Business 33001: Microeconomics

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Week 8
Today’s Class

1. Overview of Imperfect Competition

2. Introduction to Strategic Decision Making
   - Game theory example and Nash Equilibrium

3. Models of Quantity Competition
   - Simultaneous (Cournot)
   - Sequential (Stackelberg)

4. Models of Price Competition
   - Simple version with undifferentiated goods (Bertrand)
   - Differentiated goods (Hotelling)

5. Discrete Choice (Advanced)
Motivation: why should you care?

1. Understand basic concepts in game theory

2. Models behavior of oligopolistic firms
   - First-mover advantage
   - Pricing strategy

3. Strategies for Differentiation
   - Framework analyzing product differentiation

4. Demand Analysis
   - Framework for analyzing discrete consumer purchasing decisions
Overview of Imperfect Competition
Imperfect Competition

- We have discussed perfect competition and monopoly
- In reality, most industries fall somewhere in between (e.g., only a few airlines may make direct flights between Chicago and San Francisco)
- An oligopoly is an industry characterized by a small number of large suppliers
- While oligopoly is somewhere between, we will see that the economic models are very different
- The reason for this difference is strategic interaction
The figure shows

- Monopolist (or cartel) price and quantity at point A
- Competitive outcome at point C
- Under imperfect competition, we can get many solutions between A and C
- Cournot can be around point B and Bertrand can be at C
Details matter with Imperfect Competition

Small changes in model details really matter for market outcomes

- Do firms pick price, quantity, type of product?
- Timing of firm decisions
- Information about rivals actions
Equilibrium in **monopoly** markets and **competitive** markets:
- Every firm in the market is doing the best it can and has no reason to change its price or quantity.

Equilibrium in **oligopoly**:
- A given firm in the market is doing the best it can given what its competitors are doing AND every competitor is doing the best that it can given what the firm is doing.

The definition above suggests why we will often use the **nash equilibrium** concept to solve oligopoly problems.

In a nash equilibrium, all firms are choosing the **best response** to what the other firms are choosing.
Strategic Decision Making
Game Theory

- A framework developed first by mathematicians and now used by economists, military strategists, political strategists, and many others
- It describes how actors make decisions in a situation when they know their actions will affect the incentives of their rivals in the game
- We will only scratch the surface of this topic – if you are interested, you should take Emir Kamenica’s class
Prisoner’s Dilemma

- Two suspects, police think they acted together. Bring them into different rooms.
- They tell each suspect that if he provides information that is used to convict his partner, he will get a reduced sentence of 1 year in prison.
- If he does not talk and his partner rats him out, he will get 2 years in prison.
- If neither prisoner talks, they will both go free.
- Best option: talk to each other and commit not to confess.

What will happen?
Prisoner’s Dilemma

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<th>Don't Confess</th>
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<td>Don't Confess</td>
<td>-1, -1</td>
<td>-1, -2</td>
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</tbody>
</table>
Prisoner’s Dilemma

- Nash Equilibrium: set of decisions where no one wants to deviate
- What are Nash Equilibria of this game?
  - If # 2 confesses, what will # 1 want to do? (similarly for # 1)?
  - If # 2 does not confess, what will # 1 want to do? (similarly for # 1)?
  - Two Nash Equilibria: both confess or both not confess
- So what do the cops do?
  - Try to convince each guy that the other is cooperating. This makes them want to confess
Important Insight

- Lack of incentive to deviate
- When we look for a stable point in these games, it is where neither agent wants to deviate given what the other person is doing
Simultaneous Quantity Competition (Cournot)
NYTimes, October 2014 – Container Shipping

As companies look for more efficient ways to move freight from factories in China to consumers in Europe, the Maersk Mary is among the newest giants, known as the Triple-E’s. Until the late 1990s, the largest container ships could carry about 5,000 steel shipping containers, each about 20 feet long. The Triple-E’s can carry more than 18,000 containers. During a recent voyage to the Suez, the Mary’s crew sailed on a parallel course with a 10-year-old Maersk container ship that held half as much cargo, but the Mary used only 6 percent more fuel... [And] while the ships grow, the crews don’t, in another economy of scale.

Few carriers besides Maersk are profitable, too many new ships are being built, and demand for space on container ships is slowing as economies in Europe and Asia face headwinds.

“There’s too much capacity in the market and that drives down prices,” [said the global head of shipping at Boston Consulting]. “From an industry perspective, it doesn’t make any sense. But from an individual company perspective, it makes a lot of sense. It’s a very tricky thing.”
Example - Monopolist

- Wheat monopolist with marginal costs of \( MC = 4 \)
- Wheat Demand \( P = 100 - 8Q \)
  - Marginal Revenue \( 100 - 16Q \)
  - \( MR = MC \) when \( 100 - 16Q = 4 \)
- Solution: \( Q = 6, \ P = 52 \)
- Profit of \( 6 \cdot 52 - 6 \cdot 4 = $288 \)
Now let there be two identical wheat-producing firms
- Firm 1: Marginal Cost 4
- Firm 2: Marginal Cost 4

