^{c_2^I} S(Q)p_{s,2}^I(\rho, Q) \, d\rho \, f(c_2^I) \, dc_2^I \\
= u_0^I(Q) + \int_c^{c_2^I} S(Q)p_{s,2}^I(c_2^I, Q) \, F(c_2^I) \, dc_2^I.
\]

Similarly the expected second period profit \( \pi_{s,2}^E(Q) \) of an entrant is:

\[
\pi_{s,2}^E(Q) = u_0^E(Q) + \int_c^{c_2^I} S(Q)p_{s,2}^E(\rho, Q, E(c_2^I)) \, F(c_2^I) \, dc_2^I.
\]
The \textit{ex ante} profit of the buyer is equal to the expected profit of the supply chain minus the suppliers':

\[
\pi_{s,2}(Q) = \sum_{i=1}^{n_2} \left( \int_{c}^{\pi} \left( \left( r - c_2^{E,j} \right) S(Q) - sQ \right) \bar{p}_{s,2}^{E,j} \left( c_2^{E,j}, Q \right) f \left( c_2^{E,j} \right) dc_2^{E,j} - \bar{u}_{s,2}^{E,j}(Q) \right) \right. \\
+ \left. \int_{c}^{\pi} \left( r - c_2^l \right) S(Q) \bar{p}_{s,2}^l \left( c_2^{l}, Q \right) f \left( c_2^{l} \right) dc_2^{l} - \bar{u}_{s,2}^{l}(Q) \right) \\
= \sum_{i=1}^{n_2} \int_{c_2^l}^{\pi} \left( \left( r - J(c_2^{E,j}) \right) S(Q) - sQ - u_0^{E,j}(Q) \right) p_{s,2}^{E,j} \left( c_2^{E,j}, c_2^{E}, Q \right) f \left( c_2^{E} \right) dc_2^{E} f \left( c_2^{l} \right) dc_2^{l} \\
+ \int_{c}^{\pi} \left( \left( r - J(c_2^{l}) \right) S(Q) - u_0^{l}(Q) \right) p_{s,2}^l \left( c_2^l, c_2^{E}, Q \right) f \left( c_2^{E} \right) dc_2^{E} f \left( c_2^{l} \right) dc_2^{l}
\]

where \( f \left( c_2^{E} \right) \) represents the joint probability density of the profile \( c_2^{E} \) of the entrants’ types. To maximize \( \pi_{s,2}(Q) \), the buyer should set \( u_0^{l}(Q) = u_0^{E}(Q) = 0 \) so that the individual rationality constraint is binding for the least efficient supplier. Furthermore the buyer should compare the virtual profits \( \left( r - J(c_2^{E,j}) \right) S(Q) - sQ, i = 1, \ldots, n_2 \), from all entrants, and the virtual profit \( \left( r - J(c_2^{l}) \right) S(Q) \) from the incumbent in order to decide the winning probability of each supplier. Therefore, the supplier with the highest virtual profit is selected as the winner with probability 1, i.e., \( p_{s,2}^{E,j} \left( c_2^{l}, c_2^{E}, Q \right) = 1 \) if

\[
\left( r - J(c_2^{E,j}) \right) S(Q) - sQ \geq \max \left\{ \left( r - J(c_2^{l}) \right) S(Q), \left( r - J(c_2^{E,j}) \right) S(Q), i \neq j \right\},
\]

otherwise \( p_{s,2}^l \left( c_2^{l}, c_2^{E}, Q \right) = 1 \). Since \( J(c) \) is an increasing function of \( c \), the buyer selects between the incumbent and the lowest cost entrant.

(ii) Equation (4) follows (10) in (i), with the selection rule and \( u_0^l(Q) = u_0^E(Q) = 0 \) given. Also based on the optimal selection rule, \( \bar{p}_{s,2}^l \left( c_2^{l}, Q \right) = \mathcal{F}_{n_2}(\bar{c}(c_2^{l}, Q)) \), and hence

\[
\bar{u}_{s,2}^l(Q) = \int_{c}^{\pi} S(Q) \bar{p}_{s,2}^l \left( c_2^{l}, Q \right) F \left( c_2^{l} \right) dc_2^{l} = \int_{c}^{\pi} S(Q) \mathcal{F}_{n_2}(\bar{c}(c_2^{l}, Q)) \frac{F \left( c_2^{l} \right)}{f \left( c_2^{l} \right)} dc_2^{l}
\]

\textbf{■}

To prove Proposition 3, we want to show the following lemma first:

\textbf{Lemma A 2} A sourcing mechanism is incentive compatible if and only if

(1) The total expected profit of a supplier \( i \) over two periods satisfies

\[
u_0^l(c_1^i) = u + \int_{c}^{\pi} E_{c_1^{-i}} \left[ S(Q^i(\rho, c_1^{-i})) p^i_1(\rho, c_1^{-i}) \right] d\rho \tag{11}
\]

where \( u \geq 0 \) is a constant.

(2) \( E_{c_1^{-i}} \left[ S(Q^i(\rho, c_1^{-i})) p^i_1(\rho, c_1^{-i}) \right] \) is a decreasing function of \( \rho \).
Proof If the mechanism is incentive compatible, \( u^i(c) = \max_{\hat{c} \in \mathcal{C}} u^i(\hat{c}, c) \). Based on the envelop theorem,
\[
\frac{d}{dc} u^i(c) = \frac{\partial}{\partial c} u^i(\hat{c}, c)|_{\hat{c}=c} = E_{c_1^{-i}} \left[ S(Q_1(c, c_1^{-i})) p^i_{1,1}(c, c_1^{-i}) \right].
\]

Then the conclusion follows with similar proofs for Lemma A 1. ■

Proof of Proposition 3. Based on Lemma A 2, the total expected profit \( \pi^i \) of a supplier over two periods in either relationship is:
\[
\pi^i = \int_{\xi} u^i(c) f(c) dc = u_0 + \int_{\xi} \mathrm{E}_{c_1} \left[ S(Q_1(c_1, c_1^{-i})) p^i_{1,1}(c_1, c_1^{-i}) \right] dp f(c) dc
\]
\[
= u_0 + \int_{\xi} \mathrm{E}_{c_1} \left[ S(Q_1(c_1, c_1^{-i})) p^i_{1,1}(c_1, c_1^{-i}) \right] F(c_1) dc_1.
\]

Note the minimum profit of a first-period arrival supplier \( i \) is \( \pi_\ell = 0 \) in a long-term relationship, and \( \pi_s = \mathrm{E} \left[ \sum_{j=1, j \neq i}^{n_1} \pi^F_{s,2}(Q_1^j(c_1)) p^j_{1,1}(c_1) \right] \) in a short-term relationship. Then the ex ante total profit of the buyer from a long-term relationship is
\[
\pi_\ell = \sum_{i=1}^{n_1} \left[ \int_{\xi} \mathrm{E}_{c_1} \left[ \left( (r - c_1^i) S(Q_1^i(c_1)) - k Q_1^i(c_1) + \pi_{\ell,2} (Q_1^i(c_1)) \right) p^i_{1,1}(c_1) \right] f(c_1) dc_1 - \pi_\ell^i \right]
\]
\[
= \sum_{i=1}^{n_1} \left[ \int_{c_1} ((r - J(c_1^i)) S(Q_1^i(c_1)) - k Q_1^i(c_1) + \pi_{\ell,2} (Q_1^i(c_1)) + \pi_{s,2}^F (Q_1^i(c_1))) p^i_{1,1}(c_1) f(c_1) dc_1 \right]
\]
and from a short-term relationship is
\[
\pi_s = \sum_{i=1}^{n_1} \left( \int_{\xi} \mathrm{E}_{c_1} \left[ \left( (r - c_1^i) S(Q_1^i(c_1)) - k Q_1^i(c_1) + \pi_{s,2}^F (Q_1^i(c_1)) \right) p^i_{1,1}(c_1) \right] f(c_1) dc_1 - \pi_s^i \right)
\]
\[
= \sum_{i=1}^{n_1} \left[ \int_{c_1} ((r - J(c_1^i)) S(Q_1^i(c_1)) - k Q_1^i(c_1) + \pi_{s,2}^F (Q_1^i(c_1))) p^i_{1,1}(c_1) f(c_1) dc_1 \right].
\]

