Technical Appendix

to accompany

Political Uncertainty and Risk Premia

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**Proof of Lemma 1:** The same argument leading to equation (IA.8) in the Internet Appendix of Pastor and Veronesi (2012) implies that conditional on policy \( n, n = 0, 1, \ldots, N, \) being chosen at time \( \tau, \) aggregate capital is given by

\[
B_T = B_\tau e^{(\mu_g - \frac{1}{2} \sigma^2)(T-\tau) + \sigma (Z_T - Z_\tau)}
\]

Thus, exploiting \( W_T = B_T \) we have

\[
E_\tau \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \right]_{\text{policy } n} = \frac{B_\tau^{1-\gamma}}{1-\gamma} e^{(1-\gamma)(T-\tau)\mu_g + \frac{1}{2} (1-\gamma)^2 (T-\tau)^2 \sigma_{g,n}^2 + (\mu - \frac{1}{2} \sigma^2)(T-\tau)(1-\gamma)}
\]

It follows immediately that

\[
E_\tau \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \right]_{\text{policy } n} = \frac{B_\tau^{1-\gamma}}{1-\gamma} e^{(1-\gamma)(T-\tau)\mu_g + \frac{1}{2} (1-\gamma)^2 (T-\tau)^2 \sigma_{g,n}^2 + (\mu - \frac{1}{2} \sigma^2)(T-\tau)(1-\gamma)}
\]

if and only if

\[
\tilde{\mu}^n = \mu_g^n + \frac{1}{2} (1-\gamma)(T-\tau)\sigma_{g,n}^2 > \mu_g^m + \frac{1}{2} (1-\gamma)(T-\tau)\sigma_{g,m}^2 = \tilde{\mu}^m
\]

Q.E.D.

**Proof of Proposition 1.** The government chooses policy \( n \in \{0, 1, \ldots, N\} \) if and only if for all \( m \neq n, m = 0, 1, \ldots, N, \)

\[
E_\tau \left[ \frac{C_n W_T^{1-\gamma}}{1-\gamma} \right]_{\text{policy } n} > E_\tau \left[ \frac{C_m W_T^{1-\gamma}}{1-\gamma} \right]_{\text{policy } m}
\]

where recall that \( C^0 = 1. \) The same calculations as in Lemma 1 lead to the inequality

\[
\mu^n_g - \frac{\sigma_{g,n}^2}{2} (T-\tau) (\gamma - 1) - \frac{\epsilon^n}{(\gamma - 1) (T-\tau)} > \mu^m_g - \frac{\sigma_{g,m}^2}{2} (T-\tau) (\gamma - 1) - \frac{\epsilon^m}{(\gamma - 1) (T-\tau)}
\]

(B1)

The claim follows from the definitions of \( \tilde{\mu}^n \) and \( \tilde{c}^n \) in equations (15) and (23). Q.E.D.

**Proof of Corollary 1.** Immediate from Proposition 1 and equations (16) and (17).

**Proof of Corollary 2.** As of time \( t, \) we have for each \( n = 1, \ldots, N \)

\[
c^n \sim N \left( \tilde{c}_n, \tilde{\sigma}_{c,t}^2 \right)
\]

(B2)

Recall from Proposition 1 that policy \( n \in \{1, \ldots, N\} \) is chosen if and only if

\[
\tilde{\mu}^n - \tilde{c}^n > \tilde{\mu}^m - \tilde{c}^m \quad m \neq n, \ m = 1, \ldots, N
\]

(B3)

\[
\tilde{\mu}^n - \tilde{c}^n > x_\tau,
\]

(B4)

where we define

\[
x_\tau = \tilde{\mu}^0 = \tilde{g}_\tau - \frac{\tilde{\sigma}_\tau^2}{2} (T-\tau) (\gamma - 1)
\]

(B5)
Therefore, the conditional probability at \( t \) that policy \( n \) is chosen at \( \tau \) is given by

\[
p^n_t = \Pr \left( \tilde{\mu}^n - \tilde{c}^n > \tilde{\mu}^m - \tilde{c}^m \text{ for } m \neq n \right)
\]
\[
= \int_{-\infty}^{\infty} \Pr \left( \tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{c}^m \text{ for } m \neq n \bigg| \tilde{c}^n \right) \phi_{\tilde{c}^n} (\tilde{c}^n) \, d\tilde{c}^n
\]
\[
= \int_{-\infty}^{\infty} \Pi_{m \neq n} \Pr (\tilde{c}^n - \tilde{\mu}^n + \tilde{\mu}^m < \tilde{c}^m | \tilde{c}^n) \Pr (\tilde{\mu}^n - \tilde{c}^n > x_{\tau} | \tilde{c}^n) \phi_{\tilde{c}^n} (\tilde{c}^n) \, d\tilde{c}^n
\]

where we used the fact that \( \tilde{c}^m \)'s are independent of each other as well as of \( x_{\tau} \). Moreover, from the definition of \( x_{\tau} = \hat{\gamma}_\tau - \frac{\tilde{\sigma}_t^2}{2}(T - \tau)(\gamma - 1) \) (see equation (16)) we have \( x_{\tau} | \tilde{\gamma}_t \sim N \left( \hat{\gamma}_t - \frac{\tilde{\sigma}_t^2}{2}(T - \tau)(\gamma - 1), \tilde{\sigma}_t^2 - \tilde{\sigma}_\tau^2 \right) \).

