Internet Appendix for

"Income Inequality and Asset Prices under Redistributive Taxation"

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This Internet Appendix provides additional material in support of the analysis presented in Pástor and Veronesi (2016). The Internet Appendix is organized in three sections:

Section A. Proof of Theoretical Results.

In this section, we provide the proofs of the theoretical results presented in Section 2 of the paper. This section is organized as follows:

Proof of Proposition 1: Conditional on the set $\mathcal{I}$ of entrepreneurs, total capital at time T is

$$\begin{aligned} B_T &= \int_{\mathcal{I}} B_{i,T} di = B_0 \int_{\mathcal{I}} e^{\mu_i T + \varepsilon_{i,T} + \varepsilon_T} di \\ &= B_0 e^{\varepsilon_T} \int_{\mathcal{I}} e^{\mu_i T + \varepsilon_{i,T}} di \\ &= B_0 e^{\varepsilon_T} m(\mathcal{I}) E^{\mathcal{I}} \left[e^{\mu_j T} | j \in \mathcal{I} \right] \end{aligned}$$

where we used the law of large numbers,

$$\begin{aligned} \int_{\mathcal{I}} e^{\mu_i T + \varepsilon_{i,T}} di &= \left(\int_{\mathcal{I}} di \right) \int_{\mathcal{I}} e^{\mu_i T + \varepsilon_{i,T}} \frac{1}{\left(\int_{\mathcal{I}} di \right)} di \\ &= \left(\int_{\mathcal{I}} di \right) E^{\mathcal{I}} \left[e^{\mu_i T} e^{\varepsilon_{iT}} | i \in \mathcal{I} \right] \\ &= m(\mathcal{I}) E^{\mathcal{I}} \left[e^{\varepsilon_{i,T}} | i \in \mathcal{I} \right] E^{\mathcal{I}} \left[e^{\mu_i T} | i \in \mathcal{I} \right] \\ &= m(\mathcal{I}) E^{\mathcal{I}} \left[e^{\mu_i T} | i \in \mathcal{I} \right] \ , \end{aligned}$$

and the facts that $\varepsilon_{i,T}$ are i.i.d. and independent of μ_i, with $E^{\mathcal{I}} \left[e^{\varepsilon_{iT}} | i \in \mathcal{I} \right] = 1$. The measure of agents who become pensioners is $1 - m(\mathcal{I})$. Because total tax revenue at time T is

$$\tau B_T = \tau B_0 e^{\varepsilon_T} m(\mathcal{I}) E^{\mathcal{I}} \left[e^{\mu_j T} | j \in \mathcal{I} \right] \ ,$$

pensioners' consumption per capita is

$$C_{iT} = \tau B_0 e^{\varepsilon_T} E^{\mathcal{I}} \left[e^{\mu_j T} | j \in \mathcal{I} \right] \frac{m(\mathcal{I})}{1 - m(\mathcal{I})} \quad \text{for } i \notin \mathcal{I} \ .$$

The expected utility of a pensioner with risk aversion γ_i is then

$$\begin{aligned} E\left[U(C_{iT}) | \gamma_i, \text{pensioner} \right] &= E \left[\frac{\left(\tau B_0 e^{\varepsilon_T} E^{\mathcal{I}} \left[e^{\mu_j T} | j \in \mathcal{I} \right] \frac{m(\mathcal{I})}{1 - m(\mathcal{I})} \right)^{1-\gamma_i}}{1 - \gamma_i} \right] \\ &= \frac{\tau^{1-\gamma_i} B_0^{1-\gamma_i}}{1 - \gamma_i} \left(\frac{m(\mathcal{I})}{1 - m(\mathcal{I})} \right)^{1-\gamma_i} E\left[e^{(1-\gamma_i)\varepsilon_T} \right] \left(E^{\mathcal{I}} \left[e^{\mu_j T} | j \in \mathcal{I} \right] \right)^{1-\gamma_i} \end{aligned}$$

where the only ex-ante uncertainty is about the realization of the aggregate shock ε_T. Q.E.D.

Proof of Proposition 2. The claims in this proposition depend on derivations in the proof of Proposition 4, which should be read first. We only report here some additional steps not included in the proof of Proposition 4. In particular, denoting the after-tax dividend of firm i at time T by $D_T^i = (1-\tau) B_T^i$, entrepreneur i's consumption is

$$
\begin{aligned}
C_{i,T} &= \left(\theta D_T^i + \int_{\mathcal{I}\setminus i} N_0^{ij} D_T^j dj + N_0^{i0}\right) \\
&= \left(\theta \left(D_T^i - M_0^i\right) + \int_{\mathcal{I}\setminus i} N_0^{ij} \left(D_T^j - M_0^j\right) dj + M_0^i\right) \\
&= \left(\theta \left(D_T^i - M_0^i\right) + \widetilde{\alpha}^i (1-\theta) \frac{M_0^i}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk} \int_{\mathcal{I}\setminus i} \left(D_T^j - M_0^j\right) dj + M_0^i\right) ,
\end{aligned}
$$

where we used the budget equation

$$
(1-\theta) M_0^i = \int_{\mathcal{I}\setminus i} N_0^{ij} M_0^j dj + N_0^{i0}
$$

and the optimal allocation in stocks j from equation (A8)

$$
N_0^{ij} = \widetilde{\alpha}^i \omega^j \frac{M_0^i}{M_0^j} = (1-\theta) \frac{\widetilde{\alpha}^i M_0^i}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk} = (1-\theta) \frac{\widetilde{\alpha}^i e^{\mu_i T}}{\int_{\mathcal{I}} \widetilde{\alpha}^k e^{\mu_k T} dk} . \tag{A1}
$$

Factoring out M_0^i, we can rewrite entrepreneur i's consumption as

$$
\begin{aligned}
C_{i,T} &= M_0^i \left(\theta \left(\frac{D_T^i}{M_0^i} - 1\right) + \widetilde{\alpha}^i (1-\theta) \frac{1}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk} \int_{I\setminus i} M_0^j \left(\frac{D_T^j}{M_0^j} - 1\right) dj + 1\right) \\
&= M_0^i \left(\theta R^i + \widetilde{\alpha}^i (1-\theta) \Omega R^{Mkt} + 1\right) \\
&= (1-\tau) B_0 e^{\mu_i T} Z \left[\theta R^i + \widetilde{\alpha}^i (1-\theta) \Omega R^{Mkt} + 1\right] \\
&= (1-\tau) B_0 e^{\mu_i T} \left[\left(\theta \left(e^{\varepsilon_{i,T}+\varepsilon_T} - Z\right) + \widetilde{\alpha}^i (1-\theta) \Omega \left(e^{\varepsilon_T} - Z\right) + Z\right)\right] \\
&= (1-\tau) B_0 e^{\mu_i T} \left[\left(\theta \left(e^{\varepsilon_{i,T}+\varepsilon_T} - Z\right) + \alpha^i (1-\theta) \left(e^{\varepsilon_T} - Z\right) + Z\right)\right] ,
\end{aligned}
$$

where we used the definition from equation (A19)

$$
\alpha^i = \widetilde{\alpha}^i \Omega .
$$

Note that from

$$
\begin{aligned}
& \left(\theta R^i + (1-\theta) \alpha^i R^{Mkt} + 1\right) \\
= \; & \left(\theta \left(1 + R^i\right) + (1-\theta) \alpha^i \left(1 + R^{Mkt}\right) + (1-\theta) \left(1 - \alpha^i\right)\right) ,
\end{aligned}
$$

we can also write

$$
C_{i,T} = M_0^i \left(\theta \left(1 + R^i\right) + (1-\theta) \alpha^i \left(1 + R^{Mkt}\right) + (1-\theta) \left(1 - \alpha^i\right)\right) .
$$

The expected utility follows immediately from substituting the above expressions in

$$E\left[U\left(C_{i,T}\right)|\text{entrepreneur}\right] = \frac{1}{1-\gamma_i}E\left[\left(C_{i,T}\right)^{1-\gamma_i}\right] .$$

Finally, the stock and bond allocations follow from equation (A1):

$$\begin{aligned} N_0^{ij} &= (1-\theta)\frac{\widetilde{\alpha}^i e^{\mu_i T}}{\int_{\mathcal{I}}\widetilde{\alpha}^k e^{\mu_k T}dk} = (1-\theta)\frac{\alpha^i e^{\mu_i T}}{\int_{\mathcal{I}}\alpha^k e^{\mu_k T}dk} \\ &= (1-\theta)\frac{\alpha^i e^{\mu_i T}}{\int_{\mathcal{I}} e^{\mu_k T}dk} = (1-\theta)\frac{\alpha^i M_{i,0}}{M_0^P} , \end{aligned}$$

where we exploit the market-clearing condition (A23), $\int_{\mathcal{I}}\alpha^k e^{\mu_k T}dk = \int_{\mathcal{I}} e^{\mu_k T}dk$, and the stock-pricing formula (A5). Finally, from (A9) we have

$$\begin{aligned} N_0^{i0} &= (1-\theta)M_0^i - (1-\theta)\widetilde{\alpha}^i M_0^i\Omega \\ &= (1-\theta)\left(1-\widetilde{\alpha}^i\Omega\right)M_0^i \\ &= (1-\theta)\left(1-\alpha\left(\gamma_i\right)\right)M_0^i . \end{aligned}$$

Q.E.D.

Proof of Proposition 3: For convenience, we denote

$$\overline{V}^i = E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i\left(1-\theta\right)\Omega\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma_i}\right]$$

An agent decides to become an entrepreneur if and only if

$$E\left[U\left(C_{iT}\right)|\text{entrepreneur}\right] > E\left[U\left(C_{iT}\right)|\text{pensioner}\right]$$

that is, iff

$$\begin{aligned} &\frac{(1-\tau)^{1-\gamma_i}B_0^{1-\gamma_i}}{1-\gamma_i}e^{(1-\gamma_i)\mu_i T}\overline{V}^i \\ > &\frac{\tau^{1-\gamma_i}B_0^{1-\gamma_i}}{1-\gamma_i}\left(\frac{m\left(\mathcal{I}\right)}{1-m\left(\mathcal{I}\right)}\right)^{1-\gamma_i}E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)^{1-\gamma_i} \end{aligned}$$

Consider first the case $\gamma > 1$. Then this condition is satisfied if and only if

$$e^{(1-\gamma_i)\mu_i T}\overline{V}^i < \left(\frac{\tau}{1-\tau}\right)^{1-\gamma_i}\left(\frac{m\left(\mathcal{I}\right)}{1-m\left(\mathcal{I}\right)}\right)^{1-\gamma_i}E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)^{1-\gamma_i}$$

Taking logs and rearranging we obtain the cutoff rule:

$$\begin{aligned} (1-\gamma_i)\mu_i T + \log\left(\overline{V}^i\right) &< (1-\gamma_i)\log\left(\frac{\tau}{1-\tau}\right) + (1-\gamma_i)\log\left(\frac{m\left(\mathcal{I}\right)}{1-m\left(\mathcal{I}\right)}\right) \\ &\quad + \log\left(E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\right) + (1-\gamma_i)\log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right) \end{aligned}$$

or, dividing by $(1-\gamma_i)T$,

$$\begin{aligned} \mu_i + \frac{1}{(1-\gamma_i)\,T}\log\left(\overline{V}^i\right) &> \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\right] \\ &\quad + \frac{1}{(1-\gamma_i)\,T}\log\left(E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\right) \end{aligned}$$

or

$$\begin{aligned} \mu_i &> \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\right] \\ &\quad + \frac{1}{(1-\gamma_i)\,T}\left[\log\left(E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\right) - \log\left(\overline{V}^i\right)\right] \end{aligned}$$

or, finally

$$\mu_i > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\right] \tag{A2}$$

$$+\frac{1}{(1-\gamma_i)\,T}\left[\log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i(1-\theta)\,\Omega\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma_i}\right]}\right)\right] \tag{A3}$$

For $\gamma<1$, the same proof applies with obvious intermediate changes of inequalities, yielding the same cutoff rule.

For $\gamma=1$ (log utility), we have

$$E\left[U\left(C_{iT}\right)|\text{entrepreneur}\right] > E\left[U\left(C_{iT}\right)|\text{pensioner}\right]\ ,$$

that is, iff

$$\begin{aligned} & E\left[\log\left((1-\tau)\,B_0 e^{\mu_i T}\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i(1-\theta)\left(e^{\varepsilon_T}-Z\right)+Z\right)\right]\right)\right] \\ > \;& E\left[\log\left(\tau B_0 e^{\varepsilon_T}E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)\right] \end{aligned}$$

or

$$\begin{aligned} & \log(1-\tau)+\log(B_0)+\log\left(e^{\mu_i T}\right)+E\left[\log\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i(1-\theta)\left(e^{\varepsilon_T}-Z\right)+Z\right)\right]\right] \\ > \;& \log(\tau)+\log(B_0)+E\left[\log\left(e^{\varepsilon_T}\right)\right]+\log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)+\log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) \end{aligned}$$

or

$$\begin{aligned} \mu_i &> \frac{1}{T}\left\{\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\right. \\ &\quad \left. + E\left[\log\left(\frac{e^{\varepsilon_T}}{\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i(1-\theta)\left(e^{\varepsilon_T}-Z\right)+Z\right)}\right)\right]\right\} \end{aligned}$$

Q.E.D.

Proof of Proposition 4: Let $\mathcal{I}$ be the set of agents who choose to become entrepreneurs. Let M_0^i be the market value of firm i at time 0. The net-of-tax dividend paid by firm i at time T is $D_T^i = (1-\tau) B_T^i$.

Conjecture (to be verified later): The aggregate state price density depends only on ε_T:

$$\pi_T = \pi(\varepsilon_T) \ ,$$

for some function $\pi(\varepsilon_T)$.

Given the conjectured state price density, we can compute the price of each asset at time 0 as

$$\begin{aligned} M_0^i &= \frac{E\left[\pi_T (1-\tau) B_T^i\right]}{E\left[\pi_T\right]} = (1-\tau) B_0 e^{\mu_i T} \frac{E\left[\pi(\varepsilon_T) e^{\varepsilon_T + \varepsilon_{iT}}\right]}{E\left[\pi_T\right]} &\text{(A4)} \\ &= (1-\tau) B_0 e^{\mu_i T} \frac{E\left[\pi(\varepsilon_T) e^{\varepsilon_T}\right]}{E\left[\pi_T\right]} E\left[e^{\varepsilon_{iT}}\right] \\ &= (1-\tau) B_0 e^{\mu_i T} \frac{E\left[\pi(\varepsilon_T) e^{\varepsilon_T}\right]}{E\left[\pi_T\right]} \\ &= (1-\tau) B_0 e^{\mu_i T} Z &\text{(A5)} \end{aligned}$$

where we define the constant Z as

$$Z = \frac{E\left[\pi(\varepsilon_T) e^{\varepsilon_T}\right]}{E\left[\pi_T\right]} \ ,$$

which is the time-0 price of a security with payoff e^{ε_T} at time T. For later reference, note that the aggregate market value of the market portfolio is

$$M_0^P = \int_{\mathcal{I}} M_0^i di = (1-\tau) B_0 Z \int_{\mathcal{I}} e^{\mu_i T} di$$

and the total dividend

$$D_T^{Mkt} = (1-\tau) B_0 e^{\varepsilon_T} \int_{\mathcal{I}} e^{\mu_i T} di \ ,$$

so that the market return is

$$R^{Mkt} = \frac{D_T^{Mkt}}{M_0^P} - 1 = \frac{e^{\varepsilon_T}}{Z} - 1 \ .$$

In the arguments below, we will also make use of the fact that each individual stock is infinitesimal, that is, removing one stock from a continuum does not change the value of the market portfolio. In particular, we will use the following equality for $j \neq i$:

$$\int_{\mathcal{I}\backslash i} M_0^j dj = \int_{\mathcal{I}\backslash j} M_0^i di \ .$$

Consider now the budget equation of each agent i. At time 0, each agent i issues $1-\theta$ shares of his own firm i. From the proceeds, the agent purchases N_0^{ij} shares of firm j and N_0^{i0} bonds. As we show below, if unrestricted ($\theta = 0$), each agent would sell all of his firm and purchase the market portfolio, which would entail an infinitesimal position in his own firm. The θ constraint is always binding; for any given θ, each agent restricts his holdings of his own firm to exactly θ shares. All quantities are expressed in terms of our numeraire, which is the zero-coupon bond with maturity T that is a claim to one unit of capital at T. The bond price is thus equal to one at both times 0 and T. The bonds are in zero net supply. The budget equation is

$$(1-\theta) M_0^i = \int_{\mathcal{I}\backslash i} N_0^{ij} M_0^j dj + N_0^{i0} . \tag{A6}$$

Agents only trade once, at time 0, and they hold their positions until time T. At time T, agent i's consumption is

$$C_{i,T} = \theta D_T^i + \int_{\mathcal{I}\backslash i} N_0^{ij} D_T^j dj + N_0^{i0} . \tag{A7}$$

We assume that the distributions of shocks ε_T and ε_{iT} are such that the equilibrium $C_{i,T} > 0$ with probability one.

Before we move to analyze the optimal choice of each individual, consider the market-clearing condition. Each agent j issues exactly $1-\theta$ shares. Therefore, we must have that in equilibrium all shares issued are bought by somebody. That is, the sum of all the j shares bought by agents i must equal $1-\theta$

$$1-\theta = \int_{\mathcal{I}\backslash j} N_0^{ij} di .$$

Compared to the budget equation, the integral here is over i and not over j. The bond market must clear, too, and given that bonds are in zero net supply, we must have

$$\int_{\mathcal{I}} N_0^{i0} di = 0 .$$

The utility function of entrepreneur $i \in \mathcal{I}$ is*

$$\begin{aligned} E\left[U\left(C_{i,T}\right)\right] &= \frac{1}{1-\gamma_i} E\left[\left(C_{i,T}\right)^{1-\gamma_i}\right] \\ &= \frac{1}{1-\gamma_i} E\left[\left(\theta D_T^i + \int_{I\backslash i} N_0^{ij} D_T^j dj + N_0^{i0}\right)^{1-\gamma_i}\right] . \end{aligned}$$

Consider again the budget equation of agent i, now rewritten as

$$(1-\theta) M_0^i - \int_{\mathcal{I}\backslash i} N_0^{ij} M_0^j dj = N_0^{i0} .$$

*The argument below also applies to agents with $\gamma_i = 1$, that is, log utility investors, as the main equations only depend on marginal utility $C_{i,T}^{-\gamma_i}$, which are independent of whether $\gamma_i = 1$ or not.

Substitute for N_0^{i0} in the utility function to find

$$E\left[U\left(C_{i,T}\right)\right]=\frac{1}{1-\gamma_i}E\left[\left(\theta\left(D_T^i-M_0^i\right)+\int_{\mathcal{I}\setminus i}N_0^{ij}\left(D_T^j-M_0^j\right)dj+M_0^i\right)^{1-\gamma_i}\right]$$

The first-order conditions (FOC) with respect to N_0^{ij} are

$$E\left[\left(\theta\left(D_T^i-M_0^i\right)+\int_{\mathcal{I}\setminus i}N_0^{ij}\left(D_T^j-M_0^j\right)dj+M_0^i\right)^{-\gamma_i}\left(D_T^j-M_0^j\right)\right]=0\ .$$

We can rewrite this expression as

$$E\left[\left(\theta\left(\frac{D_T^i}{M_0^i}-1\right)M_0^i+M_0^i\int_{\mathcal{I}\setminus i}\frac{N_0^{ij}M_0^j}{M_0^i}\left(\frac{D_T^j}{M_0^j}-1\right)dj+M_0^i\right)^{-\gamma_i}\left(D_T^j-M_0^j\right)\right]=0\ .$$

Factoring M_0^i out of the expectation and simplifying, we can rewrite the FOC as

$$E\left[\left(\theta\left(\frac{D_T^i}{M_0^i}-1\right)+\int_{\mathcal{I}\setminus i}\frac{N_0^{ij}M_0^j}{M_0^i}\left(\frac{D_T^j}{M_0^j}-1\right)dj+1\right)^{-\gamma_i}\left(\frac{D_T^j}{M_0^j}-1\right)\right]=0\ .$$

Define ω_0^{ij} as

$$\omega^{ij}=\frac{N_0^{ij}M_0^j}{M_0^i}\ .$$

Note that for every j, the net-of-tax arithmetic return on investment is

$$R^j=\frac{D_T^j}{M_0^j}-1=\frac{(1-\tau)\,B_0e^{\mu_jT+\varepsilon_{j,T}+\varepsilon_T}}{(1-\tau)\,B_0e^{\mu_jT}Z}-1=\frac{e^{\varepsilon_{j,T}+\varepsilon_T}}{Z}-1\ .$$

That is, the return R^j is the same across firms, except for the realization of the idiosyncratic shock $\varepsilon_{j,T}$. Indeed, all stocks have the same expected return

$$E\left[R^j\right]=Z^{-1}-1\ .$$

We can rewrite the FOC then as

$$E\left[\left(\theta R^i+\int_{\mathcal{I}\setminus i}\omega^{ij}R^j dj+1\right)^{-\gamma_i}R^j\right]=0\ .$$

From the above discussion, all R^j have the same risk-return characteristics.

The FOC for each agent i and j are identical except for differences in risk aversion. We now show that the equilibrium portfolio allocation takes the multiplicative form

$$\omega^{ij}=\widetilde{\alpha}^i\omega^j\ .$$

That is, each agent i invests the same fraction ω^j in each stock j, except that is scaled by a common (across stocks) constant $\widetilde{\alpha}^i$, which is due to risk aversion. We just have to show that in equilibrium, $\omega^{ij} = \widetilde{\alpha}^i \omega^j$ satisfy the FOC for all agents i and all stocks j.

To show this, let's first express the number of shares bought N_0^{ij} as a function of ω^{ij} and thus $\widetilde{\alpha}^i \omega^j$:

$$\omega^{ij} = \widetilde{\alpha}^i \omega^j = \frac{N_0^{ij} M_0^j}{M_0^i} \text{ for } j \neq i \ .$$

Solving for N_0^{ij} we obtain the number of shares bought by each agent i:

$$N_0^{ij} = \widetilde{\alpha}^i \omega^j \frac{M_0^i}{M_0^j} \text{ for } j \neq i \ . \tag{A8}$$

We determine $\widetilde{\alpha}^i$ and ω^j separately. To determine the equilibrium value of ω^j, we exploit the market-clearing condition. Recall that we must have that the total number of shares issued by firm j satisfies

$$1 - \theta = \int_{\mathcal{I} \backslash j} N_0^{ij} di \ .$$

Substitute for N_0^{ij}:

$$1 - \theta = \int_{\mathcal{I} \backslash j} \widetilde{\alpha}^i \omega^j \frac{M_0^i}{M_0^j} di$$

or

$$(1 - \theta) \, M_0^j = \omega^j \int_{\mathcal{I} \backslash j} \widetilde{\alpha}^i M_0^i di \ .$$

That is, for every agent i,

$$\omega^j = (1 - \theta) \, \frac{M_0^j}{\int_{\mathcal{I} \backslash j} \widetilde{\alpha}^i M_0^i di} \ .$$

Consider now the budget equation of agent i:

$$(1 - \theta) \, M_0^i = \int_{\mathcal{I} \backslash i} N_0^{ij} M_0^j dj + N_0^{i0} \ .$$

Substitute for N_0^{ij} from above

$$(1 - \theta) \, M_0^i = \int_{\mathcal{I} \backslash i} \left(\widetilde{\alpha}^i \omega^j \frac{M_0^i}{M_0^j} \right) M_0^j dj + N_0^{i0} \ ,$$

which gives

$$(1 - \theta) \, M_0^i = \widetilde{\alpha}^i M_0^i \int_{\mathcal{I} \backslash i} \omega^j dj + N_0^{i0} \ .$$

Substitute for ω^j as well

$$(1 - \theta) \, M_0^i = \widetilde{\alpha}^i M_0^i \int_{\mathcal{I} \backslash i} (1 - \theta) \, \frac{M_0^j}{\int_{\mathcal{I} \backslash j} \widetilde{\alpha}^k M_0^k dk} dj + N_0^{i0} \ ,$$

which gives

$$(1-\theta)\, M_0^i = (1-\theta)\, \widetilde{\alpha}^i M_0^i \frac{\int_{\mathcal{I}\setminus i} M_0^j dj}{\int_{\mathcal{I}\setminus j} \widetilde{\alpha}^k M_0^k dk} + N_0^{i0}$$

or

$$(1-\theta)\, M_0^i = (1-\theta)\, \widetilde{\alpha}^i M_0^i \Omega + N_0^{i0}\ , \tag{A9}$$

where

$$\Omega = \frac{\int_{\mathcal{I}} M_0^j dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk}\ .$$

Each stock is infinitesimal, and therefore even if the market prices of i and j are different, we still have in the continuous limit $\int_{\mathcal{I}\setminus i} M_0^j dj = \int_{\mathcal{I}\setminus j} M_0^i di = \int_{\mathcal{I}} M_0^i di$. We obtain:

$$N_0^{i0} = (1-\theta)\, M_0^i \left(1 - \widetilde{\alpha}^i \Omega\right)\ .$$

Note that for every choice of $\{\widetilde{\alpha}^i\}$, the above construction implies that the bond market clears. In fact, integrating over both sides of equation (A9), we have

$$(1-\theta) \int_{\mathcal{I}} M_0^i di = (1-\theta) \int_{\mathcal{I}} \widetilde{\alpha}^i M_0^i di \frac{\int_{\mathcal{I}\setminus i} M_0^j dj}{\int_{\mathcal{I}\setminus j} \widetilde{\alpha}^k M_0^k dk} + \int_{\mathcal{I}} N_0^{i0} di\ . \tag{A10}$$

Simplifying, this gives

$$\int_{\mathcal{I}} N_0^{i0} di = 0\ ,$$

which is the market-clearing condition for bonds. That is, for any choice of $\widetilde{\alpha}^i$ across i, the bond market-clearing condition is satisfied.

Finally, we have to check that with this choice of $\omega^{ij} = \widetilde{\alpha}^i \omega^j$, with ω^j given above, the FOC of each agent i with respect to stock j is indeed satisfied. Recall the FOC of agent i is

$$E\left[\left(\theta R^i + \int_{\mathcal{I}\setminus i} \omega^{ij} R^j dj + 1\right)^{-\gamma_i} R^j\right] = 0\ .$$

Substitute what we found earlier as the equilibrium weight of agent i into stock j :

$$\omega^{ij} = \widetilde{\alpha}^i \omega^j = (1-\theta)\, \frac{\widetilde{\alpha}^i M_0^j}{\int_{\mathcal{I}\setminus j} \widetilde{\alpha}^k M_0^k dk}$$

to find that the FOC is

$$E\left[\left(\theta R^i + \int_{\mathcal{I}\setminus i} (1-\theta)\, \frac{\widetilde{\alpha}^i M_0^j}{\int_{\mathcal{I}\setminus j} \widetilde{\alpha}^k M_0^k dk} R^j dj + 1\right)^{-\gamma_i} R^j\right] = 0$$

or

$$E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \int_{\mathcal{I}\setminus i} \frac{M_0^j}{\int_{\mathcal{I}\setminus j} \widetilde{\alpha}^k M_0^k dk} R^j dj + 1\right)^{-\gamma_i} R^j\right] = 0$$

or, recalling the constant $\Omega = \frac{\int_{\mathcal{I}} M_0^k dk}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk}$, we can write this equation as

$$E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega \int_{\mathcal{I}\setminus i} \frac{M_0^j}{\int_{\mathcal{I}} M_0^k dk} R^j dj + 1\right)^{-\gamma_i} R^j\right] = 0$$

or

$$E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} R^j\right] = 0 \tag{A11}$$

where R^{Mkt} is the return on the market portfolio

$$\begin{aligned} R^{Mkt} &= \int_{\mathcal{I}} \frac{M_0^j}{\int_{\mathcal{I}} M_0^k dk} R^j dj \\ &= \int_{\mathcal{I}} \frac{M_0^j}{\int_{\mathcal{I}} M_0^k dk} \left(\frac{D_T^j}{M_0^j} - 1\right) dj \\ &= \frac{\int_{\mathcal{I}} D_T^j dj}{\int_{\mathcal{I}} M_0^k dk} - 1\,. \end{aligned}$$

Ex ante, all R^i, R^j have the same characteristics, as we can write

$$R^i = \frac{e^{\varepsilon_{i,T}+\varepsilon_T}}{Z} - 1 \tag{A12}$$

$$R^{Mkt} = \frac{e^{\varepsilon_T}}{Z} - 1\,. \tag{A13}$$

This implies that we can write equation (A11) as

$$\begin{aligned} 0 &= E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} R^j\right] \\ &= E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} \left(\frac{e^{\varepsilon_{j,T}+\varepsilon_T}}{Z} - 1\right)\right] \\ &= E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} \frac{e^{\varepsilon_{j,T}+\varepsilon_T}}{Z}\right] \\ &\quad -E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i}\right] \\ &= E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} \frac{e^{\varepsilon_T}}{Z}\right] \\ &\quad -E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i}\right] \end{aligned}$$

or, finally,

$$0 = E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}^i \Omega R^{Mkt} + 1\right)^{-\gamma_i} R^{Mkt}\right]\,. \tag{A14}$$

That is, the FOC with respect to any stock $j \neq i$ is the same as the FOC with respect to the market portfolio, because only the systematic part of each stock j comoves with the personal SDF, and all stocks have the same risk exposure. Thus, the only difference across agents i is the risk aversion γ_i. Therefore, we can find a solution $\widetilde{\alpha}^i$ that makes the equation equal to zero. Because this quantity depends only on risk aversion, we can denote it by

$$\widetilde{\alpha}^i = \widetilde{\alpha}(\gamma_i) \ .$$

In summary, the above calculations show that the equilibrium portfolio allocation is given by

$$\omega^{ij} = \widetilde{\alpha}(\gamma_i)\,\omega^j \ ,$$

where $\widetilde{\alpha}(\gamma_i)$ solves equation (A14), and

$$\omega^j = (1-\theta)\frac{M_0^j}{\int_{\mathcal{I}} \widetilde{\alpha}(\gamma^k)\, M_0^k dk} \ .$$

That is,

$$\omega^{ij} = \widetilde{\alpha}\left(\gamma^i\right)\omega^j = (1-\theta)\frac{\widetilde{\alpha}(\gamma^i)\, M_0^j}{\int_{\mathcal{I}} \widetilde{\alpha}(\gamma^k)\, M_0^k dk} \ .$$

We have already shown that market-clearing holds for both stocks and bonds.

