The Peso Problem Hypothesis and Stock Market Returns*

Pietro Veronesi
Graduate School of Business
University of Chicago
1101 E. 58th St.
Chicago IL 60637 USA
ph: (773) 702 6348
e-mail: pietro.veronesi@gsb.uchicago.edu

This Version: November 2002

JEL Classification Number: G12

Keywords: Peso Problem, Learning, Uncertainty

*I thank Nick Barberis, Gadi Barlevy, Domenico Cuoco, Chris Good, Andrew Metrick, Luis Viceira, Jonathan Wright, and seminar participants at Harvard, Chicago GSB, Northwestern, Columbia, Duke, Berkeley, London Business School, UCLA, Rochester, UBC and Michigan. I am especially grateful to John Y. Campbell for his useful comments while writing my Ph.D. dissertation at Harvard University, on which this article heavily draws on. Any remaining errors are my own.

Abstract

The Peso Problem Hypothesis has often been advocated in the financial literature to explain the historically puzzlingly high risk premium of stock returns. Using a dynamic model of learning, this paper shows that the implications of the Peso Problem Hypothesis are much more far reaching than the ones commonly advocated, implying most of the stylized facts about stock returns. These include high risk premia, time-varying volatility, asymmetric volatility reaction to good and bad news, excess sensitivity of price reaction to dividend changes and thus excess return volatility,

Introduction

Stock market returns have a number of features that have been puzzling financial economists for long. Among others, these include a high realized risk premium, excess volatility, changing volatility, asymmetric reaction of volatility to good and bad news.[1] The financial literature has put forward various models to explain one or more of these stylized facts. As an example, a number of papers have argued that the puzzlingly high risk premium of stock returns may be due to a "Peso problem situation" (see e.g. Rietz (1988), Brown, Goetzmann and Ross (1995), Danthine and Donaldson (1998), Goetzman and Jorion (1999a,b)): that is, since no catastrophic event ever realized during the sampling period to the US economy *ex post* realized returns are high even if *ex ante* expected returns are low.

However, the possibility that a bad event *could* happen may affect investors expectations in many other ways aside from generating higher returns *ex-post*. For example, referring to a comment by Robert C. Merton about the high volatility during the 30's, Schwert (1989) writes:

> '... the Depression was an example of the so called "Peso problem," in the sense that there was legitimate uncertainty about whether the economic system would survive.... Uncertainty about whether the "regime" had changed adds to the fundamental uncertainty reflected in past and future volatility of macroeconomic data.'

This paper builds on this intuition to explore the implications for the *ex-post* behavior of stock returns under the assumption that a bad state could happen but it did not during the sample period. Using an intertemporal, rational expectations model of learning, this paper

[1] The literature on each of these findings is immense, and I refer the reader to classic textbooks, such as Campbell, Lo and MacKinlay (1997) or Cochrane (2000) for references and discussion. Classic early references are Mehra and Prescott (1985) for the equity premium; Shiller (1981) and LeRoy and Porter (1981) for excess volatility; Engle (1982), Bollerslev (1986) and Nelson (1991) for the modeling of time-varying volatility; French et al. (1987), Schwert (1989, 1990), Hamilton and Lin (1997) for a characterization of time-varying return volatility and macro-economic factors.

shows that the Peso Problem Hypothesis has much more far reaching implications than just a high *realized* equity premium. Indeed, I show that all the stylized facts described above are implied by a model where there is a very small probability that the economy may enter into a very long recession.

Specifically, suppose that economic fundamentals – call them "dividends" – are generated by a diffusion process whose drift is not observable. For simplicity, the drift is assumed constant for most of the time. Suppose now that at every instant, there is an *ex-ante* very small probability that the economy enters into a long recession. That is to say, there is a very small probability that the drift changes to a lower value and there is also a small probability to revert back to normal. Since investors do not observe the true drift but can only learn about it by observing the past realizations of fundamentals, this model implies that investors' uncertainty about the true drift fluctuates over time. For example, suppose that at some time t investors' conditional probability of the normal state $\pi(t)$ is close to 1. A sequence of negative dividend innovations will tend to decrease $\pi(t)$ driving it closer to $\frac{1}{2}$, that is the point of maximum uncertainty. It is intuitive that when there is more uncertainty, investors' beliefs tend to react more to news. Hence, since in a rational expectations model the stock price depends on investors' conditional expectations, during period of high uncertainty investors' expect to react heavily to news and hence they also expect that returns are more volatile. As a consequence, they require a higher discount for holding the stock. This feedback effects from the sensitivity of investors' beliefs to news onto the stock price itself determines most of the results. Indeed, Veronesi (1999) shows that this model implies that the equilibrium price of the asset is an increasing and convex function of $\pi(t)$ and studies the general properties of the model. In particular, the stock price is very steep for $\pi(t)$ close to one and rather flat for $\pi(t)$ close to zero which yields to a stock-market overreaction to bad news in good times and an underreaction to good news in bad times.

Building on the results from Veronesi (1999), this paper formally studies the *ex-post* features of stock returns under the assumption that during the sampling period it never occurred that the drift of the dividend process shifted to the lower one: that is, the economy never entered a long recession. This assumption formalizes the "Peso problem hypothesis" and captures the spirit of Merton's comment reported above. Conditioning on this assumption, I show the following: first and most obviously, there is a positive bias on the mean realized returns. This bias is positively albeit not-linearly related to stock return volatility and to the degree of risk aversion. Second, returns display "excess volatility", in the sense that they are more volatile than the underlying fundamentals (dividends). This is due to an implied excess sensitivity of prices to dividend changes. Third, the volatility of returns changes over time, it is mean reverting and it is negatively correlated with realized returns, increasing after bad news and decreasing after good news. I finally perform Monte Carlo simulations to gauge the size of the effects reported in the theoretical section.

The paper is organized as follows: in section 1, I review the model and the results in Veronesi (1999). Section 2 investigates the properties of stock returns under the "Peso Problem Hypothesis." Section 3 relates the model to U.S. data and describes the results of Monte Carlo simulations. Section 4 concludes. All results are given in the appendices.

1. The Model

The model is similar to Campbell and Kyle (1993), Wang (1993) and Veronesi (1999), and thus I describe it only briefly. I consider an economy with a single physical consumption good, which can be allocated to investment or consumption. Two investment assets are available to investors/consumers: a risky asset and a riskless asset. The risky asset yields a stochastic dividend rate $D(t)$, described by the linear process:

$$dD = \theta dt + \sigma d\xi \tag{1.1}$$

where the assumptions about $\theta(t)$ are described below, σ is a constant, and $\xi(t)$ denotes a Wiener process. The supply of the risky asset is normalized to unity. Instead, the riskless asset is infinitely elastically supplied and yields a constant rate of return r.

Finally, I assume that investors/consumers are endowed with a CARA utility function over consumption $U(c,t) = -e^{-\rho t-\gamma c}$, where ρ is the parameter of time preference and γ is the coefficient of absolute risk aversion.

1.1. Modeling a Peso Problem Situation

I now capture the spirit of Merton's quote in the Introduction by assuming the following: (1) during the sample period $[0,T]$ the drift rate of dividends has been a constant $\theta(t) = \overline{\theta}$; (2) there is a small *ex-ante* chance that the drift rate of dividends shifts to a low state $\theta(t) = \underline{\theta} < \overline{\theta}$; and (3) investors do not actually observe $\theta(t)$ and hence are unaware of whether a shift ever occurred or not. This last assumption is the key ingredient to generate the additional implications of the Peso-Problem situation uncovered in this paper, as it is responsible for the additional "uncertainty about whether the "regime" had changed" that "adds to the fundamental uncertainty," to use the words of Merton.

