Stock Valuation with Uncertainty about Long-Term Profitability

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1. Stock valuation and learning about profitability
   - Learning about long-term profitability and Bayesian updating
   - The pricing relation when long-term profitability is uncertainty: How does uncertainty affect firms’ market-to-book (M/B) ratios?
   - Return volatility and M/B ratios
   - The empirical evidence

2. Explaining technological ”bubbles”
   - High prices and high volatility of new firms: the role of long-term uncertainty and learning
   - Was there a tech bubble in the late 1990s?
3. Stock price “bubbles” during technological revolutions
   • Why do stock prices of innovative firms show a “bubble-like” behavior during technological revolutions?
   • Endogenous time varying risk in technological revolutions
   • Evidence from the 1990s tech bubble
     – Time varying risk in Nasdaq
   • Evidence from the railroad revolution
     – Stock prices and risk during the railroad revolution

4. Long-term uncertainty and IPO-waves
   • Why do IPO’s come in waves?
   • What makes a hot and cold market?
   • Time varying uncertainty and expected returns and the determinants of IPO waves.
5. Entrepreneurial Learning and the Post-IPO Underperformance
   • Why do typically firms have lower profitability after the IPO?
   • Learning about long-term profitability, uncertainty, and underperformance

6. Uncertainty, leverage, and firms' cost of capital
   • How does firm’s uncertainty affect the cost of capital?
   • Learning about market betas, diversification, and the cost of capital
The late 1990s have witnessed an unprecedented surge in the number of newly listed firms on the major U.S. stock exchanges.

According to Fama and French (2001b), over 550 new firms per year appeared between years 1980 and 2000, on average, compared to less than 150 firms in the previous two decades.

In addition, they tend to be listed early in their life cycle.

They typically do not pay any dividends.

Investors attempting to value the new firms are confronted with substantial uncertainty about the firms’ future profitability.

Pastor and Veronesi (2003, JF) put forward a new model for stock valuation.

- Analytical formulas for stock prices even for stocks that
  - Pay no dividends
  - Have negative earnings

PV use this model to evaluate the effect of uncertainty on M/B and volatility, and assess the predictions empirically.
The Discounted Cash Flow (Gordon) Model

- Let $D_t$ be the annualized dividends produced at time $t$ during the time interval $\Delta$.
- Let $g =$ constant annualized expected dividend growth:
  $$\frac{D_{t+\Delta}}{D_t} = (1 + g\Delta)$$
- Let $r =$ annualized discount rate: the rate at which investors discount future dividends.
- The stock price today is the discounted value of the cash flow tomorrow:
  $$P_t = \frac{E[D_{t+\Delta} + P_{t+\Delta}]}{1 + r\Delta}$$
The Discounted Cash Flow (Gordon) Model

• From the one period pricing formula we can substitute the price iteratively forward:

\[ P_t = \frac{E[D_{t+\Delta} \Delta]}{1 + r\Delta} + \frac{E[P_{t+\Delta}]}{1 + r\Delta}; \]

\[ P_{t+\Delta} = \frac{E[D_{t+2\Delta} \Delta]}{1 + r\Delta} + \frac{E[P_{t+2\Delta}]}{1 + r\Delta}; \]

\[ P_{t+2\Delta} = \frac{E[D_{t+3\Delta} \Delta]}{1 + r\Delta} + \frac{E[P_{t+3\Delta}]}{1 + r\Delta}; \]

\[ \vdots \]

• to arrive at

\[ P_t = \frac{E[D_{t+\Delta} \Delta]}{(1 + r\Delta)} + \frac{E[D_{t+2\Delta} \Delta]}{(1 + r\Delta)^2} + \frac{E[D_{t+3\Delta} \Delta]}{(1 + r\Delta)^3} + \ldots \]
The Discounted Cash Flow (Gordon) Model

• Moreover, using

\[ D_{t+\Delta} = D_t \left(1 + g\Delta\right); \quad D_{t+2\Delta} = D_t \left(1 + g\Delta\right)^2; \quad D_{t+3\Delta} = D_t \left(1 + g\Delta\right)^3; \]

• we obtain

\[ P_t = D_t \Delta \left\{ \left(\frac{1 + g\Delta}{1 + r\Delta}\right) + \left(\frac{1 + g\Delta}{1 + r\Delta}\right)^2 + \left(\frac{1 + g\Delta}{1 + r\Delta}\right)^3 + \ldots \right\} \]

• Finally, solving for the geometric series\(^1\) we obtain that the price-dividend ratio is:

\[ \frac{P_t}{D_t} = \frac{1 + g\Delta}{r - g} \]

• As the payment frequency increases, \(\Delta \to 0\) and we obtain the standard formula

\[ \frac{P_t}{D_t} = \frac{1}{r - g} \]

\(^1\)The result follows from using \(\sum_{j=1}^{\infty} a^j = a/(1-a)\) with \(a = \left(\frac{1+g}{1+r}\right)\). We have to assume \(r > g\).
The Discounted Cash Flow (Gordon) Model with Continuous Payments

• Indeed, what happens as $\Delta \to 0$?