Finally, we need to show that the SDF satisfies the initial conjecture $\pi_T = \pi(\varepsilon_T)$. Let us rewrite the FOC in terms of dividends again:

$$E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i}\left(\frac{D_T^j}{M_0^j} - 1\right)\right] = 0$$

or, for every i,

$$E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i} D_T^j\right] = E\left[\left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i}\right] M_0^j \ .$$

Integrate across $i \in \mathcal{I}$ to obtain

$$E\left[\int_{\mathcal{I}} \left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i} di\; D_T^j\right] = E\left[\int_{\mathcal{I}} \left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i} di\right] M_0^j \ .$$

We can define the state price density as

$$\pi_T = \int_{\mathcal{I}} \left(\theta R^i + (1-\theta)\,\widetilde{\alpha}(\gamma_i)\,\Omega R^{Mkt} + 1\right)^{-\gamma_i} di \qquad \text{(A15)}$$

so that the above equation is

$$E\left[\pi_T\, D_T^j\right] = E\left[\pi_T\right] M_0^j \ ,$$

which is the standard pricing equation. We now show that this state price density only depends on ε_T as initially conjectured. We have

$$\pi_T = \int_{\mathcal{I}} \left(\theta \left(\frac{D_T^i}{M_0^i} - 1 \right) + (1-\theta)\, \widetilde{\alpha}(\gamma_i)\, \Omega \left(\frac{\int_{\mathcal{I}} D_T^j dj}{\int_{\mathcal{I}} M_0^k dk} - 1 \right) + 1 \right)^{-\gamma_i} di \tag{A16}$$

$$= \int_{\mathcal{I}} \left(\theta \left(\frac{e^{\varepsilon_{iT}+\varepsilon_T}}{Z} - 1 \right) + (1-\theta)\, \widetilde{\alpha}(\gamma_i)\, \Omega \left(\frac{e^{\varepsilon_T}}{Z} - 1 \right) + 1 \right)^{-\gamma_i} di \tag{A17}$$

$$= m(\mathcal{I}) E^{\mathcal{I}} \left[\left(\theta \left(\frac{e^{\varepsilon_{iT}+\varepsilon_T}}{Z} - 1 \right) + (1-\theta)\, \widetilde{\alpha}(\gamma_i)\, \Omega \left(\frac{e^{\varepsilon_T}}{Z} - 1 \right) + 1 \right)^{-\gamma_i} | i \in \mathcal{I} \right] \tag{A18}$$

where the expectation is taken over the random variables affecting agents i cross-sectionally, that is, $\varepsilon_{i,T}$ and γ_i (and not ε_T). Because these variables are independent, denoting by $f(\gamma)$ the distribution of $\gamma \in \Gamma$ and by $g(\varepsilon_{i,T})$ the distribution of $\varepsilon_{i,T} \in G$, we have that for every ε_T:

$$\begin{aligned}\pi_T = m(\mathcal{I}) \int_\Gamma \int_G & \left(\theta \left(\frac{e^{\varepsilon_{iT}+\varepsilon_T}}{Z} - 1 \right) \right. \\ & \left. + (1-\theta)\, \widetilde{\alpha}(\gamma)\, \Omega \left(\frac{e^{\varepsilon_T}}{Z} - 1 \right) + 1 \right)^{-\gamma} g(\varepsilon_{iT})\, d\varepsilon_{iT} f(\gamma)\, d\gamma\end{aligned}$$

We thus have $\pi_T = \pi(\varepsilon_T)$. The state price density above requires a fixed point because Z itself depends on π_T, from its definition

$$Z = \frac{E\left[\pi(\varepsilon_T)\, e^{\varepsilon_T}\right]}{E\left[\pi_T\right]}.$$

Finally, note that substituting for market prices in Ω, we obtain

$$\Omega = \frac{\int_{\mathcal{I}} M_0^j dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k dk} = \frac{\int_{\mathcal{I}} (1-\tau)\, B_0 e^{\mu_j T} Z dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k (1-\tau)\, B_0 e^{\mu_k T} Z dk} = \frac{\int_{\mathcal{I}} e^{\mu_j T} dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k e^{\mu_k T} dk}.$$

We can denote

$$\alpha(\gamma_i) = \widetilde{\alpha}(\gamma_i)\, \Omega = \widetilde{\alpha}(\gamma_i) \frac{\int_{\mathcal{I}} e^{\mu_j T} dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k e^{\mu_k T} dk} \tag{A19}$$

to obtain the simpler formula

$$\pi_T = \int_{\mathcal{I}} \left(\theta R^i + (1-\theta)\, \alpha(\gamma_i)\, R^{Mkt} + 1 \right)^{-\gamma_i} di$$

and the condition $\alpha^i = \alpha(\gamma_i)$ defined by

$$0 = E\left[\left(\theta R^i + (1-\theta)\, \alpha^i R^{Mkt} + 1 \right)^{-\gamma_i} R^{Mkt} \right].$$

With the new notation, equation (A9)

$$(1-\theta)\, M_0^i = (1-\theta)\, \widetilde{\alpha}^i M_0^i \Omega + N_0^{i0} \tag{A20}$$

becomes

$$(1-\theta)\, M_0^i = (1-\theta)\, \alpha^i M_0^i + N_0^{i0} \ . \tag{A21}$$

Integrating over i, we have

$$(1-\theta) \int_{\mathcal{I}} M_0^i di = (1-\theta) \int_{\mathcal{I}} \alpha^i M_0^i di + \int_{\mathcal{I}} N_0^{i0} di \ .$$

We know from (A10) that market clearing for bonds is satisfied, $\int_{\mathcal{I}} N_0^{i0} di = 0$, which gives the equivalent restriction on α^i as

$$(1-\theta) \int_{\mathcal{I}} M_0^i di = (1-\theta) \int_{\mathcal{I}} \alpha^i M_0^i di$$

or

$$1 = \frac{\int_{\mathcal{I}} \alpha^i M_0^i di}{\int_{\mathcal{I}} M_0^i di} \ . \tag{A22}$$

Substituting for M_0^i, we obtain the following restriction on α^i:

$$1 = \frac{\int_{\mathcal{I}} \alpha^i e^{\mu_i T} di}{\int_{\mathcal{I}} e^{\mu_i T} di} \ . \tag{A23}$$

Q.E.D.

Proof of Proposition 5. Immediate from (A12) and (A13) and $E\left[e^{\varepsilon_{i,T}}\right] = E\left[e^{\varepsilon_T}\right] = 1$. Q.E.D.

Proof of Proposition 6: For part (a), see equation (A5) in the proof of Proposition 4, together with the definition of $1 + r = \frac{1}{Z}$. Part (b) follows from part (a) after integrating over $i \in \mathcal{I}$ and recalling that $B_0^P = m(\mathcal{I})\, B_0$. Q.E.D.

Corollary A1. If $\alpha(\gamma_i) > 0$ for all γ_i, then $\alpha'(\gamma_i) < 0$. In other words, agent i's allocation to stocks declines with the agent's risk aversion.

Proof of Corollary A1: Consider two levels of risk aversion, $\gamma_H > \gamma_L$. We want to show that $\alpha(\gamma_H) < \alpha(\gamma_L)$. Consider first the FOC of the low-risk aversion agent. Let the agent's stock allocation $\alpha^*(\gamma_L)$ satisfy the first order condition

$$E\left[\left(\theta R^i + (1-\theta)\, \alpha^*(\gamma_L)\, R^{Mkt} + 1\right)^{-\gamma_L} R^{Mkt}\right] = 0 \ .$$

We know that if $\gamma_H > \gamma_L$ there exists an increasing concave function $G(.)$ such that

$$u_H = G(u_L) \ .$$

Consider the marginal utility of γ_H computed at the optimal choice of γ_L:

$$V_H = E\left[\left(\theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt} + 1\right)^{-\gamma_H} R^{Mkt}\right] .$$

From the chain rule, we have

$$u_H'(\alpha) = G'(u_L(\alpha))\,u_L'(\alpha) \ ,$$

that is,

$$\begin{aligned} V_H &= E\left[\left(\theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt} + 1\right)^{-\gamma_H} R^{Mkt}\right] \\ &= E\left[G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(\theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt}+1\right)^{1-\gamma_L}\right)\left(\theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt}+1\right)^{-\gamma_L} R^{Mkt}\right] \end{aligned}$$

If $G'(.)$ were a constant, then this expectation would be equal to zero, as it corresponds to the FOC of γ_L. Instead, $G(.)$ is increasing and concave, and therefore $G'(x) > 0$ and it is decreasing in x. Denote for convenience

$$\begin{aligned} R_P(\gamma_L) &= \theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt} + 1 \\ &= \theta\frac{\left(e^{\varepsilon_{i,T}}e^{\varepsilon_T}-1\right)}{Z} + (1-\theta)\,\alpha^*(\gamma_L)\left(\frac{e^{\varepsilon_T}}{Z}-1\right) + 1 \ . \end{aligned}$$

In particular, we can write

$$\begin{aligned} & E\left[G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(R_P(\gamma_L)\right)^{1-\gamma_L}\right)\left(R_P(\gamma_L)\right)^{-\gamma_L} R^{Mkt}\right] \\ =\ & \frac{E\left[G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(R_P(\gamma_L)\right)^{1-\gamma_L}\right)\left(R_P(\gamma_L)\right)^{-\gamma_L} R^{Mkt}\right]}{E\left[G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(R_P(\gamma_L)\right)^{1-\gamma_L}\right)\right]} E\left[G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(R_P(\gamma_L)\right)^{1-\gamma_L}\right)\right] \\ =\ & E^*\left[\left(R_P(\gamma_L)\right)^{-\gamma_L} R^{Mkt}\right] E\left[G'\left(M_{i,0}^{1-\gamma_L}\frac{\left(R_P(\gamma_L)\right)^{1-\gamma_L}}{1-\gamma_L}\right)\right] \ , \end{aligned}$$

where $E^*[]$ uses the joint density

$$f^*(\varepsilon_{i,T},\varepsilon_T) = \frac{G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(\theta\frac{\left(e^{\varepsilon_{i,T}}e^{\varepsilon_T}-1\right)}{Z} + (1-\theta)\,\alpha^*(\gamma_L)\left(\frac{e^{\varepsilon_T}}{Z}-1\right)+1\right)^{1-\gamma_L}\right) f(\varepsilon_{i,T})\,f(\varepsilon_T)}{\int\int G'\left(\frac{M_{i,0}^{1-\gamma_L}}{1-\gamma_L}\left(\theta\frac{\left(e^{\varepsilon_{i,T}}e^{\varepsilon_T}-1\right)}{Z} + (1-\theta)\,\alpha^*(\gamma_L)\left(\frac{e^{\varepsilon_T}}{Z}-1\right)+1\right)^{1-\gamma_L}\right) f(\varepsilon_{i,T})\,f(\varepsilon_T)\,d\varepsilon_{i,T}d\varepsilon_T} .$$

Assume that $\alpha^*(\gamma_L) > 0$. Then, because $G'(x)$ is decreasing in its argument, it is also decreasing in the aggregate shock ε_T. Compared to the original distribution $f(\varepsilon_{i,T},\varepsilon_T)$, the distribution

$f^*(\varepsilon_{i,T}, \varepsilon_T)$ gives more weight to low ε_T states, and less weight to high ε_T. That is, the states with $R^{Mkt} = \frac{e^{\varepsilon_T}}{Z} - 1 < 0$ get more weight while the states with $R^{Mkt} > 0$ get less weight compared to the original distribution. Therefore, if for constant $G'(x)$ we have $f^* = f$ and thus $E^*\left[(R_P(\gamma_L))^{-\gamma_L} R^{Mkt}\right] = E\left[(R_P(\gamma_L))^{-\gamma_L} R^{Mkt}\right] = 0$, with the new f^* distribution (and hence more weight to negative R^{Mkt}), we have

$$E^*\left[(R_P(\gamma_L))^{-\gamma_L} R^{Mkt}\right] < 0 \, .$$

From the definition, we thus have

$$V_H = E\left[\left(\theta R^i + (1-\theta)\,\alpha^*(\gamma_L)\,R^{Mkt} + 1\right)^{-\gamma_H} R^{Mkt}\right] < 0 \, .$$

Finally, note that the function

$$V_H(\alpha) = E\left[\left(\theta R^i + (1-\theta)\,\alpha R^{Mkt} + 1\right)^{-\gamma_H} R^{Mkt}\right]$$

is decreasing in α, as

$$V_H'(\alpha) = -\gamma_H E\left[\left(\theta R^i + (1-\theta)\,\alpha R^{Mkt} + 1\right)^{-\gamma_H - 1} \left(R^{Mkt}\right)^2 (1-\theta)\right] < 0 \, ,$$

which implies

$$0 = V_H(\alpha^*(\gamma_H)) > V_H(\alpha^*(\gamma_L))$$

if and only if $\alpha^*(\gamma_H) < \alpha^*(\gamma_L)$.

Q.E.D.

Corollary A2. The threshold on the right-hand-side of (19) in the text (or (A3) in the proof of Proposition 3) is increasing in γ when $\theta \to 1$ (Case 1 below) and also when $\varepsilon_T = 0$ (Case 2 below) as long as $\gamma > 1$.

Proof of Corollary A2. The selection condition is the following:

$$\begin{aligned}
\mu_i \;\; > \;\; & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \\
& + \frac{1}{(1-\gamma_i)\,T}\left[\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_i}\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T} - Z\right) + \alpha^i (1-\theta)\left(e^{\varepsilon_T} - Z\right) + Z\right)^{1-\gamma_i}\right]}\right)\right] \, .
\end{aligned}$$

We are interested in examples in which the right-hand-side (RHS) of this selection condition increases in γ_i.

Case 1. (Full constraints). Consider $\theta = 1$. Then

$$\begin{aligned}
\mu_i &> \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] \\
&\quad + \frac{1}{(1-\gamma_i)T}\left[\log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[e^{(1-\gamma_i)\left(\varepsilon_{i,T}+\varepsilon_T\right)}\right]}\right)\right] \\
&= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] \\
&\quad + \frac{1}{(1-\gamma_i)T}\left[\log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[e^{(1-\gamma_i)\varepsilon_{i,T}}\right]E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}\right)\right] \\
&= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] \\
&\quad - \frac{1}{(1-\gamma_i)T}\left[\log\left(E\left[e^{(1-\gamma_i)\varepsilon_{i,T}}\right]\right)\right] .
\end{aligned}$$

Adding the assumption $\varepsilon_{i,T} \sim N\left(-\frac{1}{2}\sigma^2 T, \sigma^2 T\right)$,

$$\mu_i > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] + \gamma_i \frac{1}{2}\sigma_1^2 .$$

The RHS is clearly increasing in γ_i.

Case 2. (no systematic risk). If there is no systematic risk, then $\varepsilon_T = 0$. It follows that $Z = E[\pi_T \times 1]/E[\pi_T] = 1$. We now show that if $\gamma > 1$ the RHS of selection condition is increasing in γ_i. In fact, we have

$$\begin{aligned}
\mu_i &> \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] \\
&\quad + \frac{1}{(1-\gamma_i)T}\left[\log\left(\frac{1}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]}\right)\right] \\
&= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]\right)\right] \\
&\quad - \frac{1}{T}\frac{1}{(1-\gamma_i)}\left[\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]\right)\right] .
\end{aligned}$$

Consider $\gamma > 1$ and define

$$U(\gamma) = \frac{1}{1-\gamma}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]\right) .$$

Then

$$\begin{aligned} U'(\gamma) &= (1-\gamma)^{-2}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]\right) \\ & \quad -\frac{1}{1-\gamma}\frac{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]}. \end{aligned}$$

The first term is positive, the second term is negative. We have

$$U'(\gamma) < 0$$

iff

$$\begin{aligned} & (1-\gamma)^{-2}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]\right) \\ < \; & \frac{1}{1-\gamma}\frac{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]} \end{aligned}$$

iff

$$\begin{aligned} & \frac{1}{1-\gamma}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]\right)E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right] \\ > \; & E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right] \end{aligned}$$

iff

$$\begin{aligned} & \log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]^{\frac{1}{(1-\gamma)}}\right)E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right] \\ > \; & Cov\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma},\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right] \\ & +E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]E\left[\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right] \end{aligned}$$

iff

$$\begin{aligned} & \left[\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]^{\frac{1}{(1-\gamma)}}\right)-E\left[\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right]\right]E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right] \\ > \; & Cov\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma},\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right] \end{aligned}$$

iff

$$\begin{aligned} & \left[\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]^{\frac{1}{(1-\gamma)}}\right)-E\left[\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right]\right] \\ > \; & \frac{Cov\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma},\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]} \end{aligned}$$

The RHS is negative, because the correlation between $\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}$ and $\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)$

is negative. The LHS is always positive. In fact, we have

$$\begin{aligned}
\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma}\right]^{\frac{1}{(1-\gamma)}}\right) &> \log\left(\left[\left[E\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right]^{1-\gamma}\right]^{\frac{1}{(1-\gamma)}}\right)\\
&= \log\left(\left(\theta E\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\\
&= \log\left(1\right)=0
\end{aligned}$$

and

$$E\left[\log\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)\right] < \log\left(E\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)\right)=\log\left(1\right)=0\,.$$

Hence the condition above is always satisfied.

Q.E.D.

Counterexample: We show below that it is possible for the right-hand-side of equation (19) in the paper (or (A3) in the proof of Proposition 3) to be increasing in γ.

Let $\theta=0$ and define

$$RHS\left(\gamma\right)=\frac{1}{1-\gamma}\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma}\right]}{E\left[\left(\alpha\left(\gamma\right)\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma}\right]}\right)\,,$$

which we can rewrite as

$$RHS\left(\gamma\right)=\frac{1}{1-\gamma}\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma}\right]/\left(1-\gamma\right)}{E\left[\left(\alpha\left(\gamma\right)\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma}\right]/\left(1-\gamma\right)}\right)\,.$$

Consider two agents with two different risk aversions, $\gamma_1<\gamma_2$. Choose γ_1 such that $\alpha\left(\gamma_1\right)=1$ is the optimal choice for the first agent. In this case

$$\begin{aligned}
RHS\left(\gamma_1\right) &= \frac{1}{1-\gamma_1}\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_1}\right]/\left(1-\gamma_1\right)}{E\left[\left(\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma_1}\right]/\left(1-\gamma_1\right)}\right)\\
&= \frac{1}{1-\gamma_1}\log\left(1\right)\\
&= 0\,.
\end{aligned}$$

Let $\alpha\left(\gamma_2\right)\neq 1$ be the optimal choice of the agent with the higher risk aversion (we know that we should have $\alpha\left(\gamma_2\right)<1$ but that is irrelevant). We have

$$RHS\left(\gamma_2\right)=\frac{1}{1-\gamma_2}\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_2}\right]/\left(1-\gamma_2\right)}{E\left[\left(\alpha\left(\gamma_2\right)\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma_2}\right]/\left(1-\gamma_2\right)}\right)\,.$$

Because $\alpha(\gamma_2) \neq 1$ is the optimal choice, the utility under $\alpha(\gamma_2)$ must be higher than the utility under $\alpha = 1$, that is

$$E\left[\left(\alpha(\gamma_2)(e^{\varepsilon_T} - Z) + Z\right)^{1-\gamma_2}\right] / (1-\gamma_2) > E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_2}\right] / (1-\gamma_2)$$

and hence

$$1 < \frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_2}\right] / (1-\gamma_2)}{E\left[\left(\alpha(\gamma_2)(e^{\varepsilon_T} - Z) + Z\right)^{1-\gamma_2}\right] / (1-\gamma_2)} .$$

Thus

$$\log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_2}\right] / (1-\gamma_2)}{E\left[\left(\alpha(\gamma_2)(e^{\varepsilon_T} - Z) + Z\right)^{1-\gamma_2}\right] / (1-\gamma_2)}\right) > 0$$

and hence

$$RHS(\gamma_2) = \frac{1}{1-\gamma_2} \log\left(\frac{E\left[\left(e^{\varepsilon_T}\right)^{1-\gamma_2}\right] / (1-\gamma_2)}{E\left[\left(\alpha(\gamma_2)(e^{\varepsilon_T} - Z) + Z\right)^{1-\gamma_2}\right] / (1-\gamma_2)}\right) < 0 = RHS(\gamma_1) .$$

That is, for $\theta = 0$, there exists a value of risk aversion γ_1 such that for all higher risk aversions $\gamma_2 > \gamma_1$, we have $RHS(\gamma_2) < RHS(\gamma_1)$. That is, the RHS of the selection inequality initially increases but eventually decreases with risk aversion. In other words, in the special (though unrealistic) case in which idiosyncratic risk can be completely diversified away, the selection effect based on risk aversion is in place for relatively low levels of risk aversion but not for high values.

Q.E.D.

Corollary A3: For either $\theta = 1$ (Case 1 below) or $\varepsilon_T = 0$ (Case 2 below), as long as $\gamma > 1$, the average consumption of entrepreneurs is always higher than the average consumption of pensioners. That is, mathematically,

$$\frac{1-\tau}{m} > \frac{\tau}{1-m} .$$

Proof of Corollary A3: The result follows from equation (19) in the paper (or (A3) in the proof of Proposition 3), which we can rewrite as

$$e^{\mu_i T} > \frac{\tau}{1-\tau} \frac{m(\mathcal{I})}{1-m(\mathcal{I})} E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right] e^{\frac{1}{1-\gamma_i} \log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T} - Z\right) + \alpha^i (1-\theta)\left(e^{\varepsilon_T} - Z\right) + Z\right)^{1-\gamma_i}\right]}\right)} \quad \text{(A24)}$$

or

$$\frac{1-\tau}{m(\mathcal{I})} \frac{e^{\mu_i T}}{E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]} > \frac{\tau}{1-m(\mathcal{I})} e^{\frac{1}{1-\gamma_i} \log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T} - Z\right) + \alpha^i (1-\theta)\left(e^{\varepsilon_T} - Z\right) + Z\right)^{1-\gamma_i}\right]}\right)} . \quad \text{(A25)}$$

Note that if

$$e^{\frac{1}{1-\gamma_i}\log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}+\varepsilon_T}-Z\right)+\alpha^i(1-\theta)\left(e^{\varepsilon_T}-Z\right)+Z\right)^{1-\gamma_i}\right]}\right)} > 1 \tag{A26}$$

then the above condition implies that for every $i \in \mathcal{I}$, we have

$$\frac{1-\tau}{m(\mathcal{I})}\frac{e^{\mu_i T}}{E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]} > \frac{\tau}{1-m(\mathcal{I})} .$$

If this holds for each $i \in \mathcal{I}$, then it obviously also holds for the average, so that

$$\frac{1-\tau}{m(\mathcal{I})}\frac{E^{\mathcal{I}}\left[e^{\mu_i T}|i \in \mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j \in \mathcal{I}\right]} > \frac{\tau}{1-m(\mathcal{I})} ,$$

or

$$\frac{1-\tau}{m(\mathcal{I})} > \frac{\tau}{1-m(\mathcal{I})} .$$

We now show that the condition (A26) is indeed satisfied for $\theta = 1$ (case 1) as well as for $\varepsilon_T = 0$ (case 2).

1. For $\theta = 1$, we have

$$\frac{1}{1-\gamma_i}\log\left(\frac{E\left[e^{(1-\gamma_i)\varepsilon_T}\right]}{E\left[\left(e^{\varepsilon_{i,T}+\varepsilon_T}\right)^{1-\gamma_i}\right]}\right) = \frac{1}{1-\gamma_i}\log\left(\frac{1}{E\left[\left(e^{\varepsilon_{i,T}}\right)^{1-\gamma_i}\right]}\right) > 0$$

   where the inequality stems from the fact that when $\gamma > 1$, $E\left[\left(e^{\varepsilon_{i,T}}\right)^{1-\gamma_i}\right] > \left(E\left[e^{\varepsilon_{i,T}}\right]\right)^{1-\gamma_i} = 1$. Thus $\log\left(\frac{1}{E\left[\left(e^{\varepsilon_{i,T}}\right)^{1-\gamma_i}\right]}\right) < \log(1) = 0$. The inequality also holds when $\gamma < 1$ because we then have $E\left[\left(e^{\varepsilon_{i,T}}\right)^{1-\gamma_i}\right] < \left(E\left[e^{\varepsilon_{i,T}}\right]\right)^{1-\gamma_i} = 1$ and hence $\log\left(\frac{1}{E\left[\left(e^{\varepsilon_{i,T}}\right)^{1-\gamma_i}\right]}\right) > 0$.

2. For $\varepsilon_T = 0$, we have $Z = 1$ and thus

$$\frac{1}{1-\gamma}\log\left(\frac{1}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]}\right)$$

   Denote $R = \left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)$. For $\gamma > 1$, Jensen's inequality implies

$$E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right] = E\left[(R)^{1-\gamma_i}\right] > \left(E[R]\right)^{1-\gamma_i} = 1$$

   and therefore

$$\log\left(\frac{1}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]}\right) < \log(1) = 0$$

and the result follows. Similarly, for $\gamma < 1$ we have

$$E\left[(\theta(e^{\varepsilon_{i,T}}-1)+1)^{1-\gamma_i}\right] = E\left[(R)^{1-\gamma_i}\right] < (E[R])^{1-\gamma_i} = 1$$

and hence

$$\log\left(\frac{1}{E\left[(\theta(e^{\varepsilon_{i,T}}-1)+1)^{1-\gamma_i}\right]}\right) > \log(1) = 0\ .$$

In either case,

$$\frac{1}{1-\gamma}\log\left(\frac{1}{E\left[(\theta(e^{\varepsilon_{i,T}}-1)+1)^{1-\gamma_i}\right]}\right) > 0\ .$$

Q.E.D.

