Learning about the Neighborhood: A Model of Housing Cycles

by Michael Sockin and Wei Xiong

Discussion

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What Does This Paper Do?

Figure 1: Structure of the Static Model
What Does This Paper Do?

- Proposes a model of housing choice based on *rational* learning with differentially informed agents about the quality of a location.
- Location quality affect agents’ labor productivity, and hence amount of consumption goods produced and traded.
- Housing is treated a “good” that is complementary to other consumption goods, and it is produced by special builders.
- House price aggregate information about location quality
  - Higher house price signal higher quality $\implies$ increase housing demand (in a partial sense)
- Solve for equilibrium prices and obtain several results about housing demand, supply elasticity, learning, etc.
- Dynamic extension
  - Generate short-term momentum and long-term reversal
  - U-shape relation about price variation and housing supply
Assessment and Outline of Discussion

• Interesting and somewhat plausible mechanism
  – House prices convey information about neighborhood quality

• There are some issues about interpretation and the source of the results that need to be clarified

• Paper is still a major challenge to read and even more to understand (it is indeed Preliminary). Advise to wait for first clean version.

• My discussion:
  (A) Static asymmetric information models
  (B) The mechanism in a simplified version of model
  (C) Further comments on the paper.
Static Asymmetric Information Model - I

- Consider a model a la’ Hellwig (1980).
- Risky asset with payoff $A \sim N(0, v_A)$, and random supply $H_S = \xi \sim N(0, v_\xi)$.
- Each agent $i$ observes signal $s_i = A + \epsilon_i$, chooses $H_i$ in risky asset and $B_i$ in riskless bond to maximize the expected utility from final wealth $W_i = H_iA + B_i R$:
  $$\max_{H_i, B_i} E \left[ -e^{-\eta W_i} | s_i, P_H \right] \text{ subject to } W_0 = P_H H_i + B_i$$
- Conjecture linear equilibrium:
  $$P_H = a + b A + c \xi$$
- Vector $(A, P_H, s_i)'$ is jointly normal
  $$\implies A|s_i, P_H \sim N(\mu_A(s_i, P_H), \sigma_A) \text{ where } \mu_A(s_i, P_H) = h_1 (P_H - a) + h_2 s_i$$
  ($h_1, h_2$ and $\sigma_A$ depend on parameters.)
- Expected utility is
  $$E \left[ -e^{-\eta H_i A + B_i} | s_i, P_H \right] = -e^{-\eta H_i \mu_A + B_i + \frac{1}{2} \eta^2 H_i^2 \sigma_A}$$
Static Asymmetric Information Model - II

- Maximizing this utility is equivalent to max the exponent, obtaining

\[ H_i = \frac{\mu_A(s_i, P_H) - P_H}{\eta v_A} = \frac{h_1(P_H - a) + h_2s_i - P_H}{\eta v_A} \]

- Integrate on both sides and impose market clearing \( \int H_i di = H_S = \xi \)

\[ \int H_i di = \frac{-h_1a + (h_1 - 1)P_H + h_2\int s_idi}{\eta v_A} \implies \xi = \frac{-h_1a + (h_1 - 1)P_H + h_2A}{\eta v_A} \]

- Solve for \( P_H \):

\[ P_H = \frac{-h_1a}{1 - h_1} + \frac{h_2}{1 - h_1} A - \frac{\eta v_A}{1 - h_1} \xi \]

- From Bayes formula, \( h_2 > 0 \) and \( \text{sign}(h_1) = \text{sign}(b) \).

\[ \implies b > 0 \text{ and thus } 1 - h_1 > 0. \]

* \( \implies \) information effect \( (= h_1) \) always weaker than price effect \( (= 1) \).

* \( P_H \uparrow \implies E[A|P_H] \uparrow \) but makes asset more costly. Latter effect dominates.

\[ \implies c < 0 \implies \text{higher supply decrease price} \]
Static Asymmetric Information Model with Power Utility

- Negative exponential utility is not too appealing as
  - A. No wealth effect
  - B. Relative risk aversion increases with wealth
  - B. Equilibrium prices can be negative

- What happens if we use another utility function? Consider power utility:
  \[
  \max_{H_i, B_i} E \left[ \frac{(H_i A + B_i R)^{1-\eta}}{1 - \eta} \bigg| s_i, P_H \right]
  \]

- A linear or log-linear equilibrium price function does not work in this case.