• In this case, we have:
  - Dividend growth per unit of time
    $$\frac{D_{t+\Delta} - D_t}{D_t} = g\Delta \implies \frac{D_{t+\Delta} - D_t}{\Delta} = gD_t \to \frac{dD_t}{dt} = gD_t$$
  - Thus, "$g$" is the *continuously-compounded annualized* growth rate and, given $D_t$:
    $$D_s = D_t e^{g(s-t)}$$

• Similarly, the discount rate $r$ becomes the continuously compounded rate.

• Because dividends are paid every instant, the "sum of dividends" becomes an "integral," that is:
  $$\frac{D_{t+1\Delta\Delta}}{(1+r\Delta)^{\Delta\Delta}} + \frac{D_{t+2\Delta\Delta}}{(1+r\Delta)^{2\Delta\Delta}} + \frac{D_{t+3\Delta\Delta}}{(1+r\Delta)^{3\Delta\Delta}} + \frac{D_{t+4\Delta\Delta}}{(1+r\Delta)^{4\Delta\Delta}} + \ldots \to \int_t^\infty e^{-r(s-t)} D_s ds$$

• Thus, we have
  $$P_t = \int_t^\infty e^{-r(s-t)} D_s ds = \int_t^\infty e^{-r(s-t)} D_t e^{g(s-t)} ds = D_t \int_t^\infty e^{-(r-g)(s-t)} ds = D_t \frac{1}{r-g}$$
How Do We Compute the Discount Rate $r$?

- The discount rate $r$ is given by
  \[ r = \text{risk-free rate} + \text{risk premium} \]

- The risk free rate is observable (\(=\) T-Bond rate).

- The risk premium, in general, is given by
  \[ \text{risk premium} = \text{risk aversion} \times \text{amount of systematic risk} \]

- Typically, the amount of systematic risk is computed from the covariance of the stock’s equity or cash flows with the aggregate economy.

- For instance, the Capital Asset Pricing Model’’
  \[ \text{amount of systematic risk} = Cov(r^i_t, r^M_t) \]
  - A stock $i$ that covaries with the aggregate market return is “risky”, as its risk cannot be diversified away,
  - \(\implies\) investors require (or should require) a large risk premium to hold the stock.
How Do We Compute the Discount Rate $r$?

- More generally, macro-finance people computes the amount of systematic risk as
  
  \[
  \text{amount of systematic risk} = Cov \left( r_t^i, \text{Economic Growth} \right)
  \]

- Think about savings decisions and the investment in a stock $i$.
  
  - If the stock $i$ has a large return when the economy collapses, it is a good hedge (think about T-bonds during the financial crisis)
    
    $\implies$ Small or no risk premium, as we want to buy a lot of them.
  
  - If the stock $i$ collapses exactly when the economy tanks, it is very risky
    
    $\implies$ Large risk premium (or low price, given expected cash flows), as we need to be induced to buy them.
How Do We Compute the Discount Rate $r$?

- Denote by $\gamma$ the risk aversion of agents. Then the discount of stock $i$ can be computed as
  \[ r_i = r_f + \gamma \times Cov(r_i^t, r_M^t) \]

- Because this must hold for the market as well
  \[ r_M = r_f + \gamma \times Cov(r_M^t, r_M^t) = r_f + \gamma \times Var(r_M^t) \]

- Solve for $\gamma$ and substitute
  \[ \gamma = \frac{r_M - r_f}{Var(r_M^t)} \]

- It follows the famous Capital Asset Pricing Model (CAPM) beta formula
  \[ r_i = r_f + \beta_i \times (r_M - r_f) \]
  
  - where
    \[ \beta_i = \frac{Cov(r_i^t, r_M^t)}{Var(r_M^t)} \]

- $\beta_i$ can then be estimated from data.
The Discounted Cash Flow (Gordon) Model

- Gordon growth model:
  - Dividend’s growth $g$ and discount rate $r$ are constant;
    
    When $g$ is known: \( \frac{P}{D} = \frac{1}{r - g} \)
    
    When $g$ is unknown: \( \frac{P}{D} = \mathbb{E}\left(\frac{1}{r - g}\right) \)
  
  - Since $1/(r - g)$ is convex in $g$, uncertainty about $g$ increases $P/D$
  - This holds also when uncertainty about $g$ affects $r$

- Example: Suppose $P/D = 50$ and $r = 20\%$.
  
  - When $g$ is known, need $g = 18\%$ to match $P/D$
  - When $g$ is unknown, need a lower $g$
    
    * E.g., if $g$ is uniform with $\sigma(g) = 4\%$, $P/D$ is matched with $\mathbb{E}(g) = 13.06\%$

- Same intuition holds in our model, which focuses on $M/B$ rather than $P/D$
- Note: Uncertainty matters most when the discount rate is low
The Discounted Cash Flow (Gordon) Model

Present Value of Expected Future Dividends

PV of Expected Future Dividends

growth rate g
Pastor and Veronesi (2003) Model

- The Gordon growth model is hard to use in practice, especially for young, new companies.
  - Many firms do not pay any dividends at all for a while
    \[ * \implies \text{It is hard to compute the price dividend ratio } D_t \]
  - Many firms experience negative earnings
    \[ * \implies \text{It is hard to compute the price earnings ratio } P/E \]

- Pastor and Veronesi (2003) focuses on profitability as fundamental variable
  - Firm’s profitability is strongly time varying
  - It is persistent
  - We do not know the long-term profitability of new firms

- The model is able to deal with firms
  - that pay no dividends
  - that have negative earnings
Pastor and Veronesi (2003) Model