Proof of Proposition 7: The consumption of pensioner i is

$$C_{i,T}^{no} = \tau B_0 e^{\varepsilon_T} E\left[e^{\mu_j T} | j \in \mathcal{I}\right] \frac{m(\mathcal{I})}{1-m(\mathcal{I})}\ .$$

The consumption of entrepreneur i is

$$\begin{aligned} C_{i,T}^{yes} &= M_0^i\left(\theta R^i + \alpha^i(1-\theta) R^{Mkt} + 1\right) \\ &= (1-\tau) B_0 e^{\mu_i T} Z\left(\theta R^i + \alpha^i(1-\theta) R^{Mkt} + 1\right) \\ &= (1-\tau) B_0 e^{\mu_i T}\left[\left(\theta\left(e^{\varepsilon_T+\varepsilon_{iT}} - Z\right) + (1-\theta)\alpha(\gamma_i)(e^{\varepsilon_T} - Z) + Z\right)\right]\ . \end{aligned}$$

Total average consumption is then

$$\overline{C}_T = \int_{i\notin\mathcal{I}} C_{i,T}^{no} di + \int_{i\in\mathcal{I}} C_{i,T}^{yes} di\ .$$

By market clearing, we must have that total consumption equals total capital produced:

$$\overline{C}_T = B_T = B_0 e^{\varepsilon_T} E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right] m(\mathcal{I})\ .$$

Therefore, we can write

$$\frac{C_{i,T}^{no}}{\overline{C}_T} = \frac{\tau}{1-m(\mathcal{I})}$$

and

$$\frac{C_{i,T}^{yes}}{\overline{C}_T} = \frac{(1-\tau) e^{\mu_i T} Z\left(\theta R^i + \alpha^i(1-\theta) R^{Mkt} + 1\right)}{e^{\varepsilon_T} E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right] m(\mathcal{I})}\ .$$

The main component in Var$(s_{i,T}) = E[s_{i,T}^2] - 1$ is the following quantity:

$$\begin{aligned} E[s_{i,T}^2] &= \int \left(\frac{C_{i,T}}{\overline{C}_T}\right)^2 di = \left(\tau \frac{1}{1-m(\mathcal{I})}\right)^2 (1-m(\mathcal{I})) \\ &\quad + (1-\tau)^2 \int_{i\in I} \left(\frac{e^{\mu_i T} Z \left(\theta R^i + \alpha^i (1-\theta) R^{Mkt} + 1\right)}{(e^{\varepsilon_T}) E^{\mathcal{I}} [e^{\mu_j T} | j \in I] m(\mathcal{I})}\right)^2 di \end{aligned}$$

which gives

$$\begin{aligned} E[s_{i,T}^2] &= \int \left(\frac{C_{i,T}}{\overline{C}_T}\right)^2 di = \left(\tau \frac{1}{1-m(\mathcal{I})}\right)^2 (1-m(\mathcal{I})) \\ &\quad + (1-\tau)^2 \frac{1}{E^{\mathcal{I}} [e^{\mu_j T} | j \in \mathcal{I}]^2 m(\mathcal{I})^2} \frac{\int_{i\in I} e^{2\mu_i T} Z^2 \left(\theta R^i + \alpha^i (1-\theta) R^{Mkt} + 1\right)^2 di}{(e^{\varepsilon_T})^2} \end{aligned}$$

or

$$\begin{aligned} E[s_{i,T}^2] &= \int \left(\frac{C_{i,T}}{\overline{C}_T}\right)^2 di = \left(\tau \frac{1}{1-m(\mathcal{I})}\right)^2 (1-m(\mathcal{I})) \\ &\quad + (1-\tau)^2 \frac{1}{E^{\mathcal{I}} [e^{\mu_j T} | j \in \mathcal{I}]^2 m(\mathcal{I})^2} \frac{m(I) E^{\mathcal{I}} \left[e^{2\mu_i T} Z^2 \left(\theta R^i + \alpha^i (1-\theta) R^{Mkt} + 1\right)^2 | i \in \mathcal{I}\right]}{(e^{\varepsilon_T})^2} \end{aligned}$$

$$\begin{aligned} E[s_{i,T}^2] &= \frac{\tau^2}{1-m(\mathcal{I})} + \frac{(1-\tau)^2}{m(\mathcal{I})} \frac{E^{\mathcal{I}} \left[e^{2\mu_j T} | j \in \mathcal{I}\right]}{E^{\mathcal{I}} [e^{\mu_j T} | j \in \mathcal{I}]^2} \\ &\quad \times \frac{Z^2 E^{\mathcal{I}} \left[\left(\theta R^i + \alpha^i (1-\theta) R^{Mkt} + 1\right)^2 | i \in \mathcal{I}\right]}{(e^{\varepsilon_T})^2} . \end{aligned}$$

Recall that

$$R^{Mkt} = \frac{e^{\varepsilon_T}}{Z} - 1 ,$$

which then gives

$$Z \left(1 + R^{Mkt}\right) = e^{\varepsilon_T} .$$

We thus obtain

$$\begin{aligned} E[s_{i,T}^2] &= \int \left(\frac{C_{i,T}}{\overline{C}_T}\right)^2 di \\ &= \frac{\tau^2}{1-m(\mathcal{I})} + \frac{(1-\tau)^2}{m(\mathcal{I})} \frac{E^{\mathcal{I}} \left[e^{2\mu_j T} | j \in \mathcal{I}\right]}{E^{\mathcal{I}} [e^{\mu_j T} | j \in \mathcal{I}]^2} \\ &\quad \times \frac{E^{\mathcal{I}} \left[\left(\theta R^j + \alpha^j (1-\theta) R^{Mkt} + 1\right)^2 | j \in \mathcal{I}\right]}{\left(R^{Mkt} + 1\right)^2} \end{aligned}$$

which gives the expression in the claim. Q.E.D.

The expectation above is a cross-agent population mean as of time T, so that it is dependent on the realization of the market return. However, when we calculate the same expectation as of time 0, we obtain an identical formula with the market return inside the expectation. In other words, Var$(s_{i,T})$ in Proposition 7 can be interpreted as either as of time 0 or as of time T, with the former variance including the randomness associated with the market return realization. In Figures 1 through 5 in the paper, we consider the special case of $\theta \to 1$, in which entrepreneurs hold infinitesimal positions in the market portfolio, as a result of which the randomness associated with the market return does not matter.

B. Special case: No entrepreneurial diversification ($\theta \to 1$).

In this part of the appendix, we consider the special case of $\theta \to 1$, in which entrepreneurs are allowed to sell only a negligible fraction of their firms. We preserve heterogeneity in skill, μ_i, and risk aversion, γ_i. Section B.1 provides additional figures that are related to the discussion in Section 3 of the paper. Section B.2 collects additional theoretical results for this case, while Section B.3 contains the proofs for this case.

B.1. Additional figures.

In this section we provide additional model-implied figures for the special case $\theta \to 1$.

Panel A of Figure B1 plots the distribution of realized consumption across agents. Since all pensioners consume the same amount, we plot their consumption by a vertical line whose height indicates the mass of pensioners (1.5% for $\tau = 0.1\%$, 58% for $\tau = 20\%$, and 91% for $\tau = 70\%$). The entrepreneurs' consumption is plotted by a probability density. Consumption is highly right-skewed. As explained in the paper, there are two reasons behind this skewness. First, consumption is right-skewed among entrepreneurs, due to its convexity in μ_i and random shocks. Second, most entrepreneurs consume more than pensioners, due to higher skill and larger risk exposure.

Panel B of Figure B1 plots the distribution of certainty equivalent consumption across agents. Each of the three lines plots a mixture of two distributions, one for each type of agents. Unlike in Panel A, there are no vertical lines; even though all pensioners consume the same amount, their utilities differ due to different risk aversions (see equation (44) in the paper). The distributions in Panel B are right-skewed, due to the convexity of entrepreneurs' consumption in μ_i, but less so than in Panel A because of the absence of convexity in random shocks (see equation (43) in the paper).

Figures B2 through B5 are the counterparts of the four panels of Figure 5 in the paper, adding more detail. Specifically, all four figures analyze the impact of changing σ_μ and σ_γ around their baseline values of $\sigma_\mu = 5\%$ and $\sigma_\gamma = 0.5$. These results are described in Section 3.5 in the paper.

Figure B2 plots our second measure of inequality: the income share of the top 10% of agents. We see that the pattern from Figure 5 in the paper is robust to changes in σ_μ (Panel A) and σ_γ (Panel B). While the effect of σ_γ on inequality is small, the effect of σ_μ is large: as explained in the paper, more dispersion in skill implies more inequality.

Figure B3 plots expected aggregate productivity against τ. Productivity increases with τ due to the selection effect described in the paper. Productivity also depends on σ_μ and, to a lesser extent,

σ_γ. An increase in σ_μ raises expected productivity in two ways. First, it amplifies the selection effect whereby only sufficiently skilled agents become entrepreneurs. Second, there is a convexity effect whereby more dispersion in individual growth rates increases the aggregate growth rate. In contrast, an increase in σ_γ depresses productivity because it strengthens the importance of γ_i at the expense of μ_i in the entrepreneur selection mechanism. As a result of the weaker selection on μ_i, an increase in σ_γ reduces the average μ_i among entrepreneurs, thereby reducing expected productivity.

Figure B4 plots the expected return on the market portfolio as a function of τ. The expected return falls as τ rises due to the second selection effect described in the paper. Both σ_μ and σ_γ affect the expected return. A higher σ_μ lifts the expected return because it strengthens the importance of μ_i at the expense of γ_i in the entrepreneur selection mechanism. As a result of the weaker selection on γ_i, a higher σ_μ implies a higher average γ_i among entrepreneurs, which pushes up the expected return. The effect of σ_γ on the expected return is parameter-dependent because the state price density depends on the full distribution of γ_i across entrepreneurs. On the one hand, a higher σ_γ implies a lower average γ_i among entrepreneurs through the selection effect discussed above. On the other hand, a higher σ_γ increases the mass of high-γ_i entrepreneurs who have a disproportionately high effect on the state price density. While the former effect reduces the expected return, the latter effect increases it. In Panel B of Figure B4, the latter effect is stronger. But the former effect can be stronger if, for example, σ_μ is low enough and τ high enough.

Figure B5 plots the level of stock prices, measured by the market portfolio's M/B ratio, as a function of τ. M/B exhibits a concave and mostly negative relation to τ, as explained in the paper. Stock prices are also substantially affected by both types of heterogeneity across agents. An increase in σ_μ raises M/B by increasing expected cash flow. While a higher σ_μ also increases the discount rate, the former effect prevails. An increase in σ_γ reduces M/B in two ways, by reducing expected cash flow and increasing the discount rate. But we focus on the dependence of M/B on τ, which is robust to changes in σ_μ and σ_γ.

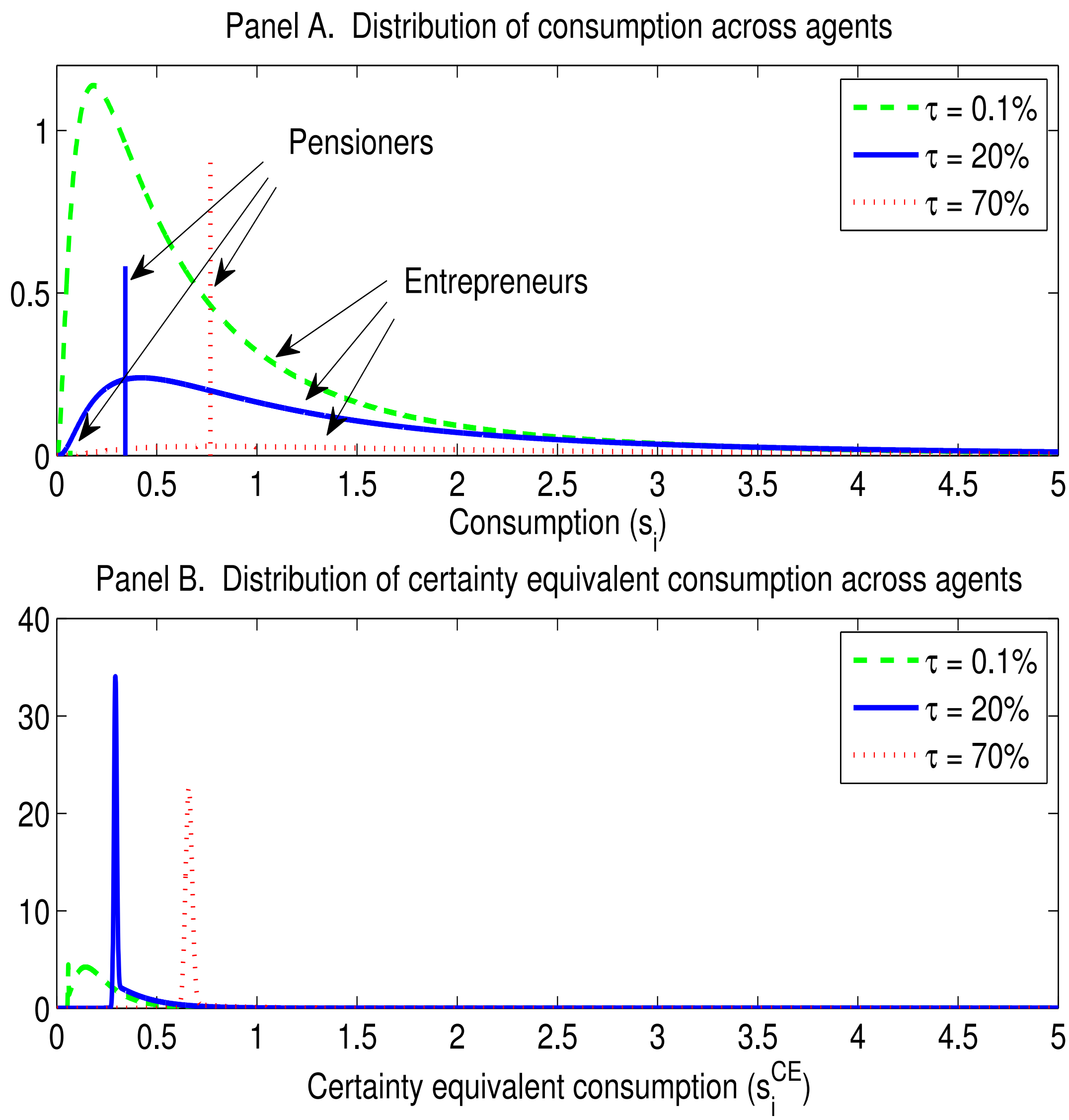

Figure B1. Distribution of consumption across agents. Panel A plots the distribution of consumption across agents. The consumption of pensioners is plotted by vertical lines whose height indicates the corresponding probability mass. The consumption of entrepreneurs is plotted by probability densities. Panel B plots the distribution of certainty equivalent consumption. We consider three tax rates τ. Both consumption and its certainty equivalent are scaled by their averages across all agents. Throughout, we normalize $B_0 = 1$.

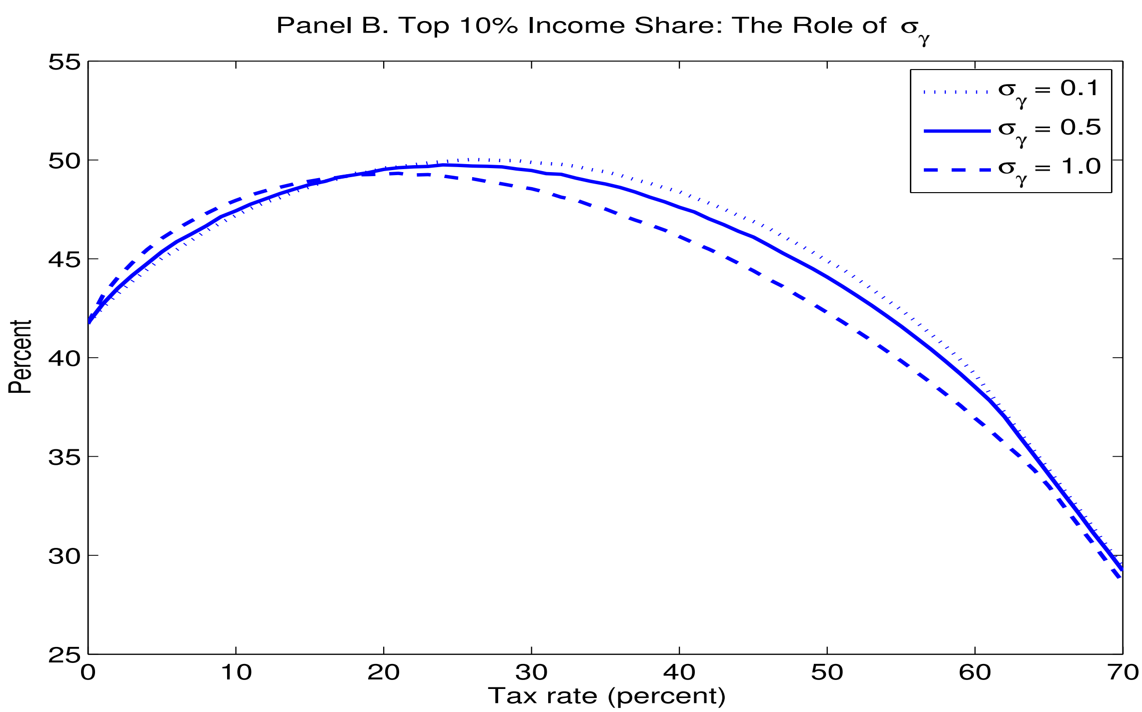

Figure B2. Model-implied inequality. This figure plots the income share of the top 10% of agents as a function of the tax rate τ. The solid lines in both panels correspond to the baseline case in which $\sigma_\mu = 5\%$ and $\sigma_\gamma = 0.5$. Panel A varies σ_μ while keeping all other parameters at their baseline values. Panel B varies σ_γ.

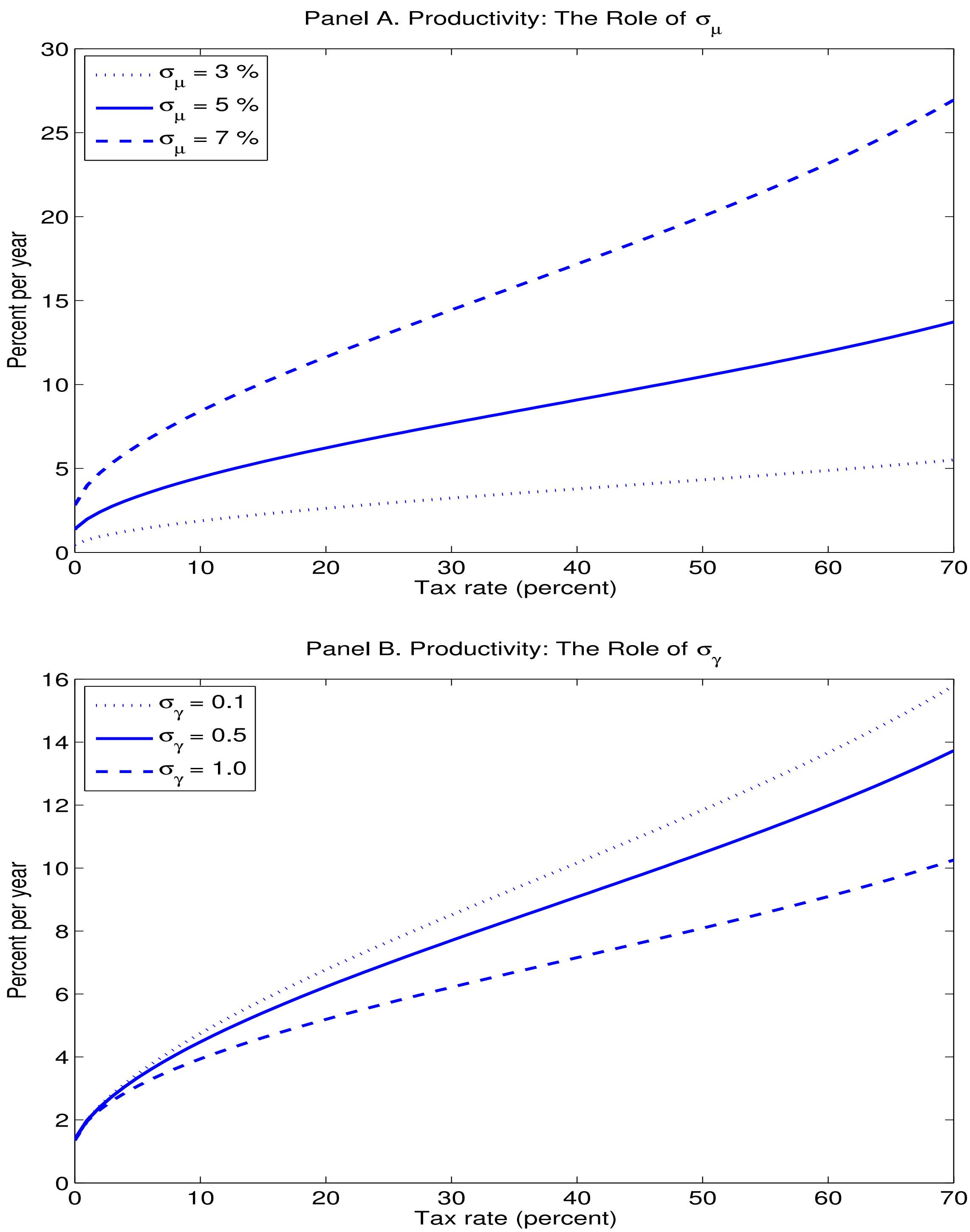

Figure B3. Model-implied aggregate productivity. This figure plots the model-implied expected aggregate productivity as a function of the tax rate τ. Expected productivity is computed as the annualized expected growth rate of total capital, or $(1/T)\mathrm{E}\left[B_T/(m(\mathcal{I})B_0)-1\right]$. The solid lines in both panels correspond to the baseline case in which $\sigma_\mu = 5\%$ and $\sigma_\gamma = 0.5$. Panel A varies σ_μ while keeping all other parameters at their baseline values. Panel B varies σ_γ.

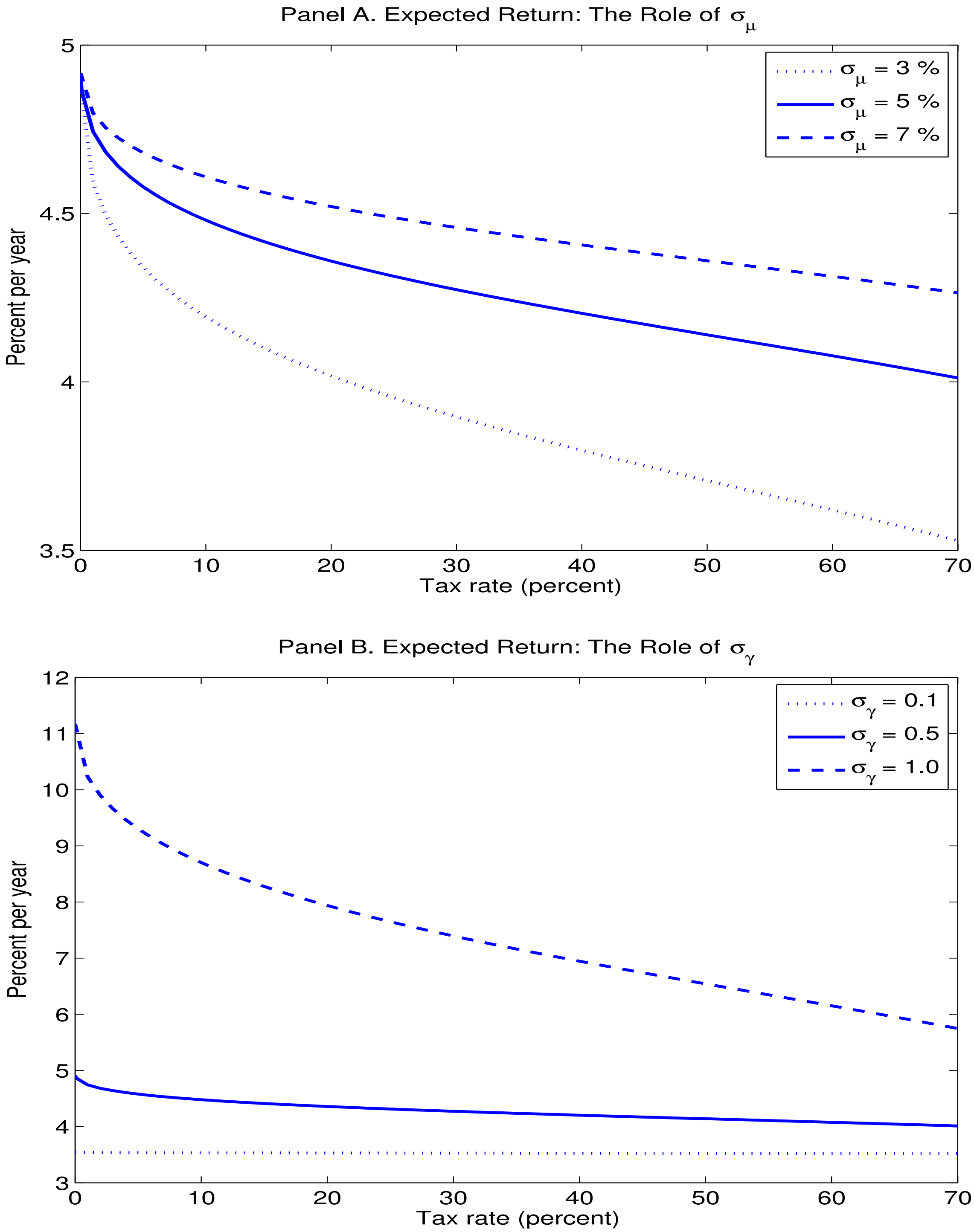

Figure B4. Model-implied expected return. This figure plots the model-implied expected rate of return on the aggregate market portfolio as a function of the tax rate τ. The solid lines in both panels correspond to the baseline case in which $\sigma_\mu = 5\%$ and $\sigma_\gamma = 0.5$. Panel A varies σ_μ while keeping all other parameters at their baseline values. Panel B varies σ_γ.

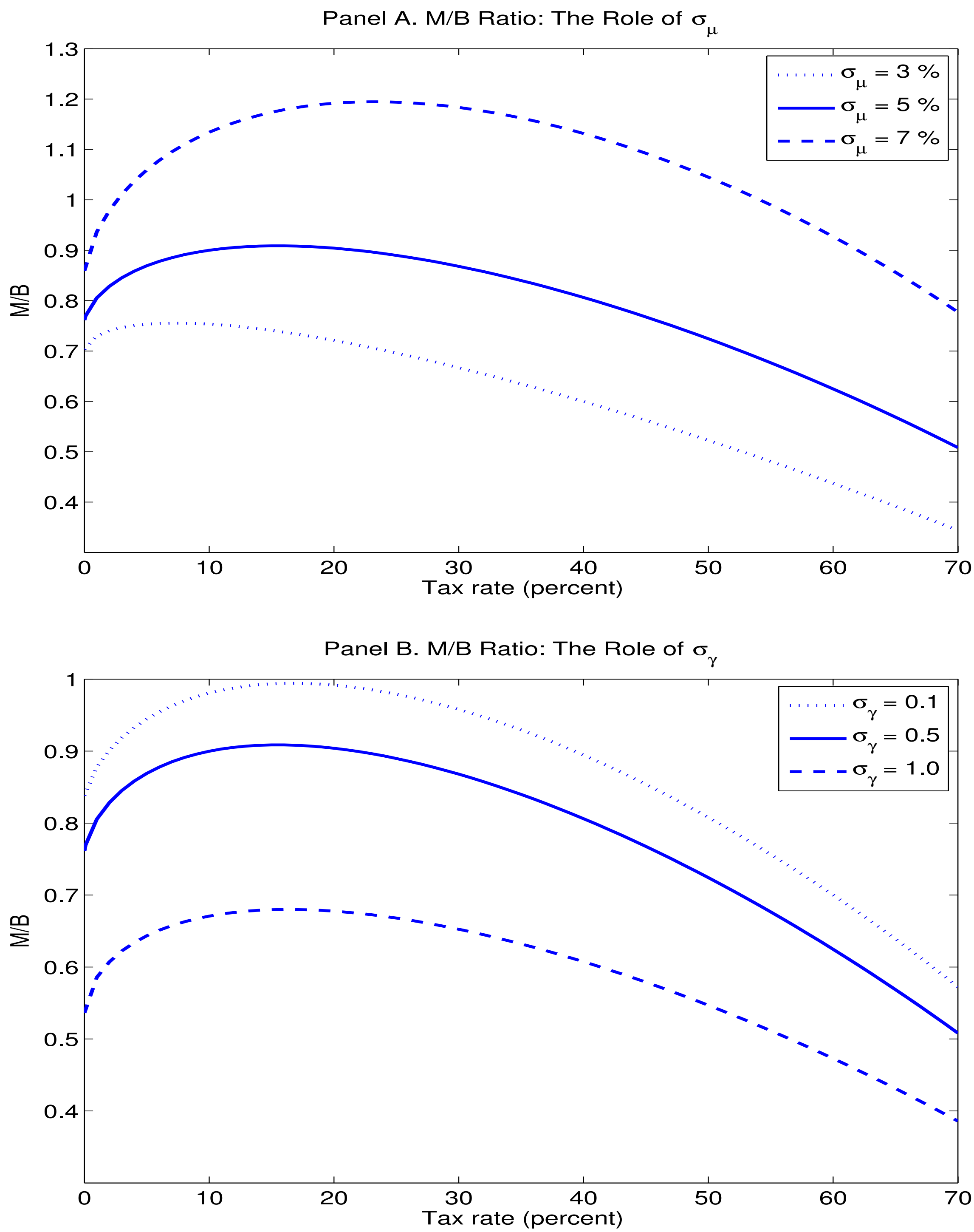

Figure B5. Model-implied M/B. This figure plots the model-implied M/B ratio for the aggregate market portfolio as a function of the tax rate τ. The solid lines in both panels correspond to the baseline case in which $\sigma_\mu = 5\%$ and $\sigma_\gamma = 0.5$. Panel A varies σ_μ while keeping all other parameters at their baseline values. Panel B varies σ_γ.

