Technical Appendix

to accompany

Political Uncertainty and Risk Premia

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**Proof of Lemma 1.** The same argument leading to equation (IA.8) in the Internet Appendix of Pastor and Veronesi (2011) implies that conditional on policy \( n, n = 0, 1, \ldots, N \), being chosen at time \( \tau \), aggregate capital is given by
\[
B_T = B_\tau e^{(\mu_n + g_n - \frac{1}{2} \sigma_n^2)(T - \tau) + \sigma (Z_T - Z_\tau)}
\]
Thus, exploiting \( W_T = B_T \) we have
\[
E_\tau \left[ \frac{W_T^{1 - \gamma}}{1 - \gamma} \middle| \text{policy } n \right] = \frac{B_\tau^{1 - \gamma}}{1 - \gamma} e^{(1 - \gamma)(T - \tau) \mu_n + \frac{1}{2}(1 - \gamma)^2(T - \tau)^2 \sigma_n^2 + (\mu - \gamma \frac{3}{2} \sigma^2)(T - \tau)(1 - \gamma)}
\]
It follows immediately that
\[
E_\tau \left[ \frac{W_T^{1 - \gamma}}{1 - \gamma} \middle| \text{policy } n \right] > E_\tau \left[ \frac{W_T^{1 - \gamma}}{1 - \gamma} \middle| \text{policy } m \right]
\]
if and only if
\[
\widetilde{\mu}^n = \mu_n + \frac{1}{2} (1 - \gamma)(T - \tau) \sigma_n^2 > \mu_m + \frac{1}{2} (1 - \gamma)(T - \tau) \sigma_m^2 = \tilde{\mu}^m
\]
Q.E.D.

**Proof of Proposition 1.** The government chooses policy \( n \in \{0, 1, \ldots, N\} \) if and only if for all \( m \neq n, m = 0, 1, \ldots, N \),
\[
E_\tau \left[ \frac{C^n W_T^{1 - \gamma}}{1 - \gamma} \middle| \text{policy } n \right] > E_\tau \left[ \frac{C^m W_T^{1 - \gamma}}{1 - \gamma} \middle| \text{policy } m \right]
\]
where recall that \( C^0 = 1 \). The same calculations as in Lemma 1 lead to the inequality
\[
\mu_n - \frac{\sigma_n^2}{2} (T - \tau)(\gamma - 1) - \frac{c^n}{(\gamma - 1)(T - \tau)} > \mu_m - \frac{\sigma_m^2}{2} (T - \tau)(\gamma - 1) - \frac{c^m}{(\gamma - 1)(T - \tau)}
\]
(B1)
The claim follows from the definitions of \( \tilde{\mu}^n \) and \( \tilde{c}^n \) in equations (15) and (23). Q.E.D.

**Proof of Corollary 1.** Immediate from Proposition 1 and equations (16) and (17).

**Proof of Corollary 2.** As of time \( t \), we have for each \( n = 1, \ldots, N \)
\[
c^n \sim N(\tilde{c}_n, \sigma_n^2)
\]
(B2)
Given the condition in Proposition 1 that policy \( n \in \{1, \ldots, N\} \) is chosen if and only if
\[
\tilde{\mu}^n - \tilde{c}^n > \tilde{\mu}^m - \tilde{c}^m \quad m \neq n, m = 1, \ldots, N
\]
(B3)
\[
\tilde{\mu}^n - \tilde{c}^n > x_\tau
\]
(B4)
the conditional probability at \( t \) that policy \( n \) is chosen at \( \tau \) is given by

\[
p_t^n = \Pr \left( \tilde{\mu}^n - \tilde{c}^n > \tilde{\mu}^m - \tilde{c}^m \text{ for } m \neq n \right)
\]

\[
= \int_{-\infty}^{\infty} \Pr \left( \tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{c}^m \text{ for } m \neq n \left| \tilde{c}^n \right. \right) \phi_{\tilde{c}^n} \left( \tilde{c}^n \right) d\tilde{c}^n
\]

\[
= \int_{-\infty}^{\infty} \Pi_{m \neq n} \Pr \left( \tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{c}^m \right) \Pr \left( \tilde{\mu}^n - \tilde{c}^n > x_\tau \left| \tilde{c}^n \right. \right) \phi_{\tilde{c}^n} \left( \tilde{c}^n \right) d\tilde{c}^n
\]

where we used the fact that \( \tilde{c}^m \)'s are independent of each other as well as of \( x_\tau \). Moreover, from the definition of \( x_\tau = \hat{g}_\tau - \frac{\sigma^2}{2} (T - \tau) (\gamma - 1) \) (see equation (16)) we have \( x_\tau \left| \tilde{g}_t \right. \sim N \left( \hat{g}_\tau - \frac{\sigma^2}{2} (T - \tau) (\gamma - 1), \tilde{\sigma}^2 - \sigma^2 \right) \).