We note two properties:

1. As \( \hat{\gamma}_t \to \infty \), then \( p^n_t \to 0 \) for all \( n \in \{1, \ldots, N\} \), as \( \Phi_x (\tilde{\mu}^n - \tilde{c}^n | \tilde{\gamma}_t) \to 0 \).

2. As \( t \to \tau \) we have

\[
\Phi_x (\tilde{\mu}^n - \tilde{c}^n | \tilde{\gamma}_t) = \int_{\tilde{\mu}^n - \tilde{c}^n}^{\infty} \phi_x (x | \tilde{\gamma}_t) \, dx \to 1_{\{x_{\tau} < \tilde{\mu}^n - \tilde{c}^n\}} \quad (\text{B6})
\]

so that

\[
p^n_t = \int_{-\infty}^{\infty} \Pi_{m \neq n} (1 - \Phi_{\tilde{c}^m} (\tilde{c}^n + \tilde{\mu}^m - \tilde{\mu}^n)) \Phi_x (\tilde{\mu}^n - \tilde{c}^n | \tilde{\gamma}_t) \phi_{\tilde{c}^n} (\tilde{c}^n) \, d\tilde{c}^n
\]
\[
= \int_{-\infty}^{\infty} \Pi_{m \neq n} (1 - \Phi_{\tilde{c}^m} (\tilde{c}^n + \tilde{\mu}^m - \tilde{\mu}^n)) \phi_{\tilde{c}^n} (\tilde{c}^n) \, d\tilde{c}^n
\]
\[
= p^n_{\tau}
\]

Q.E.D.

**Proof of Lemma A1.** Using the same arguments as to obtain equation (IA.20) in the Internet Appendix of Pastor and Veronesi (2012), after the announcement of policy \( n \) at time \( \tau^+ \), the state price density is given by

\[
E_{\tau^+} [\pi_T | \text{policy } n] = \pi^n_{\tau^+} = \lambda^{-1} B_{\tau^+} e^{-\gamma \mu^\gamma (T-\tau)} e^{-\gamma \mu^\gamma \frac{1}{2}(\gamma + 1) \sigma^2 (T-\tau) + \frac{\tilde{\sigma}_t^2}{2}(T-\tau)^2 \sigma^2_{\sigma_n}} \quad (\text{B7})
\]

Therefore, using also \( B_{\tau} = B_{\tau^+} \), the state price density at \( \tau \) is

\[
\pi_\tau = \sum_{n=0}^{N} p^n_{\tau^+} \pi^n_{\tau^+}
\]
The claim follows from taking the ratio
\[ M_{n} = \frac{E_{\tau} [B_{\tau}^{-\gamma} B_{\tau}^{i} | \text{policy } n]}{E_{\tau} [B_{\tau}^{-\gamma} B_{\tau}^{i} | \text{policy } n]} = \frac{E_{\tau} [\pi_{\tau} B_{\tau}^{i}]}{\pi_{\tau} B_{\tau}^{i}}. \]

Proof of Lemma A2. From (B7) and (B9) we obtain that if policy \( n, n = 0, 1, \ldots, N \), is selected at \( \tau + \), then
\[ M_{n}^{i} = \frac{E_{\tau+} [B_{\tau}^{-\gamma} B_{\tau}^{i} | \text{policy } n]}{E_{\tau+} [B_{\tau}^{-\gamma} B_{\tau}^{i} | \text{policy } n]} = B_{\tau}^{i} e^{(\gamma \sigma^{2} + \mu^{n}_{\tilde{g}})(\tau - T) + \frac{1}{2} \gamma^{2}(T - \tau)^{2} \sigma^{2}_{g,n}} \]
**Lemma B1.** \( \Delta b_r \) and \( \hat{g}_r \) are perfectly correlated, and we can write

\[
\Delta b_r = E_t[\Delta b_r] + (\hat{g}_r - E_t[\hat{g}_r]) \left[ \sigma^2 / \sigma^2_t + (\tau - t) \right] \\
= E_t[\Delta b_r] + (x_r - E_t[x_r]) \left[ \sigma^2 / \sigma^2_t + (\tau - t) \right]
\]