More specifically, I assume that during an infinitesimal time interval Δ, there is probability $\lambda\Delta$ that $\theta(t)$ shifts to the low state $\underline{\theta}$ from the normal state $\overline{\theta}$. Moreover, I also assume that in this event there is yet probability $\mu\Delta$ that the state would shift back to the normal state $\overline{\theta}$, with $\mu >> \lambda$. Thus, in this model a bad state is characterized by two parameters: How low the drift rate $\underline{\theta}$ is, and for how long it will last. To be consistent with the assumption of a Peso Problem situation, the probability of shifting to the bad state λ must be chosen very small, such as $\lambda = .005$, which implies a shift once every 200 years. However, in order to ensure that unconditionally the economy is growing, I will also be assuming $\mu >> \lambda$. Sections 2 and 3 will further discuss these issues and the parameter choices.

1.2. Investors' Posterior Probability

Investors only observe the realized series of dividends. Let $\{\mathcal{F}(t)\}$ be the filtration generated by the dividend stream $(D(\tau))_{\tau=0}^{t}$ and define the posterior probability of the good state $\overline{\theta}$ by

$$\pi(t) = Pr(\theta(t) = \overline{\theta}|\mathcal{F}(t)).$$

We then have:

Lemma 1.1: The posterior probability $\pi(t)$ satisfies the stochastic differential equation:

$$d\pi = (\lambda + \mu)(\pi^s - \pi)dt + h(\pi)dv \tag{1.2}$$

where $\pi^s = \mu / (\mu + \lambda)$, $h(\pi) = \left(\frac{\overline{\theta} - \underline{\theta}}{\sigma}\right) \pi(1 - \pi)$ and $dv = \frac{1}{\sigma}\left[dD - E\left(dD|\mathcal{F}(t)\right)\right]$. Moreover, dv is a Wiener Process with respect to $\mathcal{F}(t)$.

Proof: See Liptser and Shiryayev (1977, pg. 348). See also David (1997). □

Notice that $(\pi^s, 1 - \pi^s)$ is simply the stationary distribution of the two states. Also, notice that even if the drift $\theta(t)$ shifts between two discrete states, the process for the posterior distribution $\pi(t)$ is continuous.

1.3. The Equilibrium

A rational expectations equilibrium is defined as follows:

Definition 1.1: A *Rational Expectations Equilibrium* (REE) is given by $(P(D, \pi), X(W, P, D, \pi), c(W, P, D, \pi))$, where $P(D, \pi)$ is the price level for given dividend level D and belief π, $X(W, P, D, \pi)$ and $c(W, P, D, \pi)$ are the demand for the risky asset and the consumption level for given level of wealth W, price P, dividend and belief, respectively, such that

1. **Utility Maximization:** $(c(\cdot), X(\cdot))$ maximizes investors' expected intertemporal utility, i.e.

   $$\max_{c,X} E\left[\int_0^\infty U(c, s)ds|\mathcal{F}(0)\right]$$

   subject to an intertemporal budget constraint and a transversality condition;

2. **Market Clearing:** $P(\cdot, \cdot)$ adjusts so that $X(W, P(D, \pi), \pi) = 1$ for every W and every pair (D, π)

The assumption of CARA utility function has the convenient property that the demand of risky asset $X(W, P, D, \pi)$ is independent of wealth level W. Therefore, I will denote it as $X(P, D, \pi)$ only. Similarly, consumption won't depend on P and D.

1.3.1. Equilibrium Prices

The following proposition is proven in Veronesi (1999).

Proposition 1.1: (a) Let the conditional expectation of future dividends be denoted by

$$P^*(D, \pi) \equiv E\left[\int_0^\infty e^{-rs} D(t+s)ds \mid D(t) = D, \pi(t) = \pi\right].$$

Then, there exists a REE where the price function $P(D, \pi)$ is given by:

$$\begin{aligned} P(D, \pi) &= p_0 + S(\pi) + P^*(D, \pi) && (1.3) \\ &= p_0 + S(\pi) + p_D D + p_1 + p_\pi \pi && (1.4) \end{aligned}$$

where $p_0 = -\frac{\gamma\sigma^2}{r^2}$, $p_D = \frac{1}{r}$, $p_1 = \frac{\underline{\theta}}{r^2} + \left(\frac{\overline{\theta}-\underline{\theta}}{r^2(\lambda+\mu+r)}\right)\mu$, $p_\pi = \frac{(\overline{\theta}-\underline{\theta})}{r(\lambda+\mu+r)}$ and $S(\cdot)$ is a negative, convex and U-shaped function of $\pi \in [0, 1]$ which satisfies the differential equation (4.3) in the Appendix. A.

(b) Let $\lambda = \mu = 0$ and let $\theta = \overline{\theta}$. Then the solution reduces to

$$P(D, \overline{\theta}) = p_0 + p_D D + p_\theta \overline{\theta} \qquad (1.5)$$

where p_0 and p_D are in part (a) and $p_\theta = 1/r^2$.

The fact that $S(\pi)$ is negative implies that the equilibrium price function $P(D, \pi)$ in (1.3) is given by a discount $p_0 + S(\pi) < 0$ over discounted expected dividends $P^*(D, \pi)$. Since $P^*(D, \pi)$ is the price that would occur if investors were risk neutral, I will refer to it as the *risk-neutral price*. Since $S(\pi)$ is U-shaped, this discount is smaller for extreme values of π (i.e. for π close to 0 and 1) than for π close to $\frac{1}{2}$. Figure 1 plots $P^*(D, \pi)$ and $P(D, \pi)$ for the calibrated parameter in Section 3.[2] Finally, notice that part (b) contains the price function of the asset in the case where investors know that the drift rate is $\overline{\theta}$ and that it is a constant. This will allow us also to address the point of model misspecification.

Veronesi (1999) contains additional results in terms of conditional expected returns and conditional volatility. I refer the reader to my earlier work, and rather proceed to the implications of a peso-problem situation for returns.

2. Stock Returns under the "Peso Problem Hypothesis"

This section investigates the theoretical properties of returns under the "Peso Problem Hypothesis," as modeled in the previous section. Specifically, following Bossaerts (1996), I take the perspective of the econometrician and investigate how investors' conditional expectation is affected by the fact that *ex post* no change in regime actually occurred (but they didn't know). That is to say, if during the sample period $[0, T]$ the state has been $\overline{\theta}$, investors will only observe realizations of the process $dD = \overline{\theta}dt + \sigma d\xi$. This sequence of observations has a specific effect on investors posterior probability $\pi(t)$, through the updating rule (1.2), that on average will tend to be concentrated in an area close to one. These sequences of dividends and probabilities in turn have implications on the time series of *equilibrium* stock prices and therefore on the time series of returns, which is the ultimate object of the investigation. The next two sections investigate these effects.

