

Internet Appendix for “Political Cycles and Stock Returns”

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This Internet Appendix provides the proofs of all propositions in Pástor and Veronesi (2017).

Proof of Proposition 1. (a) Government workers. Consider an agent who decides at time t to be a government worker. From the government’s budget equation, for a given tax rate τ , total tax receipts available at time $t + 1$ are given by

$$\text{tax}_{t+1} = \tau \int_{j \in I_t} Y_{j,t+1} dj = \tau \left(\int_{j \in I_t} e^{\mu_j + \varepsilon_{j,t+1} + \varepsilon_{t+1}} dj \right) G_t = \tau G_t e^{\varepsilon_{t+1}} m_t E [e^{\mu_j} | j \in I_t] ,$$

where we used the law of large numbers

$$\int_{j \in I_t} e^{\mu_j + \varepsilon_{j,t+1}} dj = m_t E [e^{\mu_j + \varepsilon_{j,t+1}} | j \in I_t] = m_t E [e^{\mu_j} | j \in I_t] E_t [e^{\varepsilon_{j,t+1}} | j \in I_t] = m_t E [e^{\mu_j} | j \in I_t] .$$

Exploiting the balanced budget restriction, the consumption of a government worker is

$$C_{it+1}^{no} = \frac{\tau G_t e^{\varepsilon_{t+1}} m_t E [e^{\mu_j} | j \in I_t]}{1 - m_t} . \quad (\text{A1})$$

When $\gamma_t \neq 1$, the expected one-period utility at the time of the voting decision is

$$E_t [U (C_{it+1}^{no}) | \tau] = \frac{\tau^{1-\gamma_t}}{1 - \gamma_t} G_t^{1-\gamma_t} E_t [e^{(1-\gamma_t)\varepsilon_{t+1}}] E_t [e^{\mu_j} | j \in I_t]^{1-\gamma_t} \left(\frac{m_t}{1 - m_t} \right)^{1-\gamma_t} . \quad (\text{A2})$$

We immediately see that

$$E_t [U (C_{it+1}^{no}) | \tau^H] > E_t [U (C_{it+1}^{no}) | \tau^L]$$

if and only if

$$\tau^H > \tau^L .$$

Similarly, when $\gamma_t = 1$, then the utility function is log, and we obtain

$$E_t [U (C_{it+1}^{no}) | \tau] = \log (\tau) + E_t [\log [G_t e^{\varepsilon_{t+1}} m_t E [e^{\mu_j} | j \in I_t]]] - \log (1 - m_t) ,$$

so that the conclusion holds.

(b) **Entrepreneurs.** Consider now the consumption of an entrepreneur, under the assumption that m_t agents decide to be entrepreneurs in equilibrium. Each entrepreneur i sells $1 - \theta$ shares and retains θ shares of his own company. All shares are one-period claims to the next-period dividend, net of taxes:

$$M_{i,t} = E_t \left[\frac{\pi_{t+1}}{\pi_t} Y_{i,t+1} (1 - \tau_t) \right] ,$$

where π_t is the equilibrium state price density and τ_t is the tax rate decided at the election at time t . Each entrepreneur uses the shares sold at time t to purchase claims from other entrepreneurs. Let N_t^{ij} denote the fraction of firm j purchased by entrepreneur i at time t and let N_{it}^0 be the entrepreneur's (long or short) position in the bond. The entrepreneur's budget constraint is

$$(1 - \theta) M_{it} = \int_{j \neq i} N_t^{ij} M_{jt} dj + N_{it}^0 ,$$

where we normalize the price of bonds to one. Since there is no intertemporal consumption/saving choice, the value of a bond at time t is indeterminate. We assume it is equal to one and acts as the numeraire. If agent i chooses to be an entrepreneur, his consumption at time $t + 1$ (for given τ) is

$$C_{it+1} = \theta Y_{i,t+1} (1 - \tau_t) + \int_{j \in I} N_t^{ij} Y_{j,t+1} (1 - \tau_t) dj + N_{it}^0 .$$

From Proposition A1 below, $N_{it}^0 = 0$ and $N_t^{ij} = (1 - \theta) \frac{e^{\mu_i}}{\int_{k \in I} e^{\mu_k} di}$, so that

$$C_{it+1} = (1 - \tau) G_t e^{\mu_i} e^{\varepsilon_{t+1}} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)] . \quad (\text{A3})$$

Then, for $\gamma_t \neq 1$, the expected utility of an entrepreneur is

$$\begin{aligned} E_t [U (C_{i,t+1}^{yes}) | \tau] &= \frac{(1 - \tau)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1 - \gamma_t} E_t [e^{(1-\gamma_t)(\varepsilon_{t+1})} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma_t}] \\ &= \frac{(1 - \tau)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1 - \gamma_t} E_t [e^{(1-\gamma_t)(\varepsilon_{t+1})}] E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma_t}] . \end{aligned}$$

We clearly have

$$E_t [U (C_{i,t+1}^{yes}) | \tau^L] > E_t [U (C_{i,t+1}^{yes}) | \tau^H]$$

if and only if

$$\tau^L < \tau^H .$$

Similarly, if risk aversion $\gamma_t = 1$, then

$$E_t [U (C_{i,t+1}^{yes}) | \tau] = \log (1 - \tau) + \log [G_t e^{\mu_i}] + E_t [\log [e^{\varepsilon_{t+1}} (\theta e^{\varepsilon_{i,t+1}} + (1 - \theta))]]$$

and the same conclusion holds. Q.E.D.

Proof of Proposition 2. The argument is analogous to that in Pástor and Veronesi (2016). Consider $\gamma \neq 1$, tax rate τ^k , and let I^k be the equilibrium set of entrepreneurs and m^k be the equilibrium mass of entrepreneurs. For any agent i ,

$$V_t^{i,yes} > V_t^{i,no}$$

if and only if

$$E_t [U(C_{it+1}^{yes}) | \tau^k, m^k] > E_t [U(C_{it+1}^{no}) | \tau^k, m^k] .$$

Using expressions (A2) and (A3), we obtain

$$\begin{aligned} & \frac{(1 - \tau^k)^{1-\gamma_t} G_t^{1-\gamma_t} e^{(1-\gamma_t)\mu_i}}{1 - \gamma_t} E_t [e^{(1-\gamma_t)\varepsilon_{t+1}}] E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma_t}] \\ & > \frac{1}{1 - \gamma_t} (\tau^k)^{1-\gamma_t} G_t^{1-\gamma_t} E [e^{\mu_j} | j \in I^L]^{1-\gamma_t} \left(\frac{m^k}{1 - m^k} \right)^{1-\gamma_t} E_t [e^{(1-\gamma_t)\varepsilon_{t+1}}] . \end{aligned}$$

Deleting common terms, taking logs, and re-arranging, we obtain

$$\begin{aligned} & \mu_i + \frac{1}{1 - \gamma_t} \log (E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma_t}]) \\ & > \log \left(\frac{\tau^k}{1 - \tau^k} \right) + \log (E [e^{\mu_j} | j \in I^L]) + \log \left(\frac{m^k}{1 - m^k} \right) , \end{aligned}$$

or

$$\begin{aligned} \mu_i & > \underline{K}^k = \log \left(\frac{\tau^k}{1 - \tau^k} \right) + \log (E [e^{\mu_j} | j \in I^L]) + \log \left(\frac{m^k}{1 - m^k} \right) \\ & \quad - \frac{1}{1 - \gamma_t} \log (E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma_t}]) . \end{aligned}$$

We now derive m^k and $E [e^{\mu_j} | j \in I^L]$. From the definition of m^k and the distribution of skill $\mu_i \sim N(\bar{\mu}, \sigma_\mu^2)$, we obtain

$$m_t^k = \int_{\underline{K}^k}^{\infty} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) d\mu_i = 1 - \Phi(\underline{K}^k; \bar{\mu}, \sigma_\mu^2) .$$

In addition,

$$\begin{aligned} E [e^{\mu_j} | j \in I^k] & = \frac{1}{m_t^k} \int_{\underline{K}^k}^{\infty} e^{\mu_j} \phi(\mu_j; \bar{\mu}, \sigma_\mu^2) d\mu_j \\ & = \frac{e^{\bar{\mu} + \frac{1}{2}\sigma_\mu^2} (1 - \Phi(\underline{K}^k; \bar{\mu} + \sigma_\mu^2, \sigma_\mu^2))}{m_t^k} . \end{aligned} \tag{A4}$$

Therefore, substituting in the expression for \underline{K}^k , we obtain

$$\begin{aligned} \underline{K}^k &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \bar{\mu} + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi(\underline{K}^k; \bar{\mu} + \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}^k; \bar{\mu}, \sigma_\mu^2)}\right) \\ &\quad - \frac{1}{1-\gamma_t} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma_t}) . \end{aligned}$$

Define

$$\underline{\mu}^k = \underline{K}^k - \bar{\mu}$$

and exploit the properties of the normal distribution to obtain

$$\begin{aligned} \underline{K}^k - \bar{\mu} &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi(\underline{K}^k - \bar{\mu}; \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{K}^k - \bar{\mu}; 0, \sigma_\mu^2)}\right) \\ &\quad - \frac{1}{1-\gamma_t} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma_t}) , \end{aligned}$$

or

$$\begin{aligned} \underline{\mu}^k &= \log\left(\frac{\tau^k}{1-\tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1-\Phi(\underline{\mu}^k; \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}^k; 0, \sigma_\mu^2)}\right) \\ &\quad - \frac{1}{1-\gamma_t} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma_t}) , \end{aligned}$$

which defines the equation to solve.

Finally,

$$m_t^k = \int_{\underline{K}^k}^{\infty} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) d\mu_i = 1 - \Phi(\underline{K}^k - \bar{\mu}; 0, \sigma_\mu^2) = 1 - \Phi(\underline{\mu}^k; 0, \sigma_\mu^2) .$$

When $\gamma_t = 1$, we instead have that

$$E_t[U(C_{it+1}^{yes}) | \tau^k, m^k] > E_t[U(C_{it+1}^{no}) | \tau^k, m^k]$$

holds if and only if

$$\begin{aligned} &\log(1-\tau^k) + E_t[\log[G_t e^{\mu_i} e^{\varepsilon_{i,t+1}} (\theta e^{\varepsilon_{i,t+1}} + (1-\theta))]] \\ &> \log(\tau^k) + E_t\left[\log\left[\frac{G_t e^{\varepsilon_{i,t+1}} m_t^k E[e^{\mu_j} | j \in I_t]}{1-m_t^k}\right]\right] . \end{aligned}$$

Deleting common terms and re-arranging, we find

$$\begin{aligned} \mu_i &> \underline{K}^k = \log\left(\frac{\tau^k}{1-\tau^k}\right) + \log\left[\frac{m_t^k}{1-m_t^k}\right] + \log[E_t[e^{\mu_j} | j \in I_t]] \\ &\quad - E_t[\log(\theta e^{\varepsilon_{i,t+1}} + (1-\theta))] . \end{aligned}$$

The same argument as above establishes $m_t^k = 1 - \Phi(\underline{\mu}^k; 0, \sigma_\mu^2)$, where

$$\begin{aligned} \underline{\mu}^k = & \log\left(\frac{\tau^k}{1 - \tau^k}\right) + \frac{1}{2}\sigma_\mu^2 + \log\left(\frac{1 - \Phi(\underline{\mu}^k; \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}^k; 0, \sigma_\mu^2)}\right) \\ & - E_t[\log(\theta e^{\varepsilon_{i,t+1}} + (1 - \theta))] . \end{aligned}$$

Q.E.D.