To maximize \( \pi_s \), \( Q_1^i(c_1) \) has to maximize on \( Q \) the function \( \pi. \left( c_1^i, Q \right) \approx (r - J(c_1^i)) S(Q) - k Q + \pi_{s,2}^F (Q) + \pi_{s,2} (Q) \), which is independent of \( c_1^{-i} \). In addition, \( p^i_{1,1}(c_1) = 1 \) if the \( i \)-th supplier achieves the highest \( \pi. \left( c_1^i, Q(c_1) \right) \) among all bidders; otherwise \( p^i_{1,1}(c_1) = 0 \). \( \pi. \left( c_1^i, Q(c_1) \right) \) is a decreasing function of \( c_1^i \) due to the log-concavity of \( F(\cdot) \). Hence the supplier with the lowest cost will be selected as the winner. Then \( \pi. \) reduces to
\[
\pi. = \mathrm{E} \left[ \max_Q \left( (r - J(c_1^i)) S(Q) - k Q + \pi_{s,2}^F (Q) + \pi_{s,2} (Q) \right) \right].
\]

■

Proof of Lemma 2: Based on Propositions 3, 1 and 2,
\[
\pi_{\ell,1}(c_1, Q) = (r - J(c_1)) S(Q) - kQ + \mathrm{E} \left[ r - c_2^i \right] S(Q),
\]
\[
\pi_{s,1}(c_1, Q) = (r - J(c_1)) S(Q) - kQ + \mathbb{E} \left[ (r - c_2) S(Q) | c_2^E > \tilde{c}(c_2, Q) \right] \cdot \Pr(c_2^E > \tilde{c}(c_2, Q)) \\
+ \mathbb{E} \left[ (r - J(c_2^E)) S(Q) - sQ | c_2^E \leq \tilde{c}(c_2, Q) \right] \cdot \Pr(c_2^E \leq \tilde{c}(c_2, Q)) .
\]

Then

\[
\pi_{s,1}(c_1, Q) - \pi_{\ell,1}(c_1, Q) \\
= \mathbb{E} \left[ (r - J(c_2^E)) S(Q) - sQ - (r - c_2) S(Q) | c_2^E \leq \tilde{c}(c_2, Q) \right] \cdot \Pr(c_2^E \leq \tilde{c}(c_2, Q)) \\
= \mathbb{E} \left[ (J(c_2) - J(c_2^E)) S(Q) - sQ - (J(c_2 - c_2^E)) S(Q) | c_2^E \leq \tilde{c}(c_2, Q) \right] \cdot \Pr(c_2^E \leq \tilde{c}(c_2, Q)) .
\]

**Proof of Proposition 4:**

i) First we show \( \pi_s - \pi_\ell \geq 0 \) for \( s = 0 \). When \( s = 0, \tilde{c}(c_2, Q) = c_2 \) and hence

\[
\pi_s(c_1, Q) = E[r - J(c_1)]S(Q) - kQ + \int_\mathbb{R} \int_\mathbb{R} (r - J(c_2^E)) S(Q) f_{n_2}(c_2^E) dc_2^E f(c_2) dc_2 \\
+ \int_\mathbb{R} \int_\mathbb{R} (r - c_2) S(Q) f_{n_2}(c_2^E) dc_2^E f(c_2) dc_2 .
\]

Then

\[
\pi_s(c_1, Q) - \pi_\ell(c_1, Q) = S(Q) \int_\mathbb{R} \int_\mathbb{R} (c_2 - J(c_2^E)) f_{n_2}(c_2^E) dc_2^E f(c_2) dc_2 .
\]

But \( \int_\mathbb{R} J(c_2^E) f_{n_2}(c_2^E) dc_2^E = c_2^E f_{n_2}(c_2) \). Therefore, given \( \pi_s(c_1, Q_s(c_1)) - \pi_\ell(c_1, Q_l(c_1)) \geq \pi_s(c_1, Q_l(c_1)) - \pi_\ell(c_1, Q_l(c_1)), \pi_s(c_1, Q_s(c_1)) - \pi_\ell(c_1, Q_l(c_1)) \) is nonnegative when \( s = 0 \).

Next we show \( \frac{\partial}{\partial s} \pi_s < 0 \) when \( s \to 0 \). For \( s \to 0, \tilde{c}(c_2^E, Q) \to c_2^E, \frac{\partial}{\partial s} \tilde{c}(c_2^E, Q) = -\frac{Q}{S(Q) 1 + \frac{1 F(c_2^E)}{F(c_2)}}, \) and hence

\[
\frac{\partial}{\partial s} \pi_s(c_1, Q) \to -Q \int_\mathbb{R} F_{n_2}(c_2^E) f(c_2) dc_2 \\
+ S(Q) \int_\mathbb{R} \frac{\partial}{\partial s} \tilde{c}(c_2^E, Q) (c_2^E - J(c_2^E)) f_{n_2}(c_2) dc_2.
\]

\[
= -Q \int_\mathbb{R} \left( F_{n_2}(c_2^E) - \frac{F(c_2^E)}{F(c_2)} \right) f_{n_2}(c_2) f(c_2) dc_2.
\]

\[
< -Q \int_\mathbb{R} \left( \frac{F_{n_2}(c_2^E)}{f_{n_2}(c_2^E)} - \frac{F(c_2)}{f(c_2)} \right) f_{n_2}(c_2) f(c_2) dc_2 < 0 .
\]

ii) First we show \( \pi_s - \pi_\ell = 0 \) for \( s \) large enough. Since \( \tilde{c}(c_2^E, Q) \) decreases with \( s \), there exists \( \hat{s}(Q) \) such that \( \tilde{c}(\hat{s}(Q), Q) = c_2^E \) for \( s \geq \hat{s}(Q) \). In addition, since \( \tilde{c}(c_2^E, Q) \) decreases with \( Q \), \( \hat{s}(Q) \) decreases with \( Q \). Then for \( s > \hat{s}(Q_\ell(c_1)) \), \( \pi_s(Q_\ell(c_1), c_1) = \pi_\ell(Q_\ell(c_1), c_1) \) and \( \frac{\partial}{\partial Q} \pi_s(Q_\ell(c_1), c_1) = 0 .
\]
Given that \( Q_\ell (c_1) \) decreases with \( c_1 \) and \( \hat{s} (Q) \) decreases with \( Q \), we have \( \pi_s - \pi_\ell = 0 \) for \( s \geq \hat{s} (Q_\ell (\bar{c})) \).