- Taking FOC of Lagrangean does not help either:
  \[
  E \left[ A (H_i A + B_i)^{-\eta} \bigg| s_i, P_H \right] = \lambda_i P_H
  \]
  - Problem: \( P_H \) enters on both sides, and the non-linearity messes up aggregation
    \( \int H_i d_i = H_S \)
  - It can be solved numerically, or using approximation to small payoffs (e.g. Perez (2004, RFS)).
This paper uses power utility

- How could it solve the non-linear fixed-point problem discussed earlier?
- Value of housing depends on complementarity with another consumption good, and there are no riskless bonds to purchase.

Consider a super stripped down version of model, just to see this mechanism:

\[
E \left[ \frac{H_i^{1-\eta}}{1-\eta} C \mid s_i, P_H \right] \quad \text{subject to } W_0 = P_H H_i
\]

Assume:

- \( C = e^A \) is an exogenous random amount of good provided by a 3rd party or nature, e.g. schools, infrastructure, earthquakes
- Supply of asset \( H \) is random: \( H_S = e^\xi \).

FOC is far simpler:

\[
H_i^{-\eta} E \left[ e^A \mid P_H, s_i \right] = P_H
\]

Assume \( p_H = \log(P_H) \) is linear:

\[
p_h = a + bA + c\xi
\]
• Then the same learning result above implies $A \sim N(\mu_A(p_H, s_i), \sigma_A)$ and hence

$$E[e^A|P_H, s_i] = e^{\mu_A(p_H, s_i) + \frac{1}{2}\sigma_A}$$

• It is clear it can be solved, as we can write:

$$H_i^{-\eta} e^{h_1(p_H-a) + h_2 s_i + \frac{1}{2}\sigma_A} = e^{p_H}; \quad \Rightarrow \quad H_i = e^{h_1 s_i} e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{v_A}{2}}$$

• Integrate both sides

$$\int H_i di = \int e^{h_1 s_i} di \cdot e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{v_A}{2}} \quad \Rightarrow \quad e^\xi = e^{h_2 s_i} e^{\frac{1}{\eta^2} h_2^2 v_s} e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{v_A}{2}}$$

• and solve for the price

$$p_H = \frac{\left(\frac{h_2^2}{2\eta} v_s - h_1 a + \frac{1}{2}\sigma_A\right)}{(1-h_1)A} + \frac{h_2}{(1-h_1)} A - \frac{\eta}{(1-h_1)} \xi$$

- As before, $h_1 < 1$ and information effect is weaker than price effect.

$\Rightarrow$ Price $p_H$ is increasing in $A$ and decreasing in supply $\xi$.

$\Rightarrow$ Demand $H_i$ is increasing in $s_i$ and decreasing in $p_H$. 

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Sockin and Xiong Paper - III

• The barebone model is too “bare”. Sockin and Xiong add:
  – Labor / production at time $t = 2$ (after housing choice made at $t = 1$) with complementarity in consumption
    * Critical is a common productivity shock $A$;
    * Critical that there is no asymmetric information at $t = 2$ as consumption depend on realized productivity $A_j$ of everybody else.
  – Suppliers of housing:
    * From their profit maximization: $H_S = P^k_He^{k\xi}$
    * $k$ = supply elasticity.
    * Questions here:
      1. Household budget constraint: $P_HH_i + \int P_jC_j(i)dj = P_i e^{A_i\ell_i} + w_i$
         $w_i = \text{wage of the builder} = P_HH_i$. This suggests builder $i$ sell his home to himself (?). Shouldn’t we have $w_i = P_HH_S$ ?
      2. Builders belong to household (see budget constraint), but they have superior information (observe $\xi$), and their utility and labor costs does not enter the household utility?
To see (some) of the impact of these ingredients, let’s add to the simple model:

1. Utility with different weights

\[ E \left[ \frac{H_i^{1-\eta}}{1-\eta} C^{mc} | s_i, P_H \right] \text{ subject to } W_0 = P_H H_i \]

2. Price-dependent supply (obtained from optimality of builders)

\[ H_S = P_H^k e^{k\xi} \]

The same calculations above give:

\[ H_i = e^{\left(\frac{\eta c}{\eta} h_2\right) s_i + \left(\frac{1}{\eta} (\eta c h_1 - 1)\right) P_H + \left(\frac{1}{2} \frac{\eta c^2 v_A - \eta c h_1 a}{\eta} \right) } \]

\[ p_H = \frac{\frac{1}{2} \left(\frac{\eta c^2}{\eta} \right) h_2^2 v_s - \eta c h_1 a + \frac{1}{2} \eta c^2 v_A}{(1 + \eta k - \eta c h_1)} + \frac{\eta c h_2}{(1 + \eta k - \eta c h_1)} A - \frac{\eta k}{(1 + \eta k - \eta c h_1)} \xi \]

Note that again \( b > 0 \) and \( c < 0 \): Higher \( A \) or lower \( \xi \) results in higher price.