We note two properties:

1. As \( \hat{g}_t \rightarrow \infty \), then \( p_t^n \rightarrow 0 \) for all \( n \in \{1, \ldots, N\} \), as \( \Phi_x \left( \tilde{\mu}^n - \tilde{c}^n \left| \tilde{g}_t \right. \right) \rightarrow 0 \).

2. As \( t \rightarrow \tau \) we have

\[
\Phi_x \left( \tilde{\mu}^n - \tilde{c}^n \left| \tilde{g}_t \right. \right) = \int_{\tilde{\mu}^n - \tilde{c}^n}^{\infty} \phi_x \left( x \left| \tilde{g}_t \right. \right) dx \rightarrow 1_{\{x_\tau < \tilde{\mu}^n - \tilde{c}^n\}} \tag{B5}
\]

so that

\[
p_t^n = \int_{-\infty}^{\infty} \Pi_{m \neq n} \left( 1 - \Phi_{\tilde{c}^m} \left( \tilde{c}^n + \tilde{\mu}^m - \tilde{\mu}^n \right) \right) \Phi_x \left( \tilde{\mu}^n - \tilde{c}^n \left| \tilde{g}_t \right. \right) \phi_{\tilde{c}^n} \left( \tilde{c}^n \right) d\tilde{c}^n
\]

\[
= \int_{-\infty}^{\tilde{\mu}^n - x_\tau} \Pi_{m \neq n} \left( 1 - \Phi_{\tilde{c}^m} \left( \tilde{c}^n + \tilde{\mu}^m - \tilde{\mu}^n \right) \right) \phi_{\tilde{c}^n} \left( \tilde{c}^n \right) d\tilde{c}^n
\]

\[
= p_t^n \tag{B6}
\]

Q.E.D.

**Proof of Lemma A1.** Using the same arguments as to obtain equation (IA.20) in the Internet Appendix of Pastor and Veronese (2011), after the announcement of policy \( n \) at time \( \tau^+ \), the state price density is given by

\[
E_{\tau^+}[\pi_{T}\mid\text{policy } n] = \pi_{\tau^+}^n = \lambda^{-1} B_{\tau^+}^{-\gamma} e^{-\gamma \mu^+_\gamma} e^{-\gamma \mu^+ \gamma} (T - \tau) e^{-(\gamma \mu^+ + \frac{1}{2} \gamma^2 \gamma^2)(T - \tau) + \frac{\gamma^2}{2} (T - \tau)^2 \gamma^2_{\gamma, n}} \tag{B6}
\]

Therefore, using also \( B_{\tau} = B_{\tau^+} \), the state price density at \( \tau \) is

\[
\pi_{\tau} = \sum_{n=0}^{N} p_t^n \pi_{\tau^+}^n
\]

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Therefore, we have

\[ \lambda^{-1} \sum_{n=0}^{N} p_n^\tau B^{-\gamma}_\tau e^{-\gamma \mu_n^\tau (T-\tau)} e^{-\mu_{\gamma}^\tau (T-\tau)} \left( B_{\tau-\gamma} \right)^\tau \] 

Using the definition \( \mu_n^0 = \check{g}_\tau \) in equation (18) and the condition

\[ p_\tau^0 = 1 - \sum_{n=1}^{N} p_n^\tau \] (B7)

we can rewrite the state price density at \( \tau \) as

\[ \pi_\tau = \lambda^{-1} B^{-\gamma}_\tau e^{-\gamma \mu_\tau^\gamma (T-\tau)} \left( B_{\tau+} \right)^\tau \times \left( 1 + \sum_{n=1}^{N} p_n^\tau \left( e^{-\gamma (\mu_n^\tau - \check{g}_\tau)} \right) \left( B_{\tau+} \right)^\tau \right) \]

Similarly, after the announcement of policy \( n, n = 0, 1, ..., N \), at time \( \tau+ \), we have

\[ E_{\tau+} \left[ B^{-\gamma}_\tau B_\tau^\gamma | \text{policy n} \right] = N^{i,n}_{\tau+} B^{-\gamma}_\tau B_\tau^\gamma \times e^{\left( 1-\gamma \right) \mu_\tau^\gamma (T-\tau)} e^{\left( 1-\gamma \right) \mu_\tau^\gamma (T-\tau)} \left( B_{\tau+} \right)^\tau \] (B8)