2.1. The "Peso Problem" and the Small-Sample Bias in Expected Returns

For notational convenience, I will let $E^{\overline{\theta}}[\cdot\ |\mathcal{F}(t)]$ denote the expectation operator under the assumption that investors' information is described by $\mathcal{F}(t)$ – that is, the probability $\pi(t)$ – but dividend realizations are generated by the process (1.1) with $\theta(t) = \overline{\theta}$. As in Campbell

[2] All plots use the parameters assumed in Table 1. The reader is referred to Veronesi (1999) for other similar plots with different parameter values.

and Kyle (1993), Wang (1993) and Veronesi (1999), it is convenient to state the results about returns in terms of dollar excess returns. That is, I will let $dQ = (D - rP)\,dt + dP$ denote the return on a zero investment portfolio long one share of the asset and financed by borrowing at the risk-free rate r. As in proposition 1.1, a star "*" will denote quantities under risk-neutrality. I now obtain the implication for conditional expected returns under risk-neutrality and under risk-aversion.

Proposition 2.1: Let $\theta(t) = \overline{\theta}$ during the sample period $[0, T]$. Then:

(a) If investors are risk neutral, the conditional expected return is positive and given by:

$$E^{\overline{\theta}}[dQ^* \mid \mathcal{F}(t)] = \frac{\overline{\theta} - \underline{\theta}}{r}(1-\pi)\left(1 + \frac{\overline{\theta} - \underline{\theta}}{(\lambda + \mu + r)\sigma} h(\pi)\right) dt \tag{2.1}$$

$$= \frac{\overline{\theta} - \underline{\theta}}{\sigma}(1-\pi)\sigma_{P^*}(\pi) dt \tag{2.2}$$

where $\sigma_{P^*}(\pi) = \frac{1}{r}\left(1 + \frac{\overline{\theta} - \underline{\theta}}{(\lambda+\mu+r)\sigma} h(\pi)\right)$ is the volatility of dQ^* under risk-neutrality.

(b) If investors are risk averse, then the expected return are given by:

$$E^{\overline{\theta}}[dQ \mid \mathcal{F}_t] = \left(\gamma\sigma + (\gamma r p_\pi + f'(\pi))h(\pi) + \left(\frac{\overline{\theta} - \underline{\theta}}{\sigma}\right)(1-\pi)\right)\sigma_P(\pi) dt \tag{2.3}$$

where $\sigma_P(\pi) = \sigma_{P^*}(\pi) + S'(\pi)h(\pi)$ is the volatility of dQ, and $f(\pi)$ is a U-shaped, convex function of π that satisfies the ODE (4.1) in Appendix A.

Part (a) shows that if we suppose the state has been $\theta(t) = \overline{\theta}$ over the sample period, the time series of excess returns should display a positive drift even under risk neutrality. This is of course not surprising and it has been discussed already in the literature on the Peso Problem (see e.g. Rietz (1988), Danthine and Donaldson (1998)). However, equation (2.2) also shows that we should observe a positive relationship between excess returns and volatility, although the coefficient to the stock return volatility $\sigma_{P^*}(\pi)$ is not constant.

Part (b) shows a similar positive relationship between returns and volatility, but this time with a positive risk aversion coefficient. A more intuitive formula can be obtained through the decomposition:

$$E^{\overline{\theta}}[dQ \mid \mathcal{F}_t] = E[dQ \mid \mathcal{F}_t] + E^{\overline{\theta}}[dQ^* | \mathcal{F}_t] + S'(\pi)h(\pi)E^{\overline{\theta}}[dv \mid \mathcal{F}_t].$$

Thus, the presence of risk aversion affects the small sample bias in stock returns. In fact, we see that the expected return conditional on $\theta(t) = \overline{\theta}$ is given by the *ex-ante*, required expected return $E[dQ \mid \mathcal{F}_t]$ (which is the quantity the econometrician is interested in), plus two terms which depend on the actual state $\overline{\theta}$. The first, $E^{\overline{\theta}}[dQ^* \mid F_t]$, is the same positive bias that is realized even under risk-neutrality. The second, $S'(\pi)h(\pi)E^{\overline{\theta}}[dv \mid \mathcal{F}_t]$ is an extra term which is due to risk aversion. We find that if the state is $\theta = \overline{\theta}$ and $\pi > \hat{\pi}$ where $\hat{\pi}$ is such that $S'(\widehat{\pi}) = 0$, this is a positive term. Hence, if the state has been the normal one over the sample period, the positive bias is higher than in the case of risk neutrality. The simulation results will show the quantitative effects of this bias in returns.

2.2. The "Peso Problem" and Return Volatility

In this subsection I investigate in more detail the process for the volatility $\sigma_{P^*}(\pi) = \frac{1}{r}\left(1 + \frac{\overline{\theta}-\underline{\theta}}{(\lambda+\mu+r)\sigma}h(\pi)\right)$ introduced in (2.2), under the assumption that $\theta(t) = \overline{\theta}$.

Proposition 2.2: If $\theta(t) = \overline{\theta}$ and $\pi(t) > \frac{1}{2}$ over the sample period, then:

$$d\sigma_{P^*} = a(\sigma_{P^*})\, dt - b(\sigma_{P^*})\, d\xi \qquad (2.4)$$

where $a(\sigma_{P^*})$ and $b(\sigma_{P^*})$ are two explicit functions of σ_{P^*}, given in Appendix B. In addition, $b(\sigma_{P^*}) > 0$.

In (2.4) $d\sigma_{P^*}$ depends only on the past values of σ_{P^*}, through the two functions $a(\sigma_{P^*})$ and $b(\sigma_{P^*})$, given in the Appendix B and plotted in Figure 2 for calibrated parameters (see next Section). Moreover, the stochastic element is given by the Wiener process $\xi(t)$. Notice that since $b(\sigma_{P^*}) > 0$, the coefficient of $d\xi$ is negative, as we would expect: under the assumption that $\pi > \frac{1}{2}$ over the sample period, positive shocks to fundamentals decrease volatility while negative shocks increase it. In addition, the drift rate $a(\sigma_{P^*})$ is positive for low σ_{P^*} and negative for high σ_{P^*}, implying a (non-linear) mean reverting process for volatility σ_{P^*}.

Finally, σ_{P^*} characterized in proposition 2.2 is the "risk-neutral" volatility. But risk aversion implies that $\sigma_P(\pi) = \sigma_{P^*}(\pi) + S'(\pi)h(\pi)$ (see proposition 2.1 (b)), and thus a higher volatility when $\pi > \hat{\pi}$ and lower when $\pi < \hat{\pi}$, where $\hat{\pi}$ is such that $S'(\hat{\pi}) = 0$. Since the "Peso problem" hypothesis requires $\theta(t) = \overline{\theta}$ over the sample period, the relevant case is for π very large. Hence, we should expect to observe larger volatility than what is implied by proposition 2.2, but with the same qualitative behavior; that is, it increases after negative shocks to fundamentals and decreases after positive shocks.

2.3. The "Peso Problem" and the Survival of Markets

The above discussion is also related to Brown *et al.* (1995) and the literature on survival of markets (see Goetzman and Jorion (1999a,b)). Brown *et al.* (1995) investigate the *ex post* statistical behavior of the time series of returns which have "survived" for a sample period $[0, T]$. They assume a simple diffusion process for (log) prices and postulate that the market does not survive if the price hits an absorbing lower bound. Under these assumptions, they show that if the price series did not hit the lower bound, the implied time series of returns should display many of the features actually observed in U.S. data series, including "puzzling" risk premia and mean reversion. They also show that the bias in expected returns should increase with return volatility, because the latter increases the probability of hitting the lower bound. As an example, they often suggest that since emerging markets have highly volatile stock returns, they should display abnormal excess returns if they survived *ex post*. Their model, however, does not address the issue of the possible sources of return volatility.