Now however, demand \( H_i \) may be increasing in price if \( \eta c h_1 - 1 > 0 \)

– “Signal” effect may be stronger than “cost” effect.
Example: The Impact of Supply Elasticity
Global versus Local Information Effects

- These models however tend to have “global signal effects”
  - Demand is either always decreasing or always increasing in the price $P_H$.
  - There are cases in which ‘backward-bending” demand is reasonable.
    * Wine, Stocks, Houses (?)
- Barlevy and Veronesi (2003) show that in model with ($A$) informed and uninformed traders, and ($B$) only two regimes (good or bad)
  $\implies$ Strong local information effects $\implies$ backward-bending demand function (and stock market crash).

\[ \begin{align*}
  P'' &= \bar{P} \\
  P' &= P \\
  \frac{1}{P} \\
  x^U(P) \\
\end{align*} \]
Dynamic Model

• Much of the implications discussed in the paper are about dynamics.
  – Overlapping generations with segmentation: Each cohort trade only with itself.
  – \[ \Rightarrow \] Repeat of the same model above over and over.
  – Link between times only through dynamics of common factors and learning:
    \[
    A_t = \rho^A A_{t-1} + Z_t^A; \quad \xi_t = \rho^\xi \xi_{t-1} + Z_t^\xi
    \]
  – Information structure: \( A_t \) and \( \xi_t \) revealed with 2 periods lag. (Why 2 periods and not 1?)

• Suppose only one period lag (at \( t \) we know \( A_{t-1} \) and \( \xi_{t-1} \)), then learnings is identical, but price function at \( t \) must include \( A_{t-1} \) and \( \xi_{t-1} \), as they are known and determines agents expectations. Same calculations as above give:

  \[
  p_H (t) = a + bA_t + c\xi_t + dA_{t-1} + e\xi_{t-1}
  \]

  (coefficients are much more involved even in this simple case)

• Demand now depends on the current price “dynamics”
Dynamic Model and Momentum - I

• The price of the house now depend on lagged state-variables, which are persistent.
• Sockin and Xiong show that the persistence of state variables is critical to generate short-term momentum (short?) and long-term reversal.
• To generate momentum and long-term reversal, one normally needs time varying risk premia that depend on two factors with different frequencies (see e.g. Albuquerque and Miao (JET, 2014)).
• In this setting, it is not clear what is the source of the variation in risk premia (conditional on public information).
• Indeed, given the lack of trading across cohorts, the house prices at every $t$ has a different marginal buyer, as there is no wealth transfers over time.
• To some extent, the prices $p_H(t)$ are really prices of “different securities” as they pertain to different “neighborhoods” (cohorts).
Dynamic Model and Momentum - II

• Indeed, the interpretation of the results is a bit tricky here.
• We could consider the identical model in a “spatial” setting, still with
  \[ A_t = \rho_A A_{t-1} + Z_t^A, \quad \xi_t = \rho_\xi \xi_{t-1} + Z_t^\xi \]
  but now \( t \) denotes a different “town”, rather than time.
• Agents in town \( t \) learn about aggregate local productivity by observing the adjacent town \( t - 1 \) (or \( t - 2 \)).
• All the results about pricing would be identical, but it is pretty clear that \( p_H(t) \) are prices of a different securities,
  \[ \implies p_H(t) - p_H(t - 1) \] is not a return, but just price difference across different adjacent towns.
Dynamic Model and Supply Elasticity

• With the same caveat, it is interesting the relation between supply elasticity and price variation.

• In the paper, supply elasticity is about external supply shock.
  – Very low elasticity $\Rightarrow$ price do not respond to “supply noise” $\Rightarrow$ no asymmetric information (as price fully revealing)
  – Extremely high elasticity $\Rightarrow$ house price too noisy for non-fundamental reason $\Rightarrow$ no learning from prices

• This is an interesting channel.

• Do I believe it explain difference between New York and Las Vegas?
Other Minor Comments

• The good “house” in the model could be interpreted in any way as well
  – No lump sum investment, no durable good, no tradeoff purchase / rent

• Assumption of closed neighbors
  – It is not too clear what it means. Are these neighbors, or cities, or states?

• Large number of parameters and moving parts
  – Maybe want to simplify the model a bit?