Next we show \( \pi_s \) (weakly) increases with \( s \) for \( s \) large enough. Let \( \hat{c} (s, Q) \) be the maximum value of \( c_2' \) such that \( \partial \hat{c} (s, Q), Q = \underline{c} \), i.e., given the first period capacity \( Q \) the buyer will never switch for \( c_2' \leq \hat{c} (s, Q) \), \( \frac{\partial}{\partial s} \hat{c} (s, Q) = \frac{Q}{J(\hat{c}(s,Q))Q} \).

\[
\pi_s (c_1, Q) = E [r - J (c_1)] S (Q) - kQ + \int_\pi^{\bar{c}} \int_{\hat{c}(c_2',Q)}^{\hat{c}(\xi,Q)} ((r - J (c_2'')) (Q) - sQ) f_{n_2} (c_2''') dE_2 f (c_2') dE_2
+ \int_\pi^{\bar{c}} \int_{\hat{c}(c_2',Q)}^{\hat{c}(\xi,Q)} (r - c_2') S (Q) f_{n_2} (c_2'''') dE_2 f (c_2') dE_2 + \int_\pi^{\bar{c}} \int_{\hat{c}(c_2',Q)}^{\hat{c}(\xi,Q)} (r - c_2') S (Q) f (c_2') \frac{Q}{J' (\hat{c}(s,Q))}.
\]

When \( s \to \hat{s} (Q) , \hat{c} (s, Q) \to \bar{c} \), and hence
\[
\frac{\partial}{\partial s} \pi_s (c_1, Q) = (r - \bar{c}) f (\bar{c}) \frac{Q}{J' (\bar{c})}.
\]

Since \( J' (\bar{c}) > 0 , \frac{\partial}{\partial s} \pi_s (c_1, Q, c_2 (s)) > 0 \) for \( s \to \hat{s} (Q, c_2 (s)) \). Given \( \frac{\partial}{\partial s} \pi_s (c_1, Q, c_2 (s)) = 0 \) for \( s > \hat{s} (Q, c_2 (s)) \), we have \( \frac{\partial}{\partial s} \pi_s > 0 \) for \( s \to \hat{s} (Q, c_2 (s)) \).

iii) With certain demand at \( \mu_d \) and a uniform distribution of the supplier cost, \( \hat{c} (c_2', \mu_d) = c_2' - \frac{s}{2} \).

Then
\[
\pi_s - \pi_\ell = \mu_d \int_{c_2' + \frac{s}{2}}^{\bar{c}} \int_{c_2'}^{c_2' + \frac{s}{2}} \frac{c_2'''}{c_2'''} - J (c_2''') dF_{n_2,1} (c_2'''') dF (c_2').
\]

When \( s \geq 2\Delta , J (c_2') - s \leq J (\bar{c}) - 2\Delta = \underline{c} \). In this situation, it is not possible to have an entrant with type \( c_2^E \in [\underline{c}, \bar{c}] \) such that \( c_2^E < \hat{c} (c_2', \mu_d) \). Therefore a short-term relationship reduces to a long-term relationship if \( s \geq 2\Delta \).

Define \( \Psi = \frac{\hat{s}}{\pi_\ell} \). Now we restrict the attention to the case \( \Psi < 1 \). \( \pi_\ell - \pi_s \) and its derivatives with respect to \( s \) can be reformulated as follows:
\[
\frac{\pi_s - \pi_\ell}{\mu_d} = -\frac{\Delta}{2 (n_2 + 1) (n_2 + 2)} (6 \Psi n_2^2 + 2 (n_2 + 2) \Psi n_2 + 3 \Psi^2 (n_2 + 2) (n_2 + 1) + 2 (n_2 + 2) (2n_2 - 1) \Psi - (n_2 - 1) n_2),
\]
\[
\frac{d}{ds} (\pi_s - \pi_\ell) = -\frac{\mu_d}{2 (n_2 + 1)} (3 \Psi n_2 + (n_2 + 2) \Psi n_2 - 3 \Psi (n_2 + 1) + (2n_2 - 1)).
\]

Equation (13) implies that \( \pi_\ell - \pi_s \) is a quasi-convex function of \( s \) when \( \Psi \leq 1 \).
When \( s = 0 \) (\( \Psi = 0 \)), \( \pi_s - \pi_\ell = (\frac{(n_2-1)n_2 \Delta \mu}{2(n_2+1)(n_2+2)}) > 0 \), and \( \frac{d}{dt} (\pi_s - \pi_\ell) < 0 \). When \( \Psi = 1 \), \( \pi_s - \pi_\ell = 0 \), and \( \frac{d}{dt} (\pi_s - \pi_\ell) = 0 \). Therefore \( \pi_s - \pi_\ell < 0 \) when \( s \) approaches \( 2\Delta \) from the left. Hence there exists \( \hat{s} \in (0, 2\Delta) \) such that \( \pi_s - \pi_\ell > 0 \) for \( s < \hat{s} \) and \( \pi_s - \pi_\ell \leq 0 \) for \( s \geq \hat{s} \).

**Proof of Lemma 2.** Note that when \( \sigma_d \approx 0 \), \( Q_s (c_1) \approx Q_\ell (c_1) \approx \mu_d \) for any \( c_1 \). Then this lemma follows immediately Lemma 1.

**Proof of Proposition 5.** When \( \sigma_d \approx 0 \), \( Q_s (c_1) \approx Q_\ell (c_1) \approx \mu_d \) for any \( c_1 \). With the capacity given at \( \mu_d \), we shall suppress the parameter \( \mu_d \) in \( \bar{c} (c_2^f, \mu_d) \) and use the notation \( \bar{c} (c_2^f) \) for simplification of the presentation. \( \bar{c} (c_2^f) \) satisfies \( J (c_2^f) - J (\bar{c} (c_2^f)) \) \( S (\mu_d) = s \mu_d \). Then

\[
\frac{\partial}{\partial \sigma_d} \bar{c} (c_2^f) = \frac{\partial}{\partial \sigma_d} S (\mu_d) \frac{s}{\mu_d} \cdot J' (\bar{c} (c_2^f, \mu_d)).
\]

With uniform distributions of a supplier’s cost, \( J (c_2^f) = 2c_2^f - c, \)

\[
\bar{c} (c_2^f) = c_2^f - \frac{s \mu_d}{2S (\mu_d)} \approx c_2^f - \frac{s}{2},
\]

\[
\frac{\partial}{\partial \sigma_d} \bar{c} (c_2^f) = \frac{\partial}{\partial \sigma_d} S (\mu_d) \frac{s}{2 \mu_d},
\]

\[
\mathbb{E}_{c_2^f} [(J(c_2^f) - c_2^f) \frac{\partial}{\partial \sigma_d} F_{n_2} (\bar{c} (c_2^f))] \mu_d = -B \frac{\partial}{\partial \sigma_d} S (\mu_d),
\]

where \( B = -\frac{s}{2} \mathbb{E}_{c_2^f} [f_{n_2} (c_2^f - \frac{s}{2}) (c_2^f - c)] \). Define

\[
A_1 = \mathbb{E}_{c_2^f, c_2^{E*}} [J (c_2^{E*}) - c_2^f | c_2^{E*} \leq \bar{c} (c_2^f, \mu_d)] \cdot \Pr (c_2^{E*} \leq \bar{c} (c_2^f, \mu_d)),
\]

\[
A_2 = \mathbb{E}_{c_2^f, c_2^{E*}} [-s | c_2^{E*} \leq \bar{c} (c_2^f, \mu_d)] \cdot \Pr (c_2^{E*} \leq \bar{c} (c_2^f, \mu_d)).
\]

Then \( A_2 - A_1 = \frac{\pi_s - \pi_\ell}{\mu_d} \). Based on Lemma 2, \( \frac{d}{dt} (\pi_s - \pi_\ell) \approx (B - A_1) \frac{\partial}{\partial \sigma_d} S (\mu_d) \). Let \( \Psi \approx \frac{s}{2\Delta} \).