My model offers another explanation for the abnormal excess returns realized in emerging markets, which is also consistent with the substantial volatility of market returns. In periods of high uncertainty over the true state of the economy (which may include many factors, e.g. political ones), investors react heavily to news, and therefore stock returns should be highly volatile. If *ex post* the market survived, it means that the state of the world has been the favorable one over the sample period. Hence, proposition 2.1 applies and the expected returns should be positive and substantial.

3. Monte Carlo Simulations

In this section I use Monte Carlo simulations to study the characteristic features of the present model and compare them to the stylized facts of U.S. stock returns. The first step is to calibrate some of the parameters under the null hypothesis that $\theta(t) = \overline{\theta}$ over the sample period. These are $\overline{\theta}$, σ and the real interest rate r. We are subject to the difficulty that I have to assume a Gaussian dividend process for tractability reasons, whereas a log-normal process would probably be more appropriate. Even though this is just a rough approximation, I will use the mean and the standard deviation of dividend growth rates for $\overline{\theta}$ and σ, respectively.[3]

In addition, under the null hypothesis, no change in state ever occurred over the sample period, which implies I need to choose a very small value for λ. The choice $\lambda = .005$ implies an expected time for a shift of about 200 years. If a downward shift occurs, I assume that there is a $\underline{\theta} = 5\%$ average decrease in dividends for an expected time of 20 years ($\mu = .05$). These choices make a downward shift quite dramatic and are meant to capture the sense of the quotation in the introduction about the Peso problem that investors were facing during the 30's. However dramatic a shift would be, notice that the unconditional probability to be in the favorable state is around 0.91 and the unconditional expected θ is 0.009. Finally, the risk-free real interest rate r has been chosen to be 3% which is slightly above to the historical mean (less than 2%). This makes the estimates more conservative: low interest rates only amplify the price sensitivity to changes in dividends and to changes in beliefs, because dividends in the distant future have a greater weight in the determination of today's price, and all the effects will be more pronounced. Table 1 report the parameter values in the calibration.

[3] Annual data on real dividends from 1871 to 2000 were used. The source is Campbell and Shiller (1988) updated data series.

Finally, to have a sense of the order of magnitude of the coefficient of relative risk aversion implied by the assumptions made so far, Veronesi (1999, Proposition 3) shows that the value function for the representative agent can be written as $J(W,\pi) = -e^{\gamma r W} F(\pi)$ for some function $F(\pi)$. Thus, the relative risk aversion is simply given by $RRA = -WJ''(W,\pi)/J'(W) = \gamma r W = 0.03W$. In this economy with one unit of the risky asset, we can think of investor's financial wealth being in the order of magnitude of the price of the asset. In all simulations the price of the asset rarely exceeded the 100 level. Thus, we find that the coefficient of relative risk aversion implied by this model is generally below 3.

Given the parameters in Table 1, I generate 500 independent samples for dividends $D(t)$ using the Euler discrete approximation of the process in equation (1.1). Each sample has 900 "monthly" observations (75 years) while each month contains 22 (daily) observations. From the dividend observations I compute the posterior probabilities by approximating the process in (1.2). Finally, I use the dividend and probability series to compute the prices $P(D,\pi)$ and $P^*(D,\pi)$ by using (1.3) and (1.4). All the other variables are computed from these latter time series. Figures 3 shows the results of a particular sequence of dividends and probabilities, together with the implied price values and return volatility levels, generated by the above procedure.[4]

3.1. "GARCH" and Leverage Effects

In this subsection I fit the GARCH(1,1) model

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\eta_t^2 \tag{3.1}$$

[4]The volatility is estimate as

$$\sigma_t^2 = \sum_{i=1}^{20} (r_{i,t} - \bar{r}_t)^2$$

where $r_{i,t}$ is the return in day i in month t, and $\bar{r}_t$ is the average monthly return.

where $\eta_t \sim \mathcal{N}(0, \sigma_{t-1})$, on each of the 500 samples simulated.[5] Table 2 reports the distribution of the three parameters across the simulation. For comparison, I also include the estimates obtained using monthly data for excess returns from 1926:01-2001:12.[6] The results of the Monte Carlo simulation show that in average, the parameter estimates are almost identical to the ones observed in the data. The autoregressive parameter β equals a median .88 (mean = .87) across simulations, against a .86 in the data. Similarly, the impact of news to volatility is very similar, with $\alpha = .12$ in both cases. We can also notice that there is not much variation of the parameter estimates across samples, showing that the "Garch effect" is a genuine feature of the model, and mainly due to the Peso-Problem situation, as modeled in this paper.

The "Peso problem" hypothesis is also interesting because it entails an effect commonly referred to as the "leverage effect" (see Black (1976)), which is a negative relationship between returns and future volatility. In the above model, the distribution of π conditional on $\theta(t) = \overline{\theta}$ is concentrated in the area close to 1. As discussed, this implies that when π decreases, both volatility increases and the price decreases. Hence, we should observe a negative correlation between *ex post* returns and future volatility. This relationship between returns and future volatility is observed in U.S. data. Black (1976) explained this phenomenon as stemming from the increase in the debt-to-equity ratio of a leveraged firm following a drop in its stock price. The increase in the stock return volatility just reflects the increase in the riskiness of the leveraged firms. The model presented here provides an alternative explanation: both the price and the volatility of the stock react after bad news because the underlying uncertainty over the true state of the world increases.

In order to quantify this effect, Table 3 reports the results of the Monte Carlo simulation where an Exponential GARCH(1,1) model has been fitted on each of the 500 simulated samples.

[5] In this model it could happen that prices become negative, thereby making it impossible to compute percentage returns. For those simulated samples where this situation occured, I rescaled the dividend series to ensure positive prices. A previous version of the paper used "dollar returns" rather than percentage returns, which are free from this problem. The results were qualitatively similar, although harder to compare with the US data.

[6] Data are from the CRSP tapes at the University of Chicago.

The EGARCH(1,1) model is given by:

$$\log(\sigma_t) = \omega + \beta \log(\sigma_{t-1}) + \alpha[|\epsilon_t| - c\epsilon_t] \tag{3.2}$$

Under the leverage effect hypothesis, αc should be positive, implying that negative innovations have a greater impact on volatility than positive innovations. Moreover, $c > 1$ also implies that while negative innovations tend to increase the volatility, positive innovations tend to decrease it. As expected from equation (2.4), we see from Table 3 that the model implies an asymmetric reaction of volatility to bad and good news, as $c > 0$. Indeed, the model produces even "too much" of a leverage effect, as the parameter c has a median equal to 1.48 across simulations, while data yield the much smaller $c = .22$. Similarly, the autoregressive coefficient β results higher in the simulation (.997) than in the data (0.97). This implies that the volatility process implied by the model is more persistent than it is empirically observed. Still, one can conclude from the results in Table 2 and 3 that the model produces a good deal of time-variation in volatility, which is related with the directional movement in the stock market.

3.2. The small-sample bias in expected returns

This section discusses the quantitative implications of the Peso Problem for the estimated average returns, the standard finding in the Peso Problem literature (see e.g. Rietz (1988), Danthine and Donaldson (1998)). To quantify the effects, I will compare them to an alternative model, where agents know exactly the state $\theta(t) = \overline{\theta}$ and no shifts are possible. In this and the next section I will refer to this latter model as the Benchmark case, as it is the natural alternative to the Peso-Problem situation discussed here. In addition, it is a special case of the models by Wang (1993) and Campbell and Kyle (1993).