What happens in this market?
- Firms have market power – not “competitive”, price-taking
- Multiple firms in the market – not “monopoly”
- Oligopoly

To determine outcome, we need to specify market structure
- What does Firm 1 choose?
- Given Firm 1’s choice, what is Firm 2’s demand curve?
- What does Firm 2 choose?
- Given Firm 2’s choice...
Market Structure: Quantity Competition
- Suppose firms each choose quantity of production
- Firms then sell to a centralized market
- Price is set by demand curve

Two wheat-producing firms, each has $MC = 4$
Market Demand for wheat is $P = 100 - 8Q$
- Firms produce a total quantity $Q = q_1 + q_2$
- Markets clear at a price of $100 - 8Q$
Example - Quantity Competition

- $P = 100 - 8Q$
- 2 firms, each has MC of 4
- Firm 1’s demand curve depends on Firm 2’s production
  - For some fixed $q_2$, Firm 1’s inverse demand is
    $$P(q_1) = (100 - 8q_2) - 8q_1$$
Example - Quantity Competition

- $P = 100 - 8Q$
- 2 firms, each has MC of 4
- Firm 1’s demand curve depends on Firm 2’s production
  - For some fixed $q_2$, Firm 1’s inverse demand is $P(q_1) = (100 - 8q_2) - 8q_1$
- Suppose Firm 1 knew $q_2$ in advance
  - Marginal Revenue: $(100 - 8q_2) - 16q_1$
  - Set $MR = MC$ to get optimal choice of $q_1 = 6 - \frac{q_2}{2}$
  - Price would be $P = 100 - 8(q_2 + q_1) = 52 - 4q_2$
Example - Quantity Competition

\[ P = 100 - 8Q, \text{ 2 Firms, } MC = 4 \]

- If Firm 1 expects Firm 2 to choose \( q_2 \): Choose \( q_1 = 6 - \frac{q_2}{2} \)
- If Firm 2 expects \( q_1 \): Choose \( q_2 = 6 - \frac{q_1}{2} \)

- If Firm 1 expects \( q_2 = 0 \) : Choose \( q_1 = 6 \)
- If Firm 2 expects \( q_1 = 6 \) : Choose \( q_2 = 3 \)
- If Firm 1 expects \( q_2 = 3 \): Choose \( q_1 = 4.5 \)

\[ q_2 = 0 \rightarrow q_1 = 6 \rightarrow q_2 = 3 \rightarrow q_1 = 4.5 \]
\[ \rightarrow q_2 = 3.75 \rightarrow q_1 = 4.125 \rightarrow q_2 = 3.94 \]
\[ \rightarrow q_1 = 4.03... \]

What is the “solution”?
- Try \( q_1 = 4, q_2 = 4 \):
  - If Firm 1 expects \( q_2 = 4 \), it chooses \( q_1 = 6 - \frac{4}{2} = 4 \)
  - If Firm 2 expects \( q_1 = 4 \), it chooses \( q_2 = 6 - \frac{4}{2} = 4 \)
  - Is this the answer?
How to think about strategic behavior?

- **Strategic Interaction:**
  - My choice depends on your action
    - Firm 1’s “best reply” choice of $q_1$ depends on $q_2$
  - Your choice depends on my action
    - Firm 1’s “best reply” choice of $q_2$ depends on $q_1$

- **Solution:** Look for **Nash Equilibrium**
  Each player’s action is a best-reply to others’ actions
  - $q_1^*$ is a best-reply to $q_2^*$, and $q_2^*$ is a best-reply to $q_1^*$
  - Beliefs match reality – you do what I expect, I do what you expect
  - Self-enforcing plan: if firms talked in advance and agreed on $(q_1^*, q_2^*)$, neither would deviate
  - End point of dynamic adjustment process
Solving the Quantity Competition Example

- Firm 1: Choose \( q_1 = 6 - \frac{q_2}{2} \), given belief on \( q_2 \)
- Firm 2: Choose \( q_2 = 6 - \frac{q_1}{2} \), given belief on \( q_1 \)
- Nash Equilibrium: solve for simultaneous solution

\[
\begin{cases}
q_1^* = 6 - \frac{q_2^*}{2} \\
q_2^* = 6 - \frac{q_1^*}{2}
\end{cases}
\]

Plug in:

\[
q_1^* = 6 - \frac{q_2^*}{2} = 6 - \frac{\left(6 - \frac{q_1^*}{2}\right)}{2}
\]

\[
q_1^* = 3 + \frac{q_1^*}{4}
\]

\[
\Rightarrow \quad q_1^* = 4
\]

\[
q_2^* = 6 - \frac{q_1^*}{2} = 6 - \frac{4}{2} = 4
\]

- Solution: \( q_1^* = 4, q_2^* = 4 \)
- Firm 1 expects Firm 2 to choose \( q_2 = 4 \), so chooses \( q_1 = 4 \)
- Firm 2 expects Firm 1 to choose \( q_1 = 4 \), so chooses \( q_2 = 4 \)
  - Expectations match reality
  - Any other outcome: someone would be making a mistake
Sequential Quantity Competition (Stackelberg)
Two firms. What if Firm 1 “goes first”?