- Firm profitability is measured as the accounting return on equity (ROE),
  \[ \rho_t = \frac{\text{Earnings}_t}{\text{Book Equity}_t} = \frac{Y_t}{B_t} \]

- **Assumption 1**: mean reverting profitability.
  \[ d\rho_t = \phi(\bar{\rho} - \rho_t)dt + \sigma_{\rho,1}dW_{1,t} + \sigma_{\rho,2}dW_{2,t} \]
  - We will interpret:
    * \(dW_{1,t}\) as a systematic shock and
    * \(dW_{2,t}\) as an idiosyncratic shock

- **Assumption 2**: Dividend smoothing.
  \[ D_t = c B_t, \quad c \geq 0 \]

- **Assumption 3**: No new equity issues
  - Clean surplus relation: Capital tomorrow = Capital today + earnings − dividends.
  \[ dB_t = (Y_t - D_t) dt = (\rho_t - c) B_t dt \]

- The abnormal earnings model of Ohlson (1990, 1995):
  \[ M_t = B_t + \text{present value of future abnormal earnings} \]

- Competition \( \implies M_T = B_T \) at some future time \( T \)

- Market value of equity:
  \[
  M_t = E_t \left[ \int_t^T e^{-r(s-t)} D_s ds \right] + E_t \left[ e^{-r(T-t)} B_T \right],
  \]

- For simplicity, consider only for the case where the firm pays no dividends, \( D_t = 0 \implies c = 0 \).
- We then use
  \[
  M_t = E_t \left[ e^{-r(T-t)} B_T \right].
  \]
• **Proposition 1.** Assume $\bar{\rho}$ is known and $D_t = 0$. Then

(a) The M/B ratio is given by

$$
\frac{M_t}{B_t} = Z(\bar{\rho}; \rho_t, \tau) \equiv \exp \left\{ - (r - \bar{\rho}) \tau + Q_\phi(\tau)(\rho_t - \bar{\rho}) + Q(\tau) \right\}
$$

where $\tau = T - t$ and $Q_\phi(\tau) = \phi^{-1}(1 - e^{-\phi\tau})$ and $Q(\tau)$ is a known function of time.

(b) Excess stock returns follow

$$
dR_t = \mu_{R,t} dt + \sigma_{R,1,t} dW_{1,t} + \sigma_{R,2,t} dW_{2,t}
$$

where

$$
\mu_{R,t} = Q_\phi(\tau) \sigma_{\rho,1} \sigma_{\pi,1} \quad \text{and} \quad \sigma_{R,i,t} = Q_\phi(\tau) \sigma_{\rho,i}
$$

• **Corollary 1.** The market-to-book ratio increases if

(i) $\bar{\rho}$ increases; (ii) $\rho_t$ increases; (iii) $\sigma_{\rho,1} \sigma_{\pi,1}$ decreases;

(iv) $r$ decreases; (v) $\sigma^2_{\rho,1} + \sigma^2_{\rho,2}$ increases;

• **Corollary 2.** The market-to-book ratio is convex in $\bar{\rho}$. 
Figure 1. **M/B versus mean profitability.** The figure plots the model-implied M/B ratio against the known value of mean profitability $\bar{p}$ for various levels of the dividend yield $c$. The model parameters are specified as follows: $\phi = 0.3968$, $\rho_t = 0.11$, $\sigma_{\rho,1} = 0.0584$, $\sigma_{\rho,2} = 0.0596$, $\sigma_{\pi,1} = 0.6$, $r = 0.03$, and $\tau = 15$.

Learning about Mean Profitability

• Assume $\bar{\rho}$ is unknown
• Using the law of iterated expectations,
  \[
  \frac{M_t}{B_t} = \int_{\mathbb{R}} Z(\bar{\rho}; \rho_t, \tau) \, p_t(\bar{\rho}) \, d\bar{\rho}
  \]
• Since $Z(\rho_t, \bar{\rho}; \tau)$ is convex in $\bar{\rho}$, greater dispersion in $p_t(\bar{\rho})$ increases M/B
• What about the dynamics?

• **Lemma 1.** If the prior of $\bar{\rho}$ is normal, $\bar{\rho} \sim \mathcal{N}(\hat{\rho}_0, \hat{\sigma}_0^2)$,
  then the posterior conditional on $\mathcal{F}_t = \{(\pi_\tau, \rho_\tau) : 0 \leq \tau \leq t\}$ is also normal,
  \[
  \bar{\rho}|_{\mathcal{F}_t} \sim \mathcal{N}(\hat{\rho}_t, \hat{\sigma}^2_t)
  \]
  where
  \[
  d\hat{\rho}_t = \hat{\sigma}^2_t \frac{\phi}{\sigma_{\rho,2}} d\tilde{W}_{2,t}
  \]
  \[
  \hat{\sigma}^2_t = \frac{1}{\hat{\sigma}^2_0 + \frac{\phi^2}{\sigma^2_{\rho,2}} t}
  \]
Figure 2. Uncertainty about mean profitability over time. The figure plots the evolution over time of $\hat{\sigma}_t$, standard deviation of the posterior distribution of mean profitability $\hat{\rho}$. Three values are considered for the parameter $\phi$, which governs the speed of mean reversion in profitability. The prior standard deviation is $\hat{\sigma}_0 = 0.10$, $c = 0.0434$, and all other parameter values are as in Figure 1.