**Proof of Lemma B1:** From Lemma A5 in Pastor and Veronesi (2012), we have that \( b_r = \log(B_r) \) and \( \hat{g}_r \) have the conditional joint distribution

\[
\left( \begin{array}{c}
 b_r - b_t \\
 \hat{g}_r 
\end{array} \right) \sim N \left( \begin{array}{c}
 E_t[\Delta b_r] \\
 E_t[\hat{g}_r] 
\end{array} ; \begin{array}{c}
 V_{b, C_{g,b}} \\
 C_{g,b, V_{\hat{g}}} 
\end{array} \right)
\]

where

\[
E_t[\Delta b_r] = \left( \mu + \hat{g}_t - \frac{1}{2} \sigma^2 \right) (\tau - t) \\
E_t[\hat{g}_r] = \hat{g}_t \\
V_b = (\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t) \\
V_{\hat{g}} = \hat{\sigma}_t^2 - \hat{\sigma}_t^2 \\
C_{\hat{g}, b} = \hat{\sigma}_t^2 (\tau - t)
\]

We now see that \( b_r - b_t \) and \( \hat{g}_r \) are perfectly correlated. In fact,

\[
Corr = \frac{C_{\hat{g}, b}}{\sqrt{V_b V_{\hat{g}}}} = \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{((\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t)) (\hat{\sigma}_t^2 - \hat{\sigma}_t^2)}}
\]

Using the fact that

\[
\hat{\sigma}_t^2 = \frac{1}{\hat{\sigma}_t^2 + \frac{1}{\sigma} (\tau - t)} = \frac{\hat{\sigma}_t^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)}
\]

we find

\[
Corr = \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{((\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t)) \left( \hat{\sigma}_t^2 - \frac{\hat{\sigma}_t^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)} \right)}}
\]

\[
= \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{(\tau - t) \left( \hat{\sigma}_t^2 (\sigma^2 + \hat{\sigma}_t^2 (\tau - t)) - \hat{\sigma}_t^2 \sigma^2 \right)}}
\]

\[
= \frac{\hat{\sigma}_t^2 (\tau - t)}{\sqrt{(\tau - t) \left( \hat{\sigma}_t^2 (\tau - t) \right)}} = 1
\]

It follows that we can write

\[
\Delta b_r = E_t[\Delta b_r] + \{ \hat{g}_r - E_t[\hat{g}_r] \} \frac{C_{b, \hat{g}} V_{\hat{g}}}{V_b} = E_t[\Delta b_r] + \{ \hat{g}_r - E_t[\hat{g}_r] \} \sqrt{\frac{V_b}{V_{\hat{g}}}}
\]

\[
= E_t[\Delta b_r] + \{ \hat{g}_r - E_t[\hat{g}_r] \} \frac{\hat{\sigma}_t^2 (\tau - t)}{\hat{\sigma}_t^2 - \hat{\sigma}_t^2} = E_t[\Delta b_r] + \{ \hat{g}_r - E_t[\hat{g}_r] \} \left[ \sigma^2 / \sigma^2_t + (\tau - t) \right]
\]
where we also used the equality

\[
\hat{\sigma}_t^2 - \hat{\sigma}_\tau^2 = \frac{\hat{\sigma}_t^2 \sigma^2}{\sigma^2 + \hat{\sigma}_t^2 (\tau - t)} = \frac{(\hat{\sigma}_t^2)^2 (\tau - t)}{\sigma^2 + (\hat{\sigma}_t^2)^2 (\tau - t)}
\]

From the definition of \( x_\tau \), it also follows that \( x_\tau - E_t[x_\tau] = \hat{g}_\tau - E_t[\hat{g}_\tau] \). Q.E.D.

**Lemma B2:** The conditional distribution of \( \Delta b_\tau = b_\tau - b_t = \log (B_\tau/B_t) \) conditional on time-\( t \) information and policy \( n \) being chosen at time \( \tau \) is

\[
f (\Delta b_\tau | S_t, n \text{ at } \tau) = \frac{\phi_{\Delta b_\tau} (\Delta b_\tau)}{p_t^n} \int_{-\infty}^{\infty} \frac{\hat{\sigma}_n^2}{(\tau - t)\hat{\sigma}_t^2 + \sigma^2} \Pi_{m \neq n} (1 - \Phi_{\tilde{c}^n} (\tilde{c}^n - \tilde{\mu}_n^m + \tilde{\mu}_m^m)) \phi_{\tilde{c}^n} (\tilde{c}^n) d\tilde{c}^n
\]

(B14)

where \( \phi_{\Delta b_\tau} (\Delta b_\tau) \) is the normal density with mean \( E_t [\Delta b_\tau] = (\mu + \hat{g}_\tau - \frac{1}{2} \sigma^2) (\tau - t) \) and variance \( V_b = (\tau - t)^2 \hat{\sigma}_t^2 + \sigma^2 (\tau - t) \). In addition, \( E_t [x_\tau] = \hat{g}_\tau - \frac{\hat{\sigma}_n^2}{\tau } (T - \tau) (\gamma - 1) \).