Therefore, we have

\[ E_\tau \left[ B^{-\gamma}_\tau B_\tau^\gamma \right] = \sum_{n=0}^{N} p_n^\tau N^{i,n}_{\tau+} \]

\[ = B^{-\gamma}_\tau B_\tau^\gamma \times \left( \sum_{n=0}^{N} p_n^\tau \right) \left( B_{\tau+} \right)^\tau \times \left( e^{\left( 1-\gamma \right) \mu_\tau^\gamma (T-\tau)} \right) \left( B_{\tau+} \right)^\tau \]

The claim follows from taking the ratio \( M^\tau_\tau = \frac{E_\tau \left[ B^{-\gamma}_\tau B_\tau^\gamma \right]}{\pi_\tau} = \frac{E_\tau \left[ \tau B^{-\gamma}_\tau B_\tau^\gamma \right]}{\pi_\tau} \). Q.E.D.

**Proof of Lemma A2.** From (B6) and (B8) we obtain that if policy \( n, n = 0, 1, ..., N \), is selected at \( \tau+ \), then

\[ M^\tau_\tau = \frac{E_{\tau+} \left[ B^{-\gamma}_\tau B_\tau^\gamma | \text{policy n} \right]}{E_{\tau+} \left[ B^{-\gamma}_\tau \right] | \text{policy n}} = \left( B_{\tau+} \right)^\tau e^{\left( 1-\gamma \right) \mu_\tau^\gamma (T-\tau)} \left( B_{\tau+} \right)^\tau \] (B9)

Q.E.D.
**Proof of Proposition 2.** From Lemmas A1 and A2, the gross announcement return from announcing policy $n$ is

$$1 + R^n (\tilde{g}_\tau) = e^{(\mu^n - \tilde{g}_\tau)(T-\tau) + \frac{1}{2}\gamma^2(T-\tau)^2(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)} \times \left( 1 + \sum_{n=1}^{N} p^n \left( e^{-\gamma(\mu^n - \tilde{g}_\tau)(T-\tau) + \frac{1}{2}\gamma^2(T-\tau)^2(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)} - 1 \right) \right)$$

Similarly, recalling the notation $\mu^0_g = \tilde{g}_\tau$ and $\sigma_{g,0} = \tilde{\sigma}_\tau$, from Lemma A1 and A2 the gross announcement return from announcing no policy change is

$$1 + R^0 (\tilde{g}_\tau) = \frac{\left( 1 + \sum_{n=1}^{N} p^n \left( e^{-\gamma(\mu^n - \tilde{g}_\tau)(T-\tau) + \frac{1}{2}\gamma^2(T-\tau)^2(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)} - 1 \right) \right)}{\left( 1 + \sum_{n=1}^{N} p^n \left( e^{(1-\gamma)(\mu^n - \tilde{g}_\tau)(T-\tau) + \frac{(1-\gamma)^2(T-\tau)^2(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)}{2}} - 1 \right) \right)}$$

Therefore, we can write more compactly, for $n = 1, \ldots, N$,

$$1 + R^n (\tilde{g}_\tau) = e^{(\mu^n - \tilde{g}_\tau)(T-\tau) + \frac{1}{2}\gamma^2(T-\tau)^2(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)} \times \left( 1 + R^0 (\tilde{g}) \right)$$

We can express all the formulas in terms of $\tilde{\mu}^n$ and $x_\tau$. Using the definitions

$$\tilde{\mu}^n + \frac{\sigma_{g,n}^2}{2} (T-\tau) (\gamma - 1) = \mu^n$$

$$x_\tau + \frac{\tilde{\sigma}_\tau^2}{2} (T-\tau) (\gamma - 1) = \tilde{g}_\tau$$

we have

$$(\mu^n - \tilde{g}_\tau) = (\tilde{\mu}^n - x_\tau) + \frac{(\sigma_{g,n}^2 - \tilde{\sigma}_\tau^2)}{2} (T-\tau) (\gamma - 1)$$

The claim of Proposition 2 then follows quickly. Q.E.D.

**Proof of Corollary 3.** Immediate from Proposition 2. Q.E.D.

**Proof of Corollary 4.** Immediate from Corollary 3: for any two policies $n$ and $m$ with $\tilde{\mu}^n = \tilde{\mu}^m$, the result follows from equation (31). Q.E.D.