Table 4 shows the results of the Monte Carlo Simulations. From the first two columns, we see that indeed the small sample bias increases the mean average returns from 0.77% for the benchmark case, to above 3% for the Peso Problem, a four-fold increase. Although the latter number is still half of the equity premium, it shows that the small sample bias can induce large effects on the average return, as others have shown. The low value of the equity premium is due to the fact that as discussed in Section 3, the calibrated parameters imply a coefficient of relative risk aversion well below 3, thereby justifying also the very low equity premium in the benchmark case.

3.3. Stock price sensitivity to dividend changes and excess volatility

The model presented in this paper adds also to the debate on stock price fluctuations in response to dividend changes. In particular, starting with Shiller (1981) and LeRoy and Porter (1981), many papers challenged the efficient market, present-value model hypothesis on the basis that the stock price appeared to be too volatile, compared to *ex post* discounted dividends.[7] Indeed, by running a regression of monthly log-prices on log-dividends, we find the following:

$$\begin{array}{rccl} \log(P_t) = & 3.0687 & +1.1887 & \log(D_t) \\ & (0.0443) & (0.0386) & \end{array} \tag{3.3}$$

where standard errors are in parenthesis. This shows that a 1% change in dividend implies a change in price greater than 1%. This empirical regularity has been addressed by Barsky and De Long (1993), who only consider the long term case (around a 20 year time span). They propose a simple model where this "overreaction" of prices to dividend fluctuations stems from investors' revision of their own estimate of the long-term dividend growth rate, which they use to compute future dividends.

The model presented in this article gives a similar explanation to the excess sensitivity of prices to dividend changes. In fact, from the price function given in equation (1.3) and (1.4), we can see that a change in dividend has a direct and an indirect effect on the price of the asset: the direct effect is through the term $\frac{D}{r}$ and the indirect effect is through the revision in the probability π that the change in dividend would entail. Depending on the sensitivity of π

[7] See e.g. Mankiw, Romer and Shapiro (1985,1991), Campbell and Shiller (1988), Shiller (1989). Marsh and Merton (1986) offer an early reply to the concerns raised by Shiller (1981) and LeRoy and Porter (1981). See also West (1988) for a survey and references.

to news and the sensitivity of the price to changes in π the indirect effect may be substantial. To have a comparison, the last two columns in Table 4 report the results of the Monte Carlo simulation of the regression (3.3) both for the benchmark case, and the peso-problem case. In short, while the benchmark case yield a sensitivity parameter very close to 1, thereby justifying the concerns of the early literature started by Shiller (1981) and LeRoy and Porter (1981), the peso-problem effects are strong enough to generate a substantial excess sensitivity of price changes to dividend changes. The mean elasticity is about 2, which is quite higher than the one found empirically, but it confirms nonetheless that the "double kick" to prices stemming from learning and the Peso-Problem hypothesis can yield the effect.

To quantify the magnitude of the excess sensitivity of price reactions to dividend news, the second two columns of Table 4 show that the average volatility in the benchmark case is a small 6.5%, as the only volatility is stemming from changes in dividends, which are not very volatile (in sample, 6.5% was also the average volatility of dividend growth). In the Peso Problem Situation, instead, the average volatility in the simulations is around 21%. Thus, learning effects can have important effects on the level of the volatility of returns, as was first discussed by Timmerman (1993). The simulations in addition show that these learning effects have a rather strong impact on the volatility, even when the probability of entering into a (10-year) long recession is puny, about once every 200 years. Indeed, in this model even small movements in the updated probability of being in a recession are amplified by the fast increase in the discount when the probability π decreases, as shown in Figure 1.

4. Conclusion

This paper shows that the "Peso Problem Hypothesis" on economic fundamentals has several implications that have not previously documented. Specifically, I show that (*i*) returns should have GARCH behavior; (*ii*) there should be a negative predictive asymmetry between returns and future volatility; (*iii*) return volatility should increase during recessions; (*iv*) the time series of returns should have an upward bias due to small sample; and (*v*) price sensitivity to dividend changes should be greater than the one implied by standard present value models. In addition, Monte Carlo simulations show that the magnitude of the effects are comparable to those observed in the US data.

A concluding remark is in order: This paper shows theoretically that a "Peso Problem situation" generates a time-varying volatility with the same characteristics as the one in the data, and in particular with negative news that have a higher impact on the volatility than positive news. This is in line with the quote by Merton in the Introduction about the high volatility in the 30s. Yet, this of course does not imply that *all* "Garch effects" that we see in the data *must* be due to a Peso Problem situation. Other sources could be at play, possibly also related to uncertainty. Nonetheless, the contribution of this paper is to show that a Peso Problem situation would tend to generate *simultaneously* a number of features in returns, namely, effects (i) - (v) above, which are somewhat established feature of the data in "surviving" economies, as discussed in Section 2.3.

Appendix A

The two differential equations appearing in proposition 1.1 and 2.1 are the following:

$$-f''(\pi)Q_3(\pi) + f'(\pi)^2 Q_3(\pi) + f'(\pi)Q_2(\pi) + f(\pi)r + Q_0(\pi) = 0 \tag{4.1}$$

where $Q_3 = \frac{h(\pi)^2}{2}$, $Q_2 = h(\pi)\sigma\gamma - (\pi^s - \pi)(\lambda + \mu) + \gamma r p_\pi h(\pi)^2$ and $Q_0 = \frac{(r\gamma)^2}{2} p_\pi^2 h(\pi)^2 + r\gamma^2\sigma p_\pi h(\pi)$.

$$S''(\pi)P_3(\pi) = S'(\pi)P_2(\pi) + rS(\pi) + P_0(\pi) \tag{4.2}$$

where $P_3(\pi) = h(\pi)^2/2$, $P_2(\pi) = \gamma\sigma h(\pi) - (\pi^s - \pi)(\lambda + \mu) + \gamma r p_\pi h(\pi)^2 + f'(\pi)h(\pi)^2$and $P_0(\pi) = \gamma r p_\pi^2 h(\pi)^2 + 2\gamma\sigma p_\pi h(\pi) + f'(\pi)\frac{\sigma}{r}h(\pi) + f'(\pi)p_\pi h(\pi)^2$.

Appendix B

Proof of Proposition 2.1: (a) By definition, for $\theta(t) = \overline{\theta}$ we have that:

$$E(dQ^*|\mathcal{F}_t, \overline{\theta})/dt = (D - rP^*) + E(dP^*|\mathcal{F}_t, \overline{\theta}, \pi)$$

We can substitute the definition of P^*, to obtain (after some tedious algebraic manipulations):

$$E(dQ^*|\mathcal{F}_t, \overline{\theta})/dt = \frac{\overline{\theta} - \underline{\theta}}{r} - \frac{\Delta\theta}{r}\pi + \frac{\Delta\theta}{r(\lambda + \mu + r)} h(\pi) E(dv|\mathcal{F}_t, \overline{\theta}, \pi)$$

Notice that dv is a Wiener process with respect to $\mathcal{F}_t$, but it is not with respect to $\mathcal{F}_t \cup \overline{\theta}$. In fact, we have $E(dv|\mathcal{F}_t, \overline{\theta}, \pi) = \frac{\Delta\theta}{\sigma}(1 - \pi)$. By substituting this in the above expression, we prove the claim.