To simplify the notation, let \( n = n_2 \). With uniform distributions of a supplier’s cost, we have the following:

\[
A_1 = -\frac{\Delta}{2 (n + 1) (n + 2)} (2 \Psi^{n+2} (2n + 1) - 2 (n + 2) \Psi^{n+1} - \Psi^2 (n + 2) (n + 1) + 2 (n + 2) \Psi + (n - 1) n),
\]

\[
A_2 = -\frac{2\Delta}{1 + n} \Psi (\Psi^{n+1} - \Psi (n + 1) + n),
\]

\[
B = \frac{\Delta}{n + 1} \Psi (\Psi^{n+1} + \Psi^n (n + 1) - \Psi (n + 1) - 1).
\]

38
\[ A_2 - A_1 = -\frac{\Delta}{2(n+1)(n+2)} (6\Psi^{n+2} + 2(n+2)\Psi^{n+1} - 3\Psi^2(n+2)(n+1) \\
+ 2(n+2)(2n_1 - 1)\Psi - (n-1)n), \]
\[
\frac{d}{d\Psi} (A_2 - A_1) = -\frac{\Delta}{n+1} (3\Psi^{n+1} + (n+1)\Psi^n - 3(n+1)\Psi + 2n - 1) = \frac{A_2 - B}{\Psi}, \]
\[
A_2 - B = -\frac{\Delta}{n+1} \Psi (3\Psi^{n+1} + (n+1)\Psi^n - 3(n+1)\Psi + 2n - 1), \]
\[
\frac{d}{d\Psi} (A_2 - B) = -\frac{\Delta}{n+1} (3(n+2)\Psi^{n+1} + (n+1)^2\Psi^n - 6(n+1)\Psi + 2n - 1) \]

It is easily proven that \( \frac{d}{d\Psi} (A_2 - B) \) is a concave function of \( \Psi \), and with \( \Psi \) increasing from 0 to 1, \( A_2 - B \) decreases from 0, then increases to above zero, and finally decreases to 0. Also \( A_2 - A_1 \) is a quasi-convex function of \( \Psi \), \( A_2 - A_1 > 0 \) when \( \Psi = 0 \), and \( A_2 - A_1 = 0 \) when \( \Psi = 1 \).

Observe that when \( \Psi > 0 \) and \( A_2 - B = 0 \), \( \frac{d}{d\Psi} (A_1 - A_2) = 0 \). Therefore when \( A_1 - A_2 = 0 \), \( A_2 - B = A_1 - B < 0 \). Since \( \pi_s - \pi_\ell = \mu_d (A_2 - A_1) \) and \( \frac{\partial}{\partial \sigma_d} S (\mu_d) < 0 \), it follows that when \( \pi_s - \pi_\ell = 0 \), \( \frac{d}{d\sigma_d} (\pi_s - \pi_\ell) \approx - (A_1 - B) \frac{\partial}{\partial \sigma_d} S (\mu_d) < 0 \). ■

**Proof of Lemma 3.** Following Lemma 1,

\[
\pi_s(Q, c_1) - \pi_\ell(Q, c_1) = E_{c_2^F, c_2^I} \left[ (c_2^F - J(c_2^E)) S(Q) - sQ|c_2^E \leq \bar{c}(c_2^I, Q) \right] \cdot \Pr (c_2^E \leq \bar{c}(c_2^I, Q)).
\]

Taking the derivative with respect to \( Q \):

\[
\frac{\partial}{\partial Q} (\pi_s(Q, c_1) - \pi_\ell(Q, c_1)) \]
\[
= E_{c_2^F, c_2^I} \left[ (c_2^F - J(c_2^E)) \overline{G}(Q) - s|c_2^E \leq \bar{c}(c_2^I, Q) \right] \cdot \Pr (c_2^E \leq \bar{c}(c_2^I, Q)) \\
+ E_{c_2^F, c_2^I} \left[ (c_2^F - J(\bar{c}(c_2^I, Q))) S(Q) - sQ \right] \frac{\partial}{\partial Q} F_{n_2} (\bar{c}(c_2^I, Q)) \\
= E_{c_2^F, c_2^I} \left[ (J(c_2^F) - J(c_2^E)) |c_2^E \leq \bar{c}(c_2^I, Q) \right] \cdot \Pr (c_2^E \leq \bar{c}(c_2^I, Q)) \overline{G}(Q) \\
- E_{c_2^F, c_2^I} \left[ (J(c_2^F) - c_2^I) F_{n_2} (\bar{c}(c_2^I, Q)) \right] \overline{G}(Q) - sE_{c_2^F, c_2^I} \left[ F_{n_2} (\bar{c}(c_2^I, Q)) \right] \\
- E_{c_2^F, c_2^I} \left[ (J(c_2^F) - c_2^I) S(Q) \right] \frac{\partial}{\partial Q} F_{n_2} (\bar{c}(c_2^I, Q)).
\]

■

**Proof of Proposition 6.** Define \( \hat{c}(Q) \) such that \( (J(\hat{c}(Q)) - J(\hat{c})) S(Q) = sQ \). Then

\[
\frac{\partial}{\partial Q} (\pi_s(Q, c_1) - \pi_\ell(Q, c_1)) \]
\[
= E \left[ (c_2^F - J(c_2^E)) |c_2^E \leq \hat{c}(c_2^I, Q) \right] \cdot \Pr (c_2^E \leq \hat{c}(c_2^I, Q)) \overline{G}(Q) \\
- E \left[ F_{n_2} (\hat{c}(c_2^I, Q)) \right] \cdot s \cdot E \left[ (J(c_2^F) - c_2^I) \overline{G}(Q) - s \right] F_{n_2} (c_2^E) dc_2^E \\
= \int_{\hat{c}(Q)}^{c_2^I} \left( \int_{\hat{c}(Q)}^{c_2^I} \left( (c_2^F - J(c_2^E)) \overline{G}(Q) - s \right) f_{n_2} (c_2^E) dc_2^E \right) f (c_2^I) dc_2^I \\
- S(Q) \int_{\hat{c}(Q)}^{c_2^I} F (c_2^I) \frac{\partial}{\partial Q} F_{n_2} (\hat{c}(c_2^I, Q)) dc_2^I
\]
\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s (Q, c_1) - \pi_l (Q, c_1) \right) = - \int_{\hat{c}(Q)}^{\pi} F_{n_2} \left( \hat{c} (c_2', Q) \right) f (c_2') \, dc_2' \\
+ \int_{\hat{c}(Q)}^{\pi} \left( (c_2' - J (\hat{c} (c_2', Q))) G (Q) - s \right) \frac{\partial}{\partial s} F_{n_2} \left( \hat{c} (c_2', Q) \right) f (c_2') \, dc_2' \\
- \int_{\hat{c}(Q)}^{\pi} F (c_2') S (Q) \frac{\partial^2 F_{n_2} (\hat{c} (c_2', Q))}{\partial Q \partial s} \, dc_2' \\
+ \frac{\partial \hat{c} (c_2', Q)}{\partial s} F (\hat{c} (Q)) S (Q) \frac{\partial F_{n_2} (\hat{c} (c_2', Q))}{\partial Q} \bigg|_{c_2' = \hat{c}(Q)}.
\]

Let \(c_{m:n}\) be the \(m\)-th lowest value of \(n\) iid realizations of the random variable following distribution \(F (\cdot)\).

i) When \(s = 0\), we have \(\hat{c} (Q) = \underline{c} \cdot \hat{c} (c_2', Q) = c_2'\) and \(\frac{\partial \hat{c} (c_2', Q)}{\partial s} = 0\). But \(\int_{\underline{c}}^{c_2'} J (c_2') f_{n_2} (c_2') \, dc_2' \leq c_2'\) with the equality achieved at \(n_2 = 1\). Therefore, when \(s = 0\), \(\frac{\partial}{\partial Q} (\pi_s (Q, c_1) - \pi_l (Q, c_1)) \geq 0\) with the equality achieved at \(n_2 = 1\).