Corollary 1. The equilibrium mass of entrepreneurs m_t^k is decreasing in (a) the tax rate τ^k , (b) risk aversion γ_t (for $\gamma_t > 1$) (c) idiosyncratic volatility σ_1 , and (d) the degree of market incompleteness θ .

Proof of Corollary 1: (a) The implicit function defined by $\underline{\mu}^k = F(\tau^k, \underline{\mu}^k)$ is clearly increasing in τ^k as we can see by taking the total derivative

$$d\underline{\mu}^k = \frac{\partial F(\tau^k, \underline{\mu}^k)}{\partial \tau^k} d\tau^k + \frac{\partial F(\tau^k, \underline{\mu}^k)}{\partial \underline{\mu}^k} d\underline{\mu}^k .$$

From the total derivative, one obtains the implicit function theorem

$$\frac{d\underline{\mu}^k}{d\tau^k} = \frac{\frac{\partial F(\tau^k, K^k)}{\partial \tau^k}}{1 - \frac{\partial F(\tau^k, K^k)}{\partial K^k}} > 0 ,$$

as $\partial F(\tau^k, \underline{\mu}^k) / \partial \tau^k > 0$ and $\partial F(\tau^k, \underline{\mu}^k) / \partial \underline{\mu}^k < 0$. That is, higher taxes increase the threshold and decrease the mass of entrepreneurs $m^k = 1 - \Phi(\underline{\mu}^k, 0, \sigma_\mu^2)$.

(b) First, consider the function

$$U(\gamma) = \frac{1}{1 - \gamma} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma}) .$$

The first derivative is

$$U'(\gamma) = \frac{1}{(1 - \gamma)^2} \log(E[(\theta(e^\varepsilon - 1) + 1)^{1-\gamma}]) - \frac{1}{1 - \gamma} \frac{E[(\theta(e^\varepsilon - 1) + 1)^{1-\gamma} \log((\theta(e^\varepsilon - 1) + 1))]}{E[(\theta(e^\varepsilon - 1) + 1)^{1-\gamma}]} .$$

Define $X = (\theta(e^\varepsilon - 1) + 1)^{1-\gamma}$ for convenience, and factor out $1/(1 - \gamma)^2 > 0$ to obtain

$$U'(\gamma) = \frac{1}{(1 - \gamma)^2} \left[\log(E[X]) - \frac{E[X \log(X)]}{E[X]} \right] .$$

As $X > 0$ and the function $f(X)$ is convex ($f''(X) = 1/X > 0$), from Jensen's inequality we have $E[X \log(X)] > E[X] \log(E[X])$. Therefore,

$$\begin{aligned} U'(\gamma) &= \frac{1}{(1-\gamma^2)} \left[\log(E[X]) - \frac{E[X \log(X)]}{E[X]} \right] \\ &< \frac{1}{(1-\gamma^2)} \left[\log(E[X]) - \frac{E[X] \log(E[X])}{E[X]} \right] = 0. \end{aligned}$$

Note that this proof holds for any $\gamma \neq 1$.

We now define now the implicit function $\underline{\mu}^k = F(\gamma, \underline{\mu}^k)$ where we emphasize γ rather than τ . We then have

$$d\underline{\mu}^k = \frac{\partial F(\gamma, \underline{\mu}^k)}{\partial \gamma} d\gamma + \frac{\partial F(\gamma, \underline{\mu}^k)}{\partial \underline{\mu}^k} d\underline{\mu}^k.$$

From the total derivative, one obtains the implicit function theorem

$$\frac{d\underline{\mu}^k}{d\gamma} = \frac{\frac{\partial F(\gamma, \underline{\mu}^k)}{\partial \gamma}}{1 - \frac{\partial F(\gamma, \underline{\mu}^k)}{\partial \underline{\mu}^k}} > 0,$$

as we have shown $\partial F(\gamma, \underline{\mu}^k) / \partial \gamma = -U'(\gamma) > 0$ and $\partial F(\gamma, \underline{\mu}^k) / \partial \underline{\mu}^k < 0$. That is, higher risk aversion increases the threshold $\underline{\mu}^k$. Hence

$$\frac{d\underline{\mu}(\tau_k, \gamma)}{d\gamma} > 0.$$

Thus, higher γ_t decreases the mass of entrepreneurs $m_t^k = 1 - \Phi(\underline{\mu}^k, 0, \sigma_\mu^2)$, that is,

$$\frac{dm^k}{d\gamma} < 0.$$

(c) Let

$$U(\theta) = \frac{1}{1-\gamma} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma}).$$

Then

$$\begin{aligned} U'(\theta) &= \frac{1}{1-\gamma} \frac{1}{E[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma}} E[(1-\gamma) [\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{-\gamma} (e^{\varepsilon_{i,t+1}} - 1)] \\ &= \frac{E[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma} (e^{\varepsilon_{i,t+1}} - 1)}{E[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{1-\gamma}} < 0, \end{aligned}$$

which holds if and only if

$$E[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma} (e^{\varepsilon_{i,t+1}} - 1) < 0,$$

which holds if and only if

$$\text{Cov} \left[[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma}, (e^{\varepsilon_{i,t+1}} - 1) \right] + E \left[[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma} \right] E [e^{\varepsilon_{i,t+1}} - 1] < 0 ,$$

which holds if and only if

$$\text{Cov} \left[[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma}, e^{\varepsilon_{i,t+1}} - 1 \right] < 0 .$$

Because $[\theta (e^{\varepsilon_{i,t+1}} - 1) + 1]^{-\gamma}$ is decreasing in $\varepsilon_{i,t+1}$ and $(e^{\varepsilon_{i,t+1}} - 1)$ is increasing in $\varepsilon_{i,t+1}$, the result follows.

(d) Consider

$$U(\sigma) = \frac{1}{1-\gamma} \log \left(E \left[[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma} \right] \right)$$

(now as a function of σ). We want to show that as σ increases, $U(\sigma)$ decreases. Define $X = \theta e^{\varepsilon_{i,t+1}} + (1-\theta) = \theta (e^{\varepsilon_{i,t+1}} - 1) + 1$, so that

$$U(\sigma) = \frac{1}{1-\gamma} \log \left(E \left[[X]^{1-\gamma} \right] \right) .$$

Because this is a concave function of X , the result is shown if the cdf $F_X(x; \sigma_1)$ is a mean-preserving spread of $F_X(x; \sigma_0)$ with $\sigma_1 > \sigma_0$. First, note that $E[X] = 1$. Consider now the cdf $F_X(x; \sigma) = \Pr(X < x) = \Pr(\theta (e^{\varepsilon_{i,t+1}} - 1) + 1 < x) = \Pr(\varepsilon_{i,t+1} < \log(1 + \frac{x-1}{\theta}))$. Let $\eta_{it} \sim N(0, 1)$ so that $\varepsilon_{it} = -\frac{1}{2}\sigma^2 + \sigma\eta_{it}$. Thus

$$\begin{aligned} F_X(x; \sigma) &= \Pr \left(-\frac{1}{2}\sigma^2 + \sigma\eta_{it} < \log \left(1 + \frac{x-1}{\theta} \right) \right) \\ &= \Pr \left(\eta_{it} < \frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1-\theta)}{\theta} \right) \right) \\ &= \Phi \left(\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1-\theta)}{\theta} \right) \right) \\ &= \int_{-\infty}^{\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1-\theta)}{\theta} \right)} \frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{2\pi}} d\eta . \end{aligned}$$

We now show that if $\sigma_1 > \sigma_0$, then $F_X(x; \sigma_1, \theta)$ is a mean-preserving spread of $F_X(x; \sigma_0, \theta)$. We already know the two distributions have the same mean. Therefore, the claim is shown if for every x' ,

$$\int^{x'} F_X(x; \sigma_1) dx > \int^{x'} F_X(x; \sigma_0) dx ,$$

that is, if for every x' , the function

$$H(\sigma; x') = \int^{x'} \Phi \left(\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1-\theta)}{\theta} \right) \right) dx$$

is increasing in σ . We can write

$$\begin{aligned} H(\sigma; x') &= \int^{x'} \Phi \left(\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1 - \theta)}{\theta} \right) \right) dx \\ &= \int^{x'} \int_{-\infty}^{\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1 - \theta)}{\theta} \right)} \frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{2\pi}} d\eta dx . \end{aligned}$$

Therefore,

$$\frac{\partial H(\sigma; x')}{\partial \sigma} = \int^{x'} \frac{e^{-\frac{1}{2} \left(\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1 - \theta)}{\theta} \right) \right)^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2} \log \left(\frac{x - (1 - \theta)}{\theta} \right) \right) dx .$$

Consider the change of variable

$$\varepsilon = \frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1 - \theta)}{\theta} \right) ,$$

so that

$$d\varepsilon = \frac{1}{\sigma} \frac{1}{x - (1 - \theta)} dx ,$$

or

$$(x - (1 - \theta)) \sigma d\varepsilon = dx .$$

Moreover,

$$\theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} = (x - (1 - \theta)) ,$$

which implies

$$\theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \sigma d\varepsilon = dx$$

and

$$\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1 - \theta) = x' .$$

Thus,

$$\begin{aligned} \frac{\partial H(\sigma; x')}{\partial \sigma} &= \int^{x'} \frac{e^{-\frac{1}{2} \left(\frac{1}{2}\sigma + \frac{1}{\sigma} \log \left(\frac{x - (1 - \theta)}{\theta} \right) \right)^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2} \log \left(\frac{x - (1 - \theta)}{\theta} \right) \right) dx \\ &= \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1 - \theta)} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma^2} \log \left(e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \right) \right) \theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \sigma d\varepsilon \\ &= \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1 - \theta)} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} \left(\frac{1}{2} - \frac{1}{\sigma} \left(\varepsilon - \frac{1}{2}\sigma \right) \right) \theta e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \sigma d\varepsilon \\ &= \theta \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1 - \theta)} e^{\sigma\varepsilon - \frac{1}{2}\sigma^2} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}} (\sigma - \varepsilon) d\varepsilon \\ &= \theta \int^{\theta e^{\sigma\varepsilon' - \frac{1}{2}\sigma^2} + (1 - \theta)} e^{-\frac{1}{2}\sigma^2} \frac{e^{-\frac{1}{2}\varepsilon^2 + \sigma\varepsilon}}{\sqrt{2\pi}} (\sigma - \varepsilon) d\varepsilon . \end{aligned}$$

