Habits and Leverage

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Motivation

- Much discussion in the academic literature and in policy circles about leverage and its impact on the real economy and on financial markets

- Various related themes, such as:
  - Excess credit supply may lead to financial crisis
  - The excessive growth of household debt and the causal relation between households’ deleveraging and their low future consumption growth
  - Leverage cycle: Leverage is high when prices are high and volatility is low
  - Active deleveraging of financial institutions generate “fire sales” of risky financial assets, which further crash asset prices
  - The leverage ratio of financial institutions is a risk factor
  - Balance sheet recessions
  - ....
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  – Leverage cycle: Leverage is high when prices are high and volatility is low
  – Active deleveraging of financial institutions generate “fire sales” of risky financial assets, which further crash asset prices
  – The leverage ratio of financial institutions is a risk factor
  – Balance sheet recessions
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• But leverage is endogenous....
What we do

• Study a frictionless dynamic general equilibrium model featuring heterogeneous agents with external habit preferences
  
  – Heterogeneous time varying risk-bearing capacity $\implies$ leverage dynamics
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- Model’s predictions consistent with empirical evidence
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  - Heterogeneous time varying risk-bearing capacity $\implies$ leverage dynamics
- Model’s predictions consistent with empirical evidence
- Model aggregates to representative agent models with external habit
  $\implies$ It can be calibrated to yield reasonable asset pricing quantities.
Preferences

- Continuum of agents with external habit preferences:

\[ u(C_{i,t}, X_{i,t}, t) = e^{-\rho t} \log (C_{it} - X_{it}) \]
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• Habits’ loadings:

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  (i) heterogeneous: \[ a_i > 0 \text{ with } \int a_i d\bar{i} = 1 \]
Preferences

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(i) heterogeneous: \( a_i > 0 \) with \( \int a_i di = 1 \)

(ii) time varying: \( \boxed{Y_t} = \text{Recession Indicator} \) (next slide)

\[ \Rightarrow \text{Habits matter more in bad times.} \]
Preferences

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  (i) heterogeneous: \( a_i > 0 \) with \( \int a_i di = 1 \)
  
  (ii) time varying: \( Y_t = Recession \ Indicator \) (next slide)
  \[ \implies \text{Habits matter more in bad times.} \]

- Endowments \( w_i \) are also heterogeneous, with \( \int w_i di = 1 \)
• Aggregate output:

\[
\frac{dD_t}{D_t} = \mu_D dt + \sigma_D(Y_t) dZ_t
\]

\(- \sigma_D(Y_t) : Economic Uncertainty.\)
• Aggregate output:
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\frac{dD_t}{D_t} = \mu dD_t + \sigma_D(Y_t) dZ_t
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• Recession indicator \(Y_t\):
\[
dY_t = k(\bar{Y} - Y_t)dt - \nu Y_t \left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right]
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\(\implies\) Bad shocks: \(\left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] < 0 \implies Y_t \uparrow\)
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\(\Rightarrow\) Bad shocks: \(\left[ \frac{dD_t}{D_t} - E_t \left( \frac{dD_t}{D_t} \right) \right] < 0 \Rightarrow Y_t \uparrow\)

• Technical restrictions:
- \(Y_t > \lambda \geq 1\) for all \(t\): \(\sigma D(Y_t) \to 0\) as \(Y_t \to \lambda\). Otherwise \(\sigma D(Y_t)\) general.
- Endowments satisfy
\[
w_i > \frac{a_i(\bar{Y} - \lambda) + \lambda - 1}{\bar{Y}}
\]
Optimal Risk Sharing

- No consumption externalities $\implies$ solve planner’s problem

- Consumption shares: $s_{it} = \frac{C_{it}}{D_t} = a_i + (w_i - a_i) \frac{\bar{Y}}{Y_t}$
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**Consumption shares:**

$$s_{it} = \frac{C_{it}}{D_t} = a_i + \left( w_i - a_i \right) \frac{\bar{Y}}{Y_t}$$