1. Firm 1 publicly chooses $q_1$
2. Firm 2 sees $q_1$, chooses $q_2$
3. Price for both determined by $P(q_1 + q_2)$

Interpretations:
- Firm 1 is “stronger” – can credibly announce & commit to its output?
  - Firm 1 is the incumbent with a reputation to protect,
    Firm 2 is a new entrant
  - Firm 1 gets to make public capacity investments before Firm 2 enters
- Firm 2 is “nimbler” – can wait until it sees Firm 1’s output
  - before choosing own production
  - Firm 1 has to make public capacity investments before Firm 2 enters

Better to be Firm 1 or Firm 2?
- “First mover advantage” vs. “Second mover advantage”
Stackelberg Model - Example

Firm 1 goes first

- Same example: \( P(Q) = 100 - 8Q \), 2 firms with \( MC = 4 \)
- Old Problem – Choose based on expectations
  - If Firm 1 expects Firm 2 to choose \( q_2 \): Choose \( q_1 = 6 - \frac{q_2}{2} \)
  - If Firm 2 expects \( q_1 \): Choose \( q_2 = 6 - \frac{q_1}{2} \)
  - Look for equilibrium where expectations are correct
- New Problem: Firm 2 knows Firm 1’s choice, no guessing
  - Firm 2 responds to Firm 1’s actual choice: \( q_2 = 6 - \frac{q_1}{2} \)
  - Firm 1 doesn’t take \( q_2 \) as given – predicts how Firm 2 will respond
  - Old problem for Firm 1:
    \[
    \max_{q_1} P(q_1 + q_2) \cdot q_1 - 4q_1, \text{ with } q_2 \text{ fixed}
    \]
  - New problem for Firm 1:
    \[
    \max_{q_1} P(q_1 + q_2) \cdot q_1 - 4q_1, \text{ with } q_2 = 6 - \frac{q_1}{2}
    \]
    \[
    \Rightarrow \max_{q_1} P \left( q_1 + \left( 6 - \frac{q_1}{2} \right) \right) \cdot q_1 - 4q_1
    \]
Stackelberg Model

Firm 1 goes first

- Firm 1’s decision problem

\[
\max_{q_1} P \left( q_1 + \left( 6 - \frac{q_1}{2} \right) \right) \cdot q_1 - 4q_1 \\
\Rightarrow \max_{q_1} \left( 100 - 8 \left( q_1 + \left( 6 - \frac{q_1}{2} \right) \right) \right) \cdot q_1 - 4q_1 \\
\Rightarrow \max_{q_1} (52 - 4q_1) q_1 - 4q_1 \\
\Rightarrow \max_{q_1} 48q_1 - 4q_1^2 \\
\Rightarrow 48 - 8q_1 = 0 \\
\Rightarrow q_1 = 6
\]

Solution: \( q_1 = 6 \), so Firm 2 chooses \( q_2 = 6 - \frac{q_1}{2} = 3 \)
Comparing Cournot and Stackelberg

\[ P(Q) = 100 - 8Q, \text{ 2 firms, } MC = 4 \]

- Sequential instead of Simultaneous quantity competition:
  - First mover produces \( q_1 = 6 \) instead of \( 4 \) – higher
  - Second mover produces \( q = 3 \) instead of \( 4 \) – lower
  - Aggregate quantity of \( Q = 9 \) instead of \( 8 \) – higher
  - Price \( P = 28 \) instead of \( 36 \) – lower
  - First mover profit of \( 28 \cdot 6 - 4 \cdot 6 = 144 \) instead of \( 128 \) – higher
  - Second mover profit \( 28 \cdot 3 - 4 \cdot 3 = 72 \) instead of \( 128 \) – lower

- It’s good to go first
- First mover overproduces, second underproduces
  - Firm 1 wants Firm 2 to produce less
  - Firm 2 produces 1/2 unit less for every extra unit of Firm 1
  - This gives a new incentive to Firm 1 to produce extra
  - Increase of Firm 1 not totally offset by decrease of Firm 2
Comparing Cournot and Stackelberg

\[ P(Q) = 100 - 8Q, \text{ 2 firms, } MC = 4 \]

- Sequential instead of Simultaneous quantity competition:
  - First mover produces \( q_1 = 6 \) instead of 4 – higher
  - Second mover produces \( q = 3 \) instead of 4 – lower
  - First mover profit of \( 28 \cdot 6 - 4 \cdot 6 = 144 \) instead of 128 – higher
  - Second mover profit \( 28 \cdot 3 - 4 \cdot 3 = 72 \) instead of 128 – lower

- Stackelberg game: It’s good to go first

- In the original simultaneous game, what if one firm announces
  “I’m going ‘first’: I’m going to produce \( q = 6 \), no matter what.”
  - Not credible
  - Suppose Firm 2 believed it, and planned to produce \( q = 3 \).
    Then Firm 1 should instead produce 4.5 units, not 6
  - Production of \( q_1 = q_2 = 4 \) is the only credible outcome
  - Do this every week, and gain a reputation over time?
In many strategic settings, it’s good to go first