We have

$$e^{-\frac{1}{2}\varepsilon^2 + \sigma\varepsilon} = e^{-\frac{1}{2}(\varepsilon^2 - 2\sigma\varepsilon)} = e^{-\frac{1}{2}(\varepsilon^2 - 2\sigma\varepsilon + \sigma^2 - \sigma^2)} = e^{-\frac{1}{2}((\varepsilon - \sigma)^2 - \sigma^2)} = e^{-\frac{1}{2}(\varepsilon - \sigma)^2 + \frac{1}{2}\sigma^2} .$$

Therefore,

$$\frac{\partial H(\sigma)}{\partial \sigma} = \theta \int^{\theta e^{\sigma\varepsilon'} - \frac{1}{2}\sigma^2 + (1-\theta)} \frac{e^{-\frac{1}{2}(\varepsilon - \sigma)^2}}{\sqrt{2\pi}} (\sigma - \varepsilon) d\varepsilon .$$

Define

$$\begin{aligned} \eta &= \varepsilon - \sigma \\ d\eta &= d\varepsilon \\ \eta' + \sigma &= \varepsilon' \end{aligned}$$

to get

$$\frac{\partial H(\sigma; \eta')}{\partial \sigma} = -\theta \int^{\theta e^{\sigma\eta'} + \frac{1}{2}\sigma^2 + (1-\theta)} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}} \eta d\eta .$$

If $\eta' \rightarrow \infty$ then $\theta e^{\sigma\eta'} + \frac{1}{2}\sigma^2 + (1-\theta) \rightarrow \infty$ and hence

$$\frac{\partial H(\sigma; \infty)}{\partial \sigma} = -\theta \int^{\infty} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}} \eta d\eta = -E[\eta] = 0 .$$

Moreover, the function

$$L(\eta') = \frac{\partial H(\sigma; \eta')}{\partial \sigma} = -\theta \int^{\theta e^{\sigma\eta'} + \frac{1}{2}\sigma^2 + (1-\theta)} \frac{e^{-\frac{1}{2}(\eta)^2}}{\sqrt{2\pi}} \eta d\eta$$

is monotonically decreasing in η' because

$$\frac{\partial L}{\partial \eta'} = -\theta \frac{e^{-\frac{1}{2}(\theta e^{\sigma\eta'} + \frac{1}{2}\sigma^2 + (1-\theta))^2}}{\sqrt{2\pi}} (\theta e^{\sigma\eta'} + \frac{1}{2}\sigma^2 + (1-\theta)) < 0 .$$

Therefore, $\frac{\partial H(\sigma; \eta')}{\partial \sigma} > 0$ for every η' . It follows that the distribution under a higher σ is a mean-preserving spread of a distribution under a lower σ . Thus, every concave function is decreasing in σ , and so is $U(\sigma)$. Q.E.D.

Proof of Proposition 3: We know from Corollary 1 that both $\underline{\mu}(\tau^L, \gamma_t)$ and $\underline{\mu}(\tau^H, \gamma_t)$ are increasing in γ , and we have $\underline{\mu}(\tau^L, \gamma_t) < \underline{\mu}(\tau^H, \gamma_t)$. Let $\bar{\gamma}$ be such that

$$0.5 = 1 - \Phi(\underline{\mu}^L(\bar{\gamma}), 0, \sigma_\mu^2) .$$

Then, for $\gamma_t > \bar{\gamma}$, we have $m_t^L < 0.5$. Clearly, also $m_t^H < m_t^L$. So, regardless of the tax rate (H or L), the maximum mass of entrepreneurs is below 0.5, and therefore τ^L cannot win. When $\gamma_t > \bar{\gamma}$,

the unique equilibrium must be τ^H . As all agents expect this to be the case, the equilibrium mass of entrepreneurs is $m_t = m_t^H = 1 - \Phi(\underline{\mu}^H(\gamma_t), 0, \sigma_\mu^2)$.

Similarly, let $\underline{\gamma}$ such that

$$0.5 = 1 - \Phi(\underline{\mu}^H(\underline{\gamma}), 0, \sigma_\mu^2) .$$

Then for $\gamma_t < \underline{\gamma}$, we have $m_t^H > 0.5$. Clearly, also $m_t^L > m_t^H$, that is, even under high taxes, the majority of agents are entrepreneurs. Therefore, τ^H cannot win, and the unique equilibrium has τ^L . As all agents expect this to be the case, the equilibrium mass of entrepreneurs is $m_t = m_t^L = 1 - \Phi(\underline{\mu}^L(\gamma_t), 0, \sigma_\mu^2)$.

For $\underline{\gamma} < \gamma_t < \bar{\gamma}$, the above arguments imply that both equilibria can be supported. Q.E.D.

Proposition A1. In equilibrium,

1. The state price density is

$$\pi_{t+1} \propto e^{-\gamma \varepsilon_{t+1}} .$$

2. Entrepreneurs invest

$$N_t^{ij} = (1 - \theta) \frac{e^{\mu_i}}{\int_{k \in I} e^{\mu_k} di}; \quad N_{it}^0 = 0$$

in stocks and bonds, respectively.

3. Asset prices are

$$M_t^i = (1 - \tau_t) e^{\mu_i - \gamma_t \sigma^2} G_t .$$

4. The aggregate market value is

$$\begin{aligned} M_t^P &= (1 - \tau_t) e^{-\gamma_t \sigma^2} E[e^{\mu_i} | i \in I_t] G_t m_t \\ &= (1 - \tau_t) e^{-\gamma_t \sigma^2} \frac{\left(1 - \Phi\left(\underline{\mu}_t^k; \sigma_1^2, \sigma_1^2\right)\right)}{\left(1 - \Phi\left(\underline{\mu}_t^k; 0, \sigma_1^2\right)\right)} G_t m_t . \end{aligned}$$

5. The expected rate of return on each stock i is given by

$$E(R^i) = e^{\gamma_t \sigma^2} - 1 .$$

Proof of Proposition A1. The claims follow from Corollary C1 (a) - (e) in the technical appendix of Pástor and Veronesi (2016). The only difference is that $T = 1$ and total production is

multiplied by G_t , which is known at time t and therefore does not change any calculations. The expression for $E [e^{\mu_j} | j \in I]$ is in equation (A4).

Q.E.D.

Proof of Proposition 4. The proof follows from $E_t [R] = e^{\gamma_t \sigma^2} - 1$ being uniformly increasing γ_t , the fact that $\gamma_t > \bar{\gamma}$ selects a high-tax equilibrium, $\gamma_t < \underline{\gamma}$ selects a low-tax equilibrium, and for intermediate γ_t the high-tax equilibrium is selected with 50-50 chance. See the discussion following the proposition in the text.

Q.E.D.

Proof of Proposition 5. Consider the expected growth formula under tax regime k :

$$E [e^{\mu_i} | i \in I_t^k] = \frac{1 - \Phi \left(\underline{\mu}_t^k; \sigma^2, \sigma^2 \right)}{1 - \Phi \left(\underline{\mu}_t^k; 0, \sigma^2 \right)} e^{\bar{\mu} + \frac{1}{2} \sigma_\mu^2}.$$

In any H equilibrium, we must have $m_t^H = 1 - \Phi \left(\underline{\mu}_t^H; 0, \sigma^2 \right) < 0.5$ and in any L equilibrium, we must have $m_t^L = 1 - \Phi \left(\underline{\mu}_t^L; 0, \sigma^2 \right) > 0.5$. It follows that for any H and L equilibria, $\underline{\mu}_t^H > \underline{\mu}_t^L$ (see Figure 3). Consider any pair of equilibrium thresholds $\underline{\mu}_t^H > \underline{\mu}_t^L$. The claim follows from showing that the function

$$F(\underline{\mu}) = \frac{1 - \Phi(\underline{\mu}; \sigma^2, \sigma^2)}{1 - \Phi(\underline{\mu}; 0, \sigma^2)}$$

is increasing in $\underline{\mu}$. We have

$$F'(\underline{\mu}) = \frac{-\phi(\underline{\mu}; \sigma^2, \sigma^2) [1 - \Phi(\underline{\mu}; 0, \sigma^2)] + [1 - \Phi(\underline{\mu}; \sigma^2, \sigma^2)] \phi(\underline{\mu}; 0, \sigma^2)}{[1 - \Phi(\underline{\mu}; 0, \sigma^2)]^2} > 0$$

if and only if

$$-\phi(\underline{\mu}; \sigma^2, \sigma^2) [1 - \Phi(\underline{\mu}; 0, \sigma^2)] + [1 - \Phi(\underline{\mu}; \sigma^2, \sigma^2)] \phi(\underline{\mu}; 0, \sigma^2) > 0$$

or

$$\frac{\phi(\underline{\mu}; 0, \sigma^2)}{1 - \Phi(\underline{\mu}; 0, \sigma^2)} > \frac{\phi(\underline{\mu}; \sigma^2, \sigma^2)}{1 - \Phi(\underline{\mu}; \sigma^2, \sigma^2)} = \frac{\phi(\underline{\mu} - \sigma^2; 0, \sigma^2)}{1 - \Phi(\underline{\mu} - \sigma^2; 0, \sigma^2)},$$

where the last equality uses the properties of the normal distribution. The ratio $\phi(\underline{\mu}; 0, \sigma^2) / [1 - \Phi(\underline{\mu}; 0, \sigma^2)]$ is the hazard function of the normal distribution, which is increasing in $\underline{\mu}$. Thus, this inequality is always satisfied, which confirms the claim. Q.E.D.