B.2. Theoretical results.

In this section we use the following notation:

$$\begin{aligned} \varepsilon_T &= -\frac{1}{2}\sigma^2 T + \sigma W_T \\ \varepsilon_{i,T} &= -\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T} \end{aligned}$$

where $W_T \sim N(0,T)$ and $W_{i,T} \sim N(0,T)$ are independent from each other and across i.

Proposition B1: The utility function of a fully constrained entrepreneur (i.e., $\theta \to 1$) with skill μ_i and risk aversion γ_i is

$$\begin{aligned} V_{i,0}^{yes} &= \frac{(1-\tau)^{1-\gamma_i} B_0^{1-\gamma_i}}{1-\gamma_i} e^{(1-\gamma_i)\mu_i T - \frac{1}{2}\gamma_i(1-\gamma_i)\sigma^2 T - \frac{1}{2}\gamma_i(1-\gamma_i)\sigma_1^2 T} \text{ if } \gamma_i \neq 1 \\ V_{i,0}^{yes} &= \log(1-\tau) + \log(B_0) + \mu_i T - \frac{1}{2}\sigma_1^2 T - \frac{1}{2}\sigma^2 T \quad \text{if } \gamma_i = 1 \ . \end{aligned}$$

For all $\gamma_i > 0$, agent i becomes an entrepreneur if and only if

$$\mu_i - \gamma_i \frac{1}{2}\sigma_1^2 > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \ .$$

In this case, it is easy to see that higher skill or lower risk aversion makes it more likely for an agent to invest and become an entrepreneur. Instead, a higher tax rate τ makes it less likely for an agent with given characteristics (μ_i, γ_i) to invest in own technology. Note that also the mass of existing entrepreneurs affects the decision of each agent to invest, as such mass determines the total tax base. If nobody invests, $m(\mathcal{I}) = 0$, total tax revenue is zero, and the threshold on the RHS goes to $-\infty$. It follows that somebody always becomes an entrepreneur in equilibrium. All agents being entrepreneurs, however, is also not an equilibrium (the RHS goes to $+\infty$). The intuition here is that if everyone were to become an entrepreneur, there would be a large unallocated tax to be shared. By becoming a pensioner, an agent could shed idiosyncratic risk and enjoy a positive measure of consumption.

Turning to the state price density, we have the following corollary:

Proposition B2: In equilibrium, the state price density is

$$\pi_T = \int_{\mathcal{I}} e^{\gamma_i\left(\frac{1}{2}\sigma^2 T + \log(Z)\right) + \frac{1}{2}\gamma_i(1+\gamma_i)\sigma_1^2 T - \gamma_i \sigma W_T} di \ ,$$

where Z is a constant satifying the equation

$$Z = \frac{E^{\mathcal{I}}\left[e^{(\gamma_i - 1)\gamma_i \frac{1}{2}\sigma^2 T + \gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T + \log(Z)\gamma_i} | i \in \mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\gamma_i(1+\gamma_i)\frac{1}{2}\sigma^2 T + \gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T + \log(Z)\gamma_i} | i \in \mathcal{I}\right]} \ .$$

We now make tighter assumptions on the joint distribution of (μ_i, γ_i) in order to obtain closed-form solutions for many quantities, as well as prove the existence of the equilibrium. Assume that skill μ_i and risk aversion γ_i are independently distributed in the population according to the following marginal distributions:

$$\begin{aligned} \mu_i &\sim N\left(\overline{\mu}, \sigma_\mu^2\right) \\ \gamma_i &\sim \frac{\phi\left(\gamma, \overline{\gamma}, \sigma_\gamma^2\right)}{1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)} 1_{\{\gamma > 0\}} \ . \end{aligned}$$

That is, μ_i is normally distributed with mean $\overline{\mu}$ and variance σ_μ^2 while risk aversion is a truncated normal, with truncation at zero and the underlying normal distribution with mean $\overline{\gamma}$ and variance σ_γ^2.

With these assumptions, we obtain the following:

Proposition B3: The mass of entrepreneurs $m = m(\mathcal{I})$ and the expected economic growth $H = E\left[e^{\mu_i T} | i \in \mathcal{I}\right]$ satisfy the following two equations in two unknowns:

$$m = \frac{1}{1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)} \left\{ \int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) \left[\Phi\left(\max\left(\frac{2}{\sigma_1^2} \left[\mu - K\right], 0 \right); \overline{\gamma}, \sigma_\gamma^2 \right) \right] d\mu - \Phi\left(0; \overline{\gamma}, \sigma_\gamma^2\right) \right\} \tag{B1}$$

$$H = \frac{1}{m} \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T}}{1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)} \left(\int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu} + \sigma_\mu^2 T, \sigma_\mu^2\right) \Phi\left(\max\left(\frac{2}{\sigma_1^2} \left[\mu - K\right], 0 \right); \overline{\gamma}, \sigma_\gamma^2 \right) d\mu - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right) \right) , \tag{B2}$$

where

$$K = K(m, H) = \left[\frac{1}{T} \log\left(\frac{m}{1 - m} \right) + \frac{1}{T} \log\left(\frac{\tau}{1 - \tau} \right) + \frac{1}{T} \log\left(H\right) \right] .$$

The system of equations (B1) and (B2) has a unique fixed point.

Note that as $\tau \to 0$, $\log\left(\frac{\tau}{1-\tau}\right) \to -\infty$ which implies $m \to 1$ in order to have the first equation satisfied. In this case, we have $H \to e^{\overline{\mu} T + \frac{1}{2}\sigma_\mu^2 T^2}$, which is full potential.

Proposition B4: The mass of entrepreneurs decreases as the tax rate increases: $dm/d\tau < 0$.

Proposition B5: The average risk aversion across all entrepreneurs is given by

$$E\left[\gamma_i | i \in \mathcal{I}\right] = \frac{1}{m\left(1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)\right)} \int_0^{\infty} \gamma \phi\left(\gamma; \overline{\gamma}, \sigma_\gamma^2\right) \left(1 - \Phi\left(\frac{1}{2}\gamma \sigma_1^2 + K; \overline{\mu}, \sigma_\mu^2 \right) \right) d\gamma \ .$$

To compute asset prices, we need the following Lemma:

Lemma B1:

$$E\left[e^{a\gamma+b\gamma^2}|i\in\mathcal{I}\right]=\frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)}\int_0^\infty e^{a\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma\ ,$$

where K is defined above, $\Phi\left(,;\overline{\mu},\sigma_\mu^2\right)$ is the cumulative normal distribution with mean $\overline{\mu}$ and variance σ_μ^2, and the integral is well defined if the following condition holds:

$$b<\frac{1}{2\sigma_\gamma^2}\ .$$

The price of a security can then be computed as follows.

Proposition B6: (a) The M/B of firm i is given by

$$\frac{M_{i,0}}{B_{i,0}}=(1-\tau)e^{\mu_i T}Z\ ,$$

where Z solves the equation

$$\int_0^\infty\left[e^{\log(Z)}-e^{-\sigma^2 T\gamma}\right]e^{\gamma\log(Z)+\frac{\gamma(1+\gamma)}{2}\left(\sigma_1^2+\sigma^2\right)T}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma=0\ . \tag{B3}$$

The integral exists if

$$\left(\sigma^2+\sigma_1^2\right)T<\frac{1}{\sigma_\gamma^2}$$

(b) The M/B of the market portfolio is

$$\frac{M_0^P}{B_0^P}=(1-\gamma)\,HZ\ ,$$

where $H=E\left[e^{\mu_i T}|i\in I\right]$ is in equation (B2) and Z solves equation (B3).

(c) The excess return on any asset is

$$E\left[R_i\right]=\frac{1}{Z}-1\ .$$

Income inequality can also be easily computed in closed form.

Proposition B7: Income inequality Var$(s_{i,T})$ is given by

$$\text{Var}(s_{i,T})=\frac{\tau^2}{1-m}+\frac{(1-\tau)^2}{m}\frac{H_2}{H^2}e^{\sigma_1^2 T}-1\ ,$$

where $H = E\left[e^{\mu_i T}|i \in I\right]$ is in equation (B2) and

$$\begin{aligned} H_2 &= \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)} e^{2\sigma_\mu^2 T^2 + 2\overline{\mu}T} \\ &\times \left(\int_{-\infty}^{\infty} \phi\left(\mu;\overline{\mu}+2\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu - \Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right) . \end{aligned}$$

A second measure of dispersion is the distribution of consumption shares, in order to compute the total share of consumption of the top $\alpha\%$ of the population.

Proposition B8: The distribution of $s_{i,T} = C_{i,T}/\overline{C}_T$ across agents is given by the cumulative density function

$$F\left(s_{i,T}\right) = 1_{\left\{s_{i,T} > \frac{\tau}{1-m}\right\}}\left(1-m\right) + F\left(s_{i,T}|\mu_i > \frac{1}{2}\gamma_i\sigma_1^2 + K\right)m \tag{B4}$$

where

$$F\left(s_{i,T}|\mu_i > \frac{1}{2}\gamma_i\sigma_1^2 + K\right) = \frac{1}{m}\int_0^\infty f_\gamma\left(\gamma\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma$$

and

$$f_\gamma\left(\gamma\right) = \int_{\frac{1}{2}\gamma\sigma_1^2+K}^{\infty}\Phi\left(\frac{1}{2}\sigma_1^2 T + \log\left(s_{i,T}\right) + \log\left(\frac{Hm}{1-\tau}\right);T\mu,\sigma_1^2 T\right)\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu\ .$$

Proposition B9: Certainty equivalent inequality, Var$(s_{i,T}^{CE})$, is given by

$$\text{Var}(s_{i,T}^{CE}) = \frac{\frac{\tau^2}{(1-m(\mathcal{I}))}E\left[e^{-\gamma_i\sigma^2 T}|i\notin I\right] + \frac{(1-\tau)^2}{m(\mathcal{I})}\frac{E\left[e^{2\mu_i T}|i\in I\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2}E\left[e^{-\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in I\right]}{\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin I\right] + \left(1-\tau\right)E\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in I\right]\right]^2} - 1\ .$$

B.3. Proofs for Appendix B.

Proof of Proposition B1: Immediate from Propositions 3 and 4, by setting $\theta = 1$ and taking expectations over $W_{i,T}$ and W_T in the formulas for expected utility (point (a)) and the condition to become an entrepreneur (point (b)). For instance, from Proposition 3, for $\gamma \neq 1$, the condition with $\theta = 1$ is

$$\begin{aligned} &\frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m\left(\mathcal{I}\right)}{1-m\left(\mathcal{I}\right)}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\right] \\ <\ &\mu_i + \frac{1}{2}\gamma_i\sigma^2 + \frac{1}{T\left(1-\gamma_i\right)}\log\left(E\left[\left(e^{\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T+\sigma W_T+\sigma_1 W_{i,T}}\right)^{1-\gamma_i}\right]\right)\ . \end{aligned} \tag{B5}$$

The expectation in the last term is

$$
\begin{aligned}
E\left[\left(e^{-\frac{1}{2}\sigma^2 T-\frac{1}{2}\sigma_1^2 T+\sigma W_T+\sigma_1 W_T}\right)^{1-\gamma_i}\right] &= E\left[e^{-\frac{1}{2}(1-\gamma_i)\sigma^2 T-\frac{1}{2}(1-\gamma_i)\sigma_1^2 T+(1-\gamma_i)\sigma W_T+(1-\gamma_i)\sigma_1 W_T}\right] \\
&= e^{-\frac{1}{2}(1-\gamma_i)\sigma^2 T-\frac{1}{2}(1-\gamma_i)\sigma_1^2 T+\frac{1}{2}(1-\gamma_i)^2\sigma^2 T+\frac{1}{2}(1-\gamma_i)^2\sigma_1^2 T} \\
&= e^{-\frac{1}{2}(1-\gamma_i)\gamma_i\sigma^2 T-\frac{1}{2}(1-\gamma_i)\gamma_i\sigma_1^2 T}.
\end{aligned}
$$

Taking logs, the last term is

$$
\begin{aligned}
& \frac{1}{T(1-\gamma_i)}\log\left(E\left[\left(e^{\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T+\sigma W_T+\sigma_1 W_{i,T}}\right)^{1-\gamma_i}\right]\right) \\
=\; & \frac{1}{T(1-\gamma_i)}\left[-\frac{1}{2}(1-\gamma_i)\gamma_i\sigma^2 T-\frac{1}{2}(1-\gamma_i)\gamma_i\sigma_1^2 T\right] \\
=\; & -\frac{1}{2}\gamma_i\sigma^2-\frac{1}{2}\gamma_i\sigma_1^2.
\end{aligned}
$$

The claim follows upon substitution in the top inequality.

Similarly, for $\gamma = 1$, the condition is

$$
\mu_i > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)+\log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)+E\left[\log\left(\frac{e^{-\frac{1}{2}\sigma^2 T+\sigma W_T}}{e^{-\frac{1}{2}\sigma_1^2 T+\sigma_1 W_{i,T}-\frac{1}{2}\sigma^2 T+\sigma W_T}}\right)\right]\right]
$$

which yields

$$
\mu_i > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)+\log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)+\frac{1}{2}\sigma_1^2 T\right].
$$

Q.E.D.

Proof of Proposition B2: From Proposition 4, when we set $\theta = 1$, we have

$$
\begin{aligned}
\pi_T &= \int_{\mathcal{I}}\left(\frac{e^{\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T+\sigma W_T+\sigma_1 W_{i,T}}}{Z}\right)^{-\gamma_i} di \\
&= \int_{\mathcal{I}}\frac{e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T-\gamma_i\sigma_1 W_{i,T}}}{Z^{-\gamma_i}} di \\
&= \int_{\mathcal{I}} e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T-\gamma_i\sigma_1 W_{i,T}+\log(Z)\gamma_i} di \\
&= m(\mathcal{I})\,E^{\mathcal{I}}\left[e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T-\gamma_i\sigma_1 W_{i,T}+\log(Z)\gamma_i}|i\in\mathcal{I}\right] \\
&= m(\mathcal{I})\,E^{\mathcal{I}}\left[e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T+\frac{1}{2}\gamma_i^2\sigma_1^2 T+\log(Z)\gamma_i}|i\in\mathcal{I}\right] \\
&= \int_{\mathcal{I}} e^{\gamma_i\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T-\gamma_i\sigma W_T+\log(Z)\gamma_i} di,
\end{aligned}
$$

where we exploit the independence between γ_i and $W_{i,T}$ to integrate out $W_{i,T}$ from the expectation in the second to last step.[†]

From the definition of Z in Proposition 4 we also obtain

$$
\begin{aligned}
Z &= \frac{E\left[\pi_T e^{-\frac{1}{2}\sigma^2 T+\sigma W_T}\right]}{E\left[\pi_T\right]} \\
&= \frac{E\left[\int_{\mathcal{I}} e^{\gamma_i \frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T-\gamma_i\sigma W_T+\log(Z)\gamma_i} di \; e^{-\frac{1}{2}\sigma^2 T+\sigma W_T}\right]}{E\left[\int_{\mathcal{I}} e^{\gamma_i \frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T-\gamma_i\sigma W_T+\log(Z)\gamma_i} di\right]} \\
&= \frac{E\left[\int_{\mathcal{I}} e^{(\gamma_i-1)\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+(1-\gamma_i)\sigma W_T+\log(Z)\gamma_i} di\right]}{E\left[\int_{\mathcal{I}} e^{\gamma_i \frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T-\gamma_i\sigma W_T+\log(Z)\gamma_i} di\right]} \\
&= \frac{\int_{\mathcal{I}} e^{(\gamma_i-1)\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\frac{1}{2}(1-\gamma_i)^2\sigma^2 T+\log(Z)\gamma_i} di}{\int_{\mathcal{I}} e^{\gamma_i \frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\frac{1}{2}\gamma_i^2\sigma^2 T+\log(Z)\gamma_i} di} \\
&= \frac{\int_{\mathcal{I}} e^{(\gamma_i-1)\gamma_i\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\log(Z)\gamma_i} di}{\int_{\mathcal{I}} e^{\gamma_i(1+\gamma_i)\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\log(Z)\gamma_i} di},
\end{aligned}
$$

finally giving

$$
Z = \frac{E^{\mathcal{I}}\left[e^{(\gamma_i-1)\gamma_i\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\log(Z)\gamma_i} | i \in \mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\gamma_i(1+\gamma_i)\frac{1}{2}\sigma^2 T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2 T+\log(Z)\gamma_i} | i \in \mathcal{I}\right]}.
$$

Given a distribution of γ_i, this is one equation in one unknown. Q.E.D.

Proof of Proposition B3. Recall that the condition to invest is given by the cutoff rule

$$
\mu_i - \gamma_i \frac{1}{2}\sigma_1^2 > \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m}{1-m}\right) + \log(H)\right],
$$

where, for notational convenience, we denote $m = m(\mathcal{I})$ and $H = E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]$. That is, for

[†] To see it differently, our notational convention has

$$
\begin{aligned}
& \int_{\mathcal{I}} e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T-\gamma_i\sigma_1 W_{i,T}+\log(Z)\gamma_i} di \\
= & \int_{\Gamma}\int_{-\infty}^{\infty} e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T-\gamma_i\sigma_1 W_{i,T}+\log(Z)\gamma_i} \phi(W_{i,T},0,T) dW_{i,T} g(\gamma_i) d\gamma_i \\
= & \int_{\Gamma} e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T+\log(Z)\gamma_i} \int_{-\infty}^{\infty} e^{-\gamma_i\sigma_1 W_{i,T}} \phi(W_{i,T},0,T) dW_{i,T} g(\gamma_i) d\gamma_i \\
= & \int_{\Gamma} e^{-\gamma_i\left(-\frac{1}{2}\sigma^2-\frac{1}{2}\sigma_1^2\right)T-\gamma_i\sigma W_T+\log(Z)\gamma_i} e^{\frac{1}{2}\gamma_i^2\sigma_1^2 T} g(\gamma_i) d\gamma_i
\end{aligned}
$$

where Γ is the domain of γ_i and $g(\gamma_i)$ is the distribution of γ_i.

fixed μ, we consider all agents with risk aversion

$$0 < \gamma < \frac{2}{\sigma_1^2}\left\{\mu - \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m}{1-m}\right) + \log(H)\right]\right\} .$$

Denote

$$K = \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m}{1-m}\right) + \log(H)\right] ,$$

so that

$$0 < \gamma < \frac{2}{\sigma_1^2}\{\mu - K\} .$$

Hence, the mass of agents satisfying this condition is given by the following integral:

$$\begin{aligned}
m &= \int_{-\infty}^{\infty} \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\int_0^{\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right)} \frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} d\gamma\right] d\mu \\
&= \int_{-\infty}^{\infty} \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} - \frac{\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\right] d\mu \\
&= \int_{-\infty}^{\infty} \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}\right] d\mu - \frac{\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} \\
&= \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty} \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\Phi\left(\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right);\overline{\gamma},\sigma_\gamma^2\right)\right] d\mu - \Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\} ,
\end{aligned}$$

Similarly, the expected growth is

$$\begin{aligned}
& E^{\mathcal{I}}\left[e^{\mu_j T}|i \in \mathcal{I}\right] \\
=\ & \frac{1}{m}\int_{(\mu\times\gamma)\in\mathcal{I}} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} d\mu d\gamma \\
=\ & \frac{1}{m}\left\{\int_{-\infty}^{\infty} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\left[\int_0^{\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right)} \frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} d\gamma\right] d\mu\right\} \\
=\ & \frac{1}{m}\left\{\int_{-\infty}^{\infty} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\left[\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} - \frac{\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\right] d\mu\right\} \\
=\ & \frac{1}{m}\left\{\int_{-\infty}^{\infty} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}[\mu-K],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} d\mu - E\left[e^{\mu T}\right]\frac{\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\right\} .
\end{aligned}$$

Note that we can rewrite

$$e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right) = e^{\mu T}\frac{e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} = \frac{e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2-2\sigma_\mu^2\mu T}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} = \frac{e^{-\frac{1}{2}\frac{\mu^2+\overline{\mu}^2-2\left(\overline{\mu}+\sigma_\mu^2 T\right)\mu+\left(\overline{\mu}+\sigma_\mu^2 T\right)^2-\left(\overline{\mu}+\sigma_\mu^2 T\right)^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}}$$

$$
\begin{aligned}
&= \frac{e^{-\frac{1}{2}\frac{\left(\mu-\left(\overline{\mu}+\sigma_\mu^2 T\right)\right)^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T} \\
&= \phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right) e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T} .
\end{aligned}
$$

Substitute, to obtain

$$
\begin{aligned}
& E^{\mathcal{I}}\left[e^{\mu_j T} | i\in\mathcal{I}\right] \\
&= \frac{1}{m}\left\{\int_{-\infty}^{\infty} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\mu - E\left[e^{\mu T}\right]\frac{\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\right\} \\
&= \frac{1}{m}\left\{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\mu\right. \\
&\quad \left. -E\left[e^{\mu T}\right]\frac{\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\right\} \\
&= \frac{1}{m}\frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)} \\
&\quad \times\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right) .
\end{aligned}
$$

In conclusion, we have to solve two equations in two unknowns:

$$
\begin{aligned}
m &= \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\right]d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\} \\
H &= \frac{1}{m}\frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right) ,
\end{aligned}
$$

where, recall,

$$
K=\left[\frac{1}{T}\log\left(\frac{m}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(H\right)\right] .
$$

We finally prove the existence and uniqueness of the equilibrium. Defining $\widetilde{H}=mH$, we rewrite

$$
\begin{aligned}
\widetilde{H} &= \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right) \\
K &= K(m,\widetilde{H})=\left[\frac{1}{T}\log\left(\frac{1}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\widetilde{H}\right)\right] \\
m &= \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\left[\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\right]d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\} .
\end{aligned}
$$

Substituting for K, we can rewrite the two remaining equations as

$$\begin{aligned}\widetilde{H} &= F_H\left(m,\widetilde{H}\right) = \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)f\left(\mu;m,\widetilde{H}\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)\\ m &= F_m\left(m,\widetilde{H}\right) = \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)f\left(\mu;m,\widetilde{H}\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\}\ ,\end{aligned}$$

where

$$f\left(\mu;m,\widetilde{H}\right)=\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-\left[\frac{1}{T}\log\left(\frac{1}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\widetilde{H}\right)\right]\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\ .$$

This last function is clearly increasing in μ, for given m and $\widetilde{H}$. Because $\Phi(x)$ is increasing in x, we have that f is also (weakly) decreasing in m and $\widetilde{H}$. That is,

$$\frac{\partial f}{\partial m}\leq 0 \text{ and } \frac{\partial f}{\partial \widetilde{H}}\leq 0\ .$$

Moreover, $f(\mu)\in[0,1]$.

Define

$$\widetilde{\widetilde{H}}=\frac{\widetilde{H}}{1-m}\ .$$

Note that

$$\begin{aligned}f\left(\mu;m,\widetilde{H}\right) &= \Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-\left[\frac{1}{T}\log\left(\frac{1}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\widetilde{H}\right)\right]\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\\ &= \Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-\left[\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\frac{\widetilde{H}}{1-m}\right)\right]\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\end{aligned}$$

or, with a slight abuse of notation,

$$f\left(\mu,\widetilde{\widetilde{H}}\right)=\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-\left[\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\widetilde{\widetilde{H}}\right)\right]\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)\ .$$

In addition, we have

$$\begin{aligned}\widetilde{\widetilde{H}} &= \frac{\widetilde{H}}{1-m}\\ &= \frac{\frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)f\left(\mu;\widetilde{\widetilde{H}}\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)}{1-\frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)f\left(\mu;\widetilde{\widetilde{H}}\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\}}\\ &= \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)f\left(\mu;\widetilde{\widetilde{H}}\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)-\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)f\left(\mu;\widetilde{\widetilde{H}}\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\}}\ ,\end{aligned}$$

yielding

$$\widetilde{\widetilde{H}} = \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T} \left(\int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu} + \sigma_\mu^2 T, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}\right) d\mu - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right) \right)}{1 - \int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}\right) d\mu} .$$

We just have to show that this equation has a solution.

First, recall that

$$\widetilde{\widetilde{H}} = \frac{\widetilde{H}}{1-m} = \frac{\int_{\mathcal{I}} e^{\mu_i T} di}{1-m} .$$

Therefore, by its definition, $\widetilde{\widetilde{H}} \to 0$ as $m \to 0$ and $\widetilde{\widetilde{H}} \to \infty$ as $m \to 1$. Therefore,

$$\begin{aligned} f\left(\mu, \widetilde{\widetilde{H}}\right) &= \Phi\left(\max\left(\frac{2}{\sigma_1^2} \left[\mu - \left[\frac{1}{T} \log\left(\frac{\tau}{1-\tau} \right) + \frac{1}{T} \log\left(\widetilde{\widetilde{H}} \right) \right] \right], 0 \right); \overline{\gamma}, \sigma_\gamma^2 \right) \\ &\to 1 \qquad \text{for } \widetilde{\widetilde{H}} \to 0 \\ &\to \Phi\left(0; \overline{\gamma}, \sigma_\gamma^2\right) \qquad \text{for } \widetilde{\widetilde{H}} \to \infty . \end{aligned}$$

Hence

$$\begin{aligned} F\left(\widetilde{\widetilde{H}}\right) &= \widetilde{\widetilde{H}} - \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T} \left(\int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu} + \sigma_\mu^2 T, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}\right) d\mu - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right) \right)}{1 - \int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}\right) d\mu} \\ &\to \widetilde{\widetilde{H}} - \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T} \left(1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)\right)}{1-1} = -\infty \qquad \text{if } \widetilde{\widetilde{H}} \to 0 \\ &\to \widetilde{\widetilde{H}} - \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T} \left(\Phi\left(0; \overline{\gamma}, \sigma_\gamma^2\right) - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)\right)}{1 - \Phi\left(0; \overline{\gamma}, \sigma_\gamma^2\right)} = \infty \quad \text{if } \widetilde{\widetilde{H}} \to \infty . \end{aligned}$$

Moreover, $F\left(\widetilde{\widetilde{H}}\right)$ is monotonically increasing in $\widetilde{\widetilde{H}}$. Because it must cross zero, there exists a unique $\widetilde{\widetilde{H}}^*$ such that $F\left(\widetilde{\widetilde{H}}^*\right) = 0$ and hence such that

$$\widetilde{\widetilde{H}}^* = \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T} \left(\int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu} + \sigma_\mu^2 T, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}^*\right) d\mu - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right) \right)}{1 - \int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}^*\right) d\mu} .$$

Given $\widetilde{\widetilde{H}}^*$, we can compute back

$$\widetilde{H}^* = \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2 + \overline{\mu} T}}{1 - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right)} \left(\int_{-\infty}^{\infty} \phi\left(\mu; \overline{\mu} + \sigma_\mu^2 T, \sigma_\mu^2\right) f\left(\mu; \widetilde{\widetilde{H}}^*\right) d\mu - \Phi\left(0, \overline{\gamma}, \sigma_\gamma^2\right) \right)$$

$$
\begin{aligned}
&= \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right) f\left(\mu;\frac{\widetilde{H}^*}{1-m^*}\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)\\
m^* &= \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right) f\left(\mu;\widetilde{\widetilde{H}}^*\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\}\\
&= \frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left\{\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right) f\left(\mu;\frac{\widetilde{H}^*}{1-m^*}\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right\}
\end{aligned}
$$

We thus obtain the unique fixed point of the system, and we demonstrate the existence of the equilibrium. Q.E.D.