In addition,

\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s (Q, c_1) - \pi_l (Q, c_1) \right) = - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' - G (Q) \int_{\underline{c}}^{\pi} F (c_2') \frac{\partial}{\partial s} F_{n_2} \left( \hat{c} (c_2', Q) \right) \, dc_2' \\
- S (Q) \int_{\underline{c}}^{\pi} F (c_2') \frac{\partial^2 F_{n_2} (\hat{c} (c_2', Q))}{\partial Q \partial s} \, dc_2' \\
= - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' - \int_{\underline{c}}^{\pi} F (c_2') \frac{\partial}{\partial Q} \left( S (Q) \frac{\partial}{\partial s} F_{n_2} \left( \hat{c} (c_2', Q) \right) \right) \, dc_2'.
\]

Note \(\frac{\partial}{\partial s} F_{n_2} \left( \hat{c} (c_2', Q) \right) = - \frac{f_{n_2} (\hat{c} (c_2', Q))}{J' (\hat{c} (c_2', Q))} \frac{Q}{s (Q)}\). When \(s = 0\), \(\frac{\partial}{\partial Q} \hat{c} (c_2', Q) = 0\). Therefore,

\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s (Q, c_1) - \pi_l (Q, c_1) \right) = - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' - \int_{\underline{c}}^{\pi} F (c_2') \frac{\partial}{\partial Q} \left( - \frac{f_{n_2} (\hat{c} (c_2', Q))}{J' (\hat{c} (c_2', Q))} \right) \, dc_2' \\
= - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' + \int_{\underline{c}}^{\pi} F (c_2') \frac{f_{n_2} (c_2')}{J' (c_2')} \, dc_2' \\
< - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' + \int_{\underline{c}}^{\pi} F (c_2') \frac{f_{n_2} (c_2')}{J' (c_2')} \, dc_2' \\
= - \int_{\underline{c}}^{\pi} F_{n_2} (c_2') f (c_2') \, dc_2' + n_2 \int_{\underline{c}}^{\pi} F (c_2') \left( 1 - F (c_2') \right)^{n_2 - 1} \, dc_2' \\
= - \mathbb{E} \left[ \Pr (c_{1:n_2} \leq c_2') - \Pr (c_{1:n_2} \leq c_2', c_{2:n_2} \geq c_2') \right] < 0,
\]

where \(c_{i:n}\) means the \(i\)-th lowest supplier cost among \(n\) suppliers.

ii) When \(s = \pi\) such that \((J (\pi) - \underline{c}) = \pi \frac{Q}{s (Q)}\), we have \(\hat{c} (c_2', Q) = \underline{c} \cdot \hat{c} (Q) = \pi\), and

\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s (Q, c_1) - \pi_l (Q, c_1) \right) = S (Q) \frac{\partial \hat{c} (Q)}{\partial s} \frac{\partial F_{n_2} (\hat{c} (c_2', Q))}{\partial Q} \bigg|_{c_2' = \hat{c}(Q)}.
\]
Since $\frac{\partial \tilde{c}(Q)}{\partial s} > 0$ and $\frac{\partial \tilde{c}(c_2, Q)}{\partial Q} < 0$, we have $\frac{\partial^2}{\partial Q \partial s} (\pi_s(Q, c_1) - \pi_l(Q, c_1)) < 0$. 

iii) When $\sigma_d \approx 0$, we have $Q \approx S(Q)$, $\tilde{c}(c_2, Q) \approx c_2^T - \frac{\hat{s}}{2}$. 

If $s \geq 2\Delta$, $\tilde{c}(r, Q) \approx \tilde{c}$, and the buyer will not switch to a new supplier in a short-term relationship.

The discussion from now on is based on the condition $\xi + \frac{\hat{s}}{2} < \tilde{c}$, or $\frac{\hat{s}}{2\Delta} \leq 1$. Let $\Psi = \frac{s}{2\Delta}$.

With uniform distributions of supplier costs, $\tilde{c}(c_2, Q) = c_2^T - \frac{\hat{s}}{2} \frac{Q}{S(Q)}$,

$$\frac{d}{dQ} \tilde{c}(c_2, Q) = \frac{s}{2S(Q)} \left( \frac{Q \bar{G}(Q)}{S(Q)} - 1 \right) \approx -\frac{s}{2} \frac{G(Q)}{S(Q)}.$$ 

Therefore,

$$\frac{\partial}{\partial Q} \pi_s(c_1, Q) \approx (r - J(c_1)) \bar{G}(Q) - k + \int_{\xi}^{c_2^T + \frac{\hat{s}}{2}} (r - c_2^T) \bar{G}(Q) f(c_2^T) dc_2^T$$

$$+ \int_{\xi}^{c_2^T + \frac{\hat{s}}{2}} (r - c_2^T) \bar{G}(Q) F_{n_2} \left( c_2^T - \frac{s}{2} \right) f(c_1) dc_1$$

$$+ \int_{\xi}^{c_2^T + \frac{\hat{s}}{2}} \int_{\xi}^{c_2^T + \frac{\hat{s}}{2}} \left( (r - J(c_2^T)) \bar{G}(Q) - s \right) f_{n_2} \left( c_2^T, c_2^T \right) dc_2^T f(c_1) dc_1$$

$$+ \int_{\xi}^{c_2^T + \frac{\hat{s}}{2}} \frac{s}{2} \frac{E}{f(c_1) f_{n_2} \left( c_2^T - \frac{s}{2} \right) G_2(Q) f(c_1) dc_1}$$

$$= (r - J(c_1)) \bar{G}(Q) - k + \bar{G}(Q) \left( \frac{3\Delta}{n_2 + 2} \Psi^{n_2+2} + \frac{\Delta n_2}{n_2 + 1} \Psi^{n_2+1} \right)$$

$$- \frac{3}{2} \Delta \Psi^2 + r - c + \frac{\Delta n_2 (n_2 - 1)}{2(n_2 + 2)(n_2 + 1)}$$

$$- s \left[ \frac{3}{2(n_2 + 2)} \Psi^{n_2+1} + \frac{1}{2} \Psi^{n_2} - \frac{3}{2} \Psi + \frac{12n_2 - 1}{2n_2 + 1} \right].$$

Since $Q_s(c_1)$ satisfies $\frac{\partial}{\partial Q} \pi_s(c_1, Q) = 0$ at $Q = Q_s(c_1)$, it follows that $\bar{G}(Q_s(c_1)) = \frac{A}{B}$, where

$$A \equiv \Delta \left( \frac{3}{n_2 + 1} \Psi^{n_2+2} + \Psi^{n_2+1} - 3\Psi^2 + \frac{2n_2 - 1}{n_2 + 1} \Psi \right) + k,$$

$$B \equiv \Delta \left( \frac{3}{n_2 + 2} \Psi^{n_2+2} + \frac{n_2}{n_2 + 1} \Psi^{n_2+1} - \frac{3}{2} \Psi^2 + \frac{n_2 (n_2 - 1)}{2(n_2 + 2)(n_2 + 1)} \right) + 2r - 2c_1 - \frac{\Delta}{2}.$$ 

Then $\frac{d}{d\Psi} G(Q_s(c_1)) = \frac{1}{B^2} \left( \frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} \right)$, where

$$\frac{dA}{d\Psi} = \Delta \left( \frac{3(n_2 + 2)}{n_2 + 1} \Psi^{n_2+1} + (n_2 + 1) \Psi^{n_2} - 6\Psi + \frac{2n_2 - 1}{n_2 + 1} \right),$$

$$\frac{dB}{d\Psi} = \Delta \left( 3\Psi^{n_2+1} + n_2 \Psi^{n_2} - 3\Psi \right).$$

Note

$$G(Q_l(c_1)) = \frac{k}{-\Delta \frac{n_2}{2(n_2 + 1)} + 2r - 2c_1 - \frac{\Delta}{2}}.$$
is independent of $s$.