Proof of Proposition 6: We consider the more general version in which $G_t = (1 - m_t)^\alpha e^g$. In this case, output in tax regime k at time t is

$$Y_{t+1} = \left(1 - \Phi\left(\underline{\mu}_t^k; \sigma_\mu^2, \sigma_\mu^2\right)\right) e^{\bar{\mu} + \frac{1}{2}\sigma_\mu^2} \Phi\left(\underline{\mu}_t^k; 0, \sigma_\mu^2\right)^\alpha e^g e^{\varepsilon_{t+1}}.$$

Therefore,

$$\begin{aligned} E[Y_{t+1}|H] &= \left(1 - \Phi\left(\underline{\mu}_t^H; \sigma_\mu^2, \sigma_\mu^2\right)\right) e^{\bar{\mu} + \frac{1}{2}\sigma_\mu^2} \Phi\left(\underline{\mu}_t^H; 0, \sigma_\mu^2\right)^\alpha e^g \\ &> \left(1 - \Phi\left(\underline{\mu}_t^L; \sigma_\mu^2, \sigma_\mu^2\right)\right) e^{\bar{\mu} + \frac{1}{2}\sigma_\mu^2} \Phi\left(\underline{\mu}_t^L; 0, \sigma_\mu^2\right)^\alpha e^g = E[Y_{t+1}|L] \end{aligned}$$

holds if and only if

$$\left(\frac{\Phi\left(\underline{\mu}_t^H; 0, \sigma_\mu^2\right)}{\Phi\left(\underline{\mu}_t^L; 0, \sigma_\mu^2\right)}\right)^\alpha > \frac{1 - \Phi\left(\underline{\mu}_t^L; \sigma_\mu^2, \sigma_\mu^2\right)}{1 - \Phi\left(\underline{\mu}_t^H; \sigma_\mu^2, \sigma_\mu^2\right)}. \quad (\text{A5})$$

This condition is never satisfied for $\alpha = 0$, as $\Phi\left(\underline{\mu}_t; \sigma_\mu^2, \sigma_\mu^2\right)$ is increasing in $\underline{\mu}_t$ and $\underline{\mu}_t^H > \underline{\mu}_t^L$. We now show that it is always satisfied for $\alpha = 1$. Indeed, under the assumption of equilibrium symmetry, $\underline{\mu}_t^L = -\underline{\mu}_t^H$, that is, cutoffs are symmetric around 0, which implies $m_t^H < 0.5 < m_t^L$ are symmetric around 0.5. Hence, we can rewrite the term to the left as

$$\frac{1 - \Phi\left(\underline{\mu}_t^L; 0, \sigma_\mu^2\right)}{1 - \Phi\left(\underline{\mu}_t^H; 0, \sigma_\mu^2\right)} > \frac{1 - \Phi\left(\underline{\mu}_t^L; \sigma_\mu^2, \sigma_\mu^2\right)}{1 - \Phi\left(\underline{\mu}_t^H; \sigma_\mu^2, \sigma_\mu^2\right)}$$

or

$$F\left(\underline{\mu}_t^H\right) = \frac{1 - \Phi\left(\underline{\mu}_t^H; \sigma_\mu^2, \sigma_\mu^2\right)}{1 - \Phi\left(\underline{\mu}_t^H; 0, \sigma_\mu^2\right)} > \frac{1 - \Phi\left(\underline{\mu}_t^L; \sigma_\mu^2, \sigma_\mu^2\right)}{1 - \Phi\left(\underline{\mu}_t^L; 0, \sigma_\mu^2\right)} = F\left(\underline{\mu}_t^L\right).$$

The claim follows from the proof of Proposition 5, which shows that $F(\underline{\mu}) = \frac{1 - \Phi(\underline{\mu}; \sigma_\mu^2, \sigma_\mu^2)}{1 - \Phi(\underline{\mu}; 0, \sigma_\mu^2)}$ is an increasing function of $\underline{\mu}$ and the fact that $\underline{\mu}_t^H > \underline{\mu}_t^L$.

Finally, the above arguments show that condition (A5) holds for $\alpha = 1$ and does not hold for $\alpha = 0$. By continuity, there exists a value $\underline{\alpha} < 1$ such that the condition always holds for $\alpha > \underline{\alpha}$. This is the value of α for which condition (A5) holds with equality, that is,

$$\underline{\alpha} = \frac{\log\left(\frac{1 - \Phi(\underline{\mu}_t^L; \sigma_\mu^2, \sigma_\mu^2)}{1 - \Phi(\underline{\mu}_t^H; \sigma_\mu^2, \sigma_\mu^2)}\right)}{\log\left(\frac{\Phi(\underline{\mu}_t^H; 0, \sigma_\mu^2)}{\Phi(\underline{\mu}_t^L; 0, \sigma_\mu^2)}\right)}.$$

Q.E.D.

Proposition A2: The welfare-maximizing allocation of human capital is

$$m_t = 1 - \Phi\left(\frac{1}{2}\sigma_\mu^2, 0; \sigma_\mu^2\right) < 0.5 ,$$

which corresponds to the threshold $\underline{\mu}^* = \frac{1}{2}\sigma_\mu^2$ determining which agents become entrepreneurs.

Proof of Proposition A2. Let $\underline{\mu}^*$ be the threshold maximizing output. This is given by

$$E[Y] = (1 - \Phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)) e^{\bar{\mu} + \sigma_\mu^2} \Phi(\underline{\mu}^*; 0, \sigma_\mu^2) e^g .$$

The maximum over $\underline{\mu}^*$ can be obtained from the first order conditions:

$$\frac{\partial E[Y]}{\partial \underline{\mu}^*} = -\phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2) e^{\bar{\mu} + \sigma_\mu^2} \Phi(\underline{\mu}^*; 0, \sigma_\mu^2) e^g + (1 - \Phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)) e^{\bar{\mu} + \sigma_\mu^2} \phi(\underline{\mu}^*; 0, \sigma_\mu^2) e^g = 0 ,$$

or

$$(1 - \Phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)) \phi(\underline{\mu}^*; 0, \sigma_\mu^2) = \phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2) \Phi(\underline{\mu}^*; 0, \sigma_\mu^2) ,$$

or

$$\frac{\phi(\underline{\mu}^*; 0, \sigma_\mu^2)}{\Phi(\underline{\mu}^*; 0, \sigma_\mu^2)} = \frac{\phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)}{1 - \Phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)} .$$

The density $\phi(\underline{\mu}^*; 0, \sigma_\mu^2)$ is symmetric around zero; therefore,

$$\frac{\phi(-\underline{\mu}^*; 0, \sigma_\mu^2)}{1 - \Phi(-\underline{\mu}^*; 0, \sigma_\mu^2)} = \frac{\phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)}{1 - \Phi(\underline{\mu}^*; \sigma_\mu^2, \sigma_\mu^2)} = \frac{\phi(\underline{\mu}^* - \sigma_\mu^2; 0, \sigma_\mu^2)}{1 - \Phi(\underline{\mu}^* - \sigma_\mu^2; 0, \sigma_\mu^2)} .$$

Note that these are hazard rates, which are always strictly increasing functions. Therefore, this equality can hold if and only if

$$-\underline{\mu}^* = \underline{\mu}^* - \sigma_\mu^2 ,$$

or

$$\underline{\mu}^* = \frac{1}{2}\sigma_\mu^2 .$$

Therefore, the socially optimal allocation has

$$m_t = 1 - \Phi\left(\frac{1}{2}\sigma_\mu^2, 0; \sigma_\mu^2\right) < 0.5 .$$

Q.E.D.

Proof of Proposition 7: Because $\gamma^L < \underline{\gamma}$ and $\gamma^H > \bar{\gamma}$, there are only two equilibrium masses of agents, $m^L = 1 - \Phi(\underline{\mu}^L, 0, \sigma_\mu^2) > 0.5$ and $m^H = 1 - \Phi(\underline{\mu}^H, 0, \sigma_\mu^2) < 0.5$. Denoting $y_t = \log(Y_t)$, we have

$$E[y_{t+1}|k] = g + \log(\Phi(\underline{\mu}^k; 0, \sigma_\mu^2)) + \log(1 - \Phi(\underline{\mu}^k; \sigma_\mu^2, \sigma_\mu^2)) + \bar{\mu} + \frac{1}{2}\sigma_\mu^2 - \frac{1}{2}\sigma^2 ,$$

where we use

$$\varepsilon_{t+1} \sim N\left(-\frac{1}{2}\sigma^2, \sigma^2\right).$$

Recall that Proposition 6 shows

$$E[y_{t+1}|H] > E[y_{t+1}|L]$$

and denote the difference

$$\begin{aligned} \overline{\Delta y} &= E[y_{t+1}|H] - E[y_{t+1}|L] \\ &= \log\left(\frac{1 - \Phi(\underline{\mu}^H; \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}^H; 0, \sigma_\mu^2)}\right) - \log\left(\frac{1 - \Phi(\underline{\mu}^L; \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}^L; 0, \sigma_\mu^2)}\right) \\ &> 0. \end{aligned}$$

Let f be the fraction of time spent in the L government, on average, in equilibrium and define the unconditional average as

$$\bar{y} = E[y] - \frac{1}{2}\sigma^2 = (1-f)E[y_{t+1}|H] + fE[y_{t+1}|L] - \frac{1}{2}\sigma^2.$$

Suppose party H is in power. The probability of a regime change (from H to L) is

$$\begin{aligned} \lambda^{HL} &= \Pr(y_{t+1} > \bar{y}) = \Pr(E[y_{t+1}|H] + \varepsilon_{t+1} > \bar{y}) \\ &= \Pr(\varepsilon_{t+1} > \bar{y} - E[y_{t+1}|H]) \\ &= \Pr\left(\varepsilon_{t+1} > -f[E[y_{t+1}|H] - E[y_{t+1}|L]] - \frac{1}{2}\sigma^2\right) \\ &= 1 - \Phi\left(-f\overline{\Delta y} - \frac{1}{2}\sigma^2, -\frac{1}{2}\sigma^2, \sigma^2\right) \\ &= 1 - \Phi(-f\overline{\Delta y}, 0, \sigma^2) \\ &> 0.5. \end{aligned}$$

Now suppose party L is in power. Note that

$$E[y_{t+1}|L] < \bar{y}.$$

Therefore, the probability of a regime change (from L to H) is

$$\begin{aligned} \lambda^{LH} &= \Pr(y_{t+1} < \bar{y}) = \Pr(E[y_{t+1}|L] + \varepsilon_{t+1} < \bar{y}) \\ &= \Pr(\varepsilon_{t+1} < \bar{y} - E[y_{t+1}|L]) \\ &= \Pr\left(\varepsilon_{t+1} < (1-f)E[y_{t+1}|H] + fE[y_{t+1}|L] - \frac{1}{2}\sigma^2 - E[y_{t+1}|L]\right) \\ &= \Pr\left(\varepsilon_{t+1} < (1-f)[E[y_{t+1}|H] - E[y_{t+1}|L]] - \frac{1}{2}\sigma^2\right) \\ &= \Phi\left((1-f)\overline{\Delta y} - \frac{1}{2}\sigma^2, -\frac{1}{2}\sigma^2, \sigma^2\right) \\ &= \Phi((1-f)\overline{\Delta y}, 0, \sigma^2) > 0. \end{aligned}$$

The ergodic distribution of regime L implies

$$f = \frac{\lambda^{HL}}{\lambda^{HL} + \lambda^{LH}} = \frac{\Phi(f\overline{\Delta y}, 0, \sigma^2)}{\Phi(f\overline{\Delta y}, 0, \sigma^2) + \Phi((1-f)\overline{\Delta y}, 0, \sigma^2)}.$$

To show that $f = 0.5$ is the unique solution to this equation, rewrite the equation as

$$f\Phi((1-f)\overline{\Delta y}, 0, \sigma^2) = \Phi(f\overline{\Delta y}, 0, \sigma^2)(1-f)$$

The symmetry of the problem shows that there is only one solution, $f = 0.5$. If $f > 0.5$, then the left-hand side is greater than 0.25 while the right-hand side is smaller than 0.25, and vice versa. Substituting $f = 1/2$ in $\lambda^{H,L}$ and $\lambda^{L,H}$ yields the claim. Q.E.D.