- High endowment $w_i$ or low habit loading $a_i$ $\implies$ $s_{it} \uparrow$ when $Y_t \downarrow$ (good times)
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- **Risk aversion** (curvature):
  \[
  Curv_{it} = \frac{-C_{it} u_{cc}(C_{it}, X_{it}, t)}{u_c(C_{it}, X_{it}, t)} = 1 + \frac{a_i(Y_t - \lambda) + \lambda - 1}{w_i \bar{Y} - a_i(\bar{Y} - \lambda) - \lambda + 1}
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  - Cross-section: risk aversion \(\downarrow\) if \(w_i \uparrow\) or \(a_i \downarrow\)
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  - **Time-series:** (1) all agents’ risk aversion $\uparrow$ if $Y_t \uparrow$
    
    (2) risk aversion of $i \uparrow$ more if $w_i$ is low or $a_i$ is high
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- Less risk averse agents provide insurance to more risk averse agents
• Our model aggregates to Menzly, Santos, and Veronesi (2004):

• As in Campbell and Cochrane (1999), define

\[
Surplus\ consumption\ ratio = S_t = \frac{D_t - \int X_{it} \, di}{D_t} = \frac{1}{Y_t}
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(1)

**Proposition.** The equilibrium state price density

\[
M_t = e^{-\rho t} D_t^{-1} S_t^{-1}
\]  

(2)

– which follows

\[
dM_t/M_t = -r_t \, dt - \sigma_{M,t} \, dZ_t \quad \text{with} \quad \sigma_{M,t} = (1 + v)\sigma_D(S_t)
\]

We use \(S_t\) as state variable for notational convenience.
Proposition. The competitive equilibrium has:

Stock price: \[ P_t = \left( \frac{\rho + k\bar{Y}S_t}{\rho (\rho + k)} \right) D_t \]

Risk-free rate: \[ r_t = \rho + \mu_D - (1 + v)\sigma_D(S_t)^2 + k \left( 1 - \bar{Y}S_t \right) \]
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(Stock holdings) \[ N_{it} = a_i + (\rho + k)(1 + v)(w_i - a_i) H(S_t) \]

(Bond holdings) \[ N^0_{it}B_t = -v(w_i - a_i) H(S_t)D_t \]

where \[ H(S_t) = \frac{\overline{Y}S_t}{\rho + k(1 + v)\overline{Y}S_t} \]
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• Stock and bond holdings depend on \( w_i - a_i \) and the function \( H(S_t) \).
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Stock and bond holdings depend on \( w_i - a_i \) and the function \( H(S_t) \).

Stock price and risk-free rate are independent of distribution of \( w_i \) and \( a_i \).

\[ \Rightarrow \] Prices and quantities have no causal relation with each other.
• **Results**: Agents with $w_i - a_i > 0$:
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(ii) “over-invest” in risky assets ($\frac{N_{it} P_t}{W_{it}} > 1$)

(iii) increase their debt in good times ($H'(S_t) > 0$)
    when $S_t \uparrow$, their risk aversion $\downarrow$, take on more aggregate risk
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  (iv) enjoy high consumption share $s_{it}$ when their debt is high

      * Leverage $\implies$ higher return $\implies$ higher consumption in good times

      * Lower risk aversion $\implies$ even more debt in good times
Implications: Leverage, Consumption, and Business Cycle

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  2. “over-invest” in risky assets ($\frac{N_{it}P_t}{W_{it}} > 1$)
  3. increase their debt in good times ($H'(S_t) > 0$)
      - when $S_t \uparrow$, their risk aversion $\downarrow$, take on more aggregate risk
  4. enjoy high consumption share $s_it$ when their debt is high
      - Leverage $\implies$ higher return $\implies$ higher consumption in good times
      - Lower risk aversion $\implies$ even more debt in good times
  5. suffer consumption decline after consumption boom