Proof of Proposition B4: To see this, define $\widetilde{H}=mH$ first, and then rewrite

$$
\begin{aligned}
\widetilde{H} &= \frac{e^{\frac{1}{2}\sigma_\mu^2 T^2+\overline{\mu}T}}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)\\
K &= K(m,\widetilde{H})=\left[\frac{1}{T}\log\left(\frac{1}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log\left(\widetilde{H}\right)\right].
\end{aligned}
$$

Clearly, the partial derivatives have $\partial\widetilde{H}/\partial K\leq 0$ and $\partial m/\partial K\leq 0$. In addition, we have $\partial K/dm>0$ and $\partial K/d\widetilde{H}>0$ and $\partial K/\partial\tau>0$. Consider the total derivatives

$$
\begin{aligned}
dm &= \frac{\partial m}{\partial K}dK=\frac{\partial m}{\partial K}\left(\frac{\partial K}{\partial m}dm+\frac{\partial K}{\partial\tau}d\tau+\frac{\partial K}{\partial\widetilde{H}}d\widetilde{H}\right)\\
d\widetilde{H} &= \frac{\partial\widetilde{H}}{\partial K}dK=\frac{\partial\widetilde{H}}{\partial K}\left(\frac{\partial K}{\partial m}dm+\frac{\partial K}{\partial\tau}d\tau+\frac{\partial K}{\partial\widetilde{H}}d\widetilde{H}\right)
\end{aligned}
$$

or

$$
\begin{aligned}
dm &= \frac{\partial m}{\partial K}\frac{\partial K}{\partial m}dm+\frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}d\tau+\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}d\widetilde{H}\\
d\widetilde{H} &= \frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}dm+\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\tau}d\tau+\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}d\widetilde{H}
\end{aligned}
$$

or

$$
\begin{aligned}
\left(1-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}\right)dm+\left(-\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)d\widetilde{H} &= \frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}d\tau\\
\left(-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}\right)dm+\left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)d\widetilde{H} &= \frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\tau}d\tau\,.
\end{aligned}
$$

We can solve for dm and $d\widetilde{H}$

$$
\begin{aligned}
\begin{pmatrix} dm\\ d\widetilde{H}\end{pmatrix} &= \begin{pmatrix}\left(1-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}\right), \left(-\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\\ \left(-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}\right), \left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}\\ \frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\tau}\end{pmatrix}d\tau\\
&= \frac{1}{\left(1-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}\right)\left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)-\left(\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\left(\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}\right)}\begin{pmatrix}\left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right), \left(\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\\ \left(\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}\right), \left(1-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}\right)\end{pmatrix}\begin{pmatrix}\frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}\\ \frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\tau}\end{pmatrix}.
\end{aligned}
$$

Therefore, the total derivative is

$$\begin{aligned}\frac{dm}{d\tau} &= \frac{\left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\left(\frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}\right)+\left(\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\left(\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\tau}\right)}{\left(1-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}\right)\left(1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)-\left(\frac{\partial m}{\partial K}\frac{\partial K}{\partial\widetilde{H}}\right)\left(\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial m}\right)} \\ &= \frac{\frac{\partial m}{\partial K}\frac{\partial K}{\partial\tau}}{1-\frac{\partial\widetilde{H}}{\partial K}\frac{\partial K}{\partial\widetilde{H}}-\frac{\partial m}{\partial K}\frac{\partial K}{\partial m}} \\ &= \frac{(-)\,(+)}{1-(-)\,(+)-(-)\,(+)}=\frac{-}{+}<0\;.\end{aligned}$$

Q.E.D.

Proof of Proposition B5: From direct computation

$$\begin{aligned}E\left[\gamma_i|i\in\mathcal{I}\right] &= \frac{1}{m}\int_0^\infty\gamma\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\int_{\frac{1}{2}\gamma\sigma_1^2+\frac{1}{T}\log\left(\frac{m}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log(H)}^\infty\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu d\gamma \\ &= \frac{1}{m\left(1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)\right)}\int_0^\infty\gamma\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma\;,\end{aligned}$$

Q.E.D.

Proof of Lemma B1: We first notice that for every function of (γ,μ)

$$E\left[G\left(\gamma,\mu\right)|\left(\gamma,\mu\right)\in\mathcal{I}\right]=\int\int G\left(\gamma,\mu\right)f\left(\gamma,\mu|\gamma,\mu\in\mathcal{I}\right)d\gamma d\mu\;,$$

where the joint distribution is

$$f\left(\gamma,\mu|\gamma,\mu\in\mathcal{I}\right)=\frac{f\left(\gamma,\mu\right)1_{\{\mu,\gamma\in\mathcal{I}\}}}{\int\int f\left(\gamma,\mu\right)1_{\{\mu,\gamma\in\mathcal{I}\}}d\gamma d\mu}\;.$$

The denominator is the mass of agents who satisfy the constraint. We therefore obtain

$$\begin{aligned}& E\left[G\left(\gamma,\mu\right)|\left(\gamma,\mu\right)\in\mathcal{I}\right] \\ =& \frac{1}{m}\int\int G\left(\gamma,\mu\right)f\left(\gamma,\mu\right)1_{\{\mu,\gamma\in\mathcal{I}\}}d\gamma d\mu \\ =& \frac{1}{m}\int\int G\left(\gamma,\mu\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)1_{\{\mu,\gamma\in\mathcal{I}\}}d\gamma d\mu \\ =& \frac{1}{m}\int_{-\infty}^\infty\left[\int_0^{\max\left(\frac{2}{\sigma_1^2}\left(\mu-\frac{1}{T}[\log(m/(1-m))+\log(\tau/(1-\tau))+\log(H)]\right),0\right)}G\left(\gamma,\mu\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma\right]\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu \\ =& \frac{1}{m}\int_0^\infty\left[\int_{\frac{1}{2}\gamma\sigma_1^2+\frac{1}{T}[\log(m/(1-m))+\log(\tau/(1-\tau))+\log(H)]}^\infty G\left(\gamma,\mu\right)\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu\right]\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma\;,\end{aligned}$$

where we used the condition that $(\mu, \gamma) \in \mathcal{I}$ iff

$$\mu > \frac{1}{2}\gamma\sigma_1^2 + \frac{1}{T}\left[\log\left(m/\left(1-m\right)\right) + \log\left(\tau/\left(1-\tau\right)\right) + \log\left(H\right)\right]$$

or, equivalently,

$$\frac{2}{\sigma_1^2}\left(\mu - \frac{1}{T}\left[\log\left(m/\left(1-m\right)\right) + \log\left(\tau/\left(1-\tau\right)\right) + \log\left(H\right)\right]\right) > \gamma \, .$$

From direct computation, we have for given risk aversion γ, the mass of agents that choose to invest are given as those with $\mu > \frac{1}{2}\gamma\sigma_1^2 + \frac{1}{T}\log\left(\frac{m}{1-m}\right) + \frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right) + \frac{1}{T}\log\left(H\right)$. This implies that we can write

$$\begin{aligned}
E\left[e^{a\gamma+b\gamma^2}|i \in \mathcal{I}\right] &= \frac{1}{m}\int_0^\infty e^{a\gamma+b\gamma^2}\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}\int_{\frac{1}{2}\gamma\sigma_1^2+\frac{1}{T}\log\left(\frac{m}{1-m}\right)+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\frac{1}{T}\log(H)}^\infty \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu d\gamma \\
&= \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)}\int_0^\infty e^{a\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma \\
&= \frac{1}{m}\frac{1}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}\frac{1}{\sqrt{2\pi\sigma_\gamma^2}}\int_0^\infty e^{a\gamma+b\gamma^2}e^{-\frac{1}{2}\frac{(\gamma-\overline{\gamma})^2}{\sigma_\gamma^2}}\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\mu,\sigma_\mu\right)\right)d\gamma
\end{aligned}$$

The integral is well defined if the quadratic term in the exponent of the exponential function is negative, that is, if

$$b - \frac{1}{2\sigma_\gamma^2} < 0 \, .$$

Q.E.D.

Proof of Proposition B6: The pricing formula is the same as in Proposition 5. The only part to prove is the resulting equation for Z. From Corollary 3, we can write

$$Z = \frac{E^{\mathcal{I}}\left[e^{(\gamma_i-1)\gamma_i\frac{1}{2}\sigma^2T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2T+\log(Z)\gamma_i}|i \in \mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\gamma_i(1+\gamma_i)\frac{1}{2}\sigma^2T+\gamma_i(1+\gamma_i)\frac{1}{2}\sigma_1^2T+\log(Z)\gamma_i}|i \in \mathcal{I}\right]} = \frac{E^{\mathcal{I}}\left[e^{(a_1+\log(Z))\gamma_i+b\gamma_i^2}|i \in \mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{(a_2+\log(Z))\gamma_i+b\gamma_i^2}|i \in \mathcal{I}\right]} \, ,$$

where

$$\begin{aligned}
a_1 &= \frac{1}{2}\left(\sigma_1^2 - \sigma^2\right)T \\
a_2 &= \frac{1}{2}\left(\sigma_1^2 + \sigma^2\right)T \\
b &= \frac{1}{2}\left(\sigma^2 + \sigma_1^2\right)T \, .
\end{aligned}$$

These integrals are well defined if $b < \frac{1}{2\sigma_\gamma^2}$, which yields the restriction $\frac{1}{2}\left(\sigma^2+\sigma_1^2\right)T < \frac{1}{2\sigma_\gamma^2}$. In addition, applying Lemma 1, we obtain

$$Z = \frac{\int_0^\infty e^{(a_1+\log(Z))\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma}{\int_0^\infty e^{(a_2+\log(Z))\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma} \, .$$

We note that if $\sigma_\gamma^2 = 0$, then $\phi(\gamma; \overline{\gamma}, 0)$ spikes at $\overline{\gamma}$ and indeed we obtain the standard result with homogeneous γ :

$$\begin{aligned} Z &= \frac{e^{(a_1+\log(Z))\overline{\gamma}+b\overline{\gamma}^2}\left(1-\Phi\left(\frac{1}{2}\overline{\gamma}\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)}{e^{(a_2+\log(Z))\overline{\gamma}+b\overline{\gamma}^2}\left(1-\Phi\left(\frac{1}{2}\overline{\gamma}\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)} = \frac{e^{a_1\overline{\gamma}}}{e^{a_2\overline{\gamma}}} = e^{(a_1-a_2)\overline{\gamma}} \\ &= e^{\left(\frac{1}{2}\left(\sigma_1^2-\sigma^2\right)T-\frac{1}{2}\left(\sigma_1^2+\sigma^2\right)T\right)\overline{\gamma}} = e^{-\overline{\gamma}\sigma^2 T} . \end{aligned}$$

We can rewrite the equation as

$$\begin{aligned} & Z\int_0^\infty e^{(a_2+\log(Z))\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma \\ = & \int_0^\infty e^{(a_1+\log(Z))\gamma+b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma \end{aligned}$$

and hence

$$\int_0^\infty \left[e^{a_2\gamma+\log(Z)(1+\gamma)}-e^{a_1\gamma+\log(Z)\gamma}\right]e^{b\gamma^2}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma = 0 .$$

Substitute in a_1, a_2 and b to obtain the equation:

$$\int_0^\infty \left[e^{\log(Z)}-e^{-\sigma^2 T\gamma}\right]e^{\gamma\log(Z)+\frac{\gamma(1+\gamma)}{2}\left(\sigma_1^2+\sigma^2\right)T}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right)d\gamma = 0 .$$

Q.E.D.

Proof of Proposition B7: From the general expression, we have that inequality is

$$\text{Var}(s_{i,T}) = \frac{\tau^2}{1-m}+\frac{(1-\tau)^2}{m}\frac{E^{\mathcal{I}}\left[e^{2\mu_j T}|j\in\mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2}e^{\sigma_1^2 T}-1 .$$

The only term we do not know is

$$\begin{aligned} & E^{\mathcal{I}}\left[e^{2\mu_j T}|i\in\mathcal{I}\right] \\ = & \frac{1}{m}\int_{(\mu\times\gamma)\in\mathcal{I}} e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)2\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)d\mu d\gamma \\ = & \frac{1}{m}\left\{\int_{-\infty}^\infty e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\left[\int_0^{\max\left(\frac{2}{\sigma_1^2}\left[\mu-\frac{1}{T}\log\left(\frac{m}{1-m}\right)-\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)-\frac{1}{T}\log(H)\right],0\right)}\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}d\gamma\right]d\mu\right\} \\ = & \frac{1}{m}\left\{\int_{-\infty}^\infty e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\left[\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}-\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}\right]d\mu\right\} \\ = & \frac{1}{m}\left\{\int_{-\infty}^\infty e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\frac{\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}d\mu-\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)}E\left[e^{2\mu T}\right]\right\} . \end{aligned}$$

Note that we can rewrite

$$
\begin{aligned}
e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu\right) &= e^{2\mu T}\frac{e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} = \frac{e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2-4\sigma_\mu^2\mu T}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} = \frac{e^{-\frac{1}{2}\frac{\mu^2+\overline{\mu}^2-2\left(\overline{\mu}+2\sigma_\mu^2 T\right)\mu+\left(\overline{\mu}+2\sigma_\mu^2 T\right)^2-\left(\overline{\mu}+2\sigma_\mu^2 T\right)^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} \\
&= \frac{e^{-\frac{1}{2}\frac{\left(\mu-\left(\overline{\mu}+2\sigma_\mu^2 T\right)\right)^2}{\sigma_\mu^2}}}{\sqrt{2\pi\sigma_\mu^2}} e^{2\left(\sigma_\mu^2\right)T^2+2\overline{\mu}T} \\
&= \phi\left(\mu;\overline{\mu}+2\sigma_\mu^2 T,\sigma_\mu^2\right)e^{2\sigma_\mu^2 T^2+2\overline{\mu}T}\ .
\end{aligned}
$$

Substitute, to obtain

$$
\begin{aligned}
& E^{\mathcal{I}}\left[e^{2\mu_j T}|i\in\mathcal{I}\right] \\
= \ & \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)} \\
& \times\left\{\int_{-\infty}^{\infty} e^{2\mu T}\times\phi\left(\mu;\overline{\mu},\sigma_\mu\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)E\left[e^{2\mu T}\right]\right\} \\
= \ & \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)} \\
& \times\left\{e^{2\sigma_\mu^2 T^2+2\overline{\mu}T}\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+2\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)E\left[e^{2\mu T}\right]\right\} \\
= \ & \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)} \\
& \times\left\{e^{2\sigma_\mu^2 T^2+2\overline{\mu}T}\left(\int_{-\infty}^{\infty}\phi\left(\mu;\overline{\mu}+2\sigma_\mu^2 T,\sigma_\mu^2\right)\Phi\left(\max\left(\frac{2}{\sigma_1^2}\left[\mu-K\right],0\right);\overline{\gamma},\sigma_\gamma^2\right)d\mu-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)\right\}\ .
\end{aligned}
$$

Q.E.D.

Proof of Proposition B8: We need to compute

$$
\Pr\left(\mu_i T+\sigma_1 W_{i,T}<\frac{1}{2}\sigma_1^2 T+\log\left(x\right)+\log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]m\left(\mathcal{I}\right)}{1-\tau}\right)|\mu_i>\frac{1}{2}\gamma_i\sigma_1^2+K\right)
$$

From Bayes formula, we have

$$
\begin{aligned}
& \Pr\left(\mu_i T+\sigma_1 W_{i,T}<\frac{1}{2}\sigma_1^2 T+\log\left(x\right)+\log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]m\left(\mathcal{I}\right)}{1-\tau}\right)|\mu_i>\frac{1}{2}\gamma_i\sigma_1^2+K\right) \\
= \ & \frac{\Pr\left(\mu_i T+\sigma_1 W_{i,T}<\frac{1}{2}\sigma_1^2 T+\log\left(x\right)+\log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]m(\mathcal{I})}{1-\tau}\right)\&\mu_i>\frac{1}{2}\gamma_i\sigma_1^2+K\right)}{\Pr\left(\mu_i>\frac{1}{2}\gamma_i\sigma_1^2+K\right)} \\
= \ & \frac{1}{m}\int_0^{\infty}\Pr\left(\mu_i T+\sigma_1 W_{i,T}<\frac{1}{2}\sigma_1^2 T+\log\left(x\right)+\log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]m\left(\mathcal{I}\right)}{1-\tau}\right)\&\mu_i>\frac{1}{2}\gamma_i\sigma_1^2+K|\gamma\right) \\
& \times\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{m}\int_0^\infty \int \Pr\left(\mu_i T + \sigma_1 W_{i,T} < \frac{1}{2}\sigma_1^2 T + \log(x) + \log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] m(\mathcal{I})}{1-\tau}\right) \& \mu_i > \frac{1}{2}\gamma_i\sigma_1^2 + K|\gamma,\mu\right) \\
&\qquad \times \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma \\
&= \frac{1}{m}\int_0^\infty \int_{\frac{1}{2}\gamma_i\sigma_1^2+K}^\infty \Pr\left(\mu_i T + \sigma_1 W_{i,T} < \frac{1}{2}\sigma_1^2 T + \log(x) + \log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] m(\mathcal{I})}{1-\tau}\right)|\gamma,\mu\right) \\
&\qquad \times \phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma \\
&= \frac{1}{m}\int_0^\infty \int_{\frac{1}{2}\gamma_i\sigma_1^2+K}^\infty \Phi\left(\frac{1}{2}\sigma_1^2 T + \log(x) + \log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] m(\mathcal{I})}{1-\tau}\right), \mu T, \sigma_1^2 T\right)\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma\,.
\end{aligned}
$$

Therefore, we obtain the double integral

$$
\Pr\left(\frac{C_T^i}{\overline{C}_T} < x|\mu_i > \frac{1}{2}\gamma_i\sigma_1^2 + K\right) = \frac{1}{m}\int_0^\infty f(\gamma)\frac{\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)}{1-\Phi\left(0,\overline{\gamma},\sigma_\gamma^2\right)}d\gamma\,,
$$

where

$$
f(\gamma) = \int_{\frac{1}{2}\gamma_i\sigma_1^2+K}^\infty \Phi\left(\frac{1}{2}\sigma_1^2 T + \log(x) + \log\left(\frac{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] m(\mathcal{I})}{1-\tau}\right); T\mu, \sigma_1^2 T\right)\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu\,.
$$

Q.E.D.

Proof of Proposition B9. We begin by computing the certainty equivalent. From its definition

$$
\frac{CE_{it}^{1-\gamma_i}}{1-\gamma_i} = \frac{E\left[C_{i,T}^{1-\gamma_i}\right]}{1-\gamma_i}\,,
$$

we obtain

$$
CE_{i,T} = E\left[C_{i,T}^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}\,.
$$

We can compute the certainty equivalent for $\theta = 1$ in closed form. In fact, for pensioners,

$$
\begin{aligned}
E\left[C_{i,T}^{1-\gamma_i}\right] &= \tau^{1-\gamma_i}B_0^{1-\gamma_i}\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)^{1-\gamma_i} E\left[e^{(1-\gamma_i)\varepsilon_T}\right]\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)^{1-\gamma_i} \\
&= \tau^{1-\gamma_i}B_0^{1-\gamma_i}\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)^{1-\gamma_i} e^{-(1-\gamma_i)\frac{1}{2}\sigma^2 T + \frac{1}{2}(1-\gamma_i)^2\sigma^2 T}\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)^{1-\gamma_i}\,.
\end{aligned}
$$

Therefore,

$$
CE_{i,T}^{no} = \tau B_0\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right)E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]e^{-\frac{1}{2}\gamma_i\sigma^2 T}\,.
$$

For entrepreneurs, consumption at time T is

$$
C_{i,T} = (1-\tau)B_0 e^{\mu_i T}\left[e^{\varepsilon_{i,T}+\varepsilon_T}\right]
$$

and thus their expected utility is

$$
\begin{aligned}
\frac{E\left[C_{i,T}^{1-\gamma}\right]}{1-\gamma_i} &= \frac{(1-\tau)^{1-\gamma_i} B_0^{1-\gamma_i} e^{(1-\gamma_i)\mu_i T}}{1-\gamma_i} E\left(\left[e^{\varepsilon_{i,T}+\varepsilon_T}\right]^{1-\gamma_i}\right) \\
&= \frac{(1-\tau)^{1-\gamma_i} B_0^{1-\gamma_i} e^{(1-\gamma_i)\mu_i T}}{1-\gamma_i} \left(e^{-(1-\gamma_i)\frac{1}{2}\left(\sigma_1^2+\sigma^2\right)T+\frac{1}{2}(1-\gamma_i)^2\left(\sigma_1^2+\sigma^2\right)T}\right) .
\end{aligned}
$$

Therefore, for entrepreneurs, the certainty equivalent consumption is

$$
CE_{i,T}^{yes} = (1-\tau) B_0 e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T} .
$$

We now normalize by the aggregate certainty equivalent:

$$
s_{i,T}^{CE} = \frac{CE_{i,T}}{\int CE_{i,T} di} .
$$

The denominator is given by

$$
\begin{aligned}
\int CE_{i,T} di &= \int_{i\notin\mathcal{I}} \tau B_0 \left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] e^{-\frac{1}{2}\gamma_i\sigma^2 T} di \\
&\quad + \int_{i\in\mathcal{I}} (1-\tau) B_0 e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T} di \\
&= (1-m(\mathcal{I}))\tau B_0 \left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] \\
&\quad + m(\mathcal{I})(1-\tau) B_0 E^{\mathcal{I}}\left[e^{\mu_i T}|i\in\mathcal{I}\right] E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right] \\
&= \tau B_0 m(\mathcal{I}) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] \\
&\quad + m(\mathcal{I})(1-\tau) B_0 E^{\mathcal{I}}\left[e^{\mu_i T}|i\in\mathcal{I}\right] E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right] \\
&= B_0 m(\mathcal{I}) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] \left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau) E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right] .
\end{aligned}
$$

Therefore, the CE share for entrepreneurs is

$$
\begin{aligned}
s_{i,T}^{CE,yes} &= \frac{(1-\tau) B_0 e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}}{B_0 m(\mathcal{I}) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] \left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau) E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \\
&= \frac{(1-\tau) e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}}{m(\mathcal{I}) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] \left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau) E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} ,
\end{aligned}
$$

while the CE share for pensioners is

$$
\begin{aligned}
s_{i,T}^{CE,no} &= \frac{\tau B_0 \left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] e^{-\frac{1}{2}\gamma_i\sigma^2 T}}{B_0 m(\mathcal{I}) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right] \left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau) E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \\
&= \frac{\tau e^{-\frac{1}{2}\gamma_i\sigma^2 T}}{(1-m(\mathcal{I})) \left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau) E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} .
\end{aligned}
$$

We finally compute the variance as

$$\text{Var}(s_{i,T}^{CE}) = E\left[\left(s_{i,T}^{CE}\right)^2\right] - 1 \ .$$

Compute the second moment

$$\begin{aligned}
& E\left[\left(s_{i,T}^{CE}\right)^2\right] \\
= \ & \int_{i\in\mathcal{I}} \left(\frac{(1-\tau)\, e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}}{m\left(\mathcal{I}\right) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \right)^2 di \\
& + \int_{i\notin\mathcal{I}} \left(\frac{\tau e^{-\frac{1}{2}\gamma_i\sigma^2 T}}{\left(1-m\left(\mathcal{I}\right)\right)\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \right)^2 di \\
= \ & m\left(\mathcal{I}\right) E^{\mathcal{I}}\left[\left(\frac{(1-\tau)\, e^{\mu_i T} e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}}{m\left(\mathcal{I}\right) E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \right)^2 |i\in\mathcal{I}\right] \\
& + \left(1-m\left(\mathcal{I}\right)\right) E^{\mathcal{I}}\left[\left(\frac{\tau e^{-\frac{1}{2}\gamma_i\sigma^2 T}}{\left(1-m\left(\mathcal{I}\right)\right)\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]} \right)^2 |i\notin\mathcal{I}\right] \\
= \ & \frac{(1-\tau)^2}{m\left(\mathcal{I}\right)} \frac{E^{\mathcal{I}}\left[e^{2\mu_i T}|i\in\mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2} \frac{E^{\mathcal{I}}\left[e^{-\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]}{\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]^2} \\
& + \frac{\tau^2}{\left(1-m\left(\mathcal{I}\right)\right)} \frac{E^{\mathcal{I}}\left[e^{-\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right]}{\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]^2} \ .
\end{aligned}$$

Finally,

$$E\left[\left(s_{i,T}^{CE}\right)^2\right] = \frac{\frac{\tau^2}{\left(1-m(\mathcal{I})\right)} E^{\mathcal{I}}\left[e^{-\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + \frac{(1-\tau)^2}{m(\mathcal{I})}\frac{E^{\mathcal{I}}\left[e^{2\mu_i T}|i\in\mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2} E^{\mathcal{I}}\left[e^{-\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]}{\left[\tau E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\sigma^2 T}|i\notin\mathcal{I}\right] + (1-\tau)\, E^{\mathcal{I}}\left[e^{-\frac{1}{2}\gamma_i\left(\sigma_1^2+\sigma^2\right)T}|i\in\mathcal{I}\right]\right]^2} \ .$$

All the expectations can be computed in closed form by using Lemma B1. In particular, for any a, we obtain

$$E^{\mathcal{I}}\left[e^{a\gamma}|i\in\mathcal{I}\right] = \frac{1}{m\left(1-\Phi\left(0;\overline{\gamma},\sigma_\gamma^2\right)\right)} \int_0^\infty e^{a\gamma}\phi\left(\gamma;\overline{\gamma},\sigma_\gamma^2\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_1^2+K;\overline{\mu},\sigma_\mu^2\right)\right) d\gamma \ .$$

In addition, using the equality

$$\begin{aligned}
E\left[e^{a\gamma}\right] &= E^{\mathcal{I}}\left[e^{a\gamma}|i\in\mathcal{I}\right]\Pr\left(i\in\mathcal{I}\right) + E^{\mathcal{I}}\left[e^{a\gamma}|i\notin\mathcal{I}\right]\Pr\left(i\notin\mathcal{I}\right) \\
&= E^{\mathcal{I}}\left[e^{a\gamma}|i\in\mathcal{I}\right] m\left(\mathcal{I}\right) + E^{\mathcal{I}}\left[e^{a\gamma}|i\notin\mathcal{I}\right]\left(1-m\left(\mathcal{I}\right)\right) \ ,
\end{aligned}$$

we obtain

$$
\begin{aligned}
E^{\mathcal{I}}\left[e^{a\gamma}|i\notin\mathcal{I}\right](1-m(\mathcal{I})) &= E\left[e^{a\gamma}\right]-E^{\mathcal{I}}\left[e^{a\gamma}|i\in\mathcal{I}\right]m(\mathcal{I}) \\
&= \frac{1}{\left(1-\Phi\left(0;\overline{\gamma},\sigma_{\gamma}^{2}\right)\right)}\int_{0}^{\infty}e^{a\gamma}\phi\left(\gamma;\overline{\gamma},\sigma_{\gamma}^{2}\right)d\gamma \\
&\quad -\frac{1}{\left(1-\Phi\left(0;\overline{\gamma},\sigma_{\gamma}^{2}\right)\right)}\int_{0}^{\infty}e^{a\gamma}\phi\left(\gamma;\overline{\gamma},\sigma_{\gamma}^{2}\right)\left(1-\Phi\left(\frac{1}{2}\gamma\sigma_{1}^{2}+K;\overline{\mu},\sigma_{\mu}^{2}\right)\right)d\gamma \\
&= \frac{1}{\left(1-\Phi\left(0;\overline{\gamma},\sigma_{\gamma}^{2}\right)\right)}\int_{0}^{\infty}e^{a\gamma}\phi\left(\gamma;\overline{\gamma},\sigma_{\gamma}^{2}\right)\Phi\left(\frac{1}{2}\gamma\sigma_{1}^{2}+K;\overline{\mu},\sigma_{\mu}^{2}\right)d\gamma
\end{aligned}
$$

and thus

$$
E^{\mathcal{I}}\left[e^{a\gamma}|i\notin\mathcal{I}\right]=\frac{1}{1-m}\frac{1}{\left(1-\Phi\left(0;\overline{\gamma},\sigma_{\gamma}^{2}\right)\right)}\int_{0}^{\infty}e^{a\gamma}\phi\left(\gamma;\overline{\gamma},\sigma_{\gamma}^{2}\right)\Phi\left(\frac{1}{2}\gamma\sigma_{1}^{2}+K;\overline{\mu},\sigma_{\mu}^{2}\right)d\gamma\ .
$$

Specializing "a" in these formulas to the various cases above, we obtain a closed-form formula for the second moments.

Q.E.D.

C. Special case: Common risk aversion ($\gamma_i = \gamma$).

C.1. Theoretical Results.

Corollary C1: If $\gamma_i = \gamma$ for all i, then:

(a) The state price density at time T is proportional to a simple exponential function of the aggregate shock ε_T:

$$\pi_T = he^{-\gamma\varepsilon_T} , \tag{C1}$$

for a scaling constant h.