• What is the numeraire in the model?
  – In these models with utility from final wealth, the bond is normally the numeraire. But there is no bond here, and all consumption goods / houses have non-unit prices.
Conclusion

• Very rich model, and perhaps not everything is necessary, especially to convey intuition of results. But authors endogenize almost everything, which is interesting.

• Need some more thinking about interpretation.

• However, the mechanism per se’ seems plausible: Learning dynamics may generate some short-term momentum and long-term reversals, as others have shown already in the asset pricing literature.

• It would be interesting to push the “information story” further, generating local backward bending demand curves, which may have dramatic price effects.
Bayes Formula

- The following work both for Helwig model and for the simplified case discussed in the text. The only difference is whether we use $P$ or $\log P$. The joint distribution (from the perspective of the investor) of $(A, \log P, s_i)$ is

$$
\begin{pmatrix}
A \\
\log P \\
s_i
\end{pmatrix}
\sim N
\begin{pmatrix}
0 \\
a \\
0
\end{pmatrix},
\begin{pmatrix}
v_A & bv_A & v_A \\
v_A & b^2v_A + c^2v_\xi & bv_A \\
v_A & bv_A & v_A + v_s
\end{pmatrix}
$$

or

$$
\begin{pmatrix}
A \\
\log P \\
s_i
\end{pmatrix}
\sim N
\begin{pmatrix}
\mu_1 \\
\mu_2
\end{pmatrix},
\begin{pmatrix}
\Sigma_{11} & \Sigma_{12} \\
\Sigma_{21} & \Sigma_{22}
\end{pmatrix}
$$

- Using the properties of conditions normals, we have (let $p_H = \log (P_H)$)

$$(A) \mid_{p_H, s_i} \sim N \left( \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} \left( \begin{pmatrix} p_H \\ s_i \end{pmatrix} - \mu_2 \right), v_A - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}' \right)$$

where

$$
\Sigma_{12} = \begin{pmatrix}
bv_A & v_A \\
v_A & b^2v_A + c^2v_\xi & bv_A
\end{pmatrix}
$$

$$
\Sigma_{22} = \begin{pmatrix}
bv_A & c^2v_\xi & bv_A \\
BV_A & bv_A & v_A + v_s
\end{pmatrix}
$$
Therefore, algebra gives (this is tedious, but for completeness):

$$\Sigma_{22}^{-1} = \frac{1}{(b^2v_A + c^2v_\xi) v_s - b^2v_A^2} \begin{pmatrix} v_s & -bv_A \\ -bv_A & b^2v_A + c^2v_\xi \end{pmatrix}$$

so that

$$\Sigma_{12}\Sigma_{22}^{-1} = \frac{1}{(b^2v_A + c^2v_\xi) (v_A + v_s) - b^2v_A^2} \begin{pmatrix} bv_A & v_A + v_s \\ -bv_A & b^2v_A + c^2v_\xi \end{pmatrix} \begin{pmatrix} v_A + v_s & -bv_A \\ -bv_A & b^2v_A + c^2v_\xi \end{pmatrix}$$

$$= \frac{1}{(b^2v_A + c^2v_\xi) (v_A + v_s) - b^2v_A^2} \left[ bv_A v_A + v_s - bv_A^2, -b^2v_A^2 + b^2v_A + c^2v_\xi v_A \right]$$

$$= \frac{1}{b^2v_A (v_A + v_s - v_A) + c^2v_\xi (v_A + v_s)} \left[ bv_A (v_A + v_s - v_A), c^2v_\xi v_A \right]$$

and

$$\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}' = \frac{1}{b^2v_A (v_A + v_s - v_A) + c^2v_\xi v_s} \left[ bv_A (v_A + v_s - v_A), c^2v_\xi v_A \right] \begin{pmatrix} bv_A \\ v_A \end{pmatrix}$$

$$= \frac{v_A^2}{b^2v_A v_s + c^2v_\xi (v_A + v_s)}$$
• Finally

\[ A|_{s_i, \log P} \sim N (h_1 (p_H - a) + h_2 s_i, \overline{\nu}_A) \]

• where

\[
\begin{align*}
    h_1 &= \frac{b v_A v_s}{v_s b^2 v_A + v_\xi (v_A + v_s) c^2} \\
    h_2 &= \frac{c^2 v_A v_\xi}{v_s v_A b^2 + v_\xi (v_A + v_s) c^2} \\
    \overline{\nu}_A &= v_A - v_A^2 \frac{b^2 v_s + c^2 v_\xi}{b^2 v_A v_s + c^2 v_\xi (v_A + v_s)}
\end{align*}
\]

• Note that \( \text{sign}(h_1) = \text{sign}(b) \) and \( h_2 > 0 \).