Note that \( f (\Delta b_\tau | S_t, \kappa \text{ at } \tau) \) does not depend on the current value of log capital, \( b_t \), hence the conditional dependence only on \( S_t \) and time \( t \).

**Proof of Lemma B2.** The conditional CDF is

\[
F_{\Delta b_\tau} (\Delta b|S_t, \Delta b \in \{x_\tau - E_t[x_\tau] \} (\sigma^2/\hat{\sigma}_t^2 + (\tau - t))
\]

which implies

\[
x_\tau = E_t[x_\tau] + \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\hat{\sigma}_t^2}{(\sigma^2 + \hat{\sigma}_t^2 (\tau - t))}
\]

Thus, the joint distribution can be written as

\[
Pr \left( \Delta b_\tau < \Delta b, \frac{x_\tau - \tilde{\mu}_n - \tilde{c}^n}{\tilde{c}^n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}^m} \middle| S_t \right)
\]

\[
= Pr \left( \Delta b_\tau < \Delta b, \frac{E_t [x_\tau] + \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\hat{\sigma}_t^2}{(\sigma^2 + \hat{\sigma}_t^2 (\tau - t))}}{\tilde{c}^n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}^m} \middle| S_t \right)
\]

\[
= \int_{-\infty}^{\infty} Pr \left( \Delta b_\tau < \Delta b, \frac{\tilde{c}^m < \tilde{\mu}_n - E_t [x_\tau] - \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\hat{\sigma}_t^2}{(\sigma^2 + \hat{\sigma}_t^2 (\tau - t))}}{\tilde{c}^n - \tilde{\mu}_n + \tilde{\mu}_m < \tilde{c}^m} \middle| \tilde{c}^n, S_t \right) \phi_{\tilde{c}^n} (\tilde{c}^n) d\tilde{c}^n
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pi_{m \neq n} \left[ 1 - \Phi_{\tilde{c}^n} (\tilde{c}^n - \tilde{\mu}_n^m + \tilde{\mu}_m^m) \right] \phi_{\tilde{c}^n} (\tilde{c}^n) d\tilde{c}^n \phi_{\Delta b_\tau} (\Delta b_\tau) d\Delta b_\tau
\]

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where we exploited the independence across \( \hat{\gamma}^m \) and with respect to \( \Delta b_r \). Substituting into (B16) and taking the first derivative with respect to \( \Delta b \), we obtain the density (B15). Q.E.D.

**Lemma B3:** The distribution of \( \hat{g}_r \) conditional on time-\( t \) information and no new policy being chosen at time \( \tau \) is

\[
 f (\hat{g}_r | \text{no policy change at } \tau) = \frac{\phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) \prod_{n=1}^{N} \left( 1 - \Phi_{\hat{g}_n} (\mu^n - \hat{g}_r + \frac{\sigma_r^2}{2} (T - \tau) (\gamma - 1) \right)}{p_t^0} \tag{B17}
\]

where \( \phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) \) is the conditional normal density of \( \hat{g}_r \), namely, \( N(\hat{g}_t, \sigma_t^2 - \sigma_r^2) \).

**Proof of Lemma B3:** The conditional CDF is given by

\[
 F_{\hat{g}_r} (g| \text{no policy change at } \tau) \\
 = F_{\hat{g}_r} (g| \tau > \mu^n - \hat{\gamma}^n \text{ for all } n) \\
 = \frac{\Pr \left( \hat{g}_r < g \& \hat{g}_r > \mu^n - \hat{\gamma}^n + \frac{\sigma_r^2}{2} (T - \tau) (\gamma - 1) \text{ for all } n \right)}{\Pr (\tau > \mu^n - \hat{\gamma}^n \text{ for all } n)} \\
 = \frac{\int_{-\infty}^{\hat{g}_r} \Pr \left( \hat{g}_r < g \& \hat{g}_r > \mu^n - \hat{\gamma}^n + \frac{\sigma_r^2}{2} (T - \tau) (\gamma - 1) \right) \phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) \, d\hat{g}_r}{p_t^0} \\
 = \frac{\int_{-\infty}^{\hat{g}_r} \prod_{n=1}^{N} \left( 1 - \Phi_{\hat{g}_n} \left( \mu^n - \hat{g}_r + \frac{\sigma_r^2}{2} (T - \tau) (\gamma - 1) \right) \right) \phi_{\hat{g}_r} (\hat{g}_r | \hat{g}_t) \, d\hat{g}_r}{p_t^0}
\]

Taking the first derivative with respect to \( g \), we obtain the density (B17). Q.E.D.