- Spatial interpretation: e.g. counties with high $w_i$ or low $a_i$
  - Good times $\implies$ debt $\uparrow$ and consumption $\uparrow$ $\implies$ but lower future growth.
  - Crucial role of identification strategies to provide causal link between leverage and future consumption
• **Results (cntd.).** Agents with $w_i - a_i > 0$:

  (vi) increase stock holdings in good times (trend chasers)
**Implications: Active Trading**

- **Results (cntd.).** Agents with $w_i - a_i > 0$:

  (vi) increase stock holdings in good times (trend chasers)

  (vii) drastically decrease stock holdings in bad times ($H(S)$ concave)
• Much recent research on role of intermediaries’ leverage in asset prices
  – Households invest in risky assets through intermediaries, who issue debt
  – Empirically: leverage risk price is positive or negative depending on proxies
Implications for Intermediary Asset Pricing

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• In our model, agents with \( w_i > a_i \) leverage by issuing risk-free bonds to others
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- If habit \( S_t \) is unobservable, leverage is a proxy for habit.
- Let \( \ell_t = Q(S_t) \), and hence \( S_t = q(\ell_t) = Q^{-1}(\ell_t) \)
  \[ \Rightarrow SDF = M_t = e^{-\rho t}D_t^{-1}S_t^{-1} = e^{-\rho t}D_t^{-1}q(\ell_t)^{-1} \]
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  $$\implies SDF = M_t = e^{-\rho t} D_t^{-1} S_t^{-1} = e^{-\rho t} D_t^{-1} q(\ell_t)^{-1}$$

• The risk premium for any asset with return $dR_{it} = (dP_{it} + D_{it})/P_{it}$ is
  $$E_t[dR_{it} - r_t dt] = Cov_t \left( \frac{dD_t}{D_t}, dR_{it} \right) + \frac{q'(\ell_t)}{q(\ell_t)} Cov_t (d\ell_t, dR_{it})$$
  \underbrace{\text{Consumption CAPM}} \quad \underbrace{\text{Leverage risk premium}}
Implications for Intermediary Asset Pricing

- Two potential measures of leverage:
  
  Debt/Output Ratio: \( \ell_t = Q_{it}^{D/O}(S_t) = -\frac{N^0_{it}B_t}{D_t} = \nu (w_i - a_i) H(S_t) \)

  Debt/Equity Ratio: \( \ell_t = Q_{it}^{D/W}(S_t) = -\frac{N^0_{it}B_t}{W_{it}} = \frac{\sigma W_i(S_t)}{\sigma_P(S)} - 1 \)
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- Two potential measures of leverage:

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  - Increasing in \( S \)
  - Decreasing in \( S \)
Implications for Intermediary Asset Pricing

- Two potential measures of leverage:

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- Result: The price of leverage risk is

  (a)  \( \lambda_t^{D/O} = \frac{q_{D/O}'(\ell_t)}{q_{D/O}(\ell_t)} \geq 0 \) if \( \ell_t = \) Debt/Output Ratio (“book leverage”).

  (b)  \( \lambda_t^{D/W} = \frac{q_{D/W}'(\ell_t)}{q_{D/W}(\ell_t)} < 0 \) if \( \ell_t = \) Debt/Equity Ratio (“market leverage”).

- In bad times:
  - agents deleverage \( \Rightarrow \) debt/output ↓ \( \Rightarrow \) book leverage risk price > 0.
  - high discounts \( \Rightarrow \) debt/equity ↑ \( \Rightarrow \) market leverage risk price < 0.
Quantitative Predictions

• Previous results independent of the functional form of $\sigma_D(Y_t)$.

• Assume now a specific functional form to make model comparable to MSV and obtain reasonable asset pricing implications:

$$\sigma_D(Y_t) = \sigma^{max}(1 - \lambda Y_t^{-1})$$

• $\Rightarrow$ Economic uncertainty increases in bad times, but bounded between $[0, \sigma^{max}]$

• $\Rightarrow$ Obtain same process for $Y_t$ as in MSV $\Rightarrow$ Use their same parameters.
  