If $\Psi = 0$,

\[
\mathcal{G}(Q_s(c_1)) = \frac{k}{\Delta \frac{n_2(n_2-1)}{2(n_2+1)} + 2r - 2c_1 - \frac{\Delta}{2}} < \mathcal{G}(Q_\ell(c_1)),
\]

\[
\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} = \Delta \frac{2n_2 - 1}{n_2 + 1} \left( 2r - 2c_1 - \frac{\Delta}{2} + \frac{\Delta n_2 (n_2 - 1)}{2(n_2 + 2)(n_2 + 1)} \right) > 0,
\]

and hence $\frac{d}{ds} Q_s(c_1) < 0$.

If $\Psi = 1$,

\[
\mathcal{G}(Q_s(c_1)) = \mathcal{G}(Q_\ell(c_1)) = \frac{k}{-\Delta \frac{n_2}{2(n_2+1)} + 2r - 2c_1 - \frac{\Delta}{2}},
\]

\[
\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} = \Delta n_2 \left( 2r - 2c_1 - \frac{\Delta}{2} - k \right) > 0.
\]

and hence $\frac{d}{ds} Q_s(c_1) < 0$.

We now show that $\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi}$ is a convex function of $\Psi$. Its second order derivative with respect to $\Psi$ is:

\[
\frac{d}{d\Psi^2} \left( \frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} \right) = 3\Delta \left( 2n_2 - 1 \right) \left( n_2 + 2 \right) \left( n_2 + 1 \right) + \Psi^{n_2-2} \left( n_2 - 1 \right) \left( n_2 + 2 \right)^2
\]

\[
\left( 2(n_2 + 2)(n_2 + 1)(r - c_1) - 2\sqrt{3}c_1 \left( 2n_2 + 1 \right) + \left( n_2 + 2 \right) \right)
\]

\[
+ \Psi^{n_2-1} n_2 \left( n_2 + 1 \right) \left( n_2 + 2 \right) \left( 6 \left( n_2 + 2 \right) \left( n_2 + 1 \right) \left( r - c_1 \right) - 3k \left( n_2 + 1 \right)^2 + \Delta \left( 3 \left( n_2 + 1 \right) + n_2^2 \left( 2n_2 - 1 \right) \right) \right)
\]

\[
+ \Psi^{n_2+1} \frac{3}{2} \Delta n_2 \left( n_2 + 2 \right) \left( n_2 + 1 \right) \left( n_2 + 1 \right) \left( n_2 - 5 \right) + 4
\]

\[
+ \Psi^{n_2+1} \frac{9}{2} \Delta n_2 \left( n_2 + 2 \right) \left( n_2 + 1 \right) + 3
\]

\[
- \Psi^{n_2+1} 3\Delta \left( 2n_2 + 2 \right) \left( 2n_2 + 1 \right)
\]

where the first one term disappears when $n_2 = 2$, and the first two terms disappear when $n_2 = 1$.

$\frac{d}{d\Psi^2} \left( \frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} \right)$ is non-negative because $r - c_1 \geq \Delta + k$ (recall from Section 4.1 that we assume $r \geq J(\bar{\sigma}) + k + s$).

Now we show that there exist $\Psi_L$ and $\Psi_H$, $0 < \Psi_L < \Psi_H < 1$, such that $\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} < 0$ for $\Psi \in (\Psi_L, \Psi_H)$, and $\geq 0$ for $\Psi \in [0, \Psi_L] \cup [\Psi_H, 1]$. If $\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} \geq 0$ through $\Psi \in [0, 1]$, then $\frac{d}{d\Psi} \mathcal{G}(Q_s(c_1)) \geq 0$ for any $\Psi \in [0, 1]$, and $Q_s(c_1|\Psi = 0) \leq Q_\ell(c_1)$. This conflicts $Q_s(c_1|\Psi = 0) > Q_\ell(c_1)$. Therefore given that $\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi}$ is a convex function and $\frac{dA}{d\Psi} B - A \frac{dB}{d\Psi} > 0$ for $\Psi = 0$ and $\Psi = 1$, $\Psi_L$ and $\Psi_H$ exist. Then $s_L = 2\Delta \Psi_L$, $s_H = 2\Delta \Psi_H$. 

42
It follows that \( \frac{\partial}{\partial \Psi} Q_s(c_1) \leq 0 \) for \( \Psi \in [0, \Psi_L] \cup [\Psi_H, 1] \), and \( \frac{\partial}{\partial \Psi} Q_s(c_1) > 0 \) for \( \Psi \in (\Psi_L, \Psi_H) \). Since \( Q_s(c_1) = Q_t(c_1) = 0 \), there exists \( \delta \in (2\Delta \Psi_L, 2\Delta) \) such that \( Q_s(c_1) = Q_t(c_1) \). 

**Proof of Proposition 7.** i) The buyer switches to the entrant if and only if the profit from the entrant is higher than the profit from the incumbent, i.e., \( (r - c_2^E) S(Q) - sQ \geq (r - c_2^I) S(Q) \). It is equivalent to \( c_2^E \leq c_2^I - \frac{Q_s(Q)}{S(Q)} \).

ii) Since \( J'(c) \geq 1 \), we have \( J(c_2^I) - J(c_2^I - s\frac{Q_s(Q)}{S(Q)}) \geq s\frac{Q_s(Q)}{S(Q)} \), and hence \( J^{-1}\left(J(c_2^I) - s\frac{Q_s(Q)}{S(Q)}\right) \geq c_2^I - s\frac{Q_s(Q)}{S(Q)} \).

Therefore \( \hat{c}(c_2^I, Q) \geq \hat{c}(c_2^I, Q) \).

**Proof of Proposition 8.** If the profit from the optimal long-term relationship is higher, then the buyer can set the threshold entrant type \( \hat{c}(c_2^I, Q) = c \) in a short-term relationship so that no switching is possible in the second period, and also follow the optimal long-term relationship to set the capacity level. In this way the buyer constructs a short-term relationship that is equivalent to the optimal long-term relationship and brings a higher profit to the buyer than the optimal short-term relationship. This conflicts the optimality of the short-term relationship.

For Section 7, define \( \tau(Q) \) as the solution of \( (r - J(\tau)) \bar{G}(Q) = k \): given capacity \( Q \), a supplier (the incumbent or an entrant) will have the capacity expanded if and only if the unit production cost is less than \( \tau(Q) \). Let \( Q^*(c) \) be the capacity investment that maximizes \( (r - J(c)) S(Q) - kQ \) on \( Q \) in a single period problem. Then, with flexible second period capacity, the second period virtual profit of sourcing from the incumbent (in both short- and long-term relationships) is:

\[
v_2(Q, c_2) = \begin{cases} 
v_2^{L,1}(Q, c_2^I) := (r - J(c_2^I)) S(Q^*(c_2^I)) - k(Q^*(c_2^I) - Q) & \text{if } c_2^I < \tau(Q) \\
v_2^{L,2}(Q, c_2^I) := (r - J(c_2^I)) S(Q) & \text{else } c_2^I \geq \tau(Q) .
\end{cases}
\]

Define \( \hat{\tau}(Q) \) by \( (r - J(\hat{\tau})) \bar{G}(Q) = s \) as the threshold cost of the entrant so that the buyer will transfer all capacity to the entrant if and only if the entrant’s unit production cost \( c_2^E \) is less than \( \hat{\tau}(Q) \). For \( c_2^E > \hat{\tau}(Q) \), let \( \hat{Q}^*(c_2^E) \) be the optimal capacity to be transferred, where \( \hat{Q}^*(c_2^E) \) maximizes \( (r - J(c_2^E)) S(Q) - sQ \). The virtual profit of sourcing from an entrant with type \( c_2^E \) is:

\[
v_2(Q, c_2^E) = \begin{cases} 
v_2^{E,1}(Q, c_2^E) := (r - J(c_2^E)) S(Q^*(c_2^E)) - k(Q^*(c_2^E) - Q) - sQ & \text{if } c_2^E < \tau(Q) \\
v_2^{E,2}(Q, c_2^E) := (r - J(c_2^E)) S(Q) - sQ & \text{if } \tau(Q) < c_2^E < \hat{\tau}(Q) \\
v_2^{E,3}(Q, c_2^E) := (r - J(c_2^E)) S(\hat{Q}^*(c_2^E)) - s\hat{Q}^*(c_2^E) & \text{else } c_2^E \geq \hat{\tau}(Q) .
\end{cases}
\]

**Proof of Lemma 4.** If \( c_2^I \leq \tau(Q) \), then \( c' \) satisfies \( v_2^{L,1}(Q, c_2^I) = v_2^{E,1}(Q, c') \), with \( \frac{\partial \hat{c}}{\partial Q} = \frac{s}{S(Q(c')) J'(c')} < 0 \).