- “We won’t pay ransom to terrorists”
  - If a terrorist takes a hostage, we want to pay
    Terrorists will take lots of hostages!
  - If we commit not to pay, terrorist won’t bother taking hostages (hopefully)
- “Chicken” – two cars drive at each other, first one to swerve loses
  - Announce, “I won’t swerve no matter what”
  - Publicly remove steering wheel from the car

- Big picture issue: when is commitment credible?
Models of Quantity Competition

- **Set-up:**
  - Firms produce some kind of commodity good
  - Firms choose production levels
  - Market determines prices

- **Strategic incentives:**
  - The more others produce, the less you produce
  - Find equilibrium where everyone “best-responds” to each other

- **Conclusions:**
  - In equilibrium, more firms means...
    - Lower quantity per firm, approaching zero if many firms
    - Higher total quantity, approaching efficient level
    - Lower prices, approaching marginal costs
    - Less profit per firm, approaching zero
  - If you can go first...
    - First-move advantage
    - You produce more
    - Others produce less
Simple Price Competition (Bertrand)
Bertrand Competition

- Cournot Model:
  Firms choose *quantities*, then market determines price
- Maybe price competition is more realistic than quantity competition?

- Bertrand Model of price competition:
  Firms choose *price*, then market determines quantities
  Simplest possible model:
  - Everyone consumer who buys, buys from the lower priced seller
    - Interpretation – undifferentiated goods
    - No brands, no distinct flavors or styles, etc
  - If firms offer equal prices, firms split the market
Details of the Bertrand Competition model

- Market has some demand curve $Q(P)$
  - Previous example had inverse demand $P(Q) = 100 - 8Q$
  - Corresponds to demand of $Q(P) = \frac{100-P}{8}$
- Firms have constant marginal costs, identical to each other
  - Focus on example with two firms
  - Each has Marginal Cost of 4

Bertrand Game:

1. Firms 1 and 2 each choose prices $p_1$ and $p_2$ (simultaneously)
2. Consumers see prices and choose quantities
   - If $p_1 < p_2$: Market price of $P = p_1$
     - Firm 1 sells quantity of $Q(P)$
     - Firm 2 sells quantity of 0
   - If $p_1 > p_2$: Market price of $P = p_2$
     - Firm 1 sells quantity of 0
     - Firm 2 sells quantity of $Q(P)$
   - If $p_1 = p_2$: Market price of $P = p_1 = p_2$
     - Firm 1 sells quantity of $Q(P)/2$
     - Firm 2 sells quantity of $Q(P)/2$
Bertrand Competition

Outcome – Nash Equilibrium:
- Each firm chooses a price before observing other’s price
- Firms have accurate beliefs about other’s price
- Firm’s choice is profit-maximizing given her beliefs about other’s price

Four types of possible outcomes:

1. \( p_1 < MC \) and/or \( p_2 < MC \)
   - Why is this not an equilibrium? Someone loses money

2. \( MC < p_1 = p_2 \)
   - Why is this not an equilibrium? Each firm should undercut the other

3. \( MC \leq p_1 < p_2 \)
   - Why is this not an equilibrium? Firm 1 should raise prices

4. \( MC = p_1 = p_2 \)
   - Is this an equilibrium?
Bertrand Competition

The only possible equilibrium:

- $MC = p_1 = p_2$
- All other possibilities: at least one firm can deviate to increase profits
- Under Bertrand Competition, firms produce at marginal cost.

Math example:
- Firms choose $p_1 = p_2 = 4$
- Split the aggregate quantity of $Q(P) = \frac{100-4}{8} = 12$
- Firms make 0 profit
Price vs Quantity Competition

Compare:

- **Cournot – Quantity competition:**
  - With 2 firms, \( P > MC \) and each firm makes a profit
  - Approach competitive outcome of \( P = MC \) with many firms

- **Bertrand – Price competition:**
  - With 2 firms, \( P = MC \) and no firm makes a profit
  - Reach competitive outcome immediately with only 2 firms

- Bertrand *game* more sensible, but Cournot *predictions* more sensible?

- Key takeaway: in oligopoly markets, details matter
  - One market with given “fundamentals” (demand curve, costs)
  - Three distinct outcomes (so far):
    1. Cournot (simultaneous choice of quantity)
    2. Stackelberg (sequential choice of quantity)
    3. Bertrand (simultaneous choice of price)

  - *Need to know who-decides-what, and when*

  - Compare to competitive market and monopoly – only supply/cost and demand curves matter
Variations on Bertrand Competition

- What if firms can collude to maximize industry profit?
  - Monopoly outcome maximizes industry profit
  - Both firms set single-firm monopoly price ($52)
    Yields monopoly aggregate quantity (6)
  - Need commitment (or some other way to enforce a deal) – If Firm 2 agrees to charge $52, Firm 1 wants to charge $51.99

farecompare.com:

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Example of how price fixing might work

Transcript of 1983 antitrust suit:

- **CRANDALL** (American Airlines): “I think it’s dumb as hell ... to sit here and pound the (deleted) out of each other and neither one of us is making a (deleted) dime ... We can both live here and there ain’t no room for Delta. But there’s, ah, no reason that I can see, all right, to put both companies out of business.”