**Proof of Proposition 2:** We know that

\[
 \pi_t = E_t \left[ \pi_{\tau^+} \right] = \sum_{n=0}^{N} E_t \left[ \pi_{\tau^+} | n \text{ at } \tau \right] p_t^n \tag{B18}
\]

Note that for \( n = 1, \ldots, N \)

\[
 E_t \left[ \pi_{\tau^+} | n \text{ at } \tau \right] = E_t \left[ \lambda^{-1} B_{\tau^+}^{-\gamma} e^{-\gamma \mu^n_{\tau^+} (T - \tau)} e^{-\gamma \mu^n_{\tau^+} (T - \tau) - \gamma \mu^n_{\tau^+} (T - \tau) + \frac{\sigma_r^2}{2} (T - \tau)^2 \sigma_{g,n}^2 | n \text{ at } \tau} \right] \\
 = \lambda^{-1} e^{-\gamma \mu^n_{\tau^+} (T - \tau) - \gamma \mu^n_{\tau^+} (T - \tau) + \frac{\sigma_r^2}{2} (T - \tau)^2 \sigma_{g,n}^2} \times E_t \left[ e^{-\gamma (b_{\tau^+} - b_t)} | n \text{ at } \tau \right] \\
 = \lambda^{-1} B_t^{-\gamma} e^{-\gamma \mu^n_{\tau^+} (T - \tau) + \frac{\sigma_r^2}{2} (T - \tau)^2 \sigma_{g,n}^2} \int_{-\infty}^{\infty} e^{-\gamma \Delta b_r} f (\Delta b_r | S_t, n \text{ at } \tau) \, d\Delta b_r
\]
Similarly, for \( n = 0 \) we have

\[
E_t \left[ \pi_{\tau+} | 0 \text{ at } \tau \right] = E \left[ \lambda^{-1} B_T^{-\gamma} e^{-\gamma \hat{\theta}_r (T-\tau)} e^{-\mu \Gamma \gamma (1+\gamma) \sigma^2} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 | 0 \text{ at } \tau \right]
= \lambda^{-1} e^{-\mu \Gamma \gamma (1+\gamma) \sigma^2} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 e^{-\hat{\theta}_0} E_t \left[ e^{-\gamma \hat{\theta}_r (T-\tau)} | 0 \text{ at } \tau \right]
= \lambda^{-1} e^{-\mu \Gamma \gamma (1+\gamma) \sigma^2} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 e^{-\hat{\theta}_0} \times
E_t \left[ \gamma \left( \pi_{\tau+} + \left[ \tau e^{-\gamma \hat{\theta}_r (T-\tau)} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 \right] \right] \right] \times \int_{-\infty}^{\infty} e^{-\gamma \left( \pi_{\tau+} + \left[ \tau e^{-\gamma \hat{\theta}_r (T-\tau)} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 \right] \right]} f \left( \hat{\theta}_r | 0 \text{ at } \tau \right) d\hat{\theta}_r
\]

The result follows from comparing the terms in Equations (36) and (A3) with the ones above, and defining in this proposition \( \mu_0^0 = \hat{\theta}_t \) and \( \sigma_{g,0}^2 = \tilde{\sigma}_T^2 \). Q.E.D.

**Proof of Proposition 3:** The result follows from an application of Ito's Lemma to equation (36), and recalling that \( \pi_t \) is a martingale, and thus \( E_t [d\pi_t / \pi_t] = 0 \). Q.E.D.

**Proof of Corollary 3:** From property 1 in the proof of Corollary 2, for a given distribution of \( \tilde{c}^n \), we have \( p^i_t \to 1 \) as \( \tilde{g}_t \to \infty \). It follows that the state price density converges to one that assigns zero probability to a policy change:

\[
\pi_t \to \Omega(S_t) = E_t \left[ \pi_{\tau+} | 0 \text{ at } n \right] = \lambda^{-1} E_t \left[ B_T^{-\gamma} | 0 \text{ at } n \right]
= \lambda^{-1} B_T^{-\gamma} e^{-\gamma \hat{\theta}_r (T-t)} e^{-\mu \Gamma \gamma (1+\gamma) \sigma^2} (T-t) + \frac{\gamma}{2} (T-t)^2 \sigma^2 e^{-\hat{\theta}_0}
\]

Since this state price density does not depend on any \( \tilde{c}_t^n \), we have \( \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial \tilde{c}_t^n} = 0 \). Q.E.D.

**Proof of Proposition 4:** The proof is identical to the proof of Proposition 2, except that we have to calculate

\[
E_t \left[ \pi_{\tau+} M_{\tau+}^i \right] = \sum_{n=0}^{N} p^n_i E_t \left[ \pi_{\tau+} M_{\tau+}^i | n \text{ at } \tau \right]
\]

From (B9), for \( n = 1, \ldots, N \):

\[
E_t \left[ \pi_{\tau+} M_{\tau+}^i | n \text{ at } \tau \right] = \lambda^{-1} E_t \left[ N_{\tau+}^i | n \text{ at } \tau \right]
= \lambda^{-1} E_t \left[ B_T^{-\gamma} B_{\tau+}^i \times e^{-\mu \Gamma \gamma (1+\gamma) \sigma^2} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 n | n \text{ at } \tau \right]
= \lambda^{-1} e^{(1-\gamma) \mu \Gamma \gamma (1+\gamma) \sigma^2} (T-\tau) + \frac{\gamma}{2} (T-\tau)^2 \sigma^2 n e^{-\hat{\theta}_0} E_t \left[ e^{-\gamma \hat{\theta}_r + \hat{b}_t} | n \text{ at } \tau \right]
\]