Proof of Proposition 8: The proof for a mixed equilibrium is complicated by the fact that tax uncertainty at time t affects the state price density and hence the equilibrium price of the stock when the entrepreneur issues shares. Luckily, the same arguments as in Pástor and Veronesi (2016) go through, as we now verify.

Let \mathcal{I} be the set of agents who choose to become entrepreneurs. Let M_t^i be the market value of firm i at time t . The net-of-tax dividend paid by firm i is $D_{t+1}^i = (1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{t+1} + \varepsilon_{i,t+1}}$, where we use the subscript $t+$ to emphasize that this rate is not known at time t when agents make their occupation choice.

Proposition A4: In the mixed equilibrium, the state price density at $t + 1$ is

$$\pi_{t+1} = h(1 - \tau_{t+})^{-\gamma} e^{-\gamma\varepsilon_{t+1}}$$

for a constant h known at time t , and asset prices satisfy

$$M_t^i = \frac{E_t[\pi_{t+1} D_{t+1}^i]}{E_t[\pi_{t+1}]} \quad (\text{A6})$$

$$M_{t+}^i = \frac{E_{t+}[\pi_{t+1} D_{t+1}^i]}{E_{t+}[\pi_{t+1}]}, \quad (\text{A7})$$

where $t+$ is the announcement of the party winning the election. In addition, entrepreneur i 's consumption at time $t + 1$ is

$$C_{i,t+1}^{yes} = (1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{t+1}} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)].$$

Proof of Proposition A4: We verify below that the state price density depends on only two shocks, ε_{t+1} and τ_{t+} :

$$\pi_{t+1} = \pi(\varepsilon_{t+1}, \tau_{t+}),$$

for some function $\pi(\varepsilon_{t+1}, \tau_{t+})$.

Given the conjectured state price density, we can compute the price of each asset at time t as

$$\begin{aligned}
M_t^i &= \frac{E_t[\pi_{t+1}(1-\tau_{t+})G_t e^{\mu_i+\varepsilon_{t+1}+\varepsilon_{i,t+1}}]}{E_t[\pi_{t+1}]} = e^{\mu_i} \frac{E_t[\pi(\varepsilon_{t+1}, \tau_{t+})(1-\tau_{t+})e^{\varepsilon_{t+1}+\varepsilon_{i,t+1}}]}{E_t[\pi_{t+1}]} \quad (\text{A8}) \\
&= e^{\mu_i} \frac{E_t[\pi(\varepsilon_{t+1}, \tau_{t+})(1-\tau_{t+})e^{\varepsilon_{t+1}}]}{E_t[\pi_{t+1}]} E_t[e^{\varepsilon_{i,t+1}}] \\
&= e^{\mu_i} \frac{E_t[\pi(\varepsilon_{t+1}, \tau_{t+})(1-\tau_{t+})e^{\varepsilon_{t+1}}]}{E_t[\pi_{t+1}]} \\
&= e^{\mu_i} Z, \quad (\text{A9})
\end{aligned}$$

where we define the constant Z as

$$Z = \frac{E_t[\pi(\varepsilon_{t+1}, \tau_{t+})(1-\tau_{t+})e^{\varepsilon_{t+1}}]}{E_t[\pi_{t+1}]},$$

which is the time- t price of a security with payoff $(1-\tau_{t+})e^{\varepsilon_{t+1}}$ at time $t+1$. For later reference, note that the aggregate market value of the market portfolio is

$$M_t^P = \int_{\mathcal{I}} M_t^i di = Z \int_{\mathcal{I}} e^{\mu_i} di$$

and the total dividend

$$D_{t+1}^{Mkt} = (1-\tau_{t+})e^{\varepsilon_{t+1}} \int_{\mathcal{I}} e^{\mu_i} di,$$

so that the market return is

$$R^{Mkt} = \frac{D_{t+1}^{Mkt}}{M_t^P} - 1 = \frac{(1-\tau_{t+})e^{\varepsilon_{t+1}}}{Z} - 1.$$

In the arguments below, we will also make use of the fact that each individual stock is infinitesimal, that is, removing one stock from a continuum does not change the value of the market portfolio. In particular, we will use the following equality for $j \neq i$:

$$\int_{\mathcal{I} \setminus i} M_t^j dj = \int_{\mathcal{I} \setminus j} M_t^i di.$$

Consider the budget constraint of each entrepreneur i . At time t , entrepreneur i issues $1-\theta$ shares of his own firm i . From the proceeds, the entrepreneur purchases $N_t^{i,j}$ shares of firm j and $N_t^{i,0}$ bonds. As we show below, if unrestricted ($\theta=0$), each entrepreneur would sell all of his firm and purchase the market portfolio, which would entail an infinitesimal position in his own firm. The θ constraint is always binding; for any given θ , each entrepreneur restricts his holdings of his own firm to exactly θ shares. All quantities are expressed in terms of our numeraire, which is the

zero-coupon bond with maturity $t + 1$ that is a claim to one unit of capital at $t + 1$. The bond price is thus equal to one at both times t and $t + 1$. Bonds are in zero net supply. The budget constraint is

$$(1 - \theta) M_t^i = \int_{\mathcal{I} \setminus i} N_t^{ij} M_t^j dj + N_t^{i0}. \quad (\text{A10})$$

Within each period, agents only trade once, at time t , and they hold their positions until time $t + 1$. At time $t + 1$, agent i 's consumption is

$$C_{i,t+1} = \theta D_{t+1}^i + \int_{\mathcal{I} \setminus i} N_t^{ij} D_{t+1}^j dj + N_t^{i0}. \quad (\text{A11})$$

As we shall see, in equilibrium $C_{i,t+1} > 0$ with probability one.

Before we analyze the optimal choice of each individual, we consider the market-clearing condition. Each entrepreneur j issues exactly $1 - \theta$ shares. Therefore, we must have that in equilibrium all shares issued are bought by somebody. That is, the sum of all the j shares bought by agents i must equal $1 - \theta$:

$$1 - \theta = \int_{\mathcal{I} \setminus j} N_t^{ij} di.$$

Compared to the budget equation, the integral here is over i and not over j . The bond market must clear, too, and given that bonds are in zero net supply, we must have

$$\int_{\mathcal{I}} N_t^{i0} di = 0.$$

The utility function of entrepreneur $i \in \mathcal{I}$ is:*

$$\begin{aligned} E[U(C_{i,t+1})] &= \frac{1}{1 - \gamma} E[(C_{i,t+1})^{1-\gamma}] \\ &= \frac{1}{1 - \gamma} E\left[\left(\theta D_{t+1}^i + \int_{\mathcal{I} \setminus i} N_t^{ij} D_{t+1}^j dj + N_t^{i0}\right)^{1-\gamma}\right]. \end{aligned}$$

Consider again the budget equation of agent i , now rewritten as

$$(1 - \theta) M_t^i - \int_{\mathcal{I} \setminus i} N_t^{ij} M_t^j dj = N_t^{i0}.$$

Substitute for N_t^{i0} in the utility function to find

$$E[U(C_{i,t+1})] = \frac{1}{1 - \gamma} E\left[\left(\theta (D_{t+1}^i - M_t^i) + \int_{\mathcal{I} \setminus i} N_t^{ij} (D_{t+1}^j - M_t^j) dj + M_t^i\right)^{1-\gamma}\right].$$

*The argument below also applies to agents with $\gamma_i = 1$, that is, log utility investors, as the main equations only depend on marginal utility $C_{i,T}^{-\gamma_i}$, which are independent of whether $\gamma_i = 1$ or not.

The first-order conditions (FOC) with respect to N_t^{ij} are

$$E \left[\left(\theta (D_{t+1}^i - M_t^i) + \int_{\mathcal{I} \setminus i} N_t^{ij} (D_{t+1}^j - M_t^j) dj + M_t^i \right)^{-\gamma} (D_{t+1}^j - M_t^j) \right] = 0.$$

We can rewrite this expression as

$$E \left[\left(\theta \left(\frac{D_{t+1}^i}{M_t^i} - 1 \right) M_t^i + M_t^i \int_{\mathcal{I} \setminus i} \frac{N_t^{ij} M_t^j}{M_t^i} \left(\frac{D_{t+1}^j}{M_t^j} - 1 \right) dj + M_t^i \right)^{-\gamma} (D_{t+1}^j - M_t^j) \right] = 0.$$

Factoring M_t^i out of the expectation and simplifying, we can rewrite the FOC as

$$E \left[\left(\theta \left(\frac{D_{t+1}^i}{M_t^i} - 1 \right) + \int_{\mathcal{I} \setminus i} \frac{N_t^{ij} M_t^j}{M_t^i} \left(\frac{D_{t+1}^j}{M_t^j} - 1 \right) dj + 1 \right)^{-\gamma} \left(\frac{D_{t+1}^j}{M_t^j} - 1 \right) \right] = 0.$$

Define ω_t^{ij} as

$$\omega_t^{ij} = \frac{N_t^{ij} M_t^j}{M_t^i}.$$

Note that for every j , the net-of-tax arithmetic return on investment is

$$R_{t+1}^j = \frac{D_{t+1}^j}{M_t^j} - 1 = \frac{(1 - \tau_{t+}) e^{\mu_j + \varepsilon_{j,t+1} + \varepsilon_{t+1}}}{e^{\mu_j} Z} - 1 = \frac{(1 - \tau_{t+}) e^{\varepsilon_{j,t+1} + \varepsilon_{t+1}}}{Z} - 1.$$

That is, the return R^j is the same across firms, except for the realization of the idiosyncratic shock $\varepsilon_{j,t+1}$. All stocks have the same expected return equal to

$$E_t [R_{t+1}^j] = (1 - E_t [\tau_{t+}]) Z^{-1} - 1.$$

We can rewrite the FOC of agent i as

$$E \left[\left(\theta R_{t,t+1}^i + \int_{\mathcal{I} \setminus i} \omega^{ij} R_{t,t+1}^j dj + 1 \right)^{-\gamma} R_{t,t+1}^j \right] = 0.$$

From the above discussion, all R_{t+1}^j have the same risk-return characteristics. Therefore, the properties of the expectation are the same, and hence the FOC for each agent i or i' are identical. It follows that $\omega^{ij} = \omega^{i'j} = \omega^j$ for all i :

$$\omega_t^{ij} = \omega_t^j.$$

That is, each agent i invests the same fraction ω_t^j of their wealth in each stock j .