- **PUTNAM** (Braniff Airlines): “Do you have a suggestion for me?”

- **CRANDALL**: “Yes, I have a suggestion for you. Raise your (deleted) fares twenty percent. I’ll raise mine the next morning ... You make more money, and I will too.”

- **PUTNAM**: “We can’t talk about prices.”

- **CRANDALL**: “Oh (deleted) Howard. We can talk about any (deleted) thing we want to talk about.”
The antitrust suit was dismissed because, “Crandall’s conduct was at best unprofessional and his choice of words distasteful,” but since Putnam turned down the offer, no conspiracy to monopolize was found to exist.

Illustrates some of the difficulties with bringing suits like this. Even a “smoking gun” not necessarily enough.
Variations on Bertrand Competition

- **Bertrand Game with firm entry:**
  - If 1 firm enters, makes monopoly profit
  - If 2 firms enter, each makes 0 profit
  - With fixed costs of entry: only 1 firm enters
    - Single firm sets high prices, makes a profit
    - Threatens a price war if any other firm enters

- **Bertrand Game with asymmetric firms:**
  - Marginal costs $MC_1 < MC_2$ of firms 1 and 2
  - An Equilibrium:
    - Firm 2 charges $p_2 = MC_2$
    - Firm 1 charges $p_1$ just below $p_2$
    - Firm 2 sells nothing, no profit
    - Firm 1 sells to the whole market at price $p_1 \approx MC_2$, makes a profit
Variations on Bertrand Competition

2 Firms, $MC = 4$, Demand $P(Q) = 100 - 8Q$

What if Firm 1 sets its price first, then Firm 2 chooses a price?

- Suppose Firm 1 chooses price $p_1$
- Three cases:
  1. $p_1 < 4$
     - Then Firm 2 lets Firm 1 win
     - Firm 1 loses money, Firm 2 makes no profit
  2. $p_1 = 4$
     - Then Firm 2 either sets $p_2 = 4$ or lets Firm 1 win
     - Firms 1 and 2 each make no profit
  3. $p_1 > 4$
     - Then Firm 2 sets $p_2$ just below $p_1$
     - Firm 1 makes no profit, Firm 2 makes a profit

- So Firm 1 chooses some price $p_1 \geq 4$
  - Firm 1 makes no profit no matter what
  - If $p_1 = 4$, Firm 2 makes no profit
  - If $p_2 > 4$, Firm 2 makes a profit

- You prefer to go second (be nimble) – can undercut competitor
  - In Cournot competition, going first was better
Problems with the Bertrand Model

- Bertrand competition is very extreme:
  - If you charge a penny more than your competitor, you sell 0
  - If you charge a penny less, you sell to everyone
  - Yields pricing at marginal cost – unrealistic

- We might want a less extreme form of price competition:
  - Lower prices steal some sales from competitor
  - Higher prices give away some sales to competitor
  - Quantity sold is continuous in prices
Price Competition with Differentiated Goods (Hotelling)
Thus far, we have assumed:

1. **Firms** produce the same good
2. **People** are indifferent about which firm sells the good

In the real world:

1. **Firms** clearly produce goods with different attributes
2. **People** often have idiosyncratic preferences
Product Differentiation - Outline

1. Firms
   - What price to set?
   - Product selection

2. Differences in Preferences and Costs
   - Cost differences
   - Demand
To keep things as simple as possible for now, suppose we have two firms $A$ and $B$ with the following demands

$$q_A = \frac{1}{2} - p_A + p_B$$

$$q_B = \frac{1}{2} - p_B + p_A$$

These expressions show that a company’s demand is decreasing in its own price and increase in the price of its competitor.

---

Analytical solution:

- Given these demands and assumptions about cost, we could solve for the Nash Equilibrium for the game in which firms simultaneously pick prices.
- Profit functions will be smooth, so we could compute marginal revenues and costs.
- We could then compute best response functions.

However, rather than doing these steps, we will analyze the solution graphically for a typical Bertrand game with differentiated products.
Price Competition and Differentiated Goods

\[ P_B^* \]

\[ P_A^* \]

\[ \text{Firm B's best-response function} \]

\[ \text{Firm A's best-response function} \]

\[ \text{Nash equilibrium} \]

\[ \text{Firm B's price (P_B)} \]

\[ \text{Firm A's price (P_A)} \]

\[ 0 \]

Price Competition and Differentiated Goods

The figure shows:

- Best-response functions for each firm given the competitor’s price
- Upward sloping because higher competitor prices raise your demand and enable you to raise prices
- Note that this is different than the Cournot model in which the best-response functions were decreasing in competitor quantity
In the case we just analyzed, we thought about changing your prices given the product mix.