• **Proposition 2.** Assume investors follow the Bayes rule when learning about $\bar{\rho}$. Then if $D_t = 0$

(a) The firm’s M/B ratio is given by

$$
\frac{M_t}{B_t} = Z (\hat{\rho}_t, \rho_t, \tau) \exp \left\{ \frac{1}{2} \hat{\sigma}_t^2 (\tau - Q_\phi(\tau))^2 \right\}
$$

(b) The firm expected returns and volatility are given by

exp. ret. : $\mu_{R,t} = Q_\phi(\tau) \sigma_{\rho,1} \sigma_{\pi,2}$

sys. volat. : $\sigma_{R,1,t} = Q_\phi(\tau) \sigma_{\rho,1}$

idos. volat. : $\sigma_{R,2,t} = Q_\phi(\tau) \sigma_{\rho,2} + (\tau - Q_\phi(\tau)) \frac{\phi}{\sigma_{\rho,2}} \hat{\sigma}_t^2$

• **Corollary 3-6: Main Empirical Predictions**

  - The M/B ratio increases with uncertainty about mean profitability, $\hat{\sigma}_t^2$.
  - The effect of $\hat{\sigma}_t^2$ on $\log(M/B)$ is stronger for firms that pay no dividends.
  - Idiosyncratic return volatility increases with uncertainty about mean profitability.
  - Firms that pay no dividends have higher return volatility than dividend payers.
Figure 3. **M/B and stock return volatility versus uncertainty about mean profitability.** In the top panel, the figure plots the model-implied M/B ratio against $\sigma_t$, standard deviation of the posterior distribution of mean profitability $\bar{p}$. In the bottom panel, the figure plots the model-implied return volatility against $\sigma_t$. Three values are considered for the dividend yield $c$. All other parameter values are as in Figure 1.

Figure 4. M/B and stock return volatility over time. The figure plots the evolution over time of the model-implied M/B ratio (top two panels) and the model-implied return volatility (bottom two panels). In the left-hand panels, mean profitability \( \bar{\rho} \) is treated as unknown. In the right-hand panels, \( \bar{\rho} \) is treated as known. Three values are considered for the dividend yield \( c \). The prior standard deviation is \( \sigma_0 = 0.10 \), and all other parameter values are as in Figure 1.

Learning about Mean Profitability and Prices: Intuition

(a) Why return volatility increases with learning?
   * Good news in profitability have two effects:
     (i) Fundamental effect: Higher earnings level commands a higher price.
     (ii) Signaling effect: It increases expected $\bar{\rho}$, hence future profitability
         * $\Rightarrow$ Additional positive kick to price.

(b) Why the expected return is unaffected by learning?
   * The aggregate market provides no information on $\bar{\rho}$
     $\Rightarrow$ Innovations in expected $\bar{\rho}$ are uncorrelated with innovations in aggregate returns
     $\Rightarrow$ Additional volatility on returns is idiosyncratic.
     $\Rightarrow$ No additional risk.

(c) Why M/B increases with uncertainty on $\bar{\rho}$?
   * Given point (b) and the convex nature of compounding of profitability into book equity
     * $\Rightarrow$ More dispersed distribution of $\bar{\rho}$ translates into a higher expected value for $B_T$.
     * Since discount is unaffected, market value increases.

(d) Why dividends enter into the picture?
   * Dividends weaken convex relation $\Rightarrow$ All effects stronger for stocks that pay no dividends.
The implications of the model are robust to various generalizations, including:

1. Stochastic interest rate $r$, stochastic long-term mean $\bar{\rho}$.
2. Debt financing (so long profitability process is not affected).
   - Example: Constant Debt/Equity ratio.
3. New equity issues, so long frictions put an upper bound to the frequency/amount of new issues.
   - Pecking order: Firms tend to use own earnings, then debt, then equity.
   - Cash flow sensitivity: Firms tend to use internal cash flows to finance investments.
4. Accounting issues:
   - Conservative accounting
   - Intangibles
     - They imply $M_T > B_T$ even after competition competes away abnormal earnings.
     - We can just set $M_T = (1 + \eta) B_T$ for some constant $\eta$. 
Empirical Analysis

- Proxy for uncertainty about $\bar{p}$: firm age (number of years since listing)
- Predictions tested for M/B:
  - It is higher for younger firms, especially among dividend non-payers
  - It declines over time for a given firm, especially when learning is fast
  - It increases with volatility of profitability
  - It increases with future profitability and decreases with future returns
- Predictions tested for idiosyncratic return volatility:
  - It is higher for younger firms
  - It is higher for dividend non-payers
  - It declines over time for a given firm, especially when learning is fast
  - It increases with volatility of profitability
Figure 5. M/B ratio and return volatility in the years after listing. For each age, the figure plots the median M/B ratio (top panel) and the median idiosyncratic return volatility (bottom panel) across firms of that age, regardless of the calendar year in which that age was reached. The solid line plots the medians across all firms, the dashed line plots the medians across dividend nonpayers, and the dotted line plots the medians across dividend payers. Idiosyncratic volatility is estimated from the market model regression.