Finally, by imposing market clearing in the stock and bond market, we obtain the state price density. Express first the number of shares bought N_t^{ij} as a function of ω_t^{ij} and thus ω_t^j :

$$\omega_t^{ij} = \omega_t^j = \frac{N_t^{ij} M_t^j}{M_t^i} \text{ for } j \neq i.$$

Solving for N_t^{ij} , we obtain the number of shares bought by each agent i :

$$N_t^{ij} = \omega_t^j \frac{M_t^i}{M_t^j} \text{ for } j \neq i. \quad (\text{A12})$$

We now impose the market-clearing condition in the stock market. Recall that the total number of shares issued by firm j satisfies

$$1 - \theta = \int_{\mathcal{I} \setminus j} N_t^{ij} di.$$

Substitute for N_t^{ij} :

$$1 - \theta = \int_{\mathcal{I} \setminus j} \omega_t^j \frac{M_t^i}{M_t^j} di$$

or

$$(1 - \theta) M_t^j = \omega_t^j \int_{\mathcal{I} \setminus j} M_t^i di.$$

That is, for every agent i , their exposure to stock j , ω_t^j , must satisfy

$$\omega_t^j = (1 - \theta) \frac{M_t^j}{\int_{\mathcal{I} \setminus j} M_t^i di}, \quad (\text{A13})$$

which implies

$$N_t^{ij} = \omega_t^j \frac{M_t^i}{M_t^j} = (1 - \theta) \frac{M_t^j}{\int_{\mathcal{I} \setminus j} M_t^k dk} \frac{M_t^i}{M_t^j} = (1 - \theta) \frac{M_t^i}{\int_{\mathcal{I} \setminus j} M_t^k dk}. \quad (\text{A14})$$

That is, each agent i purchases a number of shares N_t^{ij} proportional to his wealth M_t^i .

Consider now the budget equation of agent i :

$$(1 - \theta) M_t^i = \int_{\mathcal{I} \setminus i} N_t^{ij} M_t^j dj + N_t^{i0}.$$

Substitute for N_t^{ij} from equation (A14):

$$(1 - \theta) M_t^i = \int_{\mathcal{I} \setminus i} (1 - \theta) \frac{M_t^i}{\int_{\mathcal{I} \setminus j} M_t^k dk} M_t^j dj + N_t^{i0},$$

or

$$(1 - \theta) M_t^i = (1 - \theta) M_t^i \frac{\int_{\mathcal{I} \setminus i} M_t^j dj}{\int_{\mathcal{I} \setminus j} M_t^k dk} + N_t^{i0},$$

or

$$(1 - \theta) M_t^i = (1 - \theta) M_t^i + N_t^{i0}. \quad (\text{A15})$$

This implies that, for all i ,

$$N_t^{i0} = 0.$$

That is, all agents have a zero position in bonds, which makes sense as all agents have the same risk aversion. Thus, the bond market clears.

We finally obtain the state price density that ensures that the FOC of all agents are satisfied by equation (A13). Consider again the FOC of agent i :

$$E \left[\left(\theta R_{t+1}^i + \int_{\mathcal{I} \setminus i} \omega_t^{ij} R_{t+1}^j dj + 1 \right)^{-\gamma} R_{t+1}^j \right] = 0 .$$

Substitute what we found earlier as the equilibrium weight of agent i into stock j :

$$\omega_t^{ij} = \omega_t^j = (1 - \theta) \frac{M_t^j}{\int_{\mathcal{I} \setminus j} M_t^k dk}$$

to find that the FOC is

$$E_t \left[\left(\theta R_{t+1}^i + \int_{\mathcal{I} \setminus i} (1 - \theta) \frac{M_t^j}{\int_{\mathcal{I} \setminus j} M_t^k dk} R_{t+1}^j dj + 1 \right)^{-\gamma} R_{t+1}^j \right] = 0 ,$$

or

$$E_t \left[\left(\theta R_{t+1}^i + (1 - \theta) \int_{\mathcal{I} \setminus i} \frac{M_t^j}{\int_{\mathcal{I} \setminus j} M_t^k dk} R_{t+1}^j dj + 1 \right)^{-\gamma} R_{t+1}^j \right] = 0 ,$$

or

$$E \left[(\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} R_{t+1}^j \right] = 0 , \quad (\text{A16})$$

where R_{t+1}^{Mkt} is the return on the market portfolio:

$$\begin{aligned} R_{t+1}^{Mkt} &= \int_{\mathcal{I}} \frac{M_t^j}{\int_{\mathcal{I}} M_t^k dk} R_{t+1}^j dj \\ &= \int_{\mathcal{I}} \frac{M_t^j}{\int_{\mathcal{I}} M_t^k dk} \left(\frac{D_{t+1}^j}{M_t^j} - 1 \right) dj \\ &= \frac{\int_{\mathcal{I}} D_{t+1}^j dj}{\int_{\mathcal{I}} M_t^k dk} - 1 . \end{aligned}$$

Ex ante, all R_{t+1}^i, R_{t+1}^j have the same characteristics, as we can write

$$R_{t+1}^i = \frac{(1 - \tau_{t+}) e^{\varepsilon_{i,t+1} + \varepsilon_{t+1}}}{Z} - 1 \quad (\text{A17})$$

$$R_{t+1}^{Mkt} = \frac{(1 - \tau_{t+}) e^{\varepsilon_{t+1}}}{Z} - 1 . \quad (\text{A18})$$

Let us rewrite the FOC in terms of dividends again:

$$E \left[(\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} \left(\frac{D_{t+1}^j}{M_t^j} - 1 \right) \right] = 0 .$$

For every i , we thus have

$$E \left[(\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} D_{t+1}^j \right] = E \left[(\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} \right] M_t^j .$$

Integrate across $i \in \mathcal{I}$ to obtain

$$E \left[\int_{\mathcal{I}} (\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} di D_{t+1}^j \right] = E \left[\int_{\mathcal{I}} (\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} di \right] M_t^j .$$

Define the state price density as

$$\pi_{t+1} = \int_{\mathcal{I}} (\theta R_{t+1}^i + (1 - \theta) R_{t+1}^{Mkt} + 1)^{-\gamma} di , \quad (\text{A19})$$

so that the above equation is

$$E_t [\pi_{t+1} D_T^j] = E_t [\pi_{t+1}] M_t^j ,$$

which is the standard pricing equation. We now show that this state price density only depends on ε_{t+1} and τ_{t+} as initially conjectured. We have

$$\begin{aligned} \pi_{t+1} &= \int_{\mathcal{I}} \left(\theta \left(\frac{D_{t+1}^i}{M_t^i} - 1 \right) + (1 - \theta) \left(\frac{\int_{\mathcal{I}} D_{t+1}^j dj}{\int_{\mathcal{I}} M_t^k dk} - 1 \right) + 1 \right)^{-\gamma} di \\ &= \int_{\mathcal{I}} \left(\theta \left(\frac{(1 - \tau_{t+}) e^{\varepsilon_{i,t+1} + \varepsilon_{t+1}}}{Z} - 1 \right) + (1 - \theta) \left(\frac{(1 - \tau_{t+}) e^{\varepsilon_{t+1}}}{Z} - 1 \right) + 1 \right)^{-\gamma} di \\ &= \int_{\mathcal{I}} \left(\theta \frac{(1 - \tau_{t+1}) e^{\varepsilon_{i,t+1} + \varepsilon_{t+1}}}{Z} + (1 - \theta) \frac{(1 - \tau_{t+1}) e^{\varepsilon_{t+1}}}{Z} \right)^{-\gamma} di \\ &= ((1 - \tau_{t+}) e^{\varepsilon_{t+1}})^{-\gamma} \frac{1}{Z^{-\gamma}} \int_{\mathcal{I}} (\theta e^{\varepsilon_{i,t+1}} + (1 - \theta))^{-\gamma} di \\ &= h (1 - \tau_{t+})^{-\gamma} e^{-\gamma \varepsilon_{t+1}} , \end{aligned}$$

where

$$h = \frac{1}{Z^{-\gamma}} \int_{\mathcal{I}} (\theta e^{\varepsilon_{i,t+1}} + (1 - \theta))^{-\gamma} di .$$

We can also solve for Z explicitly, from its definition

$$Z = \frac{E [\pi (\varepsilon_{t+1}, \tau_{t+}) (1 - \tau_{t+}) e^{\varepsilon_{t+1}}]}{E [\pi_{t+1}]} ,$$

but it is not necessary to do so.

We now show that prices are well defined at both times t and $t+$ and that at $t+$ agents do not wish to rebalance their portfolios. In particular, we need to show that the state price density just obtained is well defined not only at t (before the announcement) but also at $t+$ (after the

announcement), in the sense that it can still be derived from agents' first order conditions at $t+$, that it satisfies the martingale condition, and that deflated prices also satisfy the martingale conditions.