**Proof of Proposition 3:** To prove this proposition, we need three lemmas:

**Lemma B1.** $\Delta b_\tau$ and $\tilde{g}_\tau$ are perfectly correlated, and we can write

$$\Delta b_\tau = E_t [\Delta b_\tau] + (\tilde{g}_\tau - E_t [\tilde{g}_\tau]) \left[ \frac{\sigma^2}{\tilde{\sigma}_\tau^2} + (\tau - t) \right]$$

$$= E_t [\Delta b_\tau] + (x_\tau - E_t [x_\tau]) \left[ \frac{\sigma^2}{\tilde{\sigma}_\tau^2} + (\tau - t) \right]$$

**Proof of Lemma B1:** From Lemma A5 in Pastor and Veronosi (2011), we have that $b_\tau = \log (B_\tau)$ and $\tilde{g}_\tau$ have the conditional joint distribution

$$\left( \begin{array}{c} b_\tau - b_t \\ \tilde{g}_\tau \end{array} \right) \sim N \left( \begin{array}{c} E_t [\Delta b_\tau] \\ E_t [\tilde{g}_\tau] \end{array} ; \begin{array}{c} V_b, C_{g,b} \\ C_{g,b}, V_{g} \end{array} \right)$$

(B15)
We now see that \( b_r - b_t \) and \( \hat{g}_r \) are perfectly correlated. In fact,

\[
\text{Corr} = \frac{C_{\hat{g}, b}}{\sqrt{V_b V_{\hat{g}}}} = \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{((\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t)) (\hat{\sigma}_t^2 - \hat{\sigma}_r^2)}}
\]

Using the fact that

\[
\hat{\sigma}_r^2 = \frac{1}{\sigma^2} + \frac{1}{\sigma^2 (\tau - t)} = \frac{\hat{\sigma}_r^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)}
\]

we find

\[
\text{Corr} = \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{((\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t)) \left(\hat{\sigma}_t^2 - \frac{\hat{\sigma}_r^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)}\right)}}
\]

\[
= \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{(\tau - t) (\hat{\sigma}_t^2 (\sigma^2 + \hat{\sigma}_r^2 (\tau - t) - \hat{\sigma}_r^2 \sigma^2))}}
\]

\[
= \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{(\tau - t) (\hat{\sigma}_r^2 (\tau - t))}} = 1
\]

It follows that we can write

\[
\Delta b_r = E_t [\Delta b_r] + \{\hat{g}_r - E_t [\hat{g}_r]\} \frac{C_{b, \hat{g}}}{V_{\hat{g}}} = E_t [\Delta b_r] + \{\hat{g}_r - E_t [\hat{g}_r]\} \sqrt{\frac{V_b}{V_{\hat{g}}}}
\]

\[
= E_t [\Delta b_r] + \{\hat{g}_r - E_t [\hat{g}_r]\} \frac{\hat{\sigma}_t^2 (\tau - t)}{\hat{\sigma}_t^2 - \hat{\sigma}_r^2} = E_t [\Delta b_r] + \{\hat{g}_r - E_t [\hat{g}_r]\} \left[\frac{\sigma^2 / \hat{\sigma}_r^2 + (\tau - t)}{\hat{\sigma}_t^2 - \hat{\sigma}_r^2}\right]
\]

where we also used the equality

\[
\hat{\sigma}_t^2 - \hat{\sigma}_r^2 = \hat{\sigma}_t^2 - \frac{\hat{\sigma}_r^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)} = \frac{(\hat{\sigma}_r^2)^2 (\tau - t)}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)}
\]

From the definition of \( x_r \), it also follows that \( x_r - E_t [x_r] = \hat{g}_r - E_t [\hat{g}_r] \). Q.E.D.
Lemma B2: The conditional distribution of $\Delta b_\tau = b_\tau - b_t = \log(B_\tau/B_t)$ conditional on time-$t$ information and policy $n$ being chosen at time $\tau$ is

$$f(\Delta b_\tau | S_t, n \text{ at } \tau) = \frac{\phi_{\Delta b_\tau}(\Delta b_\tau)}{p_t} \int_{-\infty}^{\tilde{x}_n - E_t[x_\tau] - (\Delta b_\tau - E_t[\Delta b_\tau])} \frac{\tilde{\sigma}_n^2}{(\tau-t)^{\sigma^2+\tilde{\sigma}_t^2}} \prod_{m \neq n} (1 - \Phi_{\bar{\eta}_n}(\tilde{c}_n^m - \tilde{\mu}_n + \tilde{\mu}_m)) \phi_{\bar{\eta}_n}(\tilde{c}_n^m) d\tilde{c}_n^m \tag{B18}$$

where $\phi_{\Delta b_\tau}(\Delta b_\tau)$ is the normal density with mean $E_t[\Delta b_\tau] = (\mu + \tilde{g}_t - \frac{1}{2} \sigma^2) (\tau-t)$ and variance $V_b = (\tau-t)^2 \tilde{\sigma}_t^2 + \sigma^2 (\tau-t)$. In addition, $E_t[x_\tau] = \tilde{g}_t - \frac{\sigma^2}{2} (T - \tau) (\gamma - 1)$.