Table I

Summary Statistics

The table summarizes various statistics for groups of firms of the same age, where age measures the number of years since the firms’ listing. Panel A reports the medians across firms of the annual characteristics listed in the row label. Residual return volatility is calculated with respect to the market model. Return volatility, return on equity, stock return, and leverage are all expressed in percentage terms. Assets are in millions of dollars. Panel B shows the number of firms with valid M/B ratios, the fraction of firms that pay dividends, and the standard deviation of \( ROE \) across firms. The latter two values are expressed in percent.

<table>
<thead>
<tr>
<th>Age</th>
<th>Panel A. Medians across firms.</th>
<th>Panel B. Other statistics.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M/B</td>
<td>Number of firms</td>
</tr>
<tr>
<td></td>
<td>Total return volatility</td>
<td>Dividend payers</td>
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<tr>
<td></td>
<td>Resid. return volatility</td>
<td>Std. deviation of ( ROE )</td>
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<tr>
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Figure 6. **M/B ratios and return volatility in calendar time.** The figure plots the evolution of M/B and idiosyncratic return volatility in calendar time. Each year, all firms are categorized as young or old depending on whether their age exceeds the midpoint between the minimum and maximum age in the cross section. Firms are also separately sorted into dividend payers and nonpayers, depending on whether they paid any common stock dividends in the sorting year. Four groups of stocks are formed each year by intersecting the independent sorts on age and dividends. The top panel plots the median M/B ratios for these four groups, and the bottom panel plots the median return volatilities. Idiosyncratic volatility is estimated from the market model regression.

Regression Analysis

• The learning framework implies:

\[
\log \left( \frac{M_t}{B_t} \right) = \alpha_0 (\tau) + \alpha_1 (\tau) \hat{\rho}_t + \alpha_2 (\tau) \rho_t + \alpha_3 (\tau) \mu_{R,t} \\
+ \alpha_4 (\tau) \|\sigma\|^2 + \alpha_5 (\tau) \hat{\sigma}_t
\]

• Each year, we run the cross-sectional regression

\[
\log (M/B)_i = a + b \text{AGE}_i + c \text{DD}_i + d \text{LEV}_i + e \text{SIZE}_i + f \text{VOLP}_i \\
+ g_0 \text{ROE}(0)_i + g_1 \text{ROE}(1)_i + \ldots + g_q \text{ROE}(q)_i \ldots \\
+ h_1 \text{RET}(1)_i + \ldots + h_q \text{RET}(q)_i, \\
i = 1, \ldots, N
\]

• Fama-MacBeth (1973) approach to inference.
Table II

Determinants of Market-to-Book Ratios

For each year between 1963 and 2000, the log of the market-to-book ratio (M/B) is regressed cross-sectionally on minus the reciprocal of one plus firm age (AGE), dividend dummy (DD), leverage (LEV), the log of total assets (SIZE), the volatility of profitability (VOLP), current return on equity (ROE), and future values of ROE and stock returns (RET), up to the number of leads listed in the column headings. The reported slope coefficients and their standard errors are computed from the time-series of the estimated cross-sectional slope coefficients. The t-statistics, adjusted for any significant serial correlation in the time series, are in parentheses. The numbers of years across which the averages of the coefficients are computed are given in the last row. Also given are averages across these years of the \( R^2 \)'s and of the numbers of firms from the cross-sectional regressions. The values in the first column are obtained from the regression of log M/B on AGE only.

<table>
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<td>(1.68)</td>
<td>(1.37)</td>
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<td>(6.92)</td>
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<td>(3.63)</td>
<td>(2.42)</td>
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<tr>
<td>RET(1)</td>
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<td>−0.35</td>
<td>−0.39</td>
<td>−0.48</td>
<td>−0.50</td>
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<tr>
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<td>(−7.83)</td>
<td>(−6.37)</td>
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</table>

Average \( R^2 \) 0.05 0.21 0.30 0.42 0.51 0.61 0.69 0.78
Average N 4234 2318 2145 1840 1393 956 662 437
Years 38 38 37 33 28 23 18 13

Table III

The AGE Coefficients for Dividend Payers versus Nonpayers

For each year between 1963 and 2000, the log of the market-to-book ratio (M/B) is regressed cross-sectionally on minus the reciprocal of one plus firm age (AGE), dividend dummy (DD), the interaction term AGE*DD, leverage (LEV), the log of total assets (SIZE), the volatility of profitability (VOLP), current return on equity (ROE), and future values of ROE and stock returns (RET), up to the number of leads listed in the column headings. The reported AGE coefficients and their standard errors are computed from the time series of the estimated cross-sectional slope coefficients. The t-statistics, adjusted for any significant serial correlation in the time series, are in parentheses. The numbers of years across which the averages of the coefficients are computed are given in the last row. Also given are the averages across these years of the R²’s from the cross-sectional regressions, as well as of the numbers of dividend payers and nonpayers in each year. The values in the first column are obtained from the regression of log M/B on AGE and AGE*DD only. To obtain the t-statistics on the coefficients for dividend payers, the regression is rerun with DD redefined as its own complement.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
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<td>Non-payers</td>
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<td>-1.50</td>
<td>-1.13</td>
<td>-0.99</td>
<td>-0.89</td>
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<tr>
<td></td>
<td>(-8.88)</td>
<td>(-6.92)</td>
<td>(-7.58)</td>
<td>(-8.02)</td>
<td>(-10.76)</td>
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<td>(-4.77)</td>
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<tr>
<td>Payers</td>
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<td>-0.19</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.11</td>
<td>-0.17</td>
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<tr>
<td></td>
<td>(-6.42)</td>
<td>(-1.51)</td>
<td>(-1.87)</td>
<td>(-0.54)</td>
<td>(-0.51)</td>
<td>(-1.66)</td>
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<tr>
<td>Difference</td>
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<td>1.95</td>
<td>1.76</td>
<td>1.46</td>
<td>1.09</td>
<td>0.88</td>
<td>0.71</td>
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<tr>
<td></td>
<td>(5.32)</td>
<td>(6.32)</td>
<td>(6.58)</td>
<td>(7.49)</td>
<td>(9.94)</td>
<td>(4.93)</td>
<td>(3.82)</td>
</tr>
</tbody>
</table>