To see all this, recall the state price density at the end of period t , i.e., at $t + 1$, is given by

$$\pi_{t+1} = h (1 - \tau_{t+})^{-\gamma} e^{-\gamma \varepsilon_{t+1}},$$

where h is a constant. From the martingale condition, we have that the state price density values before time $t + 1$, at times $t+$ and t , are given by

$$\begin{aligned}\pi_{t+} &= E_{t+} [\pi_{t+1}] = h (1 - \tau_{t+})^{-\gamma} E_{t+} [e^{-\gamma \varepsilon_{t+1}}] \\ \pi_t &= E_t [\pi_{t+1}] = h E_t [(1 - \tau_{t+})^{-\gamma} E_{t+} [e^{-\gamma \varepsilon_{t+1}}]]\end{aligned}$$

The stock price must satisfy

$$\begin{aligned}M_{t+} \pi_{t+} &= E_{t+} [\pi_{t+1} M_{t+1}] \\ M_t \pi_t &= E_t [\pi_{t+1} M_{t+1}].\end{aligned}$$

The latter equation is clearly satisfied by the pricing formula, as this is how we obtained the state price density to begin with. We now show that the pricing equation at the intermediate time $t+$ determines the stock price obtained under the full information case:

$$\begin{aligned}M_{t+} &= \frac{E_{t+} [\pi_{t+1} M_{t+1}]}{\pi_{t+}} \\ &= \frac{h (1 - \tau_{t+})^{-\gamma} E_{t+} [e^{-\gamma \varepsilon_{t+1}} ((1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{t+} + \varepsilon_{i,t+1}})]}{h (1 - \tau_{t+})^{-\gamma} E_{t+} [e^{-\gamma \varepsilon_{t+1}}]} \\ &= \frac{(1 - \tau_{t+}) G_t e^{\mu_i} E_{t+} [e^{(1-\gamma) \varepsilon_{t+1}}]}{E_{t+} [e^{-\gamma \varepsilon_{t+1}}]},\end{aligned}$$

which is the same pricing formula we have for the case in which τ is known from the beginning (i.e. the pure strategy Nash equilibrium). Because also in that case the state price density is defined from agents' first order conditions, it follows that the state price density is well defined on both times. In particular, agents do not want to rebalance their portfolios after the revelation of the winning party at $t+$. To see this, we now show that each agent's wealth at $t+$ obtained from the initial investment at t also equals the wealth in the pure strategy equilibrium when the tax is announced at time t rather than $t+$. That is, their uncertainty at time t does not change the wealth position at the time of information about taxes, which in turn implies that their optimal choice

conditional on taxes is unchanged compared to the pure strategy equilibrium:

$$\begin{aligned}
W_{it} &= \theta M_{it} + (1 - \theta) M_t^P \\
&= \theta G_t e^{\mu_i} \frac{[0.5(1 - \tau^L)^{1-\gamma} + 0.5(1 - \tau^H)^{1-\gamma}]}{[0.5(1 - \tau^L)^{-\gamma} + 0.5(1 - \tau^H)^{-\gamma}]} e^{-\gamma\sigma^2} \\
&\quad + (1 - \theta) G_t m_t E[e^{\mu_i} | i \in I] \frac{[0.5(1 - \tau^L)^{1-\gamma} + 0.5(1 - \tau^H)^{1-\gamma}]}{[0.5(1 - \tau^L)^{-\gamma} + 0.5(1 - \tau^H)^{-\gamma}]} e^{-\gamma\sigma^2} \\
&= G_t \frac{[0.5(1 - \tau^L)^{1-\gamma} + 0.5(1 - \tau^H)^{1-\gamma}]}{[0.5(1 - \tau^L)^{-\gamma} + 0.5(1 - \tau^H)^{-\gamma}]} e^{-\gamma\sigma^2} [\theta e^{\mu_i} + (1 - \theta) m_t E[e^{\mu_i} | i \in I]] .
\end{aligned}$$

Agent i 's wealth at $t+$ is

$$\begin{aligned}
W_{it+} &= G_t \frac{[(1 - \tau^+)^{1-\gamma}]}{[(1 - \tau^+)^{-\gamma}]} e^{-\gamma\sigma^2} [\theta e^{\mu_i} + (1 - \theta) m_t E[e^{\mu_i} | i \in I]] \\
&= \theta M_{it+} + (1 - \theta) M_{t+}^P .
\end{aligned}$$

Because the market portfolio and every individual stock price increase or decrease by the same percentage, no rebalancing takes place at time $\tau+$. In other words, the FOC are still the same for all agents even after the information release.

Finally, we note that even with tax uncertainty, consumption of entrepreneurs at $t + 1$ is the same as in the case with tax certainty. From

$$C_{i,t+1} = \theta D_{t+1}^i + \int_{\mathcal{I} \setminus i} N_t^{ij} D_{t+1}^j dj + N_t^{i0} \quad (\text{A20})$$

and

$$\begin{aligned}
N_t^{i0} &= 0 \\
N_t^{ij} &= (1 - \theta) \frac{M_{it}}{MP} = (1 - \theta) \frac{e^{\mu_i}}{\int e^{\mu_k} dk} ,
\end{aligned}$$

we obtain

$$\begin{aligned}
C_{i,t+1} &= \theta D_{t+1}^i + \int_{\mathcal{I} \setminus i} (1 - \theta) \frac{e^{\mu_i} D_{t+1}^j}{\int e^{\mu_k} dk} dj \\
&= \theta (1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{i,t+1} + \varepsilon_{t+1}} + e^{\mu_i} \int_{\mathcal{I} \setminus i} (1 - \theta) \frac{(1 - \tau_{t+}) G_t e^{\mu_j + \varepsilon_{j,t+1} + \varepsilon_{t+1}}}{\int e^{\mu_k} dk} dj \\
&= (1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{t+1}} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)] . \quad (\text{A21})
\end{aligned}$$

Thus, consumption $C_{i,t+1} > 0$ with probability one, as claimed earlier.

Q.E.D.

Proof of Proposition 8 (cont'd). We finally construct the mixed equilibrium. First, let the equilibrium mass be $m_t = 0.5$. We keep the general notation m_t as it will be useful later to prove uniqueness. As in previous cases, given m_t , truthful voting still implies entrepreneurs (E) vote for low taxes (L) and government workers (G) vote for high taxes (H). Let $p = 0.5$ be the probability that L wins. Agents take into account this uncertainty in deciding whether to be E (and vote L) or G (and vote H). In this case, agents take into account some consumption uncertainty at time $t + 1$ which will depend on the voting outcome. In particular, if agent i chooses E , his consumption is (see equation (A21)):

$$C_{it+1}^{yes} = \begin{cases} (1 - \tau^L) G_t e^{\mu_i} e^{\varepsilon_{t+1}} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)] & \text{with probability } 1/2 \\ (1 - \tau^H) G_t e^{\mu_i} e^{\varepsilon_{t+1}} [\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)] & \text{with probability } 1/2 \end{cases}$$

If agent i chooses G , his/her consumption is

$$C_{it+1}^{no} = \begin{cases} \tau^L G_t e^{\varepsilon_{t+1}} E[e^{\mu_j} | j \in I_t] m_t / (1 - m_t) & \text{with probability } 1/2 \\ \tau^H G_t e^{\varepsilon_{t+1}} E[e^{\mu_j} | j \in I_t] m_t / (1 - m_t) & \text{with probability } 1/2 \end{cases}$$

Therefore,

$$V_t^{i,yes} > V_t^{i,no}$$

if and only if

$$\begin{aligned} & \frac{[0.5 (1 - \tau^L)^{1-\gamma^M} + 0.5 (1 - \tau^H)^{1-\gamma^M}] G_t^{1-\gamma^M} e^{(1-\gamma^M)\mu_i}}{1 - \gamma^M} E_t [e^{(1-\gamma^M)\varepsilon_{t+1}}] E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma}] \\ & > \frac{1}{1 - \gamma^M} \left(0.5 (\tau^L)^{1-\gamma^M} + 0.5 (\tau^H)^{1-\gamma^M} \right) (G_t E[e^{\mu_j} | j \in I])^{1-\gamma^M} E [e^{(1-\gamma^M)\varepsilon_{t+1}}] \left(\frac{m_t}{1 - m_t} \right)^{1-\gamma^M} \end{aligned}$$

if and only if

$$\begin{aligned} & \left[(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M} \right] e^{(1-\gamma^M)\mu_i} E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma}] \\ & < \left((\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M} \right) E [e^{\mu_j} | j \in I_t]^{1-\gamma^M} \left(\frac{m_t}{1 - m_t} \right)^{1-\gamma^M} \end{aligned}$$

if and only if

$$\begin{aligned} & \log \left((1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M} \right) + (1 - \gamma^M) \mu_i + \log \left(E [[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma}] \right) \\ & < \log \left((\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M} \right) + (1 - \gamma^M) \log \left(E [e^{\mu_j} | j \in I_t] \right) + (1 - \gamma^M) \log \left(\frac{m_t}{1 - m_t} \right) \end{aligned}$$

if and only if

$$\begin{aligned} \mu_i \geq K(\gamma^M) &= \frac{1}{(\gamma^M - 1)} \log \left(\frac{(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M}}{(\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M}} \right) + \log(E[e^{\mu_j} | j \in I_t]) \\ &+ \frac{1}{(\gamma^M - 1)} \log(E[\theta e^{\varepsilon_{i,t+1}} + (1 - \theta)]^{1-\gamma}) + \log\left(\frac{m_t}{1 - m_t}\right). \end{aligned}$$

Therefore, the mass of agents who decide to become entrepreneurs is

$$m_t(\gamma^M) = \int_{i: \mu_i \geq K(\gamma^M)} di.$$

Let γ^M be such that $m_t(\gamma^M) = 0.5$ (we show below that such $\gamma^M \in [\underline{\gamma}, \bar{\gamma}]$ exists). By construction, the median voter i^* is such that $\mu_{i^*} = K(\gamma^M)$ and hence he is indifferent between E and G . Such a median voter i^* flips a coin and decides to become E with probability 0.5, supporting the equilibrium.

We finally show that such $\gamma^M \in [\underline{\gamma}, \bar{\gamma}]$ exists. Given the distribution of $\mu_i \sim N(\bar{\mu}, \sigma_\mu^2)$, we have

$$m_t(\gamma^M) = \int_{K(\gamma^M)}^{\infty} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) di = 1 - \Phi(K(\gamma^M); \bar{\mu}, \sigma_\mu^2).$$

In addition, recalling the conditional density

$$\begin{aligned} \phi(\mu_i | i \in I_t) &= \phi(\mu_i | \mu_i \geq K(\gamma^M)) = \frac{\phi(\mu_i; \bar{\mu}, \sigma_\mu^2) 1_{\{\mu_i > K(\gamma^M)\}}}{\int_{K(\gamma^M)}^{\infty} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) d\mu_i} \\ &= \frac{\phi(\mu_i; \bar{\mu}, \sigma_\mu^2) 1_{\{\mu_i > K(\gamma^M)\}}}{1 - \Phi(K(\gamma^M); \bar{\mu}, \sigma_\mu^2)} = \frac{\phi(\mu_i; \bar{\mu}, \sigma_\mu^2) 1_{\{\mu_i > K(\gamma^M)\}}}{m_t}, \end{aligned}$$

we have

$$E[e^{\mu_j} | j \in I_t] = \frac{\int_{K(\gamma^M)}^{\infty} e^{\mu_i} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) di}{m_t} = e^{\bar{\mu} + \frac{1}{2}\sigma_\mu^2} \frac{(1 - \Phi(K(\gamma^M); \bar{\mu} + \sigma_\mu^2, \sigma_\mu^2))}{m_t}.$$