Now, recall

\[
\frac{B_t^i}{B_t} = \frac{B_t}{B_t} e^{-\frac{\gamma}{2} (T-\tau) + \sigma (Z_t^i - Z_t^i)}
\]  

(B19)
which implies
\[ e^{h_t^i} = e^{b_t^i + b_t - \frac{1}{2} \sigma^2_{\pi_t} (T - \tau) + \sigma_1 (Z_t^i - Z_i^t)} \] 
(B20)

For \( n = 1, \ldots, N \), we then have:
\[
E_t \left[ \pi_{\tau+} M_{\tau+}^i | \pi_t \right] = \lambda^{-1} B_t^{-\gamma} \pi^i_t (T - \tau) e^{\left(1 - \gamma\right) \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2 \left( T - \tau \right) + \frac{1}{2} \left( \frac{1}{2} - 2 \right) \left( T - \tau \right)^2 \sigma_{\pi_t}^2} E_t \left[ e^{(1 - \gamma) \Delta b_{\tau}} | n \right. at \tau \left. \right]\]
\[
= \lambda^{-1} B_t^{-\gamma} \pi^i_t (T - \tau) e^{\left(1 - \gamma\right) \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2 \left( T - \tau \right) + \frac{1}{2} \left( \frac{1}{2} - 2 \right) \left( T - \tau \right)^2 \sigma_{\pi_t}^2} \int e^{(1 - \gamma) \Delta b_{\tau}} f \left( \Delta b_{\tau}, S_t, n \right) d\Delta b_{\tau}
\]

Similarly, for \( n = 0 \), we have:
\[
E_t \left[ \pi_{\tau+} M_{\tau+}^i | 0 \right. at \tau \left. \right] = \lambda^{-1} E_t \left[ N_{\pi_{\tau+}}^i | 0 \right. at \tau \left. \right]\]
\[
= \lambda^{-1} E_t \left[ B_t^{-\gamma} B_t^i \times e^{\left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) e^{\left(1 - \gamma\right) \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2 \left( T - \tau \right) + \frac{1}{2} \left( \frac{1}{2} - 2 \right) \left( T - \tau \right)^2 \sigma_{\tilde{g}_{\tau}}^2} E_t \left[ e^{-\gamma b_{\tau} + b_0^t + \left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau)} | 0 \right. at \tau \left. \right]\]
\[
= \lambda^{-1} e^{\left(1 - \gamma\right) \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2 \left( T - \tau \right) + \frac{1}{2} \left( \frac{1}{2} - 2 \right) \left( T - \tau \right)^2 \sigma_{\tilde{g}_{\tau}}^2} E_t \left[ \left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) \right] E_t \left[ \left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) \right]
\]
\[
\times E_t \left[ e^{\left(1 - \gamma\right) \left( E_t [\Delta b_{\tau}] + [\tilde{g}_{\tau} - E_t [\tilde{g}_{\tau}]] \sqrt{\frac{\lambda \sigma_{\tilde{g}_{\tau}}^2}{\gamma}} \right) + \left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) | n \right. at \tau \left. \right]
\]
\[
= \lambda^{-1} B_t^{-\gamma} B_t^i e^{\left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) + \left(1 - \gamma\right) \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2 \left( T - \tau \right) + \frac{1}{2} \left( \frac{1}{2} - 2 \right) \left( T - \tau \right)^2 \sigma_{\tilde{g}_{\tau}}^2}
\]
\[
\times \int e^{\left(1 - \gamma\right) \left( E_t [\Delta b_{\tau}] + [\tilde{g}_{\tau} - E_t [\tilde{g}_{\tau}]] \sqrt{\frac{\lambda \sigma_{\tilde{g}_{\tau}}^2}{\gamma}} \right) + \left(1 - \gamma\right) \tilde{g}_{\tau} (T - \tau) | n \right. at \tau \left. \right) f \left( \tilde{g}_{\tau} | S_t, n \right) d\tilde{g}_{\tau}
\]

The result follows from comparing the terms in Equations (41) and (A4) with the ones above, and defining in this proposition \( \mu^0_{\gamma} = \tilde{g}_{\tau} \) and \( \sigma_{\tilde{g}_{\tau}, 0}^2 = \sigma_{\tau}^2 \). Q.E.D.

**Proof of Proposition 5.** The claim follows from an application of Ito’s Lemma to the price \( M_t^i \) in Proposition 4, and the equilibrium restriction \( \mu^i_M = -\text{Cov}_t \left( \frac{dM^i_t}{M^i_t}, \frac{d\pi_t}{\pi_t} \right) \). Q.E.D.