Note that $f(\Delta b_\tau | S_t, \kappa$ at $\tau)$ does not depend on the current value of log capital, $b_t$, hence the conditional dependence only on $S_t$ and time $t$.

Proof of Lemma B2. The conditional CDF is

$$F_{\Delta b_\tau}(\Delta b | S_t, \tilde{x}_n < \tilde{\mu}_n - \tilde{c}_n, \tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m \text{ for } m \neq n) = \frac{\Pr(\Delta b_\tau < \Delta b, \tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m \text{ for } m \neq n | S_t)}{\Pr(\tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m \text{ for } m \neq n | S_t)} \tag{B20}$$

The denominator is just $p_t^n$ from Corollary 2. Consider the numerator. From Lemma B1:

$$\Delta b_\tau - E_t[\Delta b_\tau] = \{x_\tau - E_t[x_\tau]\} (\sigma^2/\tilde{\sigma}_t^2 + (\tau-t))$$

which implies

$$x_\tau = E_t[x_\tau] + (\Delta b_\tau - E_t[\Delta b_\tau]) \frac{\tilde{\sigma}_n^2}{(\sigma^2 + \tilde{\sigma}_t^2 (\tau-t))}$$

Thus, the joint distribution can be written as

$$\Pr(\Delta b_\tau < \Delta b, \tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m | S_t) = \Pr(\Delta b_\tau < \Delta b, E_t[x_\tau] + (\Delta b_\tau - E_t[\Delta b_\tau]) \frac{\tilde{\sigma}_n^2}{(\sigma^2 + \tilde{\sigma}_t^2 (\tau-t))} < \tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m | S_t)$$

$$= \int_{-\infty}^{\infty} \Pr(\Delta b_\tau < \Delta b, \tilde{c}_n - \tilde{\mu}_n - E_t[x_\tau] - (\Delta b_\tau - E_t[\Delta b_\tau]) \frac{\tilde{\sigma}_n^2}{(\sigma^2 + \tilde{\sigma}_t^2 (\tau-t))} < \tilde{c}_n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}_m | S_t) \phi_{\bar{\eta}_n}(\tilde{c}_n) d\tilde{c}_n$$

$$= \int_{-\infty}^{\Delta b} \int_{-\infty}^{\tilde{c}_n - E_t[x_\tau] - (\Delta b_\tau - E_t[\Delta b_\tau])} \frac{\tilde{\sigma}_n^2}{(\sigma^2 + \tilde{\sigma}_t^2 (\tau-t))} \prod_{m \neq n} [1 - \Phi_{\bar{\eta}_n}(\tilde{c}_n^m - \tilde{\mu}_n + \tilde{\mu}_m)] \phi_{\bar{\eta}_n}(\tilde{c}_n^m) d\tilde{c}_n^m d\Delta b_\tau$$

where we exploited the independence across $\tilde{c}_m$ and with respect to $\Delta b_\tau$. Substituting into (B20) and taking the first derivative with respect to $\Delta b$, we obtain the density (B19). Q.E.D.
Lemma B3: The distribution of $\hat{g}_t$ conditional on time-$t$ information and no new policy being chosen at time $\tau$ is

$$f(\hat{g}_t|\text{no policy change at } \tau) = \frac{\phi_{\hat{g}_t}(\hat{g}_t|\hat{g}_t)\prod_{n=1}^{N} (1 - \Phi_{\mathcal{G}}(\hat{\mu}_n^n - \hat{g}_t + \frac{\hat{\sigma}_n^2}{2}(T - \tau)(\gamma - 1)))}{p_t^n}$$  

(B21)

where $\phi_{\hat{g}_t}(\hat{g}_t|\hat{g}_t)$ is the conditional normal density of $\hat{g}_t$, namely, $N(\hat{g}_t, \hat{\sigma}_t^2 - \hat{\sigma}_t^2)$.