  – Additional parameter $\sigma^{max}$ chosen to fit average consumption volatility

• All asset pricing results are similar (or stronger) than MSV.
Calibrate to Household Consumption Systematic Volatility

• $a_i \sim U(\overline{a}, \overline{a})$ and Pareto weights $\phi_i = \log N \left( -\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2 \right)$

### Table 2. Cross Sectional Parameters and Household Consumption Moments

#### Panel A. Households Quarterly Consumption Moments. Data

<table>
<thead>
<tr>
<th>Growth Rate (%)</th>
<th>Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>6.04</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>-0.59</td>
</tr>
</tbody>
</table>

#### Panel B. Households Quarterly Consumption Moments. Model

<table>
<thead>
<tr>
<th>$U[\overline{a}, \overline{a}], \sigma_\phi$</th>
<th>Arithmetic Growth Rate (%)</th>
<th>Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U[0, 2], 3$</td>
<td>0.73</td>
<td>0.52</td>
</tr>
<tr>
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<td>0.73</td>
<td>0.52</td>
</tr>
<tr>
<td>$U[1, 1], 0$</td>
<td>0.52</td>
<td>0.52</td>
</tr>
</tbody>
</table>
The Cross-Section of Agents’ Behavior: Who Levers?

Uniform Habits

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

LEVERAGED AGENTS

UNLEVERAGED AGENTS
The Cross-Section of Agents’ Behavior: Who Levers?

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

Positively skewed wealth distribution
The Cross-Section of Agents’ Behavior: Who Levers?

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

Agents missing due to endowment constraint
The Cross-Section of Agents’ Behavior: Who Levers?

A. Distribution of Habit Loadings $a_i$

B. Distribution of Endowments $w_i$

C. Relation between $w_i$ and $a_i$

D. Leveraged Agents

- Poor agents borrow``
- Rich agents borrow
Household Leverage in Good and Bad Times

Panel A. Agents' Debt/Asset: Model

- **Boom**
- **Recession**
- **Crisis**

<table>
<thead>
<tr>
<th>Net Worth</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 25</td>
<td><img src="chart1" alt="Bar Graph" /></td>
</tr>
<tr>
<td>25 - 49.5</td>
<td><img src="chart2" alt="Bar Graph" /></td>
</tr>
<tr>
<td>50 - 74.9</td>
<td><img src="chart3" alt="Bar Graph" /></td>
</tr>
<tr>
<td>75 - 89.9</td>
<td><img src="chart4" alt="Bar Graph" /></td>
</tr>
<tr>
<td>90 - 100</td>
<td><img src="chart5" alt="Bar Graph" /></td>
</tr>
</tbody>
</table>
Household Leverage in Good and Bad Times

Panel A. Agents' Debt/Asset: Model

Panel B. Agents' Debt / Assets: Data.
“Fire Sales” in a Simulation Run
“Fire Sales” in a Simulation Run

A. Surplus Consumption Ratio

B. Economic Uncertainty

C. Price / Dividend Ratio

D. Return Volatility

E. Aggregate Debt/Output and Stock Holdings

F. Aggregate Debt/Wealth
“Fire Sales” in a Simulation Run

A. Surplus Consumption Ratio

B. Economic Uncertainty

C. Price / Dividend Ratio

D. Return Volatility

E. Aggregate Debt/Output and Stock Holdings

F. Aggregate Debt/Wealth

Market Leverage

Book Leverage
### Table 3: The Market Price of Leverage Risk

<table>
<thead>
<tr>
<th></th>
<th>Panel A - Data</th>
<th></th>
<th>Panel B - Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>3.19</td>
<td>0.76</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(3.05)</td>
<td>(0.62)</td>
<td>(0.97)</td>
</tr>
<tr>
<td>Market Return</td>
<td>-0.89</td>
<td>0.97</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(-0.72)</td>
<td>(0.69)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>Market Leverage</td>
<td>-0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Book Leverage</td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 ) (%)</td>
<td>6.54</td>
<td>50.77</td>
<td>53.35</td>
</tr>
</tbody>
</table>
Table 4: The Predictability of Aggregate Stock Returns