(b) Each entrepreneur i buys the following number of shares in agent j's stock

$$N_0^{ij} = (1-\theta)\frac{e^{\mu_i T}}{\int_{\mathcal{I}} e^{\mu_k T} dk}$$

and there is no borrowing or lending

$$N_0^{0,i} = 0 .$$

(c) The market price of each individual stock is given by

$$\frac{M_{i,0}}{B_{i,0}} = (1-\tau)\, e^{\left(\mu_i - \gamma\sigma^2\right)T} .$$

(d) Assuming $B_{i,0} = B_{j,0} = B_0$ for all i, j, and renormalizing by total investment $B^P = \int_{\mathcal{I}} B_0 di = B_0 m(\mathcal{I})$, we have

$$\frac{M^P}{B^P} = \frac{1}{m(\mathcal{I})\, B_0}\int M_{j,0} dj = (1-\tau)\, E\left[e^{\mu_j T} | j \in \mathcal{I}\right] e^{-\gamma\sigma^2 T} .$$

(e) The expected rate of return on each stock i is given by

$$\mathrm{E}\left(R^i\right) = e^{\gamma\sigma^2 T} - 1 .$$

Note from (a) that θ does not affect the stochastic discount factor (π_T/π_0). All entrepreneurs hold θ in their own firm and $1-\theta$ in the market. Since all firms have the same risk exposure (see equation (1) in the paper), everyone's position is symmetric ex ante. Therefore, the risk aversion in the economy is the common risk aversion γ and the amount of idiosyncratic risk faced by each entrepreneur, as determined by θ, does not affect equilibrium asset prices.

In contrast, θ does affect asset prices in the general case with heterogeneous risk aversion. When risk aversions differ, agents insure each other by trading bonds: low-γ_i agents sell bonds to high-γ_i agents. As a result, lower-γ_i agents acquire larger positions in the market portfolio, bringing down the value-weighted average risk aversion of those holding the market. When θ increases, all agents become more exposed to idiosyncratic risk, resulting in additional demand for bonds and thus additional changes in the agents' stock market allocations. Therefore, changes in θ shift the equilibrium risk aversion of the typical agent holding the market, thereby shifting the state price density in the general case.

Note from (c) that each firm's M/B is equal to expected after-tax cash flow adjusted for risk, where the risk adjustment is particularly simple.

Proposition C2: (a) Agent i chooses to become an entrepreneur if and only if

$$\mu_i > K - \Theta \,, \tag{C2}$$

where Θ is a constant given by

$$\Theta = \frac{1}{T\left(1-\gamma\right)} \log \left(\int_{-\infty}^{\infty} \left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 \varepsilon} + (1-\theta) \right)^{1-\gamma} \phi\left(\varepsilon; 0, T\right) d\varepsilon \right)$$

and K is the unique solution to the equation

$$K = \overline{\mu} + \frac{1}{2}\sigma_\mu^2 T + \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{1-\Phi\left(K-\Theta; \overline{\mu}+T\sigma_\mu^2, \sigma_\mu^2\right)}{\Phi\left(K-\Theta; \overline{\mu}, \sigma_\mu^2\right)} \right) \right] . \tag{C3}$$

(b) The total mass of agents who become entrepreneurs is

$$m\left(\mathcal{I}\right) = 1 - \Phi\left(K - \Theta, \overline{\mu}, \sigma_\mu^2\right) \; .$$

(c) The expected growth rate of the economy is

$$H = E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right] = e^{\overline{\mu}T + \frac{1}{2}T^2\sigma_\mu^2} \frac{1-\Phi\left(K-\Theta; \overline{\mu}+T\sigma_\mu^2, \sigma_\mu^2\right)}{1-\Phi\left(K-\Theta, \overline{\mu}, \sigma_\mu^2\right)} \; .$$

(d) The aggregate stock market level is

$$\frac{M_0^P}{B_0^P} = \left(1-\tau\right) e^{\overline{\mu}T - \gamma\sigma^2 T} \left[\left(\frac{1-\Phi\left(K-\Theta; \overline{\mu}+T\sigma_\mu^2, \sigma_\mu^2\right)}{1-\Phi\left(K-\Theta, \overline{\mu}, \sigma_\mu^2\right)} \right) e^{\frac{1}{2}T^2\sigma_\mu^2} \right] . \tag{C4}$$

Part (d) highlights the channels through which the aggregate level of stock prices depends on the distribution of skill. The term outside the brackets, $(1-\tau)e^{\left(\overline{\mu}-\gamma\sigma^2\right)T}$, is the expected risk-adjusted after-tax cash flow earned by the entrepreneur with average skill. The term inside the brackets is equal to one if there is no dispersion in skill ($\sigma_\mu = 0$). If there is dispersion in skill ($\sigma_\mu > 0$), this term is a product of two terms, both of which are greater than one. The first term, the ratio in parentheses, is greater than one due to selection on skill. The second term, $e^{\frac{1}{2}T^2\sigma_\mu^2}$, is greater than one due to the convexity effect discussed in the paper.

Proposition C3: Higher taxes induce fewer agents to become entrepreneurs. Denoting the threshold by $G(K,\Theta,\tau)=K-\Theta$, we have that higher taxes increase the threshold

$$\frac{dG}{d\tau}>0\ ,$$

which implies that the mass of entrepreneurs decreases with the tax rate:

$$\frac{\partial m(\mathcal{I})}{\partial\tau}<0\ .$$

Proposition C4: If θ increases so that agents become less able to diversify idiosyncratic risk, fewer agents become entrepreneurs. That is, denoting the threshold by $G(K,\Theta)=K-\Theta$, we have

$$\frac{dG}{d\theta}>0\ ,$$

which implies that the mass of entrepreneurs decreases with θ:

$$\frac{\partial m(\mathcal{I})}{\partial\theta}<0\ .$$

Proposition C5: The variance of scaled consumption $s_{i,T}$ across agents is given by

$$\begin{aligned}\text{Var}(s_{i,t}) &= \frac{\tau^2}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}+(1-\tau)^2\left[e^{T^2\sigma_\mu^2}\frac{\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)}{\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)^2}\right] && \text{(C5)}\\ &\quad\times\left[1+\theta^2\left(e^{\sigma_1^2T}-1\right)\right]-1\ . && \text{(C6)}\end{aligned}$$

The term in the first brackets is greater than one due to cross-sectional dispersion in skill ($\sigma_\mu^2>0$). This term is a product of two terms, $e^{T^2\sigma_\mu^2}$ and a ratio, both of which are greater than one. Not surprisingly, a larger σ_μ^2 implies more income inequality. The term in the second brackets is also greater than one, due to the presence of idiosyncratic risk ($\theta>0$). If all such risk were diversifiable ($\theta=0$), it would generate no dispersion in income and this term would be equal to one. But when $\theta>0$, each entrepreneur bears idiosyncratic risk whose ex-post realizations, which are commensurate to their volatility σ_1, contribute to inequality. Higher θ implies less diversification and more inequality.

Proposition C6: The distribution of scaled consumption, $s_{i,T}=C_{i,T}/\overline{C}_T$, is given by the cumulative density function

$$F(s_{i,T})=1_{\left\{s_{i,T}>\frac{\tau}{1-m}\right\}}(1-m)+F(s_{i,T}|\mu_i>K-\Theta)\,m\ ,$$

where

$$
\begin{aligned}
& F\left(s_{i,T}|\mu_i > K-\Theta\right) \\
= \; & \frac{1}{m}\int_{K-\Theta}^{\infty}\Phi\left(\log\left(s_{i,T}\frac{Hm}{(1-\tau)\,e^{\mu T}}-(1-\theta)\right)-\log(\theta)+\frac{1}{2}\sigma_1^2T;0,\sigma_1^2T\right)\phi\left(\mu,\overline{\mu},\sigma_\mu^2\right)d\mu\,.
\end{aligned}
$$

C.2. Proofs for Appendix C.

Proof of Corollary C1: In the special case in which agents have the same risk aversion $\gamma_i = \gamma$, we have $\widetilde{\alpha}(\gamma) = \widetilde{\alpha}$ and thus $\widetilde{\alpha}\Omega = \alpha = 1$. The latter stems directly from the definition of $\Omega = \frac{\int_{\mathcal{I}} M_0^j dj}{\int_{\mathcal{I}} \widetilde{\alpha}^k M_0^k di}$. If $\widetilde{\alpha}^k = \widetilde{\alpha}$, then $\Omega = \frac{1}{\widetilde{\alpha}}$ and the result follows.

Our assumption that the shocks are normally distributed implies that we can write

$$
\begin{aligned}
\varepsilon_T &= -\frac{1}{2}\sigma^2T + \sigma W_T \\
\varepsilon_{i,T} &= -\frac{1}{2}\sigma_1^2T + \sigma_1 W_{iT}\ ,
\end{aligned}
$$

with $W_T \sim N(0,T)$ and $W_{iT} \sim N(0,T)$ and independent from W_T.

From the proof of Proposition 4, when $\alpha = \widetilde{\alpha}\Omega = 1$ and $\gamma_i = \gamma$, the SDF is given by

$$
\pi_T = \int_{\mathcal{I}}\left(\theta R^i + (1-\theta)\,R^{Mkt}+1\right)^{-\gamma}di\,.
$$

Thus, we obtain

$$
\begin{aligned}
\pi_T &= \int_{\mathcal{I}}\left(\theta R^i + (1-\theta)\,R^{Mkt}+1\right)^{-\gamma}di \\
&= \int_{\mathcal{I}}\left(\theta\left(1+R^i\right)+(1-\theta)\left(1+R^{Mkt}\right)\right)^{-\gamma}di \\
&= \int_{\mathcal{I}}\left(\theta\left(\frac{e^{-\frac{1}{2}\sigma_1^2T-\frac{1}{2}\sigma^2T+\sigma_1W_{iT}+\sigma W_T}}{Z}\right)+(1-\theta)\left(\frac{e^{-\frac{1}{2}\sigma^2T+\sigma W_T}}{Z}\right)\right)^{-\gamma}di \\
&= \left(\frac{e^{-\frac{1}{2}\sigma^2T+\sigma W_T}}{Z}\right)^{-\gamma}\int_{\mathcal{I}}\left(\theta\left(e^{-\frac{1}{2}\sigma_1^2T+\sigma_1W_{iT}}\right)+(1-\theta)\right)^{-\gamma}di \\
&= he^{-\gamma\epsilon_T}
\end{aligned}
$$

where

$$
h = \left(\frac{1}{Z}\right)^{-\gamma}\int_{\mathcal{I}}\left(\theta\left(e^{-\frac{1}{2}\sigma_1^2T+\sigma_1W_{iT}}\right)+(1-\theta)\right)^{-\gamma}di\,.
$$

It then follows from direct computations that

$$
Z = \frac{E[\pi_T e^{\epsilon_T}]}{E[\pi_T]} = e^{-\gamma\sigma^2T}\,.
$$

This yields

$$E[R^i] = \frac{1}{Z} - 1 = e^{\gamma\sigma^2 T} - 1 \ .$$

It is also immediate to show that when $\widetilde{\alpha}$ is constant and thus $\widetilde{\alpha}\Omega = 1$, then $N_0^{i0} = 0$ for all entrepreneurs, and thus there is no trading.

The remaining parts of the corollary stem immediately from the SDF.

Q.E.D

Proof of Proposition C2: From the general case with $\gamma \neq 1$ specialized to $\varepsilon_T = -\frac{1}{2}\sigma^2 T + \sigma W_T$, we have

$$\begin{aligned} & \mu_i + \frac{1}{2}\gamma\sigma^2 + \frac{1}{T(1-\gamma)} \log\left(\overline{V}^i\right) \\ > \ & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \ . \end{aligned}$$

In equilibrium, now we have $\alpha = 1$, and thus

$$\begin{aligned} \overline{V}^i &= E\left[\left(\theta\left(e^{-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma_1^2 T + \sigma W_T + \sigma_1 W_{i,T}} - Z\right) + (1-\theta)\left(e^{-\frac{1}{2}\sigma^2 T + \sigma W_T} - Z\right) + Z\right)^{1-\gamma}\right] \\ &= E\left[\left(\theta e^{-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma_1^2 T + \sigma W_T + \sigma_1 W_{i,T}} + (1-\theta) e^{-\frac{1}{2}\sigma^2 T + \sigma W_T}\right)^{1-\gamma}\right] \ . \end{aligned}$$

Substitute

$$\begin{aligned} & \mu_i + \frac{1}{2}\gamma\sigma^2 + \frac{1}{T(1-\gamma)} \log\left(E\left[\left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}} + (1-\theta)\right)^{1-\gamma}\left(e^{-\frac{1}{2}\sigma^2 T + \sigma W_T}\right)^{1-\gamma}\right]\right) \\ > \ & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \end{aligned}$$

or

$$\begin{aligned} & \mu_i + \frac{1}{2}\gamma\sigma^2 + \frac{1}{T(1-\gamma)} \log\left(E\left[\left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}} + (1-\theta)\right)^{1-\gamma}\right] E\left[\left(e^{-\frac{1}{2}\sigma^2 T + \sigma W_T}\right)^{1-\gamma}\right]\right) \\ > \ & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \end{aligned}$$

or

$$\begin{aligned} & \mu_i + \frac{1}{2}\gamma\sigma^2 + \frac{1}{T(1-\gamma)} \log\left(E\left[\left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}} + (1-\theta)\right)^{1-\gamma}\right] e^{-\frac{1}{2}(1-\gamma)\gamma\sigma^2 T}\right) \\ > \ & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \end{aligned}$$

or

$$\begin{aligned} & \mu_i + \frac{1}{2}\gamma\sigma^2 + \frac{1}{T(1-\gamma)} \log\left(E\left[\left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}} + (1-\theta)\right)^{1-\gamma}\right]\right) - \frac{1}{2}\gamma\sigma^2 \\ > \ & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \end{aligned}$$

or

$$
\begin{aligned}
& \mu_i + \frac{1}{T(1-\gamma)} \log\left(E\left[\left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}} + (1-\theta) \right)^{1-\gamma} \right] \right) \\
> \; & \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I} \right] \right) \right] .
\end{aligned}
$$

From the assumption $\mu \sim N\left(\overline{\mu}, \sigma_\mu^2\right)$, we can denote

$$
\begin{aligned}
K &= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m}{1-m}\right) + \log(H) \right] \\
H &= E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I} \right] .
\end{aligned}
$$

Therefore, the threshold is

$$
\mu_i > K - \Theta ,
$$

where

$$
\Theta = \frac{1}{T(1-\gamma)} \log\left(\int \left(\theta e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 \varepsilon} + (1-\theta) \right)^{1-\gamma} \phi(\varepsilon; 0, T)\, d\varepsilon \right)
$$

Therefore, the total measure of agents satisfying the constraint is

$$
m = \int_{K-\Theta}^{\infty} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) d\mu = 1 - \Phi\left(K - \Theta, \overline{\mu}, \sigma_\mu^2\right) .
$$

At the same time,

$$
H = E^{\mathcal{I}}\left[e^{\mu_j T} | i \in \mathcal{I} \right] = \frac{1}{m} \int_{K-\Theta}^{\infty} e^{\mu T} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) d\mu .
$$

Recall

$$
\begin{aligned}
e^{\mu T} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) &= e^{\mu T} \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2}{\sigma_\mu^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\left(\mu^2 - 2\mu\overline{\mu} + \overline{\mu}^2\right) - 2\mu T \sigma_\mu^2}{\sigma_\mu^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\mu^2 - 2\mu\left(\overline{\mu} + T\sigma_\mu^2\right) + \overline{\mu}^2 + \left(\overline{\mu} + T\sigma_\mu^2\right)^2 - \left(\overline{\mu} + T\sigma_\mu^2\right)^2}{\sigma_\mu^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\left(\mu - \left(\overline{\mu} + T\sigma_\mu^2\right)\right)^2}{\sigma_\mu^2}} e^{\frac{1}{2}\left(T^2\sigma_\mu^2 + 2\overline{\mu}T\right)} .
\end{aligned}
$$

Therefore

$$
\begin{aligned}
H &= \frac{1}{m} \int_{K-\Theta}^{\infty} e^{\mu T} \phi\left(\mu; \overline{\mu}, \sigma_\mu^2\right) d\mu \\
&= \frac{1}{m} e^{\frac{1}{2}\left(T^2\sigma_\mu^2 + 2\overline{\mu}T\right)} \int_{K-\Theta}^{\infty} \phi\left(\mu; \left(\overline{\mu} + T\sigma_\mu^2\right), \sigma_\mu^2\right) d\mu \\
&= \frac{1}{m} e^{\frac{1}{2}\left(T^2\sigma_\mu^2 + 2\overline{\mu}T\right)} \left(1 - \Phi\left(K - \Theta; \left(\overline{\mu} + T\sigma_\mu^2\right), \sigma_\mu^2\right)\right) .
\end{aligned}
$$

Note that we can write

$$
\begin{aligned}
K &= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{m}{1-m}\right)+\log\left(H\right)\right] \\
&= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{m}{1-m}\right)+\log\left(\frac{1}{m}e^{\frac{1}{2}\left(T^2\sigma_\mu^2+2\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)\right)\right] \\
&= \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}\right)+\frac{1}{2}\left(T^2\sigma_\mu^2+2\overline{\mu}T\right)\right] .
\end{aligned}
$$

We now show that the solution for K is unique. Indeed, the determination of K depends on a fixed point:

$$
K=\overline{\mu}+\frac{1}{2}\sigma_\mu^2T+\frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+f\left(K;\Theta\right) \tag{C7}
$$

with $f\left(K;\Theta\right)=\frac{1}{T}\log\left(\frac{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}\right)$. Note that $f\left(K;\Theta\right)$ is monotonically decreasing in K with

$$
\lim_{K\to\infty}f\left(K\right)=-\infty;\ \lim_{K\to-\infty}f\left(K\right)=\infty .
$$

Because the left-hand side of (C7) is increasing in K, the equation admits only one solution.

The remaining points of the corollary stem from the definitions in the proofs above.

Q.E.D.

Proof of Proposition C3: The right-hand side of (C7) is increasing in τ. Suppose that we found the fixed point K and now we increase τ. The right-hand side increases. In order for the equality (C7) be re-established, K must increase, thereby establishing the claim of the corollary. Q.E.D.

Proof of Proposition C4: Note first that Θ is decreasing in θ. In fact

$$
\frac{d\Theta}{d\theta}=\frac{1}{T}\frac{\left(\int_{-\infty}^{\infty}\left(\theta\left(e^{-\frac{1}{2}\sigma_1^2T+\sigma_1\varepsilon}-1\right)+1\right)^{-\gamma}\left(e^{-\frac{1}{2}\sigma_1^2T+\sigma_1\varepsilon}-1\right)\phi\left(\varepsilon;0,T\right)d\varepsilon\right)}{\left(\int_{-\infty}^{\infty}\left(\theta e^{-\frac{1}{2}\sigma_1^2T+\sigma_1\varepsilon}+\left(1-\theta\right)\right)^{1-\gamma}\phi\left(\varepsilon;0,T\right)d\varepsilon\right)}<0 .
$$

The inequality stems from the following argument: The numerator is like $E\left[f\left(X\right)X\right]$ where $X=\left(e^{-\frac{1}{2}\sigma_1^2T+\sigma_1\varepsilon}-1\right)$ and $f'\left(X\right)<0$. That is, $E\left[f\left(X\right)X\right]=cov\left(f\left(X\right),X\right)+E\left[f\left(X\right)\right]E\left[X\right]$. Note that E$[X]=0$, and given the negative covariance, $E\left[f\left(X\right),X\right]<0$.

In addition, we note that

$$
F\left(K,\Theta,\tau\right)=K-\overline{\mu}-\frac{1}{2}\sigma_\mu^2T-\frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}\right)\right] .
$$

The fixed point condition is

$$F(K,\Theta,\tau)=0\ .$$

From the Implicit Function theorem:

$$\frac{dK}{d\Theta}=\frac{F_\Theta}{F_K}\ .$$

We have

$$\begin{aligned}
F_\Theta &= -\frac{1}{T}\left[\frac{\phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}+\frac{\phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}\right]<0\\
F_K &= 1+\frac{1}{T}\left[\frac{\phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}+\frac{\phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}\right]\ .
\end{aligned}$$

Let

$$x=\frac{1}{T}\left[\frac{\phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}{\Phi\left(K-\Theta;\overline{\mu},\sigma_\mu^2\right)}+\frac{\phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}{1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)}\right]$$

then we have

$$\frac{dK}{d\Theta}=-\frac{x}{1+x}\in[-1,0]\ .$$

Consider now the threshold

$$\mu>G(K,\Theta)=K-\Theta\ .$$

It follows that

$$\frac{dG}{d\Theta}=\frac{dK}{d\Theta}-1<0\ .$$

That is, if Θ increases, then G decreases, and the threshold becomes less tight. Vice versa, if Θ decreases (e.g., if θ increases) the threshold becomes tigher. Formally, the chain rule implies

$$\frac{dG}{d\theta}=\frac{dG}{d\Theta}\frac{d\Theta}{d\theta}>0\ .$$

Q.E.D.

Proof of Proposition C5. From the proof of Corollary 1, the computation of H gives

$$E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]=\frac{1}{m}e^{\frac{1}{2}\left(T^2\sigma_\mu^2+2\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)\ .$$

Similar steps give the result for $E^{\mathcal{I}}\left[e^{2\mu_j T}|j\in\mathcal{I}\right]$:

$$E^{\mathcal{I}}\left[e^{2\mu_j T}|j\in\mathcal{I}\right]=\frac{1}{m}\int_{K-\Theta}^{\infty}e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu\ .$$

Recall

$$e^{2\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right) = e^{2\mu T}\frac{1}{\sqrt{2\pi\sigma_\mu^2}}e^{-\frac{1}{2}\frac{(\mu-\overline{\mu})^2}{\sigma_\mu^2}}$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\left(\mu^2-2\mu\overline{\mu}+\overline{\mu}^2\right)-4\mu T\sigma_\mu^2}{\sigma_\mu^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\mu^2-2\mu\left(\overline{\mu}+2T\sigma_\mu^2\right)+\overline{\mu}^2+\left(\overline{\mu}+2T\sigma_\mu^2\right)^2-\left(\overline{\mu}+2T\sigma_\mu^2\right)^2}{\sigma_\mu^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma_\mu^2}} e^{-\frac{1}{2}\frac{\left(\mu-\left(\overline{\mu}+2T\sigma_\mu^2\right)\right)^2}{\sigma_\mu^2}} e^{\frac{1}{2}\left(4T^2\sigma_\mu^2+4\overline{\mu}T\right)} .
\end{aligned}
$$

Therefore

$$
\begin{aligned}
E^{\mathcal{I}}\left[e^{2\mu_j T}|i\in\mathcal{I}\right] &= \frac{1}{m}\int_{K-\Theta}^{\infty} e^{\mu T}\phi\left(\mu;\overline{\mu},\sigma_\mu^2\right)d\mu \\
&= \frac{1}{m}e^{\frac{1}{2}\left(4T^2\sigma_\mu^2+4\overline{\mu}T\right)}\int_{K-\Theta}^{\infty}\phi\left(\mu;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)d\mu \\
&= \frac{1}{m}e^{\frac{1}{2}\left(4T^2\sigma_\mu^2+4\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)\right) .
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\frac{E^{\mathcal{I}}\left[e^{2\mu_j T}|j\in\mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2} &= \frac{\frac{1}{m(\mathcal{I})}e^{\frac{1}{2}\left(4T^2\sigma_\mu^2+4\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)}{\left[\frac{1}{m(\mathcal{I})}e^{\frac{1}{2}\left(T^2\sigma_\mu^2+2\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)\right]^2} \\
&= m\left(\mathcal{I}\right)\frac{e^{\frac{1}{2}\left(4T^2\sigma_\mu^2+4\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)}{e^{\frac{1}{2}\left(2T^2\sigma_\mu^2+4\overline{\mu}T\right)}\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)^2} \\
&= m\left(\mathcal{I}\right)e^{T^2\sigma_\mu^2}\frac{\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+2T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)}{\left(1-\Phi\left(K-\Theta;\left(\overline{\mu}+T\sigma_\mu^2\right),\sigma_\mu^2\right)\right)^2} .
\end{aligned}
$$

Finally, recalling that with constant γ, $\alpha=1$, the last term in $E[s_{i,T}^2]$ is

$$
\begin{aligned}
& E^{\mathcal{I}}\left[\left(\frac{1+\left(\theta R^j+(1-\theta)R^{Mkt}\right)}{1+R^{Mkt}}\right)^2|j\in\mathcal{I}\right] \\
&= E^{\mathcal{I}}\left[\left(\frac{1+R^{Mkt}+\theta\left(R^j-R^{Mkt}\right)}{1+R^{Mkt}}\right)^2|j\in\mathcal{I}\right] \\
&= E^{\mathcal{I}}\left[\left(1+\frac{\theta\left(R^j-R^{Mkt}\right)}{1+R^{Mkt}}\right)^2|j\in\mathcal{I}\right] \\
&= E^{\mathcal{I}}\left[\left(1+\frac{\theta\left(e^{-\frac{1}{2}\sigma_1^2T-\frac{1}{2}\sigma^2T+\sigma_1W_{jT}+\sigma W_T}-e^{-\frac{1}{2}\sigma^2T+\sigma W_T}\right)}{e^{-\frac{1}{2}\sigma^2T+\sigma W_T}}\right)^2|j\in\mathcal{I}\right] ,
\end{aligned}
$$

where we used

$$R^j = \frac{e^{-\frac{1}{2}\sigma_1^2 T - \frac{1}{2}\sigma^2 T + \sigma_1 W_{jT} + \sigma W_T}}{Z} - 1$$
$$R^{Mkt} = \frac{e^{-\frac{1}{2}\sigma^2 T + \sigma W_T}}{Z} - 1 \, .$$

We therefore obtain

$$\begin{aligned}
& E^{\mathcal{I}}\left[\left(\frac{1+\left(\theta R^j + (1-\theta) R^{Mkt}\right)}{1+R^{Mkt}}\right)^2 |j \in \mathcal{I}\right] \\
=\ & E^{\mathcal{I}}\left[\left(1+\theta\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{jT}} - 1\right)\right)^2 |j \in \mathcal{I}\right] \\
=\ & E^{\mathcal{I}}\left[1+\theta^2\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{jT}} - 1\right)^2 + 2\theta\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{jT}} - 1\right) |j \in \mathcal{I}\right] \\
=\ & E^{\mathcal{I}}\left[1+\theta^2\left(e^{-\frac{1}{2}2\sigma_1^2 T + 2\sigma_1 W_{jT}} + 1 - 2e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{jT}}\right) |j \in \mathcal{I}\right] \\
=\ & E^{\mathcal{I}}\left[1+\theta^2\left(e^{-\frac{1}{2}2\sigma_1^2 T + \frac{1}{2}4\sigma_1^2 T} + 1\right)\right] \\
=\ & 1+\theta^2\left(e^{\sigma_1^2 T} - 1\right) \, .
\end{aligned}$$

Q.E.D.