Proof of Lemma B3: The conditional CDF is given by

$$F_{\hat{g}_t}(g|\text{no policy change at } \tau) = F_{\hat{g}_t}(g|x_t > \hat{\mu}_n^n - \hat{\sigma}_n^2 \text{ for all } n) = \Pr(\hat{g}_t < g \& \hat{g}_t > \hat{\mu}_n^n - \hat{\sigma}_n^2 (T - \tau)(\gamma - 1) \text{ for all } n)$$

$$= \frac{\prod_{n=1}^{N} p_t^n}{p_t^n} \int_{-\infty}^{\infty} \prod_{n=1}^{N} (1 - \Phi_{\mathcal{G}}(\hat{\mu}_n^n - \hat{g}_t + \frac{\hat{\sigma}_n^2}{2}(T - \tau)(\gamma - 1))) \phi_{\hat{g}_t}(\hat{g}_t|\hat{g}_t) d\hat{g}_t$$

$$= \int_{-\infty}^{\infty} 1_{[\hat{g}_t < g]} \prod_{n=1}^{N} (1 - \Phi_{\mathcal{G}}(\hat{\mu}_n^n - \hat{g}_t + \frac{\hat{\sigma}_n^2}{2}(T - \tau)(\gamma - 1))) \phi_{\hat{g}_t}(\hat{g}_t|\hat{g}_t) d\hat{g}_t$$

Taking the first derivative with respect to $g$, we obtain the density (B21). Q.E.D.

Proof of Proposition 3: We know that

$$\pi_t = E_t[\pi_{t+}] = \sum_{n=0}^{N} E_t[\pi_{t+}|n \text{ at } \tau] p_t^n$$

(B22)

Note that for $n = 1, \ldots, N$

$$E_t[\pi_{t+}|n \text{ at } \tau] = E_t\left[\lambda^{-1} B_{\tau+}^{-\gamma} e^{-\gamma \mu_{\mathcal{G}}(T-\tau)} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]}|n \text{ at } \tau\right]$$

$$= \lambda^{-1} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]} e^{-\gamma B_x E_t[e^{-\gamma(b_{t-} - b_t)}]|n \text{ at } \tau}$$

$$= \lambda^{-1} B_{\tau+}^{-\gamma} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]} \left[\int_{-\infty}^{\infty} e^{-\gamma \Delta b_r} f(\Delta b_r|S_t, n \text{ at } \tau) d\Delta b_r\right]$$

Similarly, for $n = 0$ we have

$$E_t[\pi_{t+}|0 \text{ at } \tau] = E\left[\lambda^{-1} B_{\tau+}^{-\gamma} e^{-\gamma \hat{g}_r(T-\tau)} e^{-\gamma \mu_{\mathcal{G}}(T-\tau)} e^{-\gamma \mu_{\mathcal{G}}(T-\tau)} e^{-\gamma B_t[(T-\tau)-\delta_{\mathcal{G}} n]}|0 \text{ at } \tau\right]$$
\[ E = \lambda^{-1}e^{-\gamma(\gamma+1)\sigma^2}(T-t) + \frac{\sigma^2}{T}(T-t)^2\sigma_t^2 e^{-\gamma\eta} E_t \left[ e^{-\gamma\Delta b_T - \gamma\hat{S}_T} \right] \]
\[ = \lambda^{-1}e^{-\gamma(\gamma+1)\sigma^2}(T-t) + \frac{\sigma^2}{T}(T-t)^2\sigma_t^2 e^{-\gamma\eta} \times \]
\[ \times E_t \left[ e^{-\gamma(E_1[\Delta b_T] + (\hat{S}_T - E_1[\hat{S}_T]))\sigma_t^2} \right] \]
\[ = \lambda^{-1}B_t^{-\gamma}e^{-\gamma\hat{S}_T(T-t)} e^{-\gamma(\gamma+1)\sigma^2}(T-t) + \frac{\sigma^2}{T}(T-t)^2\sigma_t^2 \]
\[ \times \int_{-\infty}^{\infty} e^{-\gamma(E_1[\Delta b_T] + (\hat{S}_T - E_1[\hat{S}_T]))\sigma_t^2} f(\hat{S}_T | 0 \text{ at } \tau) d\hat{S}_T \]

The result follows from comparing the terms in Equations (36) and (A3) with the ones above, and defining in this proposition \( \mu_0^0 = \hat{S}_T \) and \( \sigma_{g,0}^2 = \sigma_\tau^2 \). Q.E.D.

**Proof of Proposition 4:** The result follows from an application of Ito’s Lemma to equation (36), and recalling that \( \pi_t \) is a martingale, and thus \( E_t[d\pi_t/\pi_t] = 0 \). Q.E.D.

**Proof of Corollary 5:** From property 1 in the proof of Corollary 2, for a given distribution of \( \hat{\tau}^n \), we have \( p_i^0 \to 1 \) as \( \hat{\tau} \to \infty \). It follows that the state price density converges to one that assigns zero probability to a policy change:

\[ \pi_t \to \Omega(S_t) = E_t[\pi_{T+} | 0 \text{ at } n] = \lambda^{-1}E_t[B_T^{-\gamma} | 0 \text{ at } n] \]
\[ = \lambda^{-1}B_t^{-\gamma}e^{-\gamma\hat{S}_T(T-t)} e^{-\gamma(\gamma+1)\sigma^2}(T-t) + \frac{\sigma^2}{T}(T-t)^2\sigma_t^2 \]

Since this state price density does not depend on any \( \hat{\tau}^n \), we have \( \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial n} = 0 \). Q.E.D.