| Average R² | 0.06  | 0.22  | 0.30  | 0.43  | 0.51  | 0.61  | 0.69  | 0.78  |
| Avg. N: Non-payers | 1967 | 750 | 702 | 562 | 369 | 190 | 112 | 72 |
| Avg. N: Payers | 2181 | 1567 | 1444 | 1278 | 1024 | 767 | 550 | 365 |
| Years      | 38    | 38    | 37    | 33    | 28    | 23    | 18    | 13    |

Table IV
Determinants of Return Volatility

For each year between 1963 and 2000, residual return variance from the market model is regressed cross-sectionally on various subsets of the following set of variables: Minus the reciprocal of one plus firm age \((AGE)\), the log of M/B \((M/B)\), dividend dummy \((DD)\), leverage \((LEV)\), the log of total assets \((SIZE)\), the volatility of profitability \((VOLP)\), and current return on equity \((ROE)\). The reported slope coefficients and their standard errors are computed from the time series of the estimated cross-sectional slope coefficients. The \(t\)-statistics, adjusted for any significant serial correlation in the time series, are in parentheses. Also given are averages across these years of the \(R^2\)'s and of the numbers of firms from the cross-sectional regressions. All reported coefficients are multiplied by 100 for convenience.

<table>
<thead>
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<td>Intercept</td>
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<td>1.43</td>
<td>0.94</td>
<td>0.61</td>
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<td>1.68</td>
<td>2.65</td>
<td>2.75</td>
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<td>-1.23</td>
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<tr>
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<td>(-6.73)</td>
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<td>(-2.65)</td>
<td>(-2.49)</td>
<td>(-2.19)</td>
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<td></td>
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<tr>
<td>M/B</td>
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<td>0.05</td>
<td>0.18</td>
<td>0.17</td>
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<td>(2.94)</td>
<td>(-0.10)</td>
<td>(1.22)</td>
<td>(2.64)</td>
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<td></td>
<td>(-12.56)</td>
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<td>(-12.90)</td>
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<td>(-0.77)</td>
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<tr>
<td>SIZE</td>
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<td>-0.20</td>
<td>-0.20</td>
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<tr>
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<td>(-13.23)</td>
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<td>(-9.59)</td>
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<tr>
<td>VOLP</td>
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<td>5.96</td>
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<td>(6.35)</td>
<td>(6.66)</td>
<td>(6.66)</td>
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<tr>
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<td>-0.93</td>
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<td>(-4.92)</td>
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</tbody>
</table>

Average \(R^2\): 0.02 0.01 0.04 0.06 0.08 0.15 0.18 0.18 0.18
Average \(N\): 4842 4073 4073 2533 2423 2392 2341 2311 2311

Empirical Issues

1. Are the effects on M/B economically significant?
   
   • YES: After controlling for all other determinants \((q = 20)\)

<table>
<thead>
<tr>
<th>Ages</th>
<th>Non Payer</th>
<th>All</th>
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<tbody>
<tr>
<td>1 year vs 2 years</td>
<td>12%</td>
<td>5%</td>
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<tr>
<td>1 year vs 5 years</td>
<td>27%</td>
<td></td>
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</table>

2. Survivorship:

   • Inference problem if high M/B firms or high volatility firms leave the dataset.

   – M/B:
     * Linear probability model shows that low M/B firms are significantly more likely to depart.
     * We checked: Changes in M/B are (a) in average negative, although not significantly; (b) steeper for younger firms.

   – Volatility:
     * Linear probability model shows that high volatility firms are more likely to depart.
     * Model clearly not invalidated: The ones that depart may be younger companies or having higher uncertainty (hence, the higher volatility).
     * We checked: Changes in volatility: Mixed results, probably due to the increase in idiosyncratic return volatility (see Campbell et al. (2001)) and large estimation errors.
Conclusions

• We develop an approach to valuing stocks whose average profitability is uncertain.
• Uncertainty about mean profitability increases M/B and idiosyncratic return volatility.
• We show that M/B is higher for younger firms, especially among dividend non-payers.
• We show that idiosyncratic return volatility is higher for younger firms, firms that pay no dividends, and firms with more volatile profitability.
Some immediate applications of basic model