If \( c_2^I > \tau(Q) \) and \( c' < \tau(Q) \), then \( v_2^{L,2}(Q, c_2^I) = v_2^{E,1}(Q, c') \), with \( \frac{\partial \hat{c}}{\partial Q} = \frac{k - s - (r - J(c_2^I)) \bar{G}(Q)}{S(Q(c')) J'(c')} \).

Since \( (r - J(c_2^I)) \bar{G}(Q) < k \), we have \( \frac{\partial \hat{c}}{\partial Q} < 0 \).
If \( c_2' > \tau(Q) \) and \( \tau(Q) \leq c' < \hat{\tau}(Q) \), then \( v_2^{1,2} (Q, c_2') = v_{2}^{E,2} (Q, c') \), with \( \frac{\partial \pi}{\partial Q} = \frac{(f(c_2') - f(\hat{c})) \overline{Q}(Q) - \tau}{s(Q) J(c')} < 0 \).

Else \( c_2' > \tau(Q) \) and \( c' \geq \hat{\tau}(Q) \), then \( v_2^{1,2} (Q, c_2') = v_{2}^{E,3} (Q, c') \), with \( \frac{\partial \pi}{\partial Q} = \frac{-c_2' (s - Q^* \overline{c}(Q))}{s(Q) J(c')} < 0 \).

**Proof of Proposition 9.**

i) Define \( \hat{c}(Q) \) such that \( v_2^I (\hat{c}(Q), Q) = v_{2}^E (\hat{\tau}, Q) \). Let \( \hat{u}_2^I (c_2', Q) \) be the second-period system profit of sourcing from the incumbent. Then

\[
\frac{\partial}{\partial Q} \left( \pi_s(Q, c_2) - \pi_L(Q, c_2) \right)
= \int_{\hat{c}(Q)}^{c_2} \left( \int_{\tau}^{c_2} \left( \frac{\partial}{\partial Q} v_{2}^{E} (c_2^*, Q) - \frac{\partial}{\partial Q} \hat{u}_2 (c_2^*, Q) \right) f_{n_2} (c_2^*, Q) \right) \frac{dc_2^*}{dc_2}
- \int_{\hat{c}(Q)}^{c_2} \left( \hat{u}_2 (c_2', Q) - v_2^I (c_2', Q) \right) \frac{\partial F_{n_2} (c_2^*, Q)}{\partial Q} dc_2',
\]

\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s(Q, c_2) - \pi_L(Q, c_2) \right)
= \int_{\hat{c}(Q)}^{c_2} \left( \int_{\tau}^{c_2} \left( \frac{\partial}{\partial Q} v_{2}^{E} (c_2^*, Q) - \frac{\partial}{\partial Q} \hat{u}_2 (c_2^*, Q) \right) f_{n_2} (c_2^*, Q) \right) \frac{dc_2^*}{dc_2}
- \int_{\hat{c}(Q)}^{c_2} \left( \hat{u}_2 (c_2', Q) - v_2^I (c_2', Q) \right) \frac{\partial^2 F_{n_2} (c_2^*, Q)}{\partial Q \partial s} dc_2'
+ \int_{\hat{c}(Q)}^{c_2} \left( \hat{u}_2 (\hat{c}(Q), Q) - v_2^I (\hat{c}(Q), Q) \right) \frac{\partial F_{n_2} (c_2^*, Q)}{\partial Q} \bigg|_{c_2^* = \hat{c}(Q)}.
\]

When \( s = \overline{s} \) where \( \hat{c}(Q_{\ell}(\overline{s})) = \overline{s} \), for \( Q = Q_{\ell}(\overline{s}) \) we have \( c_2' (c_2', Q) = \overline{s} \) for all \( c_2' \), \( \hat{c}(Q) = \overline{s} \), and \( \frac{\partial}{\partial Q} (\pi_s(Q, c_2) - \pi_L(Q, c_2)) = 0 \). In addition,

\[
\frac{\partial^2}{\partial Q \partial s} \left( \pi_s(Q, c_2) - \pi_L(Q, c_2) \right)
= \frac{\partial^2}{\partial Q \partial s} \left( \pi_s(Q, c_2) - \pi_L(Q, c_2) \right)
- \frac{\partial^2}{\partial Q \partial s} \left( \pi_s(Q, c_2) - \pi_L(Q, c_2) \right) \bigg|_{c_2' = \hat{c}(Q)}.
\]

Since \( \hat{u}_2^I - v_2^I > 0 \), \( \frac{\partial \hat{c}(Q)}{\partial s} > 0 \) and \( \frac{\partial c_2^* (c_2^*, Q)}{\partial Q} < 0 \), we have \( \frac{\partial^2}{\partial Q \partial s} (\pi_s(Q, c_2) - \pi_L(Q, c_2)) < 0 \). Therefore, when \( s < \overline{s} \) large enough, \( \frac{\partial}{\partial Q} (\pi_s(Q, c_2) - \pi_L(Q, c_2)) > 0 \).
ii) First we show \( \pi_s > \pi_\ell \) for \( s = 0 \). When \( s = 0 \), \( c' (c_2', Q) = c_2' \) and \( \hat{\tau} (Q) = \tau \).

\[
\pi_s (c_1, Q) - \pi_\ell (c_1, Q) = E_{c_2} \left[ \int_{c_2}^{c_2'} \left( v_2^{E,1} (c_2', Q) - ((r - c_2') S (Q^*(c_2')) - k (Q^*(c_2') - Q)) \right) f_{n_2} (c_2^E) dc_2^E | c_2' < \tau (Q) \right] \cdot \Pr (c_2' < \tau (Q)) \\
+ E_{c_2} \left[ \int_{c_2'}^{\tau (Q)} \left( v_2^{E,1} (c_2', Q) - (r - c_2) S (Q) \right) f_{n_2} (c_2^E) dc_2^E | c_2' \geq \tau (Q) \right] \cdot \Pr (c_2' \geq \tau (Q)) \\
+ E_{c_2} \left[ \int_{\tau (Q)}^{c_2} \left( v_2^{E,2} (c_2', Q) - (r - c_2') S (Q) \right) f_{n_2} (c_2^E) dc_2^E | c_2' \geq \tau (Q) \right] \cdot \Pr (c_2' \geq \tau (Q)) \\
\geq E_{c_2} \left[ \int_{c_2}^{c_2'} [(c_2' - J (c_2')) S (Q^*(c_2'))] f_{n_2} (c_2^E) dc_2^E | c_2' < \tau (Q) \right] \cdot \Pr (c_2' < \tau (Q)) \\
+ E_{c_2} \left[ \int_{c_2'}^{c_2} (c_2' - J (c_2')) S (Q) f_{n_2} (c_2^E) dc_2^E | c_2' \geq \tau (Q) \right] \cdot \Pr (c_2' \geq \tau (Q)) .
\]

The rest follows the same proof as for Proposition 4 to show \( \pi_s (c_1, Q) - \pi_\ell (c_1, Q) > 0 \) for \( s = 0 \).