Realistically, there is a two stage game:

1. Product selection
2. Pricing
Incorporating Product Selection

The figure shows:

- Two firms that initially produce fairly similar products and their initial best-response functions
- When differentiation between the firms increases, the best-response curves shift
- The new equilibrium involves higher prices for both firms
Incorporating Product Selection: Trade-offs

There are a few key trade-offs:

1. **Close to customers**: firms prefer to locate where demand is greatest
2. **Intensity of competition**: if firms have similar products, they will face fiercer price competition
3. **Entry**: if firms become too specialized, they can leave a big enough opening for a third firm to enter

We will now analyze product selection in more depth by thinking about the demand side.
Consider a given product attribute index that goes from 0 to 1.

To be concrete, suppose it is the location of a banana stand on a beach.

Suppose that the cost of going to the banana stand from any point \( x \) on the beach is:

\[
c = 10 \times (x - .5)^2
\]

Intuitively this says that the banana stand is in the middle (i.e., at .5).

Utility for each consumer depends on their location \( x \).

Define utility as follows: \( u = -c \).
Utility by consumer location
Utility if Banana stand were at .3, i.e., $u = -10 \times (x - .3)^2$
Utility with two banana stands (at .3 and .8)
Utility with two banana stands (at .3 and 1)
1. How costly would it be to switch banana stands for different people?
2. What if there was money in the banana stand?
3. Who goes to the stand at .3?
4. How much demand will the banana stands have?
What if consumers may not buy?
Takeaways on Product Differentiation

Tradeoffs:

- Obvious: Split large market vs. take 100% share of small market
- Not as obvious: Making a similar product to others could lead to price war

Making your product distinct from competitors is good for 2 reasons:

1. Sell to underserved consumers
2. Product differentiation keeps prices high
Discrete Choice (Advanced)
Consumers decide whether or not to buy
Consumers decide whether or not to buy

![Graph showing the relationship between fraction purchasing and price](image)
Consumers decide whether or not to buy

- The first graph shows the share of consumers buying a product is 50% when it’s price is $5
- The second graph shows the share of consumers buying a product is 30% when it’s price is $6
- How can we think about how responsive demand will be to changes in price when consumers are making discrete (i.e., buy or not) choices?
Suppose that individual $i$ buys if her value exceeds the price, i.e., buy if $v_i > P$.

This value can be a function of common things (e.g., income, credit conditions, etc) or idiosyncratic tastes but at this stage, specifying what is in $v_i$ doesn’t matter. The fraction of people who buy is:

\[ \text{Prob}(Q = 1) = P(v_i > P) \]
\[ = 1 - F(P) \]

where $F(x)$ is the c.d.f. of $v_i$. Note this is why the demand curve looks like a CDF rotated clockwise 90 degrees.

A c.d.f. describes the probability that a real-valued random variable $X$ with a given probability distribution will be found to have a value less than or equal to $x$.

(Note that this may make more sense after you take your stats class)
What is the elasticity of this curve?

\[ Q(P) = N(1 - F(P)) \]  (3)

where \( N \) is the size of the population (e.g., number of potential consumers in your market)

\[ \varepsilon^D = \frac{dQ(P)}{dP} \frac{Q}{P} \]  (4)
Elasticity of Demand

- What is the derivative?

\[
\frac{dQ(P)}{dP} = -Nf(P)
\]  

(5)

- where \( N \) is the size of the population (e.g., first time home buyers in an area)

- \( f(x) \) is the probability density function (p.d.f.)
Elasticity of Demand

\[ \varepsilon^D = \frac{dQ(P)}{dP} \frac{P}{Q} \quad \text{(6)} \]

\[ = -Nf(P) \frac{P}{N(1 - F(P))} \quad \text{(7)} \]

\[ = -f(P) \frac{P}{1 - F(P)} \quad \text{(8)} \]

- What matters for responsiveness is how big the density is at \( P \) relative to 1 minus the CDF.
From $5, a $1 dollar increase in price ↓ demand by 20%
From $8, a $1 dollar increase in price ↓ demand by 2%
Elasticity of Demand: In English please...

Takeaways:
- For very homogeneous populations, you’ll have very elastic demand.
- If tastes are more spread out, you’ll see smaller responses.
- At the extreme in which everyone is the same, demand will be a step function, so there is some price above which no one will buy and below which everyone will buy.
- In this case, things will be very inelastic at high prices, but very elastic near the price, and then unresponsive at very low prices.
- Thinking about consumer choice in this way will be helpful for evaluating how effective sales can be.
Conclusion

- Oligopolies – in between monopoly and competitive market
- We looked at different models of Oligopolistic competition
  - Price vs. Quantity competition
  - Firms move at same time, vs. sequentially
  - Identical goods vs. Differentiated goods
- Big picture conclusion:
  - More firms, more similar firms $\Rightarrow$ lower profits, closer to competitive outcome with $P = MC$
  - But details of the market may matter
- Introduction to concept of *Strategic Interaction*
  - My decisions affect your decisions
  - Your decisions affect my decisions
  - What happens?
    - Basis for “Game Theory”
Simplest model of price competition with continuous demand

