Heterogeneity and Asset Prices: A Different Approach

by Nicolae Garleanu and Stavros Panageas

Discussion

Pietro Veronesi

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Main Contribution and Outline of Discussion

• Main contribution of the paper:

1. Develop a macro-asset pricing framework that links volatile asset prices and high risk premiums to non-volatile, but persistent movements in the cross-sectional income and consumption distributions.

2. Propose a novel empirical approach to infer low frequency, time-series movements in the marginal agent’s consumption [...] by utilizing [...] cross-sectional information.

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Outline of discussion

1. Locally riskless consumption and volatile asset prices
2. Limited Risk Sharing
3. Comments
Locally Riskless Consumption and Stochastic Prices. Power Utility

- Standard endowment economy but with riskless dividend (≡ consumption):
  \[
  \frac{dD_t}{D_t} = g_t \, dt
  \]
Locally Riskless Consumption and Stochastic Prices. Power Utility

- Standard endowment economy but with riskless dividend (= consumption):
  \[ \frac{dD_t}{D_t} = g_t \, dt \]
- Let
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- Then:
  
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  \[ r(g_t) = \rho + \gamma g_t \]

  **P/D ratio:**
  \[ \frac{P_t}{D_t}(g_t) = \int_0^\infty e^{A_0(s) + (1-\gamma)k^{-1}(1-e^{-ks})(g_t-\bar{g})} \, ds \]
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• ⇒ Volatile prices with locally deterministic consumption
  – Because the riskless rate is time varying
Locally Riskless Consumption and Stochastic Prices. EZ Utility

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- EZ utility with RRA $\gamma$ and EIS $\psi$:

$$f(C_t, V_t) = \frac{\rho(1 - \gamma)}{1 - 1/\psi} V_t \left( \left( \frac{C_t}{((1 - \gamma)V_t)^{1/(1-\gamma)}} \right)^{\frac{1}{\psi}} - 1 \right)$$
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- Volatile prices and a risk premium with locally deterministic consumption
  - Because the riskless rate is time varying and agents care about future utility
Locally Riskless Consumption and Stochastic Prices

- More generally, one could choose a generic process for economic growth

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• From this, one could reverse engineer (if he/she is very good!)

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- Examples:
  - Affine models
  - Affine-Quadratic Models
  - Gabaix Linearity Generating Models
Share Process and Complete Markets

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- Agents do have different wealth over time, because their total endowment is different and this difference persists.

- But with complete markets, all agents’ consumption plans have identical marginal rates of substitution.

\[
\iff \quad \text{Aggregation gives the result.}
\]

\[
\implies \quad \text{With complete markets, income distribution has no impact on equilibrium}
\]
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• Therefore, the change in aggregate consumption:

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dC_t = -\lambda C_t \, dt + \lambda \int_{-\infty}^{t} e^{-\lambda(t-s)} dC_{t,s} + \lambda C_{t,t} \, dt
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Death Survivors New agents
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- And \( C_{t,t} \) depend on \( t \) new endowments:

\[ \left( \frac{C_{t,t}}{Y_t} \right) = \frac{\rho + \lambda}{\lambda} \text{[PV Human and Financial Capital of Cohort } t] \]
Share Processes and Incomplete Markets – 2

• Note the achievement:

  1. Output growth is literally constant $dY_t/Y_t = gdt$
Share Processes and Incomplete Markets – 2

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– (Recall in complete markets, \( \lambda = 0 \) and \( r_t = g + \rho \))
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• This is pretty cool.
Comments

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  - Evidence:
    - Survey of 1 agent (me): Smooth income growth.
      Sample 2010 - 2017: Quarterly consumption volatility = 11%
An Individual Monthly Consumption Growth
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<table>
<thead>
<tr>
<th></th>
<th>Individual Growth Rate (%)</th>
<th>Individual Volatility (%)</th>
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<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
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<td>Arithmetic</td>
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</tr>
<tr>
<td>Logarithmic</td>
<td>-0.59</td>
<td>-0.66</td>
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</tbody>
</table>

(Source: Santos and Veronesi “Habits and Leverage”, 2017)
• Interest rate dynamics is critical in the model
  – Relation between cohort effects and interest rates?
  – (Too) tight relation between interest rates and prices?
    ∗ Question: Why is high interest rate \textit{positively} related to high P/D ratio? (Figure 8)
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    * Question: Why is high interest rate *positively* related to high P/D ratio?
      (Figure 8)
    – Term structure implications?

Interesting read for sure. Hopefully, the first version (this is the preliminary one) will contain more intuition behind the results to clarify the main forces.