**Proof of Proposition 6.** From Lemmas A1 and A2, the gross announcement return from announcing policy \( n \) is
\[
1 + R^n (\tilde{g}_{\tau}) = e^{\left(\mu^n - \tilde{g}_{\tau}\right) (T - \tau) + \frac{1}{2} \sigma^n_{\tilde{g}_{\tau}} (T - \tau)^2 \left( \sigma_{\tilde{g}_{\tau}, n}^2 - \sigma_{\tilde{g}_{\tau}}^2 \right)} \times
\]
\[
\left( \frac{1 + \sum_{n=1}^{N} P^n_{\tau} \left( (1 - \gamma) \left( \mu^n_{\gamma} - \tilde{g}_{\tau} \right) (T - \tau) + \frac{1}{2} \sigma^n_{\tilde{g}_{\tau}} (T - \tau)^2 \left( \sigma_{\tilde{g}_{\tau}, n}^2 - \sigma_{\tilde{g}_{\tau}}^2 \right) - 1 \right) }{1 + \sum_{n=1}^{N} P^n_{\tau} \left( (1 - \gamma) \left( \mu^n_{\gamma} - \tilde{g}_{\tau} \right) (T - \tau) + \frac{1}{2} \sigma^n_{\tilde{g}_{\tau}} (T - \tau)^2 \left( \sigma_{\tilde{g}_{\tau}, n}^2 - \sigma_{\tilde{g}_{\tau}}^2 \right) - 1 \right) } \right)
\]
Similarly, recalling the notation $\mu_g^n = \tilde{g}_r$ and $\sigma_{g,0} = \hat{g}_r$, from Lemma A1 and A2 the gross announcement return from announcing no policy change is

$$1 + R^0 (\tilde{g}_r) = \frac{\left( 1 + \sum_{n=1}^N p^n_r \left( e^{-\gamma (\mu_g^n - \tilde{g}_r) (T - \tau) + \frac{\sigma_{g,n}^2}{2} (T - \tau)^2 (\sigma_{g,n}^2 - \tilde{g}_r^2) - 1} \right) \right)}{\left( 1 + \sum_{n=1}^N p^n_r \left( e^{(1-\gamma) (\mu_g^n - \tilde{g}_r) (T - \tau) + \frac{\sigma_{g,n}^2}{2} (T - \tau)^2 (\sigma_{g,n}^2 - \tilde{g}_r^2) - 1} \right) \right)}$$

(B21)

Therefore, we can write more compactly, for $n = 1, \ldots, N$,

$$1 + R^n (\tilde{g}_r) = e^{(\mu_g^n - \tilde{g}_r) (T - \tau) + \frac{1}{2} \frac{\sigma_{g,n}^2}{\tau} (T - \tau)^2 (\sigma_{g,n}^2 - \tilde{g}_r^2)} \times (1 + R^0 (\tilde{g})) \quad (B22)$$

We can express all the formulas in terms of $\tilde{\mu}^n$ and $x_\tau$. Using the definitions

$$\tilde{\mu}^n + \frac{\sigma_{g,n}^2}{2} (T - \tau) (\gamma - 1) = \mu_g^n \quad (B23)$$

$$x_\tau + \frac{\sigma_{g}^2}{2} (T - \tau) (\gamma - 1) = \tilde{g}_r \quad (B24)$$

we have

$$(\mu^n - \tilde{g}_r) = (\tilde{\mu}^n - x_\tau) + \frac{(\sigma_{g,n}^2 - \tilde{\sigma}_r^2)}{2} (T - \tau) (\gamma - 1) \quad (B25)$$

The claim of Proposition 6 then follows quickly. Q.E.D.

**Proof of Corollary 4.** Immediate from Proposition 6. Q.E.D.

**Proof of Corollary 5.** Immediate from Corollary 4: for any two policies $n$ and $m$ with $\tilde{\mu}^n = \tilde{\mu}^m$, the result follows from equation (31). Q.E.D.

**Proof of Proposition 7.** The expression for the jump risk premium follows immediately from

$$J (S_r) = \sum_{n=1}^N p^n_r R^n (x_\tau)$$

where $R^n (x_\tau)$ are given in Proposition 6. We now see that

$$J (S_r) = -\text{Cov}_\tau \left( \frac{M^n_{r+}}{M^n_r} - 1, \frac{\pi^{r+}}{\pi^r} - 1 \right) = - \{ E_r [J_M J_\pi] - E_r [J_M] E_r [J_\pi] \}$$

where, if policy $n$ is chosen, we denote $J^n_M = \frac{M^n_{r+}}{M^n_r}$ and $J^n_\pi = \frac{\pi^{r+}}{\pi^r}$. Recall from Proposition 6 that