| Panel A. Predictability with Book Leverage. Data |  
|-------------------------------------------------|---|
| Coef (×100) | 1 year | 2 year | 3 year | 4 year | 5 year |
|-------------------------------------------------|---|---|---|---|
| Coef (×100) | -1.78 | -1.79 | -2.17 | -3.13 | -9.89 |
| (-0.83) | (-0.72) | (-0.89) | (-1.03) | (-3.29) |
| $R^2$ | 0.01 | 0.01 | 0.01 | 0.01 | 0.07 |

| Panel B. Predictability with Market Leverage. Data |  
|-------------------------------------------------|---|
| Coef (×100) | 3.66 | 6.21 | 8.56 | 10.03 | 13.06 |
| (1.57) | (1.50) | (2.18) | (2.51) | (3.84) |
| $R^2$ | 0.04 | 0.07 | 0.10 | 0.12 | 0.19 |

| Panel C. Predictability with Book Leverage. Model |  
|-------------------------------------------------|---|
| Coef (×100) | -3.57 | -7.28 | -10.49 | -12.62 | -14.13 |
| (-3.45) | (-3.09) | (-3.18) | (-3.34) | (-3.52) |
| $R^2$ | 0.02 | 0.05 | 0.08 | 0.10 | 0.12 |

| Panel D. Predictability with Market Leverage. Model |  
|-------------------------------------------------|---|
| Coef (×100) | 5.86 | 10.91 | 14.69 | 17.35 | 19.36 |
| (8.08) | (7.69) | (7.55) | (7.54) | (7.74) |
| $R^2$ | 0.06 | 0.12 | 0.17 | 0.20 | 0.22 |
Conclusions

• A frictionless dynamic general equilibrium model with heterogeneous agents and external habits seem consistent with many stylized facts.

• Risk sharing motives generate endogenous leverage dynamics

• Our model predicts:
  1. Aggregate debt ↑ in good times when prices ↑ and volatility ↓
  2. Poorer agents borrow more than richer agents
  3. Leveraged agents enjoy a “consumption boom” in good times, followed by a consumption slump
  4. Crisis time ⇒ leveraged agents delever by “fire-selling” stocks, but their debt/wealth ratio ↑ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion ↑ in good times

• Leverage dynamics is due to the differential impact of aggregate shocks on agents’ risk aversion.
Table 1. Parameters and Moments

<table>
<thead>
<tr>
<th>Panel A. Parameters (MSV)</th>
<th>ρ</th>
<th>k</th>
<th>Y</th>
<th>λ</th>
<th>v</th>
<th>μ</th>
<th>σ_{max}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0416</td>
<td>0.1567</td>
<td>34</td>
<td>20</td>
<td>1.1194</td>
<td>0.0218</td>
<td>0.0641</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Moments (1952 – 2014)</th>
<th>(E[R])</th>
<th>(Std(R))</th>
<th>(E[r_f])</th>
<th>(Std(r_f))</th>
<th>(E[P/D])</th>
<th>(Std[P/D])</th>
<th>(SR)</th>
<th>(E[\sigma_i])</th>
<th>(Std(\sigma_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>7.13%</td>
<td>16.55%</td>
<td>1.00%</td>
<td>1.00%</td>
<td>38</td>
<td>15</td>
<td>43%</td>
<td>1.41%</td>
<td>0.52%</td>
</tr>
<tr>
<td>Model</td>
<td>8.19%</td>
<td>25.08%</td>
<td>0.54%</td>
<td>3.77%</td>
<td>30.30</td>
<td>5.80</td>
<td>32.64%</td>
<td>1.43%</td>
<td>1.18%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C. P/D Predictability (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
</tr>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Model</td>
</tr>
</tbody>
</table>

- Model matches asset pricing moments well.
A. Stationary Distribution

B. Price-Consumption Ratio

C. Risk Premium, Volatility, and Risk Free Rate

D. Sharpe Ratio

Conditional Moments