Proof of Proposition C6. We want to compute the cumulative distribution of $s_{i,T} = C_{i,T}/\overline{C}$ for entrepreneurs, that is, conditional on $\mu^i \geq K - \Theta$,

$$F\left(s_{i,T}|\mu_i \geq K - \Theta\right) \, .$$

Recall that for entrepreneurs, we have

$$\begin{aligned}
s_{i,T} &= \frac{(1-\tau) B_0 e^{\mu_i T}\left[\left(\theta\left(e^{-\frac{1}{2}\sigma^2 T - \frac{1}{2}\sigma_1^2 T + \sigma W_T + \sigma_1 W_{i,T}} - Z\right) + (1-\theta)\left(e^{-\frac{1}{2}\sigma^2 T + \sigma W_T} - Z\right) + Z\right)\right]}{B_0 e^{-\frac{1}{2}\sigma^2 T + \sigma W_T} E\left[e^{\mu_j T}|j \in \mathcal{I}\right] m(\mathcal{I})} \\
&= \frac{(1-\tau) e^{\mu_i T}\left[\theta\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}}\right) + (1-\theta)\right]}{Hm(\mathcal{I})} \, .
\end{aligned}$$

Therefore

$$\begin{aligned}
& F\left(s_{i,T}|\mu_i > K - \Theta\right) \\
=\ & \Pr\left(\widetilde{s}_{i,T} < s_{i,T}|\mu_i > K - \Theta\right) \\
=\ & \Pr\left(\frac{(1-\tau) e^{\mu_i T}\left[\theta\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}}\right) + (1-\theta)\right]}{Hm(\mathcal{I})} < s_{i,T}|\mu_i > K - \Theta\right) \\
=\ & \frac{\Pr\left(\frac{(1-\tau) e^{\mu_i T}\left[\theta\left(e^{-\frac{1}{2}\sigma_1^2 T + \sigma_1 W_{i,T}}\right) + (1-\theta)\right]}{Hm(\mathcal{I})} < s_{i,T} \,\&\, \mu_i > K - \Theta\right)}{\Pr\left(\mu_i > K - \Theta\right)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\Pr\left(\sigma_1 W_{i,T} < \log\left(s_{i,T}\frac{Hm(\mathcal{I})}{\theta(1-\tau)e^{\mu_i T}} - \frac{(1-\theta)}{\theta}\right) + \frac{1}{2}\sigma_1^2 T \ \&\ \mu_i > K - \Theta\right)}{\Pr\left(\mu_i > K - \Theta\right)} \\
&= \frac{\int_{K-\Theta}^{\infty} \Pr\left(\sigma_1 W_{i,T} < \log\left(s_{i,T}\frac{Hm(\mathcal{I})}{(1-\tau)e^{\mu T}} - (1-\theta)\right) - \log(\theta) + \frac{1}{2}\sigma_1^2 T | \mu\right) \phi\left(\mu, \overline{\mu}, \sigma_\mu^2\right) d\mu}{\Pr\left(\mu_i > K - \Theta\right)} \\
&= \frac{1}{m(\mathcal{I})} \int_{K-\Theta}^{\infty} \Phi\left(\log\left(s_{i,T}\frac{Hm(\mathcal{I})}{(1-\tau)e^{\mu T}} - (1-\theta)\right) - \log(\theta) + \frac{1}{2}\sigma_1^2 T; 0, \sigma_1^2 T\right) \phi\left(\mu, \overline{\mu}, \sigma_\mu^2\right) d\mu\ .
\end{aligned}
$$

Q.E.D.

Section D. Special case: No systematic risk ($\varepsilon_T = 0$).

Section D.1. Theoretical Results.

The case without any systematic risk considerably simplifies the formulas. The next corollary collects the results that obtain by setting systematic risk to zero.

Corollary D1: If $\varepsilon_T = 0$, then:

(a) The state price density at T is constant: $\pi_T =$ constant;

(b) The value of the security paying $\varepsilon_T = 0$ at T is $Z = 1$ and thus $r = 0$.

(c) The aggregate stock market becomes risk-free with return equal to the risk-free rate and hence equal to zero:

$$R^{Mkt} = 0;$$

(d) Agents' investments in the market are $\alpha(\gamma_i) = 1$ independently of risk aversion. Likewise, investors' investment in bonds is zero.

(e) The market price of individual stocks is

$$\frac{M_{i,0}}{B_{i,0}} = (1-\tau)\, e^{\mu_i\, T}$$

(f) Assuming $B_{i,0} = B_{j,0} = B_0$ for all i, j, and renormalizing by the total investment $B^P = \int_{\mathcal{I}} B_0 di = B_0 m(\mathcal{I})$, we have

$$\frac{M^P}{B^P} = \frac{1}{m(\mathcal{I})\, B_0} \int M_{j,0} dj = (1-\tau)\, E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I} \right] .$$

(g) Entrepreneur i's consumption $C^{yes}_{i,T}$ and pensioner j' consumption $C^{no}_{j,t}$ at T are given, respectively, by:

$$\begin{aligned} C^{yes}_{i,T} &= M_{i,0}\left(\theta e^{\varepsilon_{i,T}} + 1\right) \\ C^{no}_{j,T} &= \tau B_0 E^{\mathcal{I}}\left[e^{\mu_i T} | i \in \mathcal{I} \right] \frac{m(\mathcal{I})}{1 - m(\mathcal{I})} \end{aligned}$$

(h) Each agent i becomes an entrepreneur at time 0 if and only if

$$\begin{aligned} \mu_i &> \frac{1}{T}\left[\log\left(\frac{\tau}{1-\tau}\right) + \log\left(\frac{m(\mathcal{I})}{1-m(\mathcal{I})}\right) + \log\left(E^{\mathcal{I}}\left[e^{\mu_j T} | j \in \mathcal{I}\right]\right)\right] \\ &\quad - \frac{1}{T}\frac{1}{(1-\gamma_i)}\left[\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}} - 1\right) + 1\right)^{1-\gamma_i}\right]\right)\right] . \end{aligned}$$

(i) Income inequality is

$$\text{Var}\left(s_{i,T}\right)=\frac{\tau^2}{1-m\left(\mathcal{I}\right)}+\frac{\left(1-\tau^2\right)}{m\left(\mathcal{I}\right)}\frac{E^{\mathcal{I}}\left[e^{2\mu_j T}|j\in\mathcal{I}\right]}{E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]^2}E^{\mathcal{I}}\left[\left(1+\theta e^{\varepsilon_{j,T}}\right)^2|j\in\mathcal{I}\right]-1$$

Point (a) shows that without systematic risk, the state price density becomes constant, which is intuitive as there is no systematic risk to price. Therefore, a security that pays the "systematic risk" has unitary price, which equals its payoff is maturity. Important, point (b) shows that the aggregate market portfolio fully diversifies away the idiosyncratic risk of individual stocks and thus becomes itself risk free. It follows that the market portfolio and the risk free bond are the same security when $\varepsilon_T=0$. Because in general the level of risk aversion γ_i impacts the amount of systematic risk that each entrepreneur is willing to take through the investment $\alpha(\gamma_i)$ in the market portfolio, the lack of systematic risk implies $\alpha(\gamma_i)=\alpha(\gamma_j)=\alpha$. Moreover, market clearing requires $\alpha=1$ which implies $(1-\alpha)=0$ and therefore agents do not invest in risk free bonds.

Results (e) and (f) on market prices stem directly from $r=0$ in point (a), and the consumption of entrepreneur i in point (g) immediately follows from the general result when $\alpha=1$. Similarly, selection (h) also follows immediately from point (a). Finally, the formula for income inequality also specializes from its more general formula.

Proposition D1. Let μ and γ be independent with cumulative densities $F_\mu(\mu)$ and $F_\gamma(\gamma)$. Then the equilibrium exists and it is unique.

Proposition D2. (a) The mass of entrepreneurs decreases with the tax rate θ:

$$\frac{dm}{d\tau}<0$$

(b) The mass of entrepreneurs decreases as their diversification opportunities decrease, that is, θ increases:

$$\frac{dm}{d\theta}<0$$

Additional results for the case $\varepsilon_T=0$ are provided in Corollary A2 and Corollary A3.

D.2. Proofs for Appendix D.

Proof of Proposition D1. The condition is to become an entrepreneur is:

$$\begin{aligned}\mu_i \quad > \quad & \frac{1}{T}\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{m}{1-m}\right)+\log\left(E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]\right)\\ & -\frac{1}{T}\frac{1}{1-\gamma_i}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]\right)\end{aligned}$$

Define for convenience

$$U\left(\gamma_i\right)=\frac{1}{1-\gamma_i}\log\left(E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]\right) \tag{D1}$$

and rewrite the selection condition equivalently as

$$\mu_i \;>\; \frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{\widetilde{H}}{1-m}\right)-U(\gamma_i)\right)$$

where

$$\widetilde{H}=\int_{\mathcal{I}} e^{\mu_j T}dj = m\; E^{\mathcal{I}}\left[e^{\mu_j T}|j\in\mathcal{I}\right]$$

The mass of agents who become entrepreneurs is then

$$m \;=\; \int_{\underline{\gamma}}^{\overline{\gamma}}\int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{\widetilde{H}}{1-m}\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} dF_{\mu}\left(\mu\right)dF_{\gamma}\left(\gamma\right) \tag{D2}$$

where $\underline{\gamma}>0$, $\overline{\gamma}$, and $\overline{\mu}$ define the domain of the cdfs, with $\overline{\gamma}$ and $\overline{\mu}$ potentially infinite. In addition, $\widetilde{H}$ now becomes

$$\widetilde{H} \;=\; \int_{\underline{\gamma}}^{\overline{\gamma}}\int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\frac{\widetilde{H}}{1-m}\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} e^{\mu T}dF_{\mu}\left(\mu\right)dF_{\gamma}\left(\gamma\right) \tag{D3}$$

These are two equations in two unknowns. We can simplify the system by defining

$$\widetilde{\widetilde{H}}=\frac{\widetilde{H}}{1-m}$$

Substituting $\widetilde{H}$ and m in the definition of $\widetilde{\widetilde{H}}$ we obtain:

$$\widetilde{\widetilde{H}}=G\left(\widetilde{\widetilde{H}}\right)\equiv\frac{\int_{\underline{\gamma}}^{\overline{\gamma}}\int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\widetilde{\widetilde{H}}\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} e^{\mu T}dF_{\mu}\left(\mu\right)dF_{\gamma}\left(\gamma\right)}{1-\int_{\underline{\gamma}}^{\overline{\gamma}}\int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\widetilde{\widetilde{H}}\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} dF_{\mu}\left(\mu\right)dF_{\gamma}\left(\gamma\right)}$$

This is one equation is one unknown ($\widetilde{\widetilde{H}}$). It is easy to see that $G\left(\widetilde{\widetilde{H}}\right)$ is monotonically decreasing (as $\widetilde{\widetilde{H}}$ increases, the inner integrals at numerator and denominator become smaller). Moreover:

If $\widetilde{\widetilde{H}}\to 0$, then $\log\left(\widetilde{\widetilde{H}}\right)\to-\infty$ and hence $G\left(\widetilde{\widetilde{H}}\right)\to+\infty$

If $\widetilde{\widetilde{H}}\to \infty$, then $\log\left(\widetilde{\widetilde{H}}\right)\to+\infty$ and hence $G\left(\widetilde{\widetilde{H}}\right)\to 0$

Therefore, there must exist a unique one $\widetilde{\widetilde{H}}^*$ such that

$$\widetilde{\widetilde{H}}^*=G\left(\widetilde{\widetilde{H}}^*\right)$$

This proves the existence and uniqueness of the equilibrium. Moreover, for this $\widetilde{\widetilde{H}}^*$, we then have the unique values:

$$
\begin{aligned}
m^* &= \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\widetilde{\widetilde{H}}^*\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} dF_\mu(\mu)\, dF_\gamma(\gamma) \\
\widetilde{H}^* &= \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\frac{1}{T}\left(\log\left(\frac{\tau}{1-\tau}\right)+\log\left(\widetilde{\widetilde{H}}^*\right)\right)-\frac{1}{T}U(\gamma)}^{\overline{\mu}} e^{\mu T} dF_\mu(\mu)\, dF_\gamma(\gamma)
\end{aligned}
$$

Q.E.D.

Proof of Proposition D2: (a) Consider again m in equation (D2) and $\widetilde{H}$ in equation (D3). Define

$$
K(\gamma; m, \widetilde{H}) = \frac{1}{T}\left(\log\left(\frac{1}{1-m}\right) + \log\left(\frac{\tau}{1-\tau}\right) + \log\left(\widetilde{H}\right)\right) - \frac{1}{T}U(\gamma) \tag{D4}
$$

and rewrite

$$
\begin{aligned}
m &= \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{K(\gamma; m, \widetilde{H})}^{\overline{\mu}} dF_\mu(\mu)\, dF_\gamma(\gamma) \\
\widetilde{H} &= \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{K(\gamma; m, \widetilde{H})}^{\overline{\mu}} e^{\mu T} dF_\mu(\mu)\, dF_\gamma(\gamma)
\end{aligned}
$$

Clearly, the partial derivatives have $\partial \widetilde{H}/\partial K \leq 0$ and $\partial m/\partial K \leq 0$. In addition, we have $\partial K/dm > 0$ and $\partial K/d\widetilde{H} > 0$ and $\partial K/\partial \tau > 0$. The same proof of Proposition B4 then applies, yielding the result.

(b) In addition to the partial derivatives in point (a), from (D1) we also have

$$
\frac{\partial U(\gamma)}{\partial \theta} = \frac{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{-\gamma_i}\left(e^{\varepsilon_{i,T}}-1\right)\right]}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]} = \frac{cov\left(\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{-\gamma_i}, \left(e^{\varepsilon_{i,T}}-1\right)\right)}{E\left[\left(\theta\left(e^{\varepsilon_{i,T}}-1\right)+1\right)^{1-\gamma_i}\right]} < 0
$$

Therefore

$$
\frac{\partial K}{\partial \theta} = -\frac{1}{T}\frac{\partial U(\gamma)}{\partial \theta} > 0
$$

The proof is then the same as in Proposition B4, except that we use $\frac{\partial K}{\partial \theta}$ instead of $\frac{\partial K}{\partial \tau}$ everywhere.

Q.E.D

E. Additional Data Information

Total returns on stock market indices come from Global Financial Data (GFD). Table E1 lists those indices country by country. While there are 34 OECD countries, the table contains 33 countries; for Slovenia, we could not find any return data in GFD.

Table E1
Stock market indices

This table lists the country-level stock market indices whose returns we use in our empirical analysis.

Country	Stock Index
Australia	ASX All-Ordinaries
Austria	Vienna SE ATX
Belgium	Brussels All Share Index
Canada	Canada S&P TSX 300 Index
Chile	IPSA Index
Czech Republic	Prague SE PX Index
Denmark	OMX Copenhagen All-Share Gross Index
Estonia	OMX Talinn SE
Finland	OMX Helsinki All Share Index
France	CAC 40 Index
Germany	CDAX
Greece	ASE General Index
Hungary	Budapest Stock Exchange Index
Iceland	OMX Iceland All Share Gross Index
Ireland	Ireland ISEQ Index
Israel	Tel Aviv SE Index
Italy	FTSE MIB Index
Japan	TOPIX
Korea	KOSPI
Luxembourg	Luxembourg LuxX Index
Mexico	Mexico SE Mexbol Index
Netherlands	Netherlands Stock Index
Norway	Oslo SE OBX-25 Index
New Zealand	New Zealand SE Gross Share Index
Poland	Warsaw SE General Index
Portugal	Lisbon BVL General Index
Slovak Republic	SAX
Spain	Barcelona SE-30 Index
Sweden	OMX Stockholm Benchmark Gross Index
Switzerland	Swiss Performance Index
Turkey	Turkey ISE-100 Index
UK	UK FTSE All Share Index
United States	S&P 500 Index

F. Additional Empirical Results

In this section, we report additional empirical results that are only verbally summarized in the paper. Here is the full list of tables and figures reported in this section:

- **Figure F1: Alternative measure of inequality: Gini coefficient**
  - Corresponds to Figure 6 in the paper; the Gini coefficient replacing the top 10% share
- **Figure F2: Alternative measures of average stock market return**
  - Corresponds to Figure 6 in the paper; nominal U.S. dollar and local currency returns
- **Figures F3 through F7: Year-by-year cross-sectional slope estimates**
  - Plots of year-by-year estimates of slope coefficients from cross-sectional regressions
- **Tables F1 and F2: Alternative measure of inequality: Gini coefficient**
  - Corresponds to Tables 1 and 2 in the paper; Gini coefficient replacing top 10% share
- **Table F3: Alternative measures of average stock market return**
  - Corresponds to Table 1 in the paper; nominal U.S. dollar and local currency returns
- **Tables F4 through F11: Panel regressions with time fixed effects**
  - Corresponds to Tables 1, 2, and F1 through F3

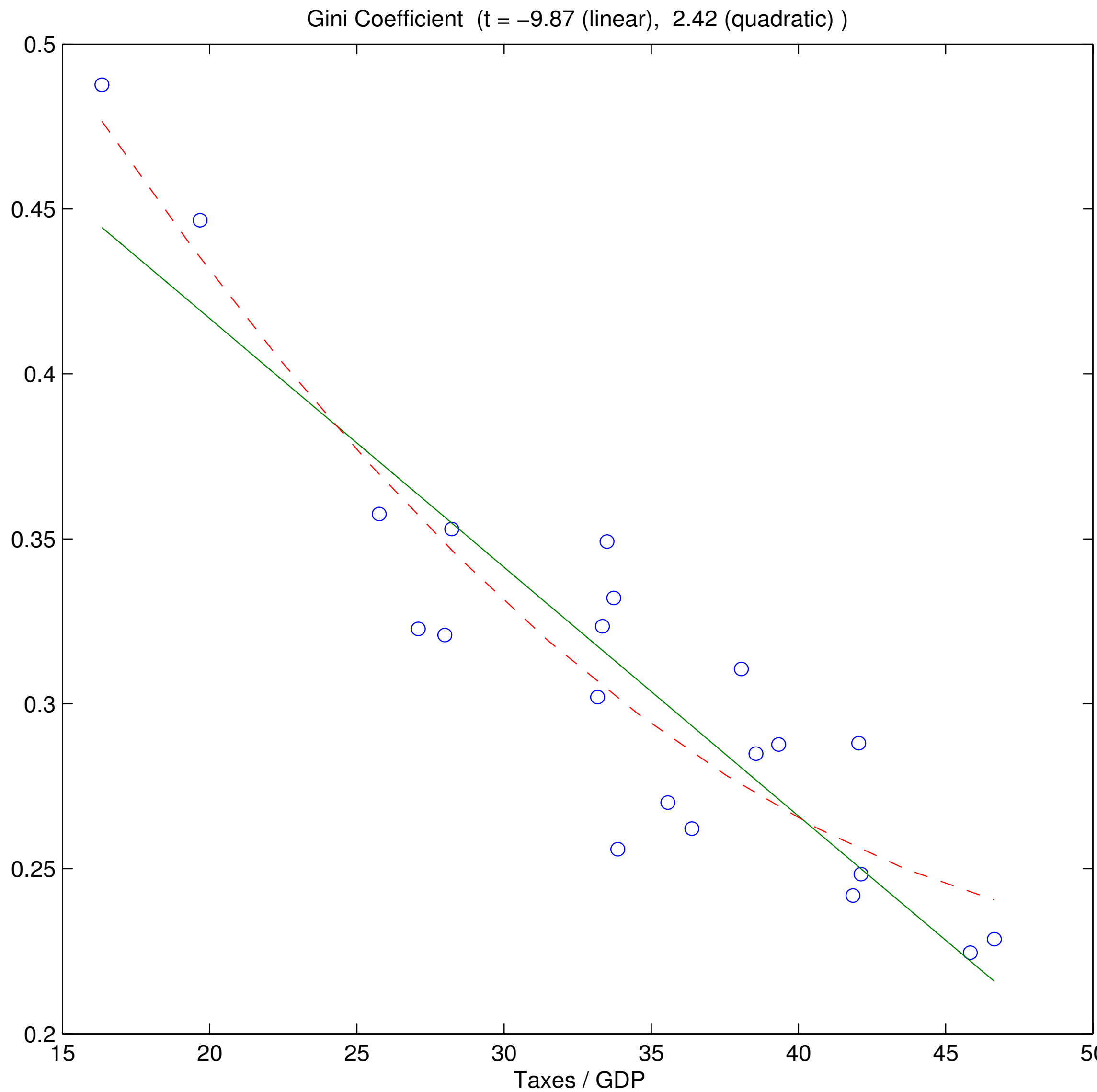

Figure F1. Income inequality vs. taxes. This figure corresponds to Figure 6 in the paper, with the Gini coefficient replacing the top 10% share.

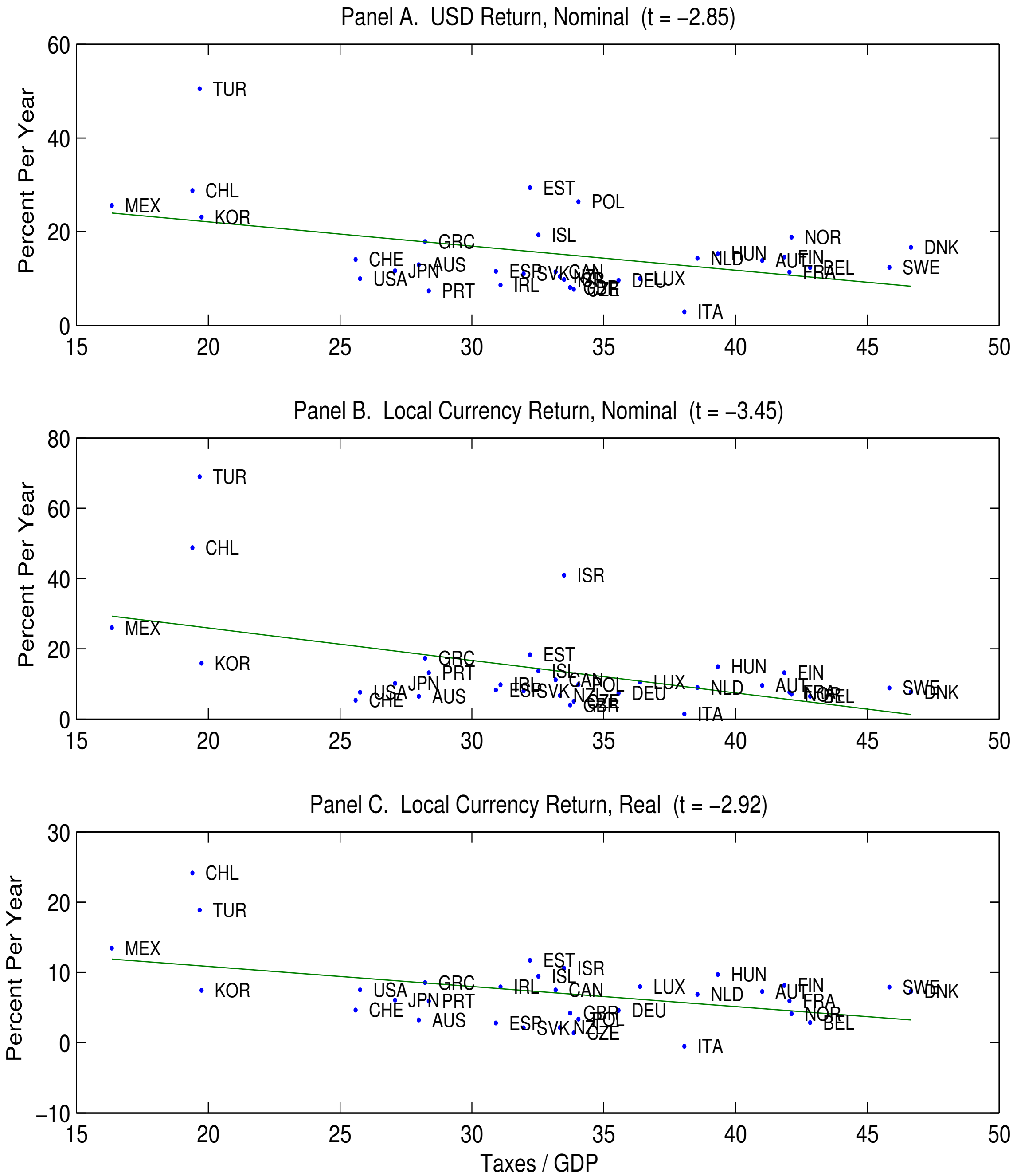

Figure F2. Average stock market return vs. taxes. This figure corresponds to Figure 6 in the paper, with three different ways of calculating average stock market returns. Only Panel C is reported in the paper.

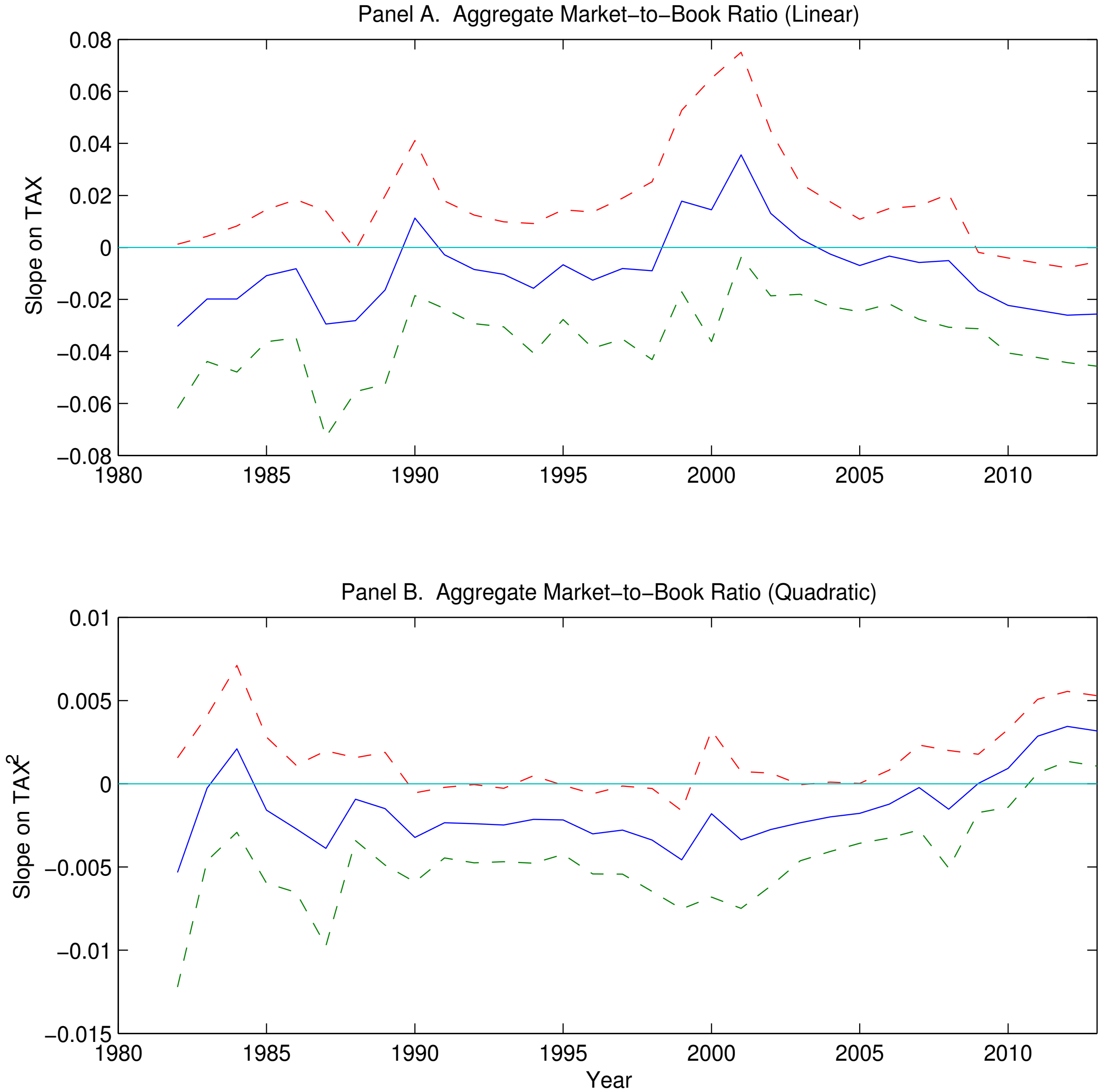

Figure F3. Stock price level vs. taxes: Time series of cross-sectional slopes. For a given year, the solid line in Panel A plots the estimated slope coefficient from the cross-country regression of the country's aggregate market-to-book ratio (M/B) at the beginning of that year on the country's tax-to-GDP ratio (TAX) in the same year. In Panel B, the line plots the slope on TAX^2 from the regression of M/B on TAX and TAX^2. The dashed lines plot the corresponding 95% confidence intervals. To run the cross-country regression in any given year, we require at least 10 country-level observations; to compute the country's M/B in any given year, we require at least 10 firm-level observations in that country. The first valid cross-country slope appears in 1982.

