Low Risk Anomalies?

by Schneider, Wagners, and Zechner

Discussion

Pietro Veronesi

The University of Chicago Booth School of Business
Main Contribution and Outline of Discussion

• Main contribution of the paper:

  – Proposes a skew-based explanation of several low-risk anomalies
    * Use approximate stochastic discount factor that loads on “skewness”
    * Use Merton (1974) model to justify several implications for levered equity
      · Levered equity returns are negatively skewed
      · Levered equity has higher market beta
      · Levered equity returns have less co-skewness with aggregate return
        ⇒ risk premia less than implied by CAPM

  – Test the model’s implications in the data
    * Use ex-ante option-implied skewness as proxy for co-skewness
    * Explain several low-risk strategies:
      (i) Bet-against-beta; (ii) high idiosyncratic risk; (iii) distress anomalies are implied by investors’ preference for low skewness

• Outline of discussion

  2. Comments
Merton (1974) model

- Firm $i$’s assets are lognormally distributed

$$A_{i,T} = A_{i,0} \times e^{(\mu_A - 1/2 \sigma_A^2)T + \sigma_A \sqrt{T} \epsilon_{i,T}}$$

- Firm issues zero coupon bond with face value $K$.

**Equity holders Payoff at $T$**

- Levered equity is

$$S_t = \text{Call Option}$$

- or, equivalently

$$S_t = A_t + \text{Put Option} - \text{Bonds}$$
Merton (1974) model: Levered Equity and Implicit Put Protection

- Implicit put protection (limited liability) is valuable if aversion to skewness
Merton (1974) model: Levered Equity is Negatively Skewed

A. Levered Equity vs. Leverage

B. Expected Return vs Leverage

C. Skewness vs Leverage

D. Betas vs Leverage
Data: Individual Stocks’ Equity Returns are *Positively Skewed*

- Aggregate stock returns are negatively skewed.
- Individual stock returns are *positively* skewed, on average.

Data: Individual Stocks’ Equity Returns are *Positively Skewed*

**Table. Skewness and Leverage**

Annual portfolio sort on leverage. The sample is individual stocks that are or used to be in the S&P500 index sampled at daily frequency. The sample is 1964 to 2014 (COMPUSTAT Sample).

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- But this paper is about co-skewness.
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Skewness by firm size decile and by firm $R^3$ decile. Reported for each decile are mean firm size, $R^3$, risk-neutral skewness, and realized return skewness at daily, monthly, and quarterly horizons.

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Higher Co-Skewness $\Rightarrow$ Higher Risk Neutral Skewness?
A Simple Model of Co-Skewness – 1

• We want:
  – Aggregate negative skewness
  – Positive average skewness

• Aggregate Factor (Market):

\[ F_T = F_0 \times e^{(\mu - 1/2\sigma^2)T + \sigma_F \sqrt{T} \epsilon_T} \times (1 - \delta_F J_{F,T}) \]

– where \( J_T = 1 \) with probability \( P(T) = e^{-\lambda T} \), and \( \delta_F > 0 \)
A Simple Model of Co-Skewness – 1

- We want:
  - Aggregate negative skewness
  - Positive average skewness

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- Individual firm’s assets at \( T \):
  
  \[ A_{i,T} = F_T \times e^{(\mu_A - 1/2\sigma^2)T + \sigma_F \sqrt{T} \epsilon_{i,T}}(1 + \delta_A J_{i,T}) \]
  
  - where \( J_{i,T} = 1 \) with probability \( P(T) = e^{-\lambda T} \), and \( \delta_A > \delta_F \)
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• With a large number of firms, aggregate wealth at \( T \) is

\[ W_T = \int A_{i,T} di = F_T \]
A Simple Model of Co-Skewness. – 2

- Pricing Kernel (= marginal CRRA utility at $T$ – assume zero risk free rate)

$$\pi_t = E_t \left[ W_T^{-\gamma} \right]$$
A Simple Model of Co-Skewness. – 2

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• Levered equity at time $t$ of firm $i$ is

$$S_t = \frac{E_t[\pi_T \max(A_{i,T} - K, 0)]}{\pi_t}$$
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- If $\delta_F = \delta_A = 0 \implies$ Black-Scholes model.
- If $0 < \delta_F < \delta_A \implies (i)$ $\log(F_T)$ is neg. skewed; (ii) $\log(A_{i,T})$ is pos. skewed.
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• Questions:

  – Can we find parameters so that levered equity $S_t$ is also positively skewed?
  – What is the expected return of levered equity? How does it depend on (i) market beta; (ii) SDF beta?

$$E[R^S_i] = \beta^{Mkt} E[R^F]; \quad E[R^S_i] = \beta^{SDF} E[R^F]$$

$$\frac{Cov(R^S_i, R^F)}{Var(R^F)} \quad \frac{Cov(R^S_i, R^\pi)}{Cov(R^F, R^\pi)}$$
Simple Model ($\lambda = 1, \delta_A = 0.4, \delta_F = 0.1$)

A. Levered Equity vs. Leverage

B. Expected Return vs Leverage.

C. Skewness vs Leverage

D. Betas vs Leverage

Positive Skewness of Levered Equity
Simple Model \((\lambda = 1, \delta_A = .4, \delta_F = .1)\)
Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

A. Average Return vs Mkt Beta Expected Return.

B. Average Return vs SDF–beta Expected Return

C. Average Return vs Idiosyncratic Volatility.

D. Average Return vs Total Volatility
Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

- Higher leverage
  - $\implies$ Higher market beta and SDF beta
  - $\implies$ $\beta^{Mkt} > \beta^{SDF}$
Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

- Higher leverage
  - $\implies$ Higher market beta and SDF beta
  - $\implies \beta_{Mkt} > \beta_{SDF}$

- Strategy: Bet against beta
  1. Pick a high market beta ($H$) and a low market beta ($L$) stock
  2. Long $w_L = 1/\beta_L^{Mkt}$ in $L$ stock; short $w_H = 1/\beta_H^{Mkt}$ in $H$ stock
**Simple Model** ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

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- By construction: $R_p = w_L R_L - w_H R_H$ has zero market beta.

\[ E[R^p] = \left( \frac{\beta^{SDF}_L}{\beta^L_{Mkt}} - \frac{\beta^{SDF}_H}{\beta^H_{Mkt}} \right) E[R^{Mkt}] > 0 \]

\[ \approx 1 \quad < 1 \]
**Simple Model** \((\lambda = 1, \delta_A = .4, \delta_F = .1)\)

- Higher leverage
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\]

\[
\approx 1 - \frac{\beta^{SDF}_H}{\beta^{Mkt}_H} < 1
\]

- Of course, in this model, long “low leverage” stocks and short “high leverage” stocks should also work
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\approx 1 < 1
\)

- Of course, in this model, long “low leverage” stocks and short “high leverage” stocks should also work

- How about idiosyncratic volatility and return?
Simple Model \((\lambda = 1, \delta_A = .4, \delta_F = .1)\)

**A. Average Return vs Mkt Beta Expected Return.**

**B. Average Return vs SDF-beta Expected Return**

**C. Average Return vs Idiosyncratic Volatility.**

**D. Average Return vs Total Volatility**

High Idio Vol \(\Rightarrow\) High Average Return
Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

- Now fix leverage $K = 0.9$ and change idiosyncratic asset volatility $\sigma_A$. 

![Graph A: Levered Equity](image)

- Expected Return

- Skewness

- Betas
Simple Model \((\lambda = 1, \delta_A = .4, \delta_F = .1)\)

- Now fix leverage \(K = 0.9\) and change idiosyncratic asset volatility \(\sigma_A\).
Concluding Remarks

1. Mechanism, paper, and especially empirical results are interesting.
   – Need to fix the “negative skeweness” issue for individual securities
     * Is *ex-ante* skewness still the proper measure of co-skewness in the model?
   – Need to relate it to Engle and Mistry (Journal of Econometrics 2014)
   – Need to relate it to Tim Johnson (JF, 2004)
     * Use a Merton’s model to show that high idio vol $\implies$ low risk premia.
   – Note on idio volatility
     * High leverage $\implies$ high idio vol and high risk premia
     * High asset vol $\implies$ high idio vol and low risk premia
       $\implies$ need to study interaction effects.

2. If you take the mechanism seriously, need to sort on credit risk (under $P$).
   – How big are the effects for reasonable parameters?

3. Consider other “leverage” mechanisms
   – Operating leverage
   – Labor leverage