- Suppose 100 consumers live on a street
- Street is of length 100 (miles)
- Consumers are spread out uniformly on the street
- Stores $A$ and $B$ are on the endpoints of the street

Consumers will have different travel costs to $A$ and $B$
- Travel costs depend on distance

Each store sets a price
Each consumer chooses the cheapest item, including travel costs
- Simplifying assumption: All consumers buy something, from $A$ or $B$
Suppose Firm A sets price $p_A$, Firm B sets price $p_B$

- Travel costs: $2$ per mile

Consider a consumer at point $x \in [0, 100]$

- Total cost of buying from A: $p_A + 2 \cdot x$
- Total cost of buying from B: $p_B + 2 \cdot (100 - x)$
- Buy from cheapest place, no matter the price
  - Buy from A if $p_A + 2x < p_B + 2(100 - x)$,
    Otherwise buy from B.

(We can revisit this assumption later)
Prices $p_A$ and $p_B$, travel costs $2$ per mile

- Buy from $A$ if $p_A + 2x < p_B + 2(100 - x)$  
  Otherwise buy from $B$.
- Buy from $A$ if:
  
  $$p_A + 2x < p_B + 2(100 - x)$$
  $$4x < p_B - p_A + 200$$
  $$x < \frac{p_B - p_A}{4} + 50$$

- Cutoff type $c$ who is indifferent between $A$ and $B$: $c = \frac{p_B - p_A}{4} + 50$

<table>
<thead>
<tr>
<th>A</th>
<th>Buy A</th>
<th>Buy B</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>c</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

- Every $x < c$ buys from $A$
  - There are $c$ such people (1 per mile)
- Every $x > c$ buys from $B$
  - There are $100 - c$ such people (1 per mile)
Cutoff point \( c = \frac{p_B - p_A}{4} + 50 \)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( c )</td>
<td>100</td>
</tr>
</tbody>
</table>

Recall: 100 people, 1 person per mile

- Demand for A’s product: \( q_A = c = \frac{p_B - p_A}{4} + 50 \)
- Demand for B’s product: \( q_B = 100 - c = \frac{p_A - p_B}{4} + 50 \)
  - At equal prices, both sell 50 units
  - Higher \( p_A \) lowers demand for A and increases demand for B
    - Raising price by $4 loses just 1 customer.
  - Higher \( p_B \) raises demand for A and reduced demand for B

Say that each firm has a marginal cost of \( MC \) per unit

\[ \text{Profit of Firm A: } q_A \cdot (p_A - MC) = \left( \frac{p_B - p_A}{4} + 50 \right) \cdot (p_A - MC) \]

\[ \text{Profit of Firm B: } q_B \cdot (p_B - MC) = \left( \frac{p_A - p_B}{4} + 50 \right) \cdot (p_B - MC) \]
Solve for Nash Equilibrium of Hotelling Game

Take other’s price as given, maximize over own price

- Profit of Firm A:
  \[ q_A \cdot (p_A - MC) = \left( \frac{p_B - p_A}{4} + 50 \right) \cdot (p_A - MC) \]

- Maximize by setting derivative wrt \( p_A \) to 0:
  \[
  \left( \frac{p_B - p_A}{4} + 50 \right) - \frac{1}{4} (p_A - MC) = 0
  \]
  \[ \Rightarrow p_A = \frac{p_B}{2} + 100 + \frac{MC}{2} \]

- Profit of Firm B:
  \[ q_B \cdot (p_B - MC) = \left( \frac{p_A - p_B}{4} + 50 \right) \cdot (p_B - MC) \]

- Maximizing gives
  \[ p_B = \frac{p_A}{2} + 100 + \frac{MC}{2} \]
If both A and B choose prices to maximize profits, taking other firm’s choice as given, then

\[
\begin{align*}
  p_A &= \frac{p_B}{2} + 100 + \frac{MC}{2} \\
  p_B &= \frac{p_A}{2} + 100 + \frac{MC}{2}
\end{align*}
\]

Solving jointly gives

\[
\begin{align*}
  p_A &= MC + 200 \\
  p_B &= MC + 200
\end{align*}
\]

\[\implies\]

\[
\begin{align*}
  q_A &= 50 \\
  q_B &= 50
\end{align*}
\]

Each sells to \(q = 50\) consumers

Profit of 200 per consumer, or $10,000 per firm

- Reasonable answer: With 2 firms, charge above MC and make profit

What if we added a willingness to pay of \(V\) for each consumer?

- As long as \(V\) is large enough \((V > MC + 200 + 100)\), no difference
- For lower \(V\), solution gets complicated
How does the market change with different transportation costs?