- Uncertainty, stream of new young firms, and increase in no dividend payers have contributed to the increase in average idiosyncratic return volatility in 1980 - 2000
  - Big literature, started with Campbell, Lettau, Malkiel, and Zu (JF, 2001)
- Time varying uncertainty may explain time variation in average M/B
  - Bubble in Nasdaq stocks in late 1990s? See Pastor and Veronesi (JFE, 2006)
- Uncertainty about long-term profitability may explain the diversification discount
  - Multiple-segment firm’s value is below the value imputed using single segment firms
  - Hund, Monk, Tice, (JFE 2010) show that focus firms are characterized by higher uncertainty about long-term profitability than multiple-segment firms.
  - Moreover, diversified firms have lower idiosyncratic volatility, and that the diversification discount is higher in good times, which is an implication of Pastor and Veronesi (2003, 2006).
Figure 7. Return volatility, volatility of profitability, the number of new lists, and the fraction of dividend nonpayers. In the top panel, the figure plots the evolution of average idiosyncratic return volatility in calendar time. The evolution of the cross-sectional standard deviation of ROE is plotted in the second panel. The third panel plots the number of new lists. The bottom panel plots the fraction of all firms that pay no dividends (solid line) and the fraction of firms that neither pay dividends nor repurchase any shares in the current year.

Some immediate applications of basic model

- IPO waves
  - There are periods characterized by an extraordinary number of new firms flowing to the market
  - Higher uncertainty due to possibly a technological revolutions may increase value of “new idea” and increase the incentive to go public (Pastor and Veronesi, JF, 2005)

- Post-IPO underperformance in profitability
  - Learning about long-term profitability imply that profitability declines after IPO (Pastor, Taylor, and Veronesi, RFS, 2009)

- Uncertainty, convexity, and the cost of capital
  - Johnson (JF, 2004): Uncertainty about firm value decreases the cost of capital of levered firms.
  - Armstrong, Banerjee, Corona (2011): Uncertainty about firm’s beta decreases the required cost of capital.
It is common to talk about a Nasdaq “bubble” in the late 1990s

- Popular press
- Academic articles

The term “bubble” is appropriate to describe, ex post, a rise in prices followed by a decline.

But common interpretation: prices of tech stocks exceeded fundamentals in the late 1990s.

Question:

*Is it obvious that tech stock prices exceeded their fundamental values?*
The Need of a Model for Fundamentals

- In order to claim that prices exceeded fundamentals, we need a model for fundamental value.
  - Such a model should include *uncertainty* about average profitability
    (i) This uncertainty increases firm value (PV, 2003)
    (ii) This uncertainty was high for tech stocks in the late 1990s
    \[(i)+(ii) \implies \text{Tech stock fundamental values were high in the late 1990s}\]

- We develop and calibrate a valuation model that includes this uncertainty

- We find that tech stock prices in March 2000 can be rationalized with plausible uncertainty
  - Our model also explains many other *facts* surrounding the end of the 1990s
Why Was Uncertainty about Average Profitability High in the Late 1990s?

• Technological revolution; “new era”


• Firms went public earlier in their life-cycles (Schultz and Zaman, 2001)

• Tech stock profitability was highly volatile

• Tech stock prices were highly volatile

• Anecdotal evidence

  “…the projections of revenue growth were, by and large, wild guesses.”
  *Investment Dealers Digest, 23 October 2000.*

  “Internet firms’ highly unpredictable growth rates make historical information less useful.”
  *TIAA-CREF Investment Forum, March 2001.*

  “...being wrong isn’t very costly, and being right has a high payoff... With Amazon, we believe the payoff for being right is high.”
  *Bill Miller, portfolio manager of the Legg Mason Value Trust, in Barron’s, 15 Nov 1999.*
Number of patent applications and grants

- Patent Applications
- Patent Grants

Year

1900 to 2000

Number of applications and grants multiplied by 10^5
Figure 9. Cross-sectional standard deviation of profitability.
Figure 8. Return volatility.
Related Literature

- On the valuation of tech stocks
  - Ofek and Richardson (2003): short-sale constraints
  - Cochrane (2002): convenience yield
  - Schwartz and Moon (2000)

- On stock valuation with a focus on cash flow

- On investor behavior during the Nasdaq “bubble”
  - Cooper, Dimitrov, and Rau (2001), Lamont and Thaler (2003), Brunnermeier and Nagel (2003), Rau et al. (2003), etc.

- On bubbles in general
Some Facts About the Nasdaq “Bubble”

A. What “went up” at the end of the 1990s?
   1. Nasdaq Index.

![Cumulative Return: NYSE and NASDAQ graph]

- The Nasdaq Index experienced a significant increase during the late 1990s, peaking around 2000.
- The graph shows the cumulative return for both NYSE and NASDAQ over the years 1997 to 2004.
Some Facts About the Nasdaq “Bubble”

A. What “went up” at the end of the 1990s?

1. Nasdaq Index.
2. Nasdaq Volatility.
Some Facts About the Nasdaq “Bubble”

A. What “went up” at the end of the 1990s?

1. Nasdaq Index.
2. Nasdaq Volatility.
3. Nasdaq Beta.
B. Whatever happened to Nasdaq also happened to NYSE High-Tech Stocks.
   – NYSE Hi Tech Stocks went up compared to NYSE Lo Tech Stocks
B. Whatever happened to Nasdaq also happened to NYSE High-Tech Stocks.