(b) By definition, $E[dQ\ |\mathcal{F}_t, \overline{\theta}] = (D - rP)dt + E[dP\ |\mathcal{F}_t, \overline{\theta}]$. By substituting for $P(D, \pi)$, we obtain:

$$\begin{aligned} E[dQ\ |\mathcal{F}_t, \overline{\theta}] &= (D - rP^*)dt + (-rp_0 - rS(\pi))dt + E[dP^*\ |\mathcal{F}_t, \overline{\theta}] + S'(\pi)E[d\pi\ |\mathcal{F}_t, \overline{\theta}] \\ &\quad + \frac{1}{2}S''(\pi)E[(d\pi)^2 \mathcal{F}_t, \overline{\theta}] \\ &= E[dQ^*\ |\mathcal{F}_t, \overline{\theta}] + (-rp_0 - rS(\pi) + S'(\pi)(\pi^s - \pi)(\lambda + \mu) + \frac{1}{2}S''(\pi)h(\pi)^2)dt \\ &\quad + S'(\pi)h(\pi)E(dv\ |\mathcal{F}_t, \overline{\theta}) \end{aligned}$$

We now substitute $E(dv\ |\mathcal{F}_t, \overline{\theta}) = \frac{\Delta\theta}{\sigma}(1 - \pi)$ and from Veronesi (1999) (appendix A) we also have:

$$-rp_0 - rS(\pi) + S'(\pi)(\pi^s - \pi)(\lambda + \mu) + \frac{1}{2}S''(\pi)h(\pi)^2 = (f'(\pi) - \gamma r S'(\pi))h(\pi)\sigma_P(\pi) + \gamma r \sigma_P(\pi)^2 \tag{4.3}$$

We see that by using the definition of $\sigma_P(\pi)$, we can rewrite the RHS as $\sigma_P(\pi)(\gamma\sigma + (\gamma r p_\pi + f'(\pi))h(\pi)) = E[dQ\ |\mathcal{F}_t]$. Hence, by substituting all this back we obtain:

$$\begin{aligned} E[dQ\ |\mathcal{F}_t, \overline{\theta}] &= E[dQ^*\ |\mathcal{F}_t, \overline{\theta}] + E[dQ\ |\mathcal{F}_t] + S'(\pi)h(\pi)E[dv\ |\mathcal{F}_t \overline{\theta}] \\ &= \frac{\Delta\theta}{r}(1 - \pi)(\sigma_P(\pi) - S'(\pi)h(\pi)) + (\gamma\sigma + (\gamma r p_\pi + f'(\pi))h(\pi))\sigma_P(\pi)dt \\ &\quad + S'(\pi)h(\pi)\frac{\Delta\theta}{\sigma}(1 - \pi)dt \\ &= \left(\gamma\sigma + (\gamma r p_\pi + f'(\pi))h(\pi) + \frac{\Delta\theta}{r}(1 - \pi)\right)\sigma_P(\pi)dt \end{aligned}$$

concluding the proof. □

Proof of Proposition 2.2: We show that if $\theta(t) = \overline{\theta}$ and $\pi > \frac{1}{2}$ during the sample period, we have:

$$\begin{aligned} d\sigma_{P^*} &= [-a_0 + a_1\sqrt{1+s_0-s_1\sigma_{P^*}} + a_2\sigma_{P^*} - a_3\sigma_{P^*}^2 - a_4\sigma_{P^*}\sqrt{1+s_0-s_1\sigma_{P^*}}]dt \\ &\quad -\frac{\Delta\theta}{2\sigma}(\sigma_{P^*} - \frac{\sigma}{r})\sqrt{1+s_0-s_1\sigma_{P^*}}d\xi \end{aligned}$$

where, defining $\tilde{p} = \frac{(\overline{\theta}-\underline{\theta})^2}{\sigma r(\lambda+\mu+r)}$, $s_0 = 4\frac{\sigma}{r\tilde{p}_\pi}$, $s_1 = \frac{4}{\tilde{p}_\pi}$, $a_0 = \frac{(\Delta\theta)^2}{2\sigma(\lambda+\mu+r)} + 3\sigma + \frac{\sigma(\lambda+\mu)}{r}$, $a_1 = \frac{(2\lambda+r)(\Delta\theta)^2}{2\sigma r(\lambda+\mu+r)}$, $a_2 = \frac{(\Delta\theta)^2}{2\sigma^2} + 6r + 4(\lambda+\mu)$, $a_3 = \frac{3r(\lambda+\mu+r)}{\sigma}$ and $a_4 = \frac{(\Delta\theta)^2}{2\sigma^2}$.

Use the definition of dv in Lemma 1.1 with $dD = \overline{\theta}dt + \sigma d\xi$ to obtain $dv = \frac{1}{\sigma}\Delta\theta(1-\pi)dt + d\xi$. This can be substituted into the process for $d\sigma_{P^*}$ obtained by Ito's Lemma to $\sigma_{P^*}(\pi) = \frac{1}{r}\left(1 + \frac{\overline{\theta}-\underline{\theta}}{(\lambda+\mu+r)\sigma}h(\pi)\right)$ to obtain:

$$\begin{aligned} d\sigma_{P^*}(\pi) &= \tilde{p}_\pi[(1-2\pi)(\pi^s-\pi)(\lambda+\mu) - h(\pi)^2]dt \\ &\quad +\tilde{p}_\pi(1-2\pi)h(\pi)\frac{\Delta\theta}{\sigma}(1-\pi)dt + \tilde{p}_\pi(1-2\pi)h(\pi)d\xi \end{aligned} \tag{4.4}$$

Since $\sigma_{P^*} = \frac{\sigma}{r} + p_\pi h(\pi)$ implies the relation $\sigma_{P^*} - \frac{\sigma}{r} - \tilde{p}_\pi\pi + \tilde{p}_\pi\pi^2 = 0$. Under the assumption that $\pi > \frac{1}{2}$, we obtain a solution of π in terms of σ_P^*, given by $\pi = \frac{1}{2} + \frac{1}{2}\sqrt{1+s_0-s_1\sigma_{P^*}}$. By substituting for π, $h(\pi)$ and $h(\pi)^2$ in (4.4), tedious algebraic manipulations show the claim. □

References

Barsky, R.B., B. J. DeLong, 1993, "Why Does the Stock Market Fluctuate?" *Quarterly Journal of Economics*, 108, 2, 291-311.

Black F., 1976, "Studies of Stock, Price Volatility Changes," Proceedings of the 1976 meetings of the Business and Economics Statistics Section, American Statistical Association.

Bollerslev, T. , 1986, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307 - 327.

Bossaerts P., 1996, "Martingale Restrictions On Equilibrium Prices of Arrow-Debreau Securities Under Rational Expectations and Consistent Beliefs," working paper, CalTech.

Brown, S.J., W.N. Goetzmann, and S.A. Ross, 1995, "Survival," *Journal of Finance*, 50,3.

Campbell, J.Y., and A.S. Kyle, 1993, "Smart Money, Noise Trading and Stock Price Behavior," *Review of Economic Studies*, 60, 1, 1-34.

Campbell, J. Y., A. W. Lo, and A. C. MacKinlay, 1997, *The Econometrics of Financial Markets*. Princeton University Press, Princeton, New Jersey.

Cochrane, J. H., 2000, *Asset Pricing*, Princeton University Press, Princeton, NJ.