$$J^n_M = 1 + R^n (x_\tau)$$

$$= e^{(\tilde{\mu}^n - x_\tau) (T - \tau) - \frac{1}{2} (T - \tau)^2 (\sigma_{g,n}^2 - \tilde{\sigma}_r^2)} \times \left( 1 + \sum_{\kappa=1}^N p^n_\kappa \left( e^{-\gamma (T - \tau) (\tilde{\mu}^n - x_\tau) + \frac{\sigma_{g,n}^2}{2} (T - \tau)^2 (\sigma_{g,n}^2 - \tilde{\sigma}_r^2) - 1} \right) \right)$$

$$\left( 1 + \sum_{n=1}^N p^n_r \left( e^{(1-\gamma) (T - \tau) (\tilde{g}^n - x_\tau) - 1} \right) \right)$$
We can compute a similar expression now for the stochastic discount factor. From the expressions for $\pi_{\tau+}$ and $\pi_{\tau}$ in the proof of Lemma A1, it follows that for $n = 1, \ldots, N$

$$J^n_{\pi}(x_{\tau}) = \frac{\pi^n_{\tau+}}{\pi_{\tau}} = e^{-\gamma(\bar{\mu}^n - x_{\tau})(T-\tau) + \frac{1}{2}(\sigma^2_{g,n} - \tilde{\sigma}^2)(T-\tau)^2} J^0_{\pi}(x_{\tau})$$

where

$$J^0_{\pi}(x_{\tau}) = \frac{\pi^0_{\tau+}}{\pi_{\tau}} = \frac{1}{1 + \sum_{k=1}^{N} p_{T}^k (e^{-\gamma(\bar{\mu}^0 - x_{\tau})(T-\tau) + \frac{1}{2}(\sigma^2_{g,n} - \tilde{\sigma}^2)(T-\tau)^2} - 1))}$$

This implies that

$$J^n_{\pi}(x_{\tau}) J^n_{M}(x_{\tau}) = \frac{e^{(1-\gamma)(\bar{\mu}^n - x_{\tau})(T-\tau)}}{1 + \sum_{k=1}^{N} p_{T}^k (e^{(1-\gamma)(T-\tau)(\bar{\mu}^n - x_{\tau})} - 1))} \quad \text{for } n = 1, \ldots, N$$

$$J^0_{\pi}(x_{\tau}) J^0_{M}(x_{\tau}) = \frac{1}{1 + \sum_{k=1}^{N} p_{T}^k (e^{(1-\gamma)(T-\tau)(\bar{\mu}^n - x_{\tau})} - 1))}$$

It follows that

$$E_{T} [J^n_{\pi}(x_{\tau}) J^n_{M}(x_{\tau})] = \sum_{k=1}^{N} p_{T}^k \left\{ \frac{e^{(1-\gamma)(\bar{\mu}^n - x_{\tau})(T-\tau)}}{1 + \sum_{k=1}^{N} p_{T}^k (e^{(1-\gamma)(T-\tau)(\bar{\mu}^n - x_{\tau})} - 1))} \right\}
+ \left(1 - \sum_{k=1}^{N} p_{T}^k \right) \frac{1}{1 + \sum_{k=1}^{N} p_{T}^k (e^{(1-\gamma)(T-\tau)(\bar{\mu}^n - x_{\tau})} - 1))}$$

$$= \frac{1}{1 + \sum_{k=1}^{N} p_{T}^k (e^{(1-\gamma)(\bar{\mu}^n - x_{\tau})(T-\tau)} + 1 - \sum_{k=1}^{N} p_{T}^k)}$$

Similarly,

$$E_{T} [J^n_{\pi}(x_{\tau})] = \sum_{k=1}^{N} p_{T}^k \left\{ \frac{e^{-\gamma(\bar{\mu}^n - x_{\tau})(T-\tau) + \frac{1}{2}(\sigma^2_{g,n} - \tilde{\sigma}^2)(T-\tau)^2}}{e^{-\gamma(\mu_{\tau+} - x_{\tau})(T-\tau) + \frac{1}{2}(\sigma^2_{g,n} - \tilde{\sigma}^2)(T-\tau)^2} J^0_{\pi}(x_{\tau})} \right\}
+ \left(1 - \sum_{k=1}^{N} p_{T}^k \right) J^0_{\pi}(x_{\tau})$$

$$= \left[ \sum_{k=1}^{N} p_{T}^k e^{-\gamma(\bar{\mu}^n - x_{\tau})(T-\tau) + \frac{1}{2}(\sigma^2_{g,n} - \tilde{\sigma}^2)(T-\tau)^2} + 1 - \sum_{k=1}^{N} p_{T}^k \right] J^0_{\pi}(x_{\tau})$$

$$= 1$$

Thus, we finally obtain

$$J(x_{\tau}) = -Cov_{\tau} (J_M, J_{\pi}) = - \{ E_{T} [J_M J_{\pi}] - E_{T} [J_M] E_{T} [J_{\pi}] \}
= E_{T} [J_M] - 1$$
Q.E.D.

REFERENCES