In fact,

$$\begin{aligned} \int_{K(\gamma^M)}^{\infty} e^{\mu_i} \phi(\mu_i; \bar{\mu}, \sigma_\mu^2) di &= \int_{K(\gamma^M)}^{\infty} \frac{e^{\mu_i - \frac{(\mu_i - \bar{\mu})^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} di = \int_{K(\gamma^M)}^{\infty} \frac{e^{-\frac{\mu_i^2 + \bar{\mu}^2 - 2\mu_i\bar{\mu} - 2\sigma_\mu^2\mu_i}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} di \\ &= \int_{K(\gamma^M)}^{\infty} \frac{e^{-\frac{\mu_i^2 + \bar{\mu}^2 - 2\mu_i(\bar{\mu} + \sigma_\mu^2) + (\bar{\mu} + \sigma_\mu^2)^2 - (\bar{\mu} + \sigma_\mu^2)^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} di \\ &= e^{\frac{1}{2}\sigma_\mu^2 + \bar{\mu}} \int_{K(\gamma^M)}^{\infty} \frac{e^{-\frac{(\mu_i - (\bar{\mu} + \sigma_\mu^2))^2}{2\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} di \\ &= e^{\frac{1}{2}\sigma_\mu^2 + \bar{\mu}} (1 - \Phi(K(\gamma^M); \bar{\mu} + \sigma_\mu^2, \sigma_\mu^2)). \end{aligned}$$

Substituting everything inside the threshold, we find

$$\begin{aligned}
K(\gamma^M) &= \frac{1}{(\gamma^M - 1)} \log \left(\frac{(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M}}{(\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M}} \right) \\
&\quad + \bar{\mu} + \frac{1}{2} \sigma_\mu^2 + \log \left(\frac{(1 - \Phi(K(\gamma^M); \bar{\mu} + \sigma_\mu^2, \sigma_\mu^2))}{m_t} \right) \\
&\quad + \frac{1}{(\gamma^M - 1)} \log \left(E \left[[\theta e^{\varepsilon_i, t+1} + (1 - \theta)]^{1-\gamma^M} \right] \right)
\end{aligned}$$

or, defining

$$\underline{\mu}(\gamma^M) = K(\gamma^M) - \bar{\mu},$$

we obtain

$$\begin{aligned}
\underline{\mu}(\gamma^M) &= \frac{1}{2} \sigma_\mu^2 + \frac{1}{(\gamma^M - 1)} \log \left(\frac{(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M}}{(\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M}} \right) \\
&\quad + \log \left(\frac{(1 - \Phi(\underline{\mu}(\gamma^M); \sigma_\mu^2, \sigma_\mu^2))}{(1 - \Phi(\underline{\mu}(\gamma^M); 0, \sigma_\mu^2))} \right) \\
&\quad + \frac{1}{(\gamma^M - 1)} \log \left(E \left[[\theta e^{\varepsilon_i, t+1} + (1 - \theta)]^{1-\gamma^M} \right] \right) + \log \left(\frac{(1 - \Phi(\underline{\mu}(\gamma^M); 0, \sigma_\mu^2))}{\Phi(\underline{\mu}(\gamma^M); 0, \sigma_\mu^2)} \right),
\end{aligned}$$

so that

$$\begin{aligned}
\underline{\mu}(\gamma^M) &= \frac{1}{2} \sigma_\mu^2 + \frac{1}{(\gamma^M - 1)} \log \left(\frac{(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M}}{(\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M}} \right) + \log \left(\frac{1 - \Phi(\underline{\mu}(\gamma^M); \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}(\gamma^M); 0, \sigma_\mu^2)} \right) \\
&\quad + \frac{1}{(\gamma^M - 1)} \log \left(E \left[[\theta e^{\varepsilon_i, t+1} + (1 - \theta)]^{1-\gamma^M} \right] \right).
\end{aligned}$$

Finally, γ^M is chosen such that

$$\underline{\mu}(\gamma^M) = 0,$$

so that

$$m_t = 1 - \Phi(\underline{\mu}(\gamma^M); 0, \sigma_\mu^2) = 1 - \Phi(0; 0, \sigma_\mu^2) = 0.5,$$

which implies

$$\begin{aligned}
0 &= \frac{1}{2} \sigma_\mu^2 + \frac{1}{(\gamma^M - 1)} \log \left(\frac{(1 - \tau^L)^{1-\gamma^M} + (1 - \tau^H)^{1-\gamma^M}}{(\tau^L)^{1-\gamma^M} + (\tau^H)^{1-\gamma^M}} \right) + \log \left(\frac{1 - \Phi(0; \sigma_\mu^2, \sigma_\mu^2)}{0.5} \right) \\
&\quad + \frac{1}{(\gamma^M - 1)} \log \left(E \left[[\theta e^{\varepsilon_i, t+1} + (1 - \theta)]^{1-\gamma^M} \right] \right).
\end{aligned}$$

The existence and uniqueness of a solution for $\gamma_M \in [\underline{\gamma}, \bar{\gamma}]$ can be obtained as follows. Define

$$\begin{aligned} \underline{\mu}(p, \gamma) &= \frac{1}{2}\sigma_\mu^2 + \frac{1}{(\gamma-1)} \log \left(\frac{p(1-\tau^L)^{1-\gamma} + (1-p)(1-\tau^H)^{1-\gamma}}{p(\tau^L)^{1-\gamma} + (1-p)(\tau^H)^{1-\gamma}} \right) + \log \left(\frac{1 - \Phi(\underline{\mu}(p, \gamma); \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}(p, \gamma); 0, \sigma_\mu^2)} \right) \\ &\quad + \frac{1}{(\gamma-1)} \log (E [[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma}]) . \end{aligned}$$

We know that for any $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ and $p = 1$, the equation is

$$\begin{aligned} \underline{\mu}(1, \gamma) &= \frac{1}{2}\sigma_\mu^2 + \log \left(\frac{\tau^L}{1-\tau^L} \right) + \log \left(\frac{1 - \Phi(\underline{\mu}(1, \gamma); \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}(1, \gamma); 0, \sigma_\mu^2)} \right) \\ &\quad + \frac{1}{(\gamma-1)} \log (E [[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma}]) \\ &= \underline{\mu}^L < 0 , \end{aligned}$$

whereas for $p = 0$, the equation is

$$\begin{aligned} \underline{\mu}(0, \gamma) &= \frac{1}{2}\sigma_\mu^2 + \log \left(\frac{\tau^H}{1-\tau^H} \right) + \log \left(\frac{1 - \Phi(\underline{\mu}(0, \gamma); \sigma_\mu^2, \sigma_\mu^2)}{\Phi(\underline{\mu}(0, \gamma); 0, \sigma_\mu^2)} \right) \\ &\quad + \frac{1}{(\gamma-1)} \log (E [[\theta e^{\varepsilon_{i,t+1}} + (1-\theta)]^{1-\gamma}]) \\ &= \underline{\mu}^H > 0 . \end{aligned}$$

Thus, for any $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ there exists a $p \in [0, 1]$ such that

$$\underline{\mu}(p, \gamma) = 0 . \tag{A22}$$

We also know that both $\underline{\mu}(1, \gamma)$ and $\underline{\mu}(0, \gamma)$ are increasing in γ . It then follows that there is unique value of γ for which equation (A22) is satisfied for $p = 0.5$. This concludes the proof of the existence of a mixed equilibrium.

Announcement Returns. We finally obtain the results about announcement returns. Given the state price density obtained in Proposition A4, we can finally compute the equilibrium under

uncertainty. Let $\text{Prob}(L \text{ wins}) = 0.5$ and denote by γ^M the corresponding risk aversion. We have

$$\begin{aligned}
M_{it} &= \frac{E_t[\pi_{t+1} D_{it+1}]}{E_t[\pi_{t+1}]} = \frac{E_t\left[(1 - \tau_{t+})^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}} (1 - \tau_{t+}) G_t e^{\mu_i + \varepsilon_{it+1} + \varepsilon_{t+1}}\right]}{E_t\left[(1 - \tau_{t+})^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}}\right]} \\
&= G_t e^{\mu_i} \frac{E_t\left[(1 - \tau_{t+})^{1-\gamma^M} e^{(1-\gamma^M)\varepsilon_{t+1}}\right]}{E_t\left[(1 - \tau_{t+})^{-\gamma^M} e^{-\gamma^M \varepsilon_{t+1}}\right]} \\
&= G_t e^{\mu_i} \frac{\left[0.5 (1 - \tau^L)^{1-\gamma^M} + 0.5 (1 - \tau^H)^{1-\gamma^M}\right] e^{(1-\gamma^M)(-\frac{1}{2}\sigma^2) + \frac{1}{2}(1-\gamma^M)^2\sigma^2}}{\left[0.5 (1 - \tau^L)^{-\gamma^M} + 0.5 (1 - \tau^H)^{-\gamma^M}\right] e^{-\gamma^M(-\frac{1}{2}\sigma^2) + \frac{1}{2}(\gamma^M)^2\sigma^2}} \\
&= G_t e^{\mu_i} \frac{\left[0.5 (1 - \tau^L)^{1-\gamma^M} + 0.5 (1 - \tau^H)^{1-\gamma^M}\right] e^{(1-\gamma^M)(-\frac{1}{2}\sigma^2) + \frac{1}{2}(1+\gamma^M)^2 - 2\gamma^M)\sigma^2 + \gamma^M(-\frac{1}{2}\sigma^2) - \frac{1}{2}(\gamma^M)^2\sigma^2}}{\left[0.5 (1 - \tau^L)^{-\gamma^M} + 0.5 (1 - \tau^H)^{-\gamma^M}\right]} \\
&= G_t e^{\mu_i} \frac{\left[0.5 (1 - \tau^L)^{1-\gamma^M} + 0.5 (1 - \tau^H)^{1-\gamma^M}\right]}{\left[0.5 (1 - \tau^L)^{-\gamma^M} + 0.5 (1 - \tau^H)^{-\gamma^M}\right]} e^{-\gamma^M \sigma^2} \\
&= G_t e^{\mu_i - \gamma^M \sigma^2} [\omega (1 - \tau^L) + (1 - \omega) (1 - \tau^H)] ,
\end{aligned}$$

where

$$\omega = \frac{(1 - \tau^L)^{-\gamma^M}}{(1 - \tau^L)^{-\gamma^M} + (1 - \tau^H)^{-\gamma^M}} .$$

The results follow from the fact that after the announcement, the price is $M_{it+} = G_t e^{\mu_i - \gamma^M \sigma^2} (1 - \tau_{t+})$, where τ_{t+} denotes the tax rate realized at time $t+$. Q.E.D.

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