Danthine, J.P. and J. Donaldson, 1998, "Non-Falsified Expectations and General Equilibrium Asset Pricing: The Power of the Peso," PaineWebber Working Paper, Columbia University.

David, A., 1997, "Fluctuating Confidence in Stock Markets: Implications for Returns and Volatility," *Journal of Financial and Quantitative Analysis*, 32, 4, 427-462.

Engle, R.F., 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 50, 987 - 1007

French, K.R., W.G. Schwert, and R.F. Stambaugh, 1987, "Expected Stock Returns and Volatility," *Journal of Financial Economics*, 19, 1, 3-29.

Goetzmann, W. and P. Jorion, 1999a, "Global Stock Markets in the Twentieth Century" *Journal of Finance*.

Goetzmann, W. and P. Jorion, 1999b, "Re-emerging Markets" *Journal of Financial and Quantitative Analysis,* 34, 1.

Hamilton, J.D. and G. Lin, 1997, "Stock Market Volatility and the Business Cycle, *Journal* *of Applied Econometrics*, 11, 573 - 593.

LeRoy, S.F. and R.D. Porter, 1981, "Stock Price Volatility: Tests Based on Implied Variance Bounds," *Econometrica* 49, 97-113.

Liptser, R.S., A.N. Shiryayev, 1977, *Statistics of Random Processes: I, II*, Springer-Verlag, New York.

Mankiw, G.N, D. Romer, and M.D. Shapiro, 1985, "An Unbiased Reexamination of Stock Market Volatility" *Journal of Finance*, 40,3, 677-687.

Mankiw, G.N, D. Romer and M.D. Shapiro, 1991, "Stock Market Forecastability and Volatility: A Statistical Appraisal," *Review of Economic Studies*, 58,3, 455-477.

Marsh, T. and R.C. Merton, 1986, "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," *American Economic Review*, 76, 483 - 498.

Mehra, R. and E.C. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145-161.

Mehra, R. and E.C. Prescott, 1988, "The Equity Premium: A Solution?," *Journal of Monetary Economics*, 22, 133-136.

Nelson, D., 1991, "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59, 347 - 370.

Rietz, T.A., 1988, "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, 22, 117-131.

Schwert, G.W., 1989, "Why Does Stock Market Volatility Change over Time?", *Journal of Finance*, 44, 5, 1115-53.

Schwert, G.W., 1990b, "Stock Returns and Real Activity: A Century of Evidence", *Journal* *of Finance*, 45, 1237 - 1257.

Shiller, R.J., 1981, "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71, 421-435.

Shiller, R.J., 1984, "Stock Price and Social Dynamics," *Brookings Papers on Economic Activity*,2,457-498.

Shiller, R.J., 1989, *Market Volatility*, the MIT Press, Cambridge, Massachusetts.

Timmermann, A. (1993) "How learning in financial markets generates excess volatility and predictability of stock returns" *Quarterly Journal of Economics*, 108, 1135-1145.

Veronesi, P., 1999 "Stock Market Overreaction to Bad News in Good Times: A Rational Expectations Equilibrium Model," *Review of Financial Studies*, v. 5, n.2, 975-1007.

Veronesi, P., 2000 "How Does Information Quality Affect Stock Returns," *Journal of Finance*, v. 35, n.2, 807-837.

Wang, J., 1993, "A Model of Intertemporal Asset Prices under Asymmetric Information," *Review of Economic Studies*, 60, 2, 249-82.

West, K.D., 1988, "Bubbles, Fads and Stock Price Volatility Tests: A Partial Evaluation," *Journal of Finance* 43, 639-655.

Table 1

Calibration

$\overline{\theta}$	$\underline{\theta}$	σ	r	γ	λ	μ
.015	–.05	.12	.03	1	.005	.05

Table 2

GARCH Effects

U.S. Stock Market			
	ω ($\times 10^3$)	β	α
estimate	0.0685	0.8588	0.1205
asymptotic s.e.	(0.0153)	(0.0183)	(0.0212)
Monte Carlo Simulations			
	ω ($\times 10^3$)	β	α
mean	0.0322	0.8723	0.1239
sd	0.0474	0.0414	0.0471
min	0.0004	0.697	0.0486
5%	0.0028	0.789	0.0713
25%	0.0086	0.8526	0.0927
50%	0.018	0.883	0.1132
75%	0.0364	0.9007	0.1417
95%	0.1096	0.9225	0.215
max	0.6496	0.943	0.3511

GARCH Model:

$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\eta_t^2$

$\eta_t = (r_t - \bar{r}) \sim \mathcal{N}(0, \sigma_{t-1})$

Table 3

The Leverage Effect

U.S. Stock Market				
	ω	β	α	c
estimate	-0.1646	0.9740	0.1109	0.2689
asymptotic s.e.	(0.0130)	(0.0048)	(0.0143)	(0.0947)
Monte Carlo Simulations				
	ω	β	α	c
mean	-0.0485	0.997	0.0453	4.2742
sd	0.0296	0.0037	0.0277	10.6926
min	-0.2149	0.9726	0.0005	-0.0352
5%	-0.1003	0.9903	0.0041	0.4441
25%	-0.0661	0.9957	0.0229	0.9212
50%	-0.0448	0.998	0.0426	1.4888
75%	-0.0257	0.999	0.0637	2.8508
95%	-0.0079	1.000	0.0945	20.0874
max	-0.0029	1.000	0.1581	137.853

EGARCH(1,1) Model:

$\log(\sigma_t) = \omega + \beta \log(\sigma_{t-1}) + \alpha[|\epsilon_t| - c\epsilon_t]$

$\epsilon_t = \frac{\eta_t}{\sigma_{t-1}}$, $\eta_t = (r_t - \bar{r}) \sim \mathcal{N}(0, \sigma_{t-1})$

Table 4

Small Sample Bias and Excess Volatility

US Stock Market

Average Returns	Average Volatility	Price Sensitivity
6.45%	19.66%	1.1887

Monte Carlo Simulations

	Average Returns		Average Volatility		Price Sensitivity	
	Benchmark	Peso	Benchmark	Peso	Benchmark	Peso
mean	0.0076	0.0313	0.0658	0.2207	1.0255	2.0147
sd	0.0071	0.0126	0.0107	0.0951	0.0042	0.4280
min	-0.0109	-0.0055	0.0417	0.0872	1.0154	1.3612
5%	-0.0037	0.0101	0.0495	0.1251	1.0194	1.4834
25%	0.0023	0.0222	0.0582	0.1584	1.0226	1.6616
50%	0.0075	0.0307	0.0648	0.1893	1.0251	1.9056
75%	0.0127	0.0416	0.0724	0.2516	1.0282	2.2832
95%	0.0194	0.0516	0.0845	0.4138	1.0334	2.8765
max	0.0243	0.065	0.105	0.7182	1.0405	3.3375

Average return and average volatility are given by the time-series annualized mean and standard deviation of log-returns in the US sample 1926 - 2001, and in simulated data. The price-sensitivity refers to the slope coefficient of the regression $\log(P_t) = \alpha + \beta \log(D_t) + \epsilon_t$ in the data and in simulations. The benchmark model is the one where dividend growth is fixed and known to investors, while the Peso column refers to the effect of the Peso problem situation.

