1. (Stein’s Identity). If \( X \sim N(\mu, \sigma^2) \) show that
\[
E((X - \mu)g(X)) = \sigma^2 E(g'(X))
\]
when both sides exist.

2. (Pareto). The Pareto distribution with parameters \( \alpha \) and \( \beta \) has probability density
\[
f_x(x|\alpha, \beta) = \frac{\alpha \beta^\alpha}{x^{\alpha+1}} \quad \text{where} \quad \beta < x < \infty \quad \text{and} \quad \alpha, \beta > 0.
\]
Find the mean and variance.
Show that the variance does not exist if \( \alpha \leq 2 \).

3. (Transformations). Derive the following distributions
(i) If \( X \sim Exp(\beta) \) then the Weibull distribution is \( Y = X^{1/\gamma} \).
(ii) If \( X \sim Exp(\beta) \) then the Rayleigh distribution is \( Y = \sqrt{2X/\beta} \).
(iii) If \( X \sim Exp(1) \) then the Gumbell distribution is \( Y = \alpha - \gamma \log X \).
(iv) If \( X \sim Gamma(1.5, \beta) \) then the Maxwell distribution is \( Y = \sqrt{X/\beta} \).

4. (Cauchy). Let \( C \) have a standard Cauchy distribution.
Show that \( 1/C \) has the same distribution.

5. (Poisson). Suppose that \( X \sim Poi(\mu) \) and independently \( Y \sim Poi(\lambda) \).
Show that \( X + Y \sim Poi(\mu + \lambda) \).
Suppose that \( X|Y \sim Poi(Y) \) and \( Y \sim Poi(\lambda) \).
Find the moment generating function of \( X + Y \).