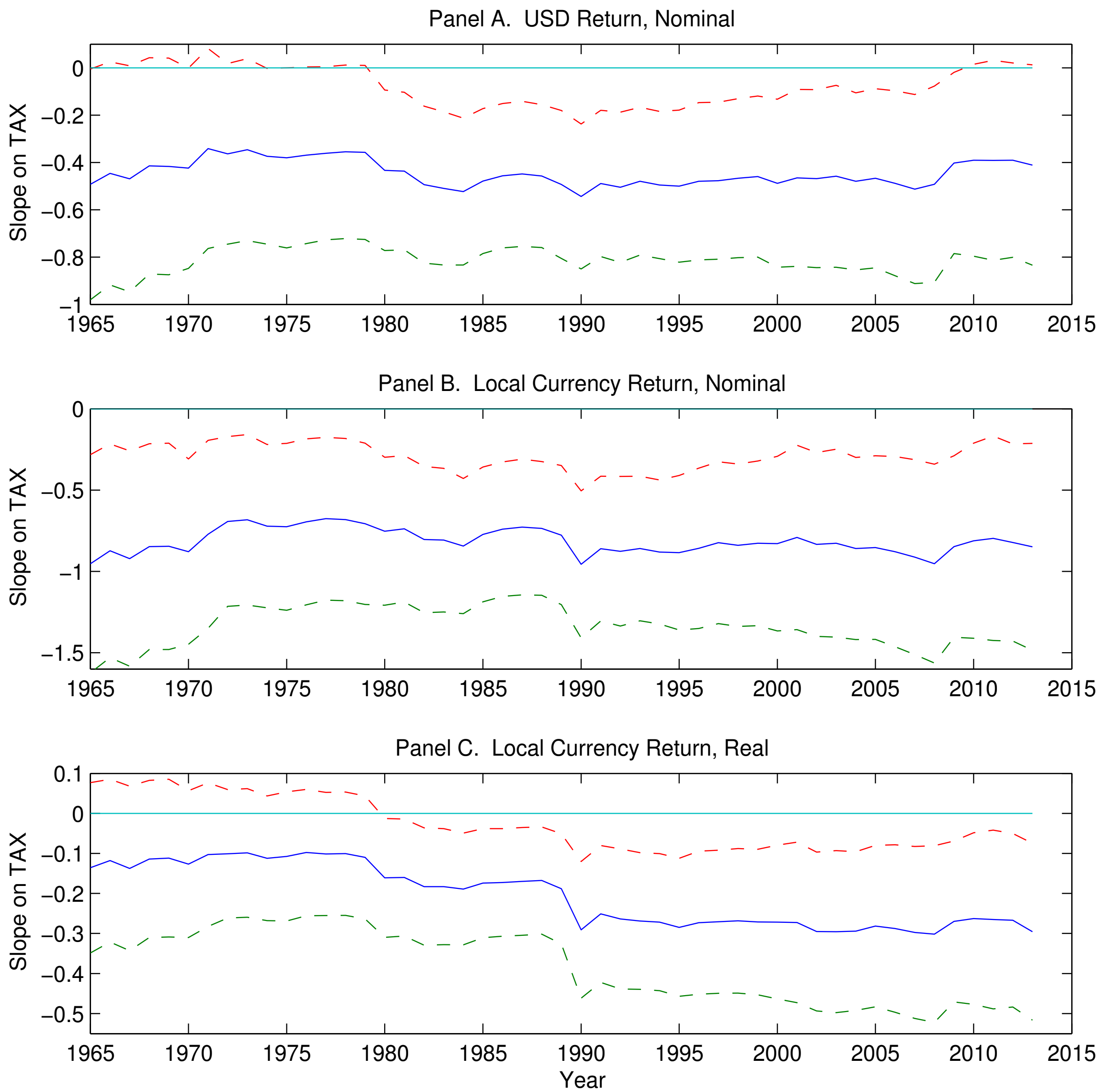

Figure F4. Average stock market return vs. taxes: Time series of cross-sectional slopes. For a given year, the solid line plots the estimated slope coefficient from the cross-country regression of the country's average stock market return on the country's tax-to-GDP ratio in that year. The three panels plot returns calculated in different ways, as explained in the panel titles. The dashed lines plot the corresponding 95% confidence intervals. To run the cross-country regression in any given year, we require at least 10 country-level observations. The first valid cross-country slope appears in 1965.

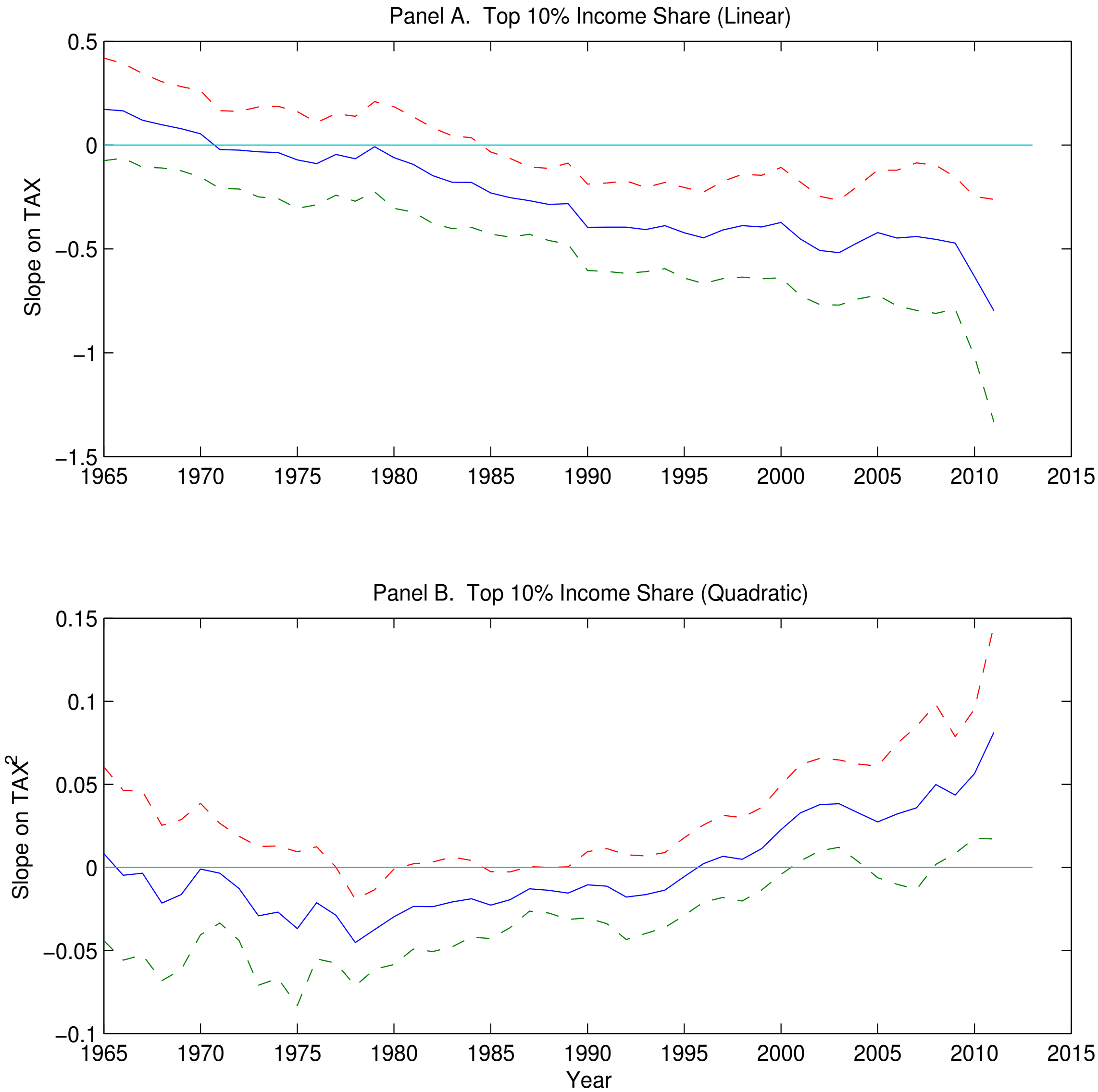

Figure F5. Income inequality (top 10%) vs. taxes: Time series of cross-sectional slopes. For a given year, the solid line in Panel A plots the estimated slope coefficient from the cross-country regression of the country's top 10% income share (TOP) in that year on the country's tax-to-GDP ratio (TAX) in the same year. In Panel B, the line plots the slope on TAX^2 from the regression of TOP on TAX and TAX^2. The dashed lines plot the corresponding 95% confidence intervals. To run the cross-country regression in any given year, we require at least 10 country-level observations. The first valid cross-country slope appears in 1965.

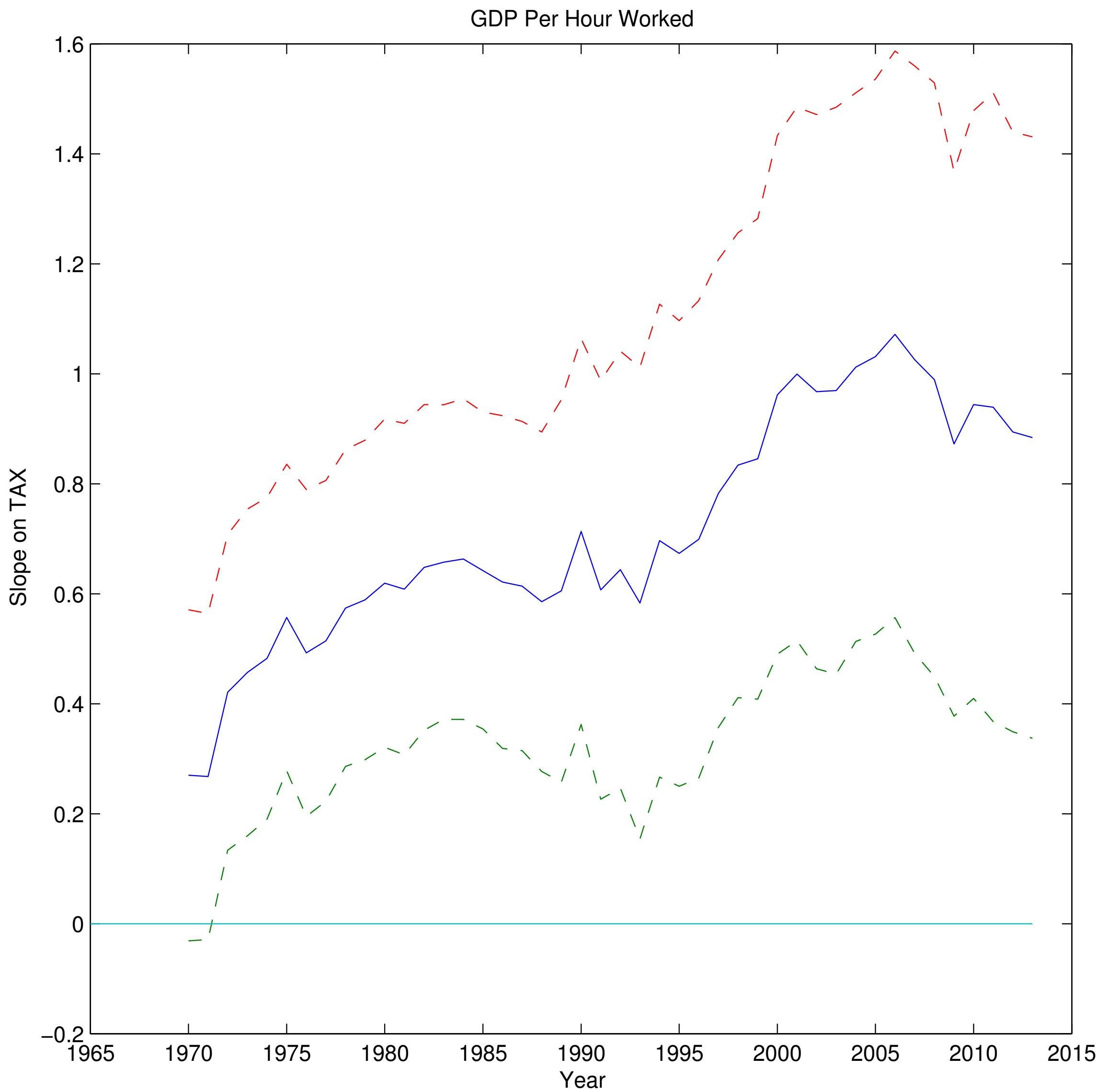

Figure F6. Productivity vs. taxes: Time series of cross-sectional slopes. For a given year, the solid line plots the estimated slope coefficient from the cross-country regression of the country's GDP per hour worked in that year on the country's tax-to-GDP ratio in the same year. The dashed lines plot the corresponding 95% confidence intervals. To run the cross-country regression in any given year, we require at least 10 country-level observations. The first valid cross-country slope appears in 1970.

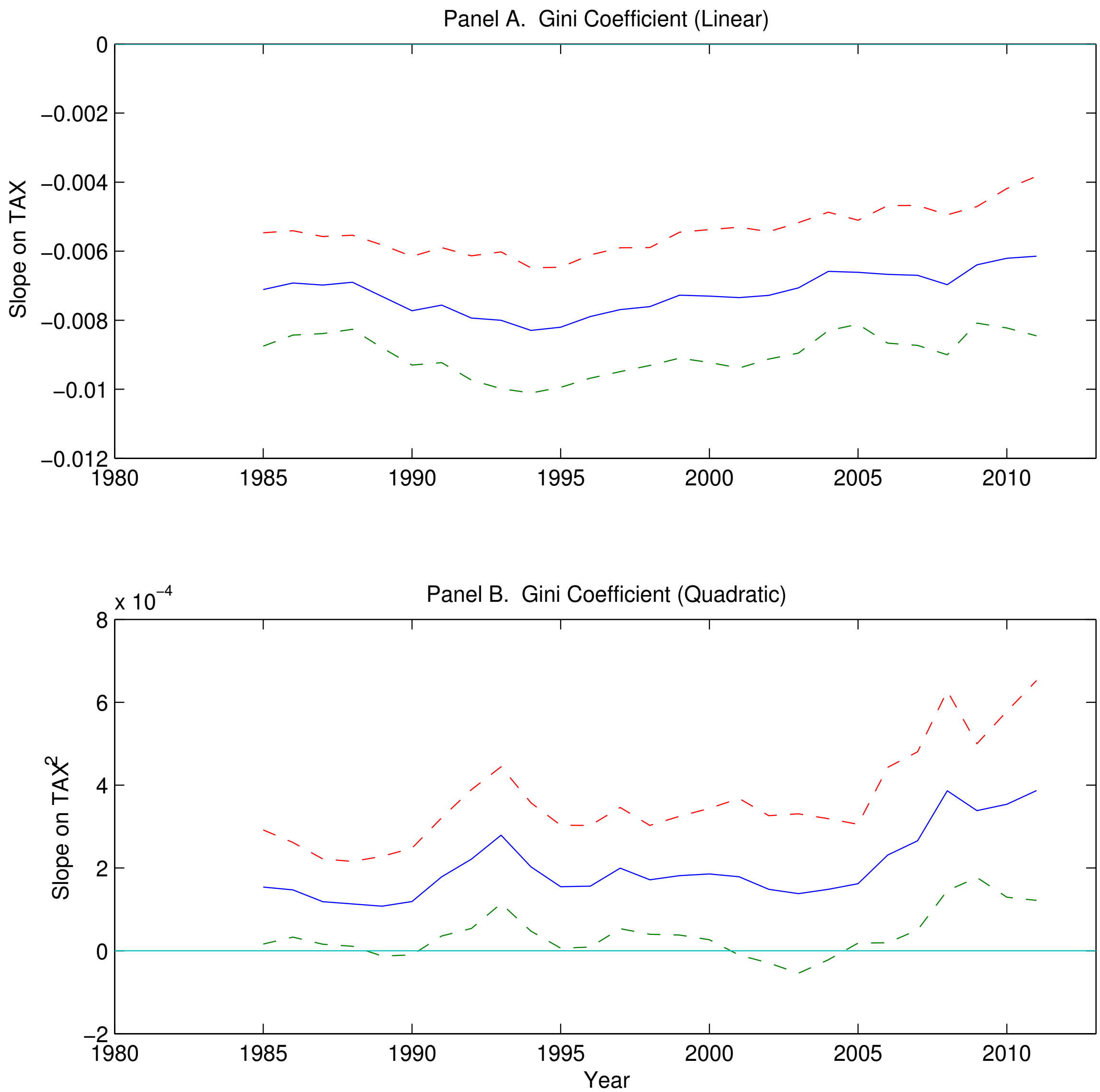

Figure F7. Income inequality (Gini coefficient) vs. taxes: Time series of cross-sectional slopes. For a given year, the solid line in Panel A plots the estimated slope coefficient from the cross-country regression of the country's Gini coefficient ($GINI$) in that year on the country's tax-to-GDP ratio (TAX) in the same year. In Panel B, the line plots the slope on TAX^2 from the regression of $GINI$ on TAX and TAX^2. The dashed lines plot the corresponding 95% confidence intervals. To run the cross-country regression in any given year, we require at least 10 country-level observations. The first valid cross-country slope appears in 1985; the last one in 2011.

Table F1
Alternative inequality measure: Gini coefficient (linear)

This table corresponds to Table 1 in the paper, replacing the top 10% share by the Gini coefficient as the dependent variable. The independent variables are in the row labels. The sample period is 1980–2013. t-statistics are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.0075^{***}	-0.0074^{***}	-0.0062^{***}	-0.0068^{***}	-0.0061^{***}
	(-9.87)	(-9.39)	(-8.84)	(-9.15)	(-8.95)
$GDPGRO$		0.0055			-0.0057
		(0.76)			(-0.67)
$INFL$			0.0016^{***}		0.0017^{**}
			(3.58)		(2.24)
$GDPPC$				-0.0000^{**}	-0.0000
				(-2.51)	(-0.52)
Constant	0.5676^{***}	0.5492^{***}	0.5080^{***}	0.5827^{***}	0.5282^{***}
	(21.17)	(15.32)	(18.88)	(24.02)	(17.56)
Observations	21	21	21	21	21
R^2	0.82	0.83	0.89	0.86	0.90

Table F2
Alternative inequality measure: Gini coefficient (quadratic)

This table corresponds to Table 1 in the paper, replacing the top 10% share by the Gini coefficient as the dependent variable. The independent variables are in the row labels. The sample period is 1980–2013. t-statistics are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.0186^{***}	-0.0182^{***}	-0.0134^{***}	-0.0152^{***}	-0.0125^{***}
	(-4.02)	(-3.96)	(-3.16)	(-3.31)	(-2.98)
TAX^2	0.0002^{**}	0.0002^{**}	0.0001^{*}	0.0001^{*}	0.0001
	(2.42)	(2.39)	(1.71)	(1.86)	(1.53)
$GDPGRO$		0.0044			-0.0053
		(0.68)			(-0.66)
$INFL$			0.0013^{***}		0.0015^{**}
			(3.02)		(2.07)
$GDPPC$				-0.0000^{**}	-0.0000
				(-1.97)	(-0.35)
Constant	0.7346^{***}	0.7166^{***}	0.6223^{***}	0.7037^{***}	0.6249^{***}
	(10.06)	(9.31)	(8.71)	(10.21)	(9.02)
Observations	21	21	21	21	21
R^2	0.86	0.86	0.90	0.88	0.91

Table F3
Alternative measures of average stock market return vs. taxes

This table reports the average stock market results from Table 1 in the paper for three different ways of calculating returns. Only a subset of Panel C results is reported in the paper. The independent variables are in the row labels. The sample period is 1980–2013. t-statistics are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	Panel A. USD returns, nominal				
TAX	-0.5157^{***}	-0.3119^{*}	-0.3090^{*}	-0.3065^{*}	-0.1329
	(-2.85)	(-1.74)	(-1.86)	(-1.67)	(-0.80)
$GDPGRO$		3.0845^{***}			2.1673^{**}
		(2.76)			(2.15)
$INFL$			0.3828^{***}		0.2564^{**}
			(3.45)		(2.22)
$GDPPC$				-0.0004^{**}	-0.0002
				(-2.59)	(-1.30)
Observations	33	33	33	33	33
R^2	0.20	0.35	0.41	0.33	0.51
	Panel B. Local currency returns, nominal				
TAX	-0.9235^{***}	-0.7110^{**}	-0.5213^{**}	-0.6285^{**}	-0.4041^{*}
	(-3.45)	(-2.54)	(-2.46)	(-2.29)	(-1.78)
$GDPGRO$		3.2160^{*}			1.3523
		(1.84)			(0.98)
$INFL$			0.7447^{***}		0.6584^{***}
			(5.27)		(4.17)
$GDPPC$				-0.0006^{**}	-0.0001
				(-2.45)	(-0.70)
Observations	33	33	33	33	33
R^2	0.27	0.33	0.60	0.38	0.62
	Panel C. Local currency returns, real				
TAX	-0.2856^{***}	-0.2143^{**}	-0.2148^{**}	-0.2234^{**}	-0.1604
	(-2.92)	(-2.08)	(-2.16)	(-2.10)	(-1.51)
$GDPGRO$		1.0788^{*}			0.7794
		(1.67)			(1.21)
$INFL$			0.1310^{**}		0.0946
			(1.97)		(1.28)
$GDPPC$				-0.0001	-0.0000
				(-1.33)	(-0.45)
Observations	33	33	33	33	33
R^2	0.20	0.27	0.29	0.25	0.33

Table F4
Panel regressions with time fixed effects: Market-to-book ratio (linear)

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is the aggregate market-to-book ratio; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. $^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.0068	-0.0046	-0.0101	-0.0122	-0.0096
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$GDPGRO$		0.0210			0.0201
		(0.02)			(0.02)
$INFL$			-0.0094^{***}		-0.0101
			(0.00)		(0.01)
$GDPPC$				0.0000	0.0000
				(0.00)	(0.00)
Constant	1.9430^{***}	1.8757^{***}	2.0974^{***}	1.7877^{***}	1.8680^{***}
	(0.29)	(0.34)	(0.33)	(0.25)	(0.36)
Observations	788	673	788	788	673
R^2	0.358	0.376	0.371	0.381	0.395

Table F5
Panel regressions with time fixed effects: Market-to-book ratio (quadratic)

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is the aggregate market-to-book ratio; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. $^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	0.1133^{***}	0.0973^{**}	0.0996^{**}	0.0918^{**}	0.0767^{*}
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
TAX^2	-0.0018^{***}	-0.0015^{**}	-0.0017^{***}	-0.0016^{**}	-0.0013^{*}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$GDPGRO$		0.0258			0.0244
		(0.02)			(0.02)
$INFL$			-0.0056^{*}		-0.0075
			(0.00)		(0.01)
$GDPPC$				0.0000	0.0000
				(0.00)	(0.00)
Constant	0.0976	0.2822	0.3703	0.2533	0.5301
	(0.57)	(0.66)	(0.67)	(0.58)	(0.77)
Observations	788	673	788	788	673
R^2	0.393	0.400	0.397	0.405	0.411

Table F6
Panel regressions with time fixed effects: Stock market returns

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is the average aggregate stock market return from all available data; the independent variables are in the row labels. The constant intercept term, which is suppressed, is always positive and significant. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
			Panel A. USD returns, nominal		
TAX	-0.4728^{**}	-0.3151^{*}	-0.2858^{**}	-0.3059^{*}	-0.1041
	(0.22)	(0.16)	(0.14)	(0.15)	(0.11)
$GDPGRO$		0.6966^{***}			0.7983^{***}
		(0.23)			(0.23)
$INFL$			0.3257^{***}		0.2431^{*}
			(0.12)		(0.12)
$GDPPC$				-0.0003^{**}	-0.0002^{**}
				(0.00)	(0.00)
Observations	1028	766	1025	1028	764
R^2	0.190	0.179	0.380	0.313	0.369
			Panel B. Local currency returns, nominal		
TAX	-0.8337^{**}	-0.5376^{**}	-0.5813^{**}	-0.5740^{**}	-0.2298^{**}
	(0.35)	(0.20)	(0.22)	(0.24)	(0.10)
$GDPGRO$		0.7863^{**}			0.9033^{**}
		(0.38)			(0.38)
$INFL$			0.4392^{**}		0.3181^{*}
			(0.20)		(0.18)
$GDPPC$				-0.0005^{**}	-0.0004^{**}
				(0.00)	(0.00)
Observations	1028	766	1025	1028	764
R^2	0.255	0.210	0.401	0.381	0.395
			Panel C. Local currency returns, real		
TAX	-0.2445^{**}	-0.1784^{*}	-0.1860^{*}	-0.1939^{**}	-0.1083
	(0.11)	(0.09)	(0.10)	(0.09)	(0.07)
$GDPGRO$		0.2042			0.2703^{**}
		(0.14)			(0.13)
$INFL$			0.1019^{**}		0.1141^{***}
			(0.04)		(0.04)
$GDPPC$				-0.0001	-0.0001
				(0.00)	(0.00)
Observations	1028	766	1025	1028	764
R^2	0.192	0.164	0.262	0.234	0.250

Table F7
Panel regressions with time fixed effects: Top 10% income share (linear)

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is the top 10% income share; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.3755^{***}	-0.3703^{***}	-0.4209^{***}	-0.4681^{***}	-0.4385^{***}
	(0.09)	(0.10)	(0.10)	(0.10)	(0.12)
$GDPGRO$		-0.4125^{***}			-0.2156
		(0.14)			(0.14)
$INFL$			-0.1179^{**}		-0.0948
			(0.05)		(0.11)
$GDPPC$				0.0003^{**}	0.0003^{**}
				(0.00)	(0.00)
Constant	43.0425^{***}	44.3357^{***}	45.3140^{***}	39.7176^{***}	39.6251^{***}
	(3.49)	(4.19)	(3.75)	(4.00)	(4.27)
Observations	764	582	761	764	580
R^2	0.281	0.228	0.322	0.397	0.376

Table F8
Panel regressions with time fixed effects: Top 10% income share (quadratic)

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is the top 10% income share; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.4038	-0.5665	-0.7371	-0.7032	-0.5891
	(0.71)	(0.73)	(0.79)	(0.69)	(0.62)
TAX^2	0.0004	0.0029	0.0048	0.0036	0.0022
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$GDPGRO$		-0.4309^{***}			-0.2306
		(0.15)			(0.15)
$INFL$			-0.1265^{**}		-0.0946
			(0.06)		(0.10)
$GDPPC$				0.0003^{**}	0.0003^{**}
				(0.00)	(0.00)
Constant	43.4806^{***}	47.5389^{***}	50.3297^{***}	43.2950^{***}	42.0924^{***}
	(12.06)	(12.53)	(13.95)	(12.54)	(11.06)
Observations	764	582	761	764	580
R^2	0.281	0.229	0.325	0.399	0.376

Table F9
Panel regressions with time fixed effects: Gini (linear)

This table corresponds to Table F1. It reports results from panel regressions with time fixed effects. The dependent variable is the Gini coefficient; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.0072^{***}	-0.0072^{***}	-0.0065^{***}	-0.0066^{***}	-0.0064^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$GDPGRO$		-0.0006			-0.0001
		(0.00)			(0.00)
$INFL$			0.0009^{***}		0.0007^{**}
			(0.00)		(0.00)
$GDPPC$				-0.0000^{***}	-0.0000^{**}
				(0.00)	(0.00)
Constant	0.5557^{***}	0.5624^{***}	0.5282^{***}	0.5704^{***}	0.5577^{***}
	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)
Observations	648	545	648	648	545
R^2	0.728	0.702	0.752	0.761	0.740

Table F10
Panel regressions with time fixed effects: Gini (quadratic)

This table corresponds to Table F2. It reports results from panel regressions with time fixed effects. The dependent variable is the Gini coefficient; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	-0.0190^{***}	-0.0190^{***}	-0.0170^{***}	-0.0167^{***}	-0.0164^{***}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
TAX^2	0.0002^{***}	0.0002^{***}	0.0002^{**}	0.0002^{***}	0.0001^{**}
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$GDPGRO$		-0.0004			-0.0002
		(0.00)			(0.00)
$INFL$			0.0004		0.0003
			(0.00)		(0.00)
$GDPPC$				-0.0000^{**}	-0.0000^{**}
				(0.00)	(0.00)
Constant	0.7364^{***}	0.7429^{***}	0.6970^{***}	0.7194^{***}	0.7153^{***}
	(0.05)	(0.05)	(0.08)	(0.05)	(0.06)
Observations	648	545	648	648	545
R^2	0.775	0.747	0.779	0.793	0.767

Table F11
Panel regressions with time fixed effects: GDP per hour worked

This table corresponds to Table 1 in the paper. It reports results from panel regressions with time fixed effects. The dependent variable is GDP per hour worked; the independent variables are in the row labels. The sample period is 1980–2013. Clustered standard errors are in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

	(1)	(2)	(3)	(4)	(5)
TAX	0.7827^{***}	0.7923^{***}	0.6889^{***}	0.3132^{***}	0.3695^{***}
	(0.17)	(0.21)	(0.19)	(0.08)	(0.08)
$GDPGRO$		-0.4961			-0.1738
		(0.36)			(0.13)
$INFL$			-0.2130^{*}		0.0209
			(0.12)		(0.09)
$GDPPC$				0.0010^{***}	0.0010^{***}
				(0.00)	(0.00)
Constant	6.5034	9.3491	10.9751	-4.1418	-5.2757
	(5.78)	(7.85)	(6.91)	(2.80)	(3.43)
Observations	1011	763	1011	1011	763
R^2	0.368	0.330	0.396	0.894	0.901

REFERENCES

Pástor, Ľuboš, and Pietro Veronesi, 2016, Income inequality and asset prices under redistributive taxation, Working paper, University of Chicago.