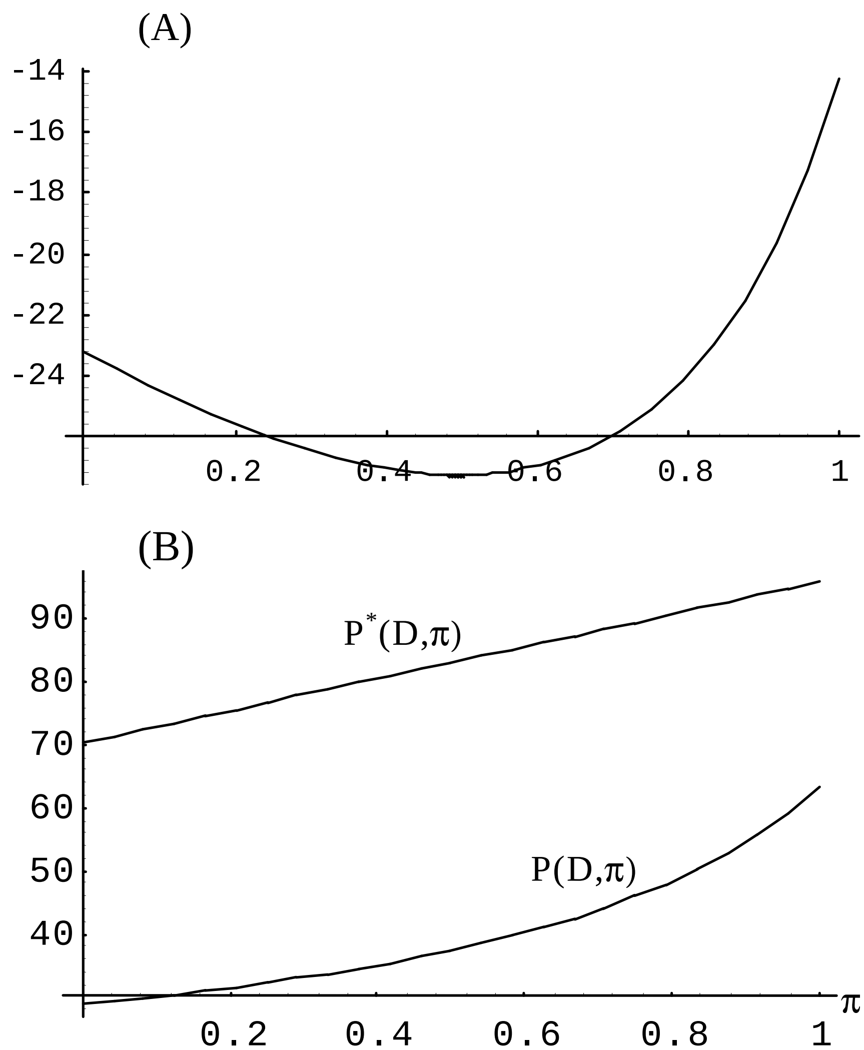

Figure 1: (A) The function S(π). (B) The risk-neutral price $P^*(D,\pi)$ and the price function $P(D,\pi)$

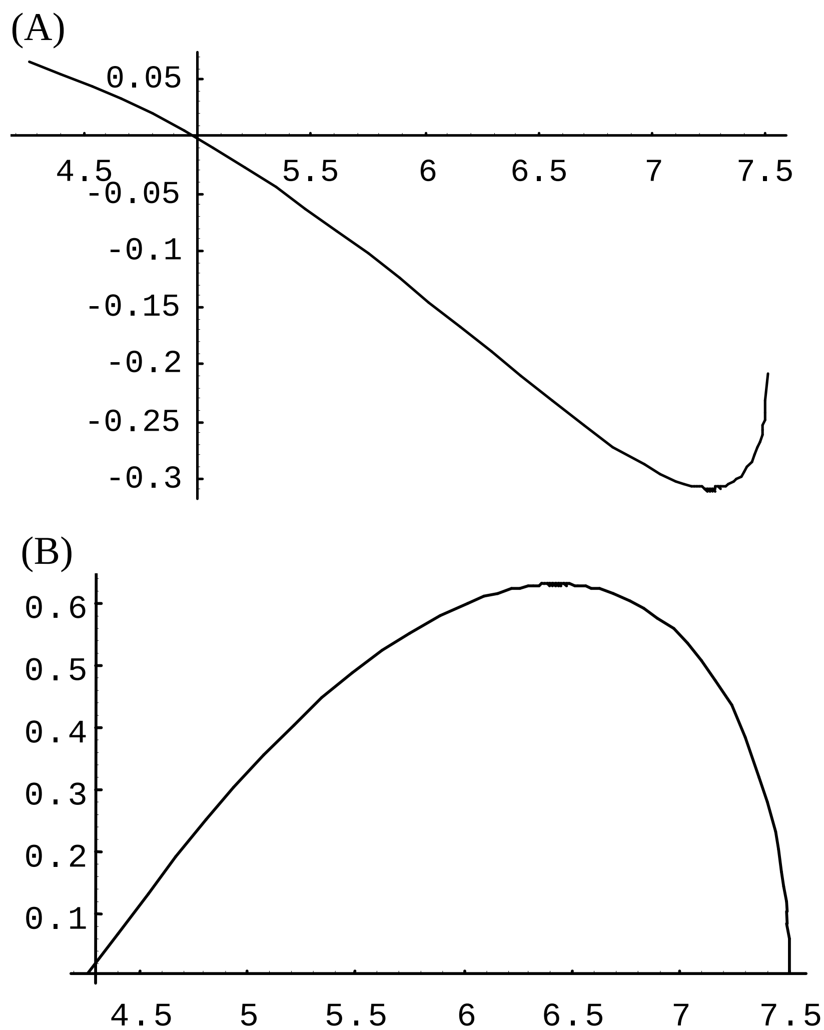

Figure 2: (A) The drift $a(\sigma_P *)$ of the volatility process. (B) The diffusion $b(\sigma_P *)$ of the volatility process.

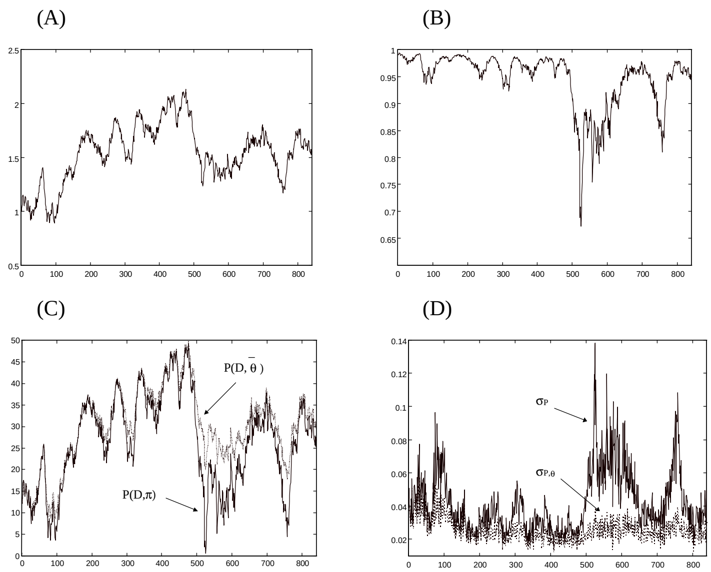

Figure 3: (A) A simulated dividend series; (B) The updated probability π; (C) The implies prices $P(D,\pi)$ and $P(D,\theta)$; (D) The estimated monthly volatility : σ_P and $\sigma_{P,\theta}$.