**Proof of Proposition 5:** The proof is identical to the proof of Proposition 3, except that we have to calculate

\[ E_t[\pi_{T+} M_{T+}^i | n \text{ at } \tau] = \sum_{n=0}^{N} p_i^0 E_t[\pi_{T+} M_{T+}^i | n \text{ at } \tau] \]

From (B8), for \( n = 1, \ldots, N \):

\[ E_t[\pi_{T+} M_{T+}^i | n \text{ at } \tau] = \lambda^{-1}E_t[N_{T+}^i | n \text{ at } \tau] \]
\[ = \lambda^{-1}E_t[B_T^{-\gamma}B_{T+}^i \times e^{(1-\gamma)\mu_0^0(T-t)} e^{(1-\gamma)\mu_0^0(\gamma-1)\sigma^2}(T-t) + \frac{(1-\gamma)^2}{2}(T-t)^2\sigma_{0,n}^2 | n \text{ at } \tau] \]
\[ = \lambda^{-1}e^{(1-\gamma)\mu_0^0(T-t)} e^{(1-\gamma)\mu_0^0(\gamma-1)\sigma^2}(T-t) + \frac{(1-\gamma)^2}{2}(T-t)^2\sigma_{0,n}^2 E_t[e^{-\gamma b_T + b_t} | n \text{ at } \tau] \]

Now, recall

\[ \frac{B_T^i}{B_T^i} = \frac{B_T}{B_T} e^{-\frac{1}{2}\sigma_t^2(T-t) + \sigma_1(Z_t - Z_t^i)} \]

(B23)

which implies

\[ e^{b_t^i} = e^{b_t^0 + b_t - \frac{1}{2}\sigma_t^2(T-t) + \sigma_1(Z_t - Z_t^i)} \]

(B24)
For \( n = 1, \ldots, N \), we then have:

\[
E_t \left[ \pi_{\tau+} M^t_{\tau+} \mid n \text{ at } \tau \right] = \lambda^{-1} B_t^{-\gamma} B^t \left[ e^{(1-\gamma)\mu_{n}^\phi(T-\tau)} e^{\left((1-\gamma)\mu + \frac{1}{2}(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2} (T-\tau)^2 \sigma_{\phi,n}^2} \mid n \text{ at } \tau \right] 
\]

\[
E_t \left[ e^{(1-\gamma)\Delta \tau_{\tau+}} \mid n \text{ at } \tau \right] = \lambda^{-1} B_t^{-\gamma} B^t \left[ e^{(1-\gamma)\mu_{n}^\phi(T-\tau)} e^{\left((1-\gamma)\mu + \frac{1}{2}(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2} (T-\tau)^2 \sigma_{\phi,n}^2} \right] 
\]

\[
\int e^{(1-\gamma)\Delta \tau_{\tau+}} f \left( \Delta \tau_{\tau+}, n \text{ at } \tau \right) d\Delta \tau_{\tau+}
\]

Similarly, for \( n = 0 \), we have:

\[
E_t \left[ \pi_{\tau+} M^t_{\tau+} \mid 0 \text{ at } \tau \right] = \lambda^{-1} E_t \left[ M^t_{\tau+} \mid 0 \text{ at } \tau \right] 
\]

\[
= \lambda^{-1} e^{(1-\gamma)\mu + \frac{1}{2}(\gamma-1)\sigma^2(T-\tau) + \frac{(1-\gamma)^2}{2} (T-\tau)^2 \sigma_{\phi,n}^2} \left[ e^{(1-\gamma)\Delta \tau_{\tau+} + (1-\gamma)\hat{\gamma} e(T-\tau)} \mid 0 \text{ at } \tau \right] 
\]

\[
\times E_t \left[ e^{(1-\gamma) \left( E_t[\Delta \tau_{\tau+}] + \hat{\gamma} e - E_t[\hat{\gamma} e] \right) \sqrt{\frac{\sigma^2}{\sigma^2}}} \right] \left[ (1-\gamma)\hat{\gamma} e(T-\tau) \mid n \text{ at } \tau \right] 
\]

\[
= \lambda^{-1} B_t^{-\gamma} B^t \left[ e^{(1-\gamma)\hat{\gamma} e(T-\tau) + \left((1-\gamma)\mu + \frac{1}{2}(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2} (T-\tau)^2 \sigma_{\phi,n}^2} \right] 
\]

\[
\times \int e^{(1-\gamma) \left( E_t[\Delta \tau_{\tau+}] + \hat{\gamma} e - E_t[\hat{\gamma} e] \right) \sqrt{\frac{\sigma^2}{\sigma^2}}} + (1-\gamma)(\hat{\gamma} e(T-\tau)) f \left( \hat{\gamma} e, S_t, n \text{ at } \tau \right) d\hat{\gamma} e
\]

The result follows from comparing the terms in Equations (41) and (A4) with the ones above, and defining in this proposition \( \mu_{\phi}^0 = \hat{\gamma} e \) and \( \sigma_{\phi,n}^2 = \hat{\gamma} e^2 \). Q.E.D.