- Say there is a cost of $t$ per mile instead of $2$
- Quantities (i.e., demand curves):
  
  \[
  q_A = \frac{p_B - p_A}{2t} + 50 \\
  q_B = \frac{p_A - p_B}{2t} + 50
  \]

- Profit maximization condition:
  \[
  \begin{align*}
  p_A &= \frac{p_B}{2} + 50t + \frac{MC}{2} \\
  p_B &= \frac{p_A}{2} + 50t + \frac{MC}{2}
  \end{align*}
  \]

- Equilibrium outcome:
  \[
  p_A = p_B = 100t + MC \\
  q_A = q_B = 50
  \]

- Profits per firm: \( q \cdot (p - MC) = 5000t \)
  - Higher transportation costs imply less competition, higher profits
  - Low costs: approach \( p = MC \), competitive (Bertrand) outcome
Meaning of “Transportation Costs”

- “Transportation Costs”:
  - Transportation Costs Model:
    - Firms A and B choose prices (simultaneously)
    - People live on a line between Firms A and B
    - Consumer location determines travel costs to go to A or B
    - Consumer chooses store with lowest Price + Travel Cost
    - In equilibrium, people go to the closer store
  - Simplified model of competition between gas stations, supermarkets
    - Key simplification – inelastic aggregate demand
  - How to model...
    - Competing brands in a supermarket (Coke vs Pepsi)?
    - Competing products sold online or by phone (Comcast vs DirecTV)?
    - Competing stores in a mall (H&M vs Forever 21)?
  - Reinterpret “transportation costs” as “preferences”
    - Location determines difference in willingness-to-pay for A vs B
    - $x = 50$: willing to pay same amount for A or B
    - $x = 0$: willing to pay up to $100t$ more for A than B
    - Smaller $t$: products more similar, profits lower
Application – Product Differentiation

Reducing “transportation costs” makes pricing more aggressive

- Apple makes tablets without styluses, keyboards – sells a lot
- Competitor sees a small market niche for tablets with alternative inputs
- Can compete in the large market for iPad-wannabes, or niche market

Tradeoffs:
- Obvious: Split large market vs. take 100% share of small market
- Not as obvious: Making a similar product to others could lead to price war

- Making your product distinct from competitors is good for 2 reasons:
  1. Sell to underserved consumers
  2. Product differentiation keeps prices high
Sequential Version of Hotelling Game

\[ q_A = \frac{1}{4}(p_B - p_A) + 50 \]

\[ q_B = \frac{1}{4}(p_A - p_B) + 50 \]

In this price competition game, would you want to go first or second? We know first mover does at least as well as in simultaneous game. Why?

- Say A goes first. B’s best response is as before:

\[ p_B = \frac{p_A}{2} + 100 + \frac{MC}{2} \]

- When determining \( p_A \), Firm A takes B’s response into account

\[ q_A(p_A) = \frac{1}{4}(p_B - p_A) + 50 = \frac{1}{4} \left( \left( \frac{p_A}{2} + 100 + \frac{MC}{2} \right) - p_A \right) + 50 \]

- Solution: \( p_A = 300 + MC \), \( p_B = 250 + MC \) (compared to 200 + MC)

- Profits: $11,250 for A, $15,625 for B (compared to $10,000)

  - Given \( p_B = 250 + MC \), A wants to switch to \( p_A = 225 + MC \)
  - New use for coupons – trick competitors into raising prices?
Sequential Version of Hotelling Game

- **Quantity competition, sequential vs. simultaneous:**
  - First mover increases quantity, gets higher profits
  - Second mover decreases quantity, gets lower profits
    - Want to move first
    - Want to prevent opponent from moving first

- **Price competition, sequential vs. simultaneous:**
  - First mover increases price, gets higher profits
  - Second mover increases price *by less*, gets *even* higher profits
    - Want your opponent to move first, then undercut him
    - Still good: move first if your opponent doesn’t
    - Price advertising in advance?
      - If you can – advertise a high price early (so competitor sees it)
      - Then lower prices at the last minute (so consumers see it)

- One idea: maybe we can use coupons to move later
  - Publicly set high prices \((MC + 300)\)
  - Competitor responds by undercutting you \((MC + 250)\)
  - Secretly mail your customers with low price coupons \((MC + 225)\)
Bertrand Competition over Differentiated Goods

- **Hotelling model:**
  - Simple model of oligopolistic price competition
  - Consumers have different physical or “preference” locations
  - Gives rise to Demand curves of following form:
    
    A’s demand increases in $p_B$ and decreases in $p_A$, at the same rate
    \[ q_A = \frac{1}{4}(p_B - p_A) + 50 \quad q_B = \frac{1}{4}(p_A - p_B) + 50 \]

- Problem: Inelastic aggregate demand
  \[ q_A + q_B = 100, \text{ independent of prices} \]

- Collusive outcome – charge infinitely high prices

- More general oligopolistic demand curves:
  
  A’s demand increases in $p_B$ and decreases in $p_A$, different rates
  \[ q_A = \frac{1}{4}p_B - \frac{1}{2}p_A + 50 \quad q_B = \frac{1}{4}p_A - \frac{1}{2}p_B + 50 \]

- Want A’s demand to fall faster in $p_A$ than it rises in $p_B$ – why?
- “Reduced form” approach to a price competition game