Next we show \( \frac{\partial}{\partial s} \pi_s (c_1, Q) < 0 \) when \( s \to 0 \).

If \( c_2' \leq \tau (Q) \), then \( c' \) satisfies \( v_2^{I,1} (Q, c_2') = v_2^{E,1} (Q, c') \) with \( \frac{\partial v}{\partial s} = -\frac{Q}{S(Q^*(c'))J'(c')} < 0 \).

If \( c_2' > \tau (Q) \) and \( c' < \tau (Q) \), then \( v_2^{I,2} (Q, c_2') = v_2^{E,1} (Q, c') \) with \( \frac{\partial v}{\partial s} = -\frac{Q}{S(Q^*(c'))J'(c')} < 0 \).

If \( c_2' > \tau (Q) \) and \( \tau (Q) \leq c' < \hat{\tau} (Q) \), then \( v_2^{I,2} (Q, c_2') = v_2^{E,2} (Q, c') \) with \( \frac{\partial v}{\partial s} = -\frac{Q}{S(Q^*(c'))J'(c')} < 0 \).

Else \( c_2' > \tau (Q) \) and \( c' \geq \hat{\tau} (Q) \), then \( v_2^{I,2} (Q, c_2') = v_2^{E,3} (Q, c') \) with \( \frac{\partial v}{\partial s} = -\frac{Q}{S(Q^*(c'))J'(c')} < 0 \).

When \( s \to 0 \), \( c' (c_2', Q) \to c_2' > \hat{\tau} (Q) \), and
\[ \frac{\partial}{\partial s} \pi_s (c_1, Q) \rightarrow -Q \int_\Sigma F_{n_2} (c_2') f (c_2') dc_2' + \int_\Sigma -s (Q^* (c_2')) \frac{\partial}{\partial s} \tilde{c} (c_2', Q) (c_2' - J (c_2')) f_{n_2} (c_2') f (c_2') dc_2' + S (Q) \int_{\tau (Q)} -S (Q) J' (c_2') \left( c_2' - J (c_2') \right) f_{n_2} (c_2') f (c_2') dc_2' = -Q \int_\Sigma F_{n_2} (c_2') f (c_2') dc_2' + \int_\Sigma \frac{Q}{S (Q) J' (c_2')} (c_2' - J (c_2')) f_{n_2} (c_2') f (c_2') dc_2' = -Q \int_\Sigma \left( F_{n_2} (c_2') - \frac{F (c_2')}{f (c_2')} \frac{n_2 (c_2')}{f (c_2')} \right) f (c_2') dc_2' < -Q \int_\Sigma \left( \frac{F_{n_2} (c_2')}{f (c_2')} \right) f_{n_2} (c_2') f (c_2') dc_2' < 0. \]

iii) First we show \( \pi_s - \pi_\ell = 0 \) for \( s \) large enough. Since \( \tilde{c} (c_2', Q) \) decreases with \( s \), when \( k \) large enough, there exists \( \hat{s} (Q) < k \) such that \( v_1^1 (Q, \pi) = v_2^E (Q, \pi) \), i.e., \( \tilde{c} (c_2', Q) = \xi \), for \( s \geq \hat{s} (Q) \). Since \( \tilde{c} (c_2', Q) \) decreases with \( Q \) and \( s \), \( \hat{s} (Q) \) decreases with \( Q \). Let \( Q = Q_\ell (c_1) \). Then for \( s > \hat{s} (Q_\ell (c_1)) \), \( \pi_s (Q_\ell (c_1), c_1) = \pi_\ell (Q_\ell (c_1), c_1) \) and \( \frac{\partial}{\partial Q} \pi_s (Q_\ell (c_1), c_1) = 0 \). Given that \( Q_\ell (c_1) \) decreases with \( c_1 \) and \( \hat{s} (Q) \) decreases with \( Q \), we have \( \pi_s - \pi_\ell = 0 \) for \( s \geq \hat{s} (Q_\ell (\pi)) \).

Second we show \( \pi_s \) (weakly) increases with \( s \) for \( s \) large enough. Let \( \hat{c} (s, Q) \) be the maximum value of \( c_2' \) such that \( \hat{c} (\hat{c} (s, Q), Q) = \xi \), i.e., given the first period capacity \( Q \) the buyer will never switch for \( c_2' \leq \hat{c} (s, Q) \).

If \( \hat{c} \leq \tau (Q) \), then \( \hat{c} \) satisfies \( v_2^{I, 1} (Q, \hat{c}) = v_2^{E, 1} (Q, \xi) \) with \( \frac{\partial \xi}{\partial s} = \frac{Q}{S (Q) J' (\xi)} > 0 \).

If \( \hat{c} > \tau (Q) \) and \( \xi < \tau (Q) \), then \( v_2^{I, 2} (Q, \hat{c}) = v_2^{E, 1} (Q, \xi) \) with \( \frac{\partial \xi}{\partial s} = \frac{Q}{S (Q) J' (\xi)} > 0 \).

If \( \hat{c} > \tau (Q) \) and \( \tau (Q) \leq \xi < \hat{\tau} (Q) \), then \( v_2^{I, 2} (Q, \hat{c}) = v_2^{E, 2} (Q, \xi) \) with \( \frac{\partial \xi}{\partial s} = \frac{Q}{S (Q) J' (\xi)} > 0 \).

Else \( \hat{c} > \tau (Q) \) and \( \xi \geq \hat{\tau} (Q) \), then \( v_2^{I, 2} (Q, \hat{c}) = v_2^{E, 3} (Q, \xi) \) with \( \frac{\partial \xi}{\partial s} = \frac{Q}{S (Q) J' (\xi)} > 0 \).

Similar as in the proof of Proposition 4, point ii), we can show that when \( s \rightarrow \hat{s} (Q) \), \( \hat{c} (s, Q) \rightarrow \tau \), and hence

\[ \frac{\partial}{\partial s} \pi_s (c_1, Q) \rightarrow \begin{cases} f (\hat{c} (s, Q), Q) \frac{\partial \hat{c} (s, Q)}{\partial s} \cdot v_2^{I, 1} (Q, \hat{c}) & \text{if } \hat{c} (s, Q) < \tau (Q), \\ f (\hat{c} (s, Q), Q) \frac{\partial \hat{c} (s, Q)}{\partial s} \cdot v_2^{I, 2} (Q, \hat{c}) & \text{else } \hat{c} (s, Q) \geq \tau (Q). \end{cases} \]
Given $\frac{\partial c}{\partial s} > 0$, we have $\frac{\partial}{\partial s} \pi_s (c_1, Q_s (c_1)) > 0$ when $s \to \hat{s}(Q_s (c_1))$. Given $\frac{\partial}{\partial s} \pi_s (c_1, Q_s (c_1)) = 0$ for $s > \hat{s}(Q_s (c_1))$, we have $\frac{\partial}{\partial s} \pi_s > 0$ for $s \to \hat{s}(Q_s (\bar{c}))$. ■

Figure 5: Expected profit difference $\pi_s - \pi_\ell$ and capacity difference $E[Q_s (c_1^*) - Q_\ell (c_1^*)]$ between the short-term and long-term relationships when capacity is flexible in the second period. $r = 3$, $n_1 = 1$, $k = 0.5$, and supplier production costs are drawn from truncated normal distribution $\mathcal{N}(0.5, 1)$ on the support $[0.35, 0.65]$. For the left plots, $n_2 = \{1, 3, 6\}$, and the demand follows a truncated normal distribution $\mathcal{N}(0.5, 1)$ on the support $[0.2, 0.8]$. For the right plots, $n_2 = 3$ and the demand follows a truncated normal distribution $\mathcal{N}(0.5, \sigma_d)$, $\sigma_d \in \{0.01, 0.1, 0.2, 1\}$ on the support $[0, 1]$. 

47