- NYSE Hi Tech Stocks went up compared to NYSE Lo Tech Stocks
- NYSE Hi-Tech Volatility has the same properties of Nasdaq Volatility
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- NYSE Hi Tech Stocks went up compared to NYSE Lo Tech Stocks
- NYSE Hi-Tech Volatility has the same properties of Nasdaq Volatility
- NYSE Hi-Tech Beta has the same properties of Nasdaq Beta.
C. Is it really true that trading volume in tech stocks went up in the late 1990s?
   - Nasdaq Turnover increased somewhat.
     * But it peaked well after the “bubble” burst.
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- Nasdaq Turnover increased somewhat.
  * But it peaked well after the “bubble” burst.
- NYSE Hi Tech Stocks’ Turnover did not move at all
C. Is it really true that trading volume in tech stocks went up in the late 1990s?

- Nasdaq Turnover increased somewhat.
  * But it peaked well after the “bubble” burst.
- NYSE Hi Tech Stocks’ Turnover did not move at all
- Nasdaq Hi Tech Stocks’ Turnover went down.
Overview

1. Outlay of the Model (simplified version)
   • Valuation formula
2. Calibration
3. Matching Nasdaq valuations on March 10, 2000
   • For Nasdaq as a whole
   • For selected individual firms
4. Why did the “Bubble” burst?
5. Technological Revolutions and Asset Prices
6. Conclusion
Firm profitability is measured as the accounting return on equity (ROE),

\[ \rho_t = \frac{\text{Earnings}_t}{\text{Book Equity}_t} = \frac{Y_t}{B_t} \]

Mean reversion in firm profitability:

\[ d\rho^i_t = \phi^i (\overline{\rho}^i_t + \overline{\psi}^i_t - \rho^i_t) \, dt + \sigma_{i,0} \, dW_{0,t} + \sigma_{i,i} \, dW_{i,t} \]

- \( \overline{\rho} \) ... Average aggregate profitability

\[ d\overline{\rho}_t = k_L (\overline{\rho}_L - \overline{\rho}_t) \, dt + \sigma_{L,0} dW_{0,t} + \sigma_{L,L} dW_{L,t} \]

- \( \overline{\psi} \) ... Average firm-specific excess profitability

* Investors do not observe this parameter
Model: Dividends

- Dividends are proportional to book equity:
  \[ D_t = c \, B_t, \quad c \geq 0 \]

- Clean surplus relation (assuming no new equity issues/withdrawals):
  \[ dB_t = (Y_t - D_t) \, dt = (\rho_t - c) \, B_t \, dt \]

Note: Since \( \frac{dB_t}{B_t} = (\rho_t - c) \, dt \),
uncertainty about average \( \rho_t \) = uncertainty about the average growth rate of \( B_t \)
The abnormal earnings model of Ohlson (1990, 1995):

\[ M_t = B_t + \text{present value of future abnormal earnings} \]

- Competition \( \implies M_T = B_T \) at some future time \( T \)
  - \( T \) is random, exponentially distributed with density \( h(T; p) \)
  - At any point in time, there is probability \( p \) that \( T \) arrives in the next period

Market value of equity:

\[
M_t = E_t \left[ \int_t^\infty \left( \int_t^T \text{PV}(D_s) \, ds + \text{PV}(B_T) \right) h(T; p) \, dT \right],
\]

- The present value formula \( \text{PV}(.) \) is exogenously given but it allows for time varying risk aversion;
  - the equity premium varies due to time-varying risk aversion of the representative investor
  - Sometimes investors require a higher discount rate to hold stocks, and sometimes a lower discount rate, possibly due to the state of the economy.
Valuation Formula

- **Proposition 1.** Suppose $\bar{\psi}_t^i$ is known.

\[
\frac{M_t}{B_t} = G\left(\bar{\psi}_t^i; y_t, \bar{\rho}_t, \rho_t^i\right) = (c + p) \int_0^\infty Z\left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i, s\right) ds
\]

- When $\bar{\psi}_t^i$ is unknown, the law of iterated expectations yields

\[
\frac{M_t}{B_t} = E\left[G\left(\bar{\psi}_t^i; y_t, \bar{\rho}_t, \rho_t^i\right)\right] = \int G\left(\bar{\psi}_t^i; y_t, \bar{\rho}_t, \rho_t^i\right) f_t\left(\bar{\psi}_t^i\right) d\bar{\psi}_t^i
\]

Note: Since $G$ is convex in $\bar{\psi}_t^i$, greater dispersion in $f_t\left(\bar{\psi}_t^i\right)$ increases M/B

- **Proposition 2.** Suppose that $\bar{\psi}_t^i$ is unknown, and that $f_t\left(\bar{\psi}_t^i\right) = N\left(\hat{\psi}_t^i, \sigma_{i,t}^2\right)$.

\[
\frac{M_t}{B_t} = (c + p) \int_0^\infty Z\left(y_t, \bar{\rho}_t, \rho_t^i, \hat{\psi}_t^i, s\right) e^{\frac{1}{2}Q_4(s)\sigma_{i,t}^2} ds
\]

- **Note:** M/B increases if

(i) expected profitability increases ($\hat{\psi}_t^i \uparrow$, $\bar{\rho}_t \uparrow$, $\rho_t^i \uparrow$)

(ii) the discount rate decreases ($y_t \uparrow \Rightarrow$ equity premium $\downarrow$)

(iii) uncertainty about $\bar{\psi}_t^i$ increases ($\hat{\sigma}_{i,t} \uparrow$)
Calibration

- Two sectors:
  - “New economy” (Nasdaq): described above
  - “Old economy” (NYSE/Amex): pays dividends $D_t^O = cO B_