**Proof of Proposition 6.** The claim follows from an application of Ito’s Lemma to the price \( M^t_i \) in Proposition 5, and the equilibrium restriction \( \mu_M = \text{Cov}_t \left( \frac{dM^t_i}{M^t_i}, \frac{dP}{P} \right) \). Q.E.D.

**Proof of Proposition 7.** The expression for the jump risk premium follows immediately from

\[
J(S_t) = \sum_{n=0}^{N} p^n_t R^n(x_t)
\]

where \( R^n(x_t) \) are given in Proposition 2. We now see that

\[
J(S_t) = -\text{Cov}_t \left( \frac{M^n_{\tau+}}{M^t_i} - 1, \frac{\pi_{\tau+}}{\pi_t} - 1 \right) = - \left\{ E_t \left[ J_M J_{\pi^n} \right] - E_t \left[ J_M \right] E_t \left[ J_{\pi^n} \right] \right\}
\]

where, if policy \( n \) is chosen, we denote \( J^n_m = \frac{M^n_{\tau+}}{M^t_i} \) and \( J^n = \frac{\pi_{\tau+}}{\pi_t} \). Recall from Proposition 2 that

\[
J^n_M = 1 + R^n(x_t)
\]
\[
J^n_\pi (x_\tau) = \frac{\pi^n_\tau}{\pi_\tau} = \frac{1}{\left(1 + \sum_{k=1}^{N} p_k^\pi \left( e^{-\gamma (\tilde{\mu}_n - x_\tau)} + \frac{1}{2}(T-\tau) e^\pi e(T-\gamma (\tilde{\mu}_n - x_\tau))^2 - 1 \right) \right)}
\]

This implies that

\[
J^n_\pi (x_\tau) J^0_\pi (x_\tau) = \frac{e^{(1-\gamma)(\tilde{\mu}_n - x_\tau)}(T-\tau)}{\left(1 + \sum_{k=1}^{N} p_k^\pi \left( e^{(1-\gamma)(T-\tau)}(\tilde{\mu}_n - x_\tau) - 1 \right) \right)}
\] for \(n = 1, \ldots, N\)

\[
J^0_\pi (x_\tau) J^0_\pi (x_\tau) = \frac{1}{\left(1 + \sum_{k=1}^{N} p_k^\pi \left( e^{(1-\gamma)(T-\tau)}(\tilde{\mu}_n - x_\tau) - 1 \right) \right)}
\]

It follows that

\[
E_\tau \left[ J_\pi (x_\tau) J_\pi (x_\tau) \right] = \sum_{k=1}^{N} p_k^\pi \left\{ \frac{e^{(1-\gamma)(\tilde{\mu}_n - x_\tau)(T-\tau)}}{\left(1 + \sum_{k=1}^{N} p_k^\pi \left( e^{(1-\gamma)(T-\tau)}(\tilde{\mu}_n - x_\tau) - 1 \right) \right)} \right\}
\]

\[
= \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \]

Similarly,

\[
E_\tau \left[ J_\pi (x_\tau) \right] = \sum_{k=1}^{N} p_k^\pi \left\{ \frac{e^{-\gamma \tilde{\mu}_n (T-\tau)} e^{\frac{1}{2}(x_n - x_\tau)^2}}{e^{-\gamma x_\tau (T-\tau)} e^{\frac{1}{2}(x_n - x_\tau)^2} J^0_\pi (x_\tau)} \right\} + \left(1 - \sum_{k=1}^{N} p_k^\pi \right) J^0_\pi (x_\tau)
\]

\[
= \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \left[ \sum_{k=1}^{N} p_k^\pi \right] \]
Thus, we finally obtain

\[ J(x_\tau) = -\text{Cov}_\tau(J_M, J_\pi) = -\left\{ E_\tau [J_M J_\pi] - E_\tau [J_M] E_\tau [J_\pi] \right\} \]
\[ = E_\tau [J_M] - 1 \]

Q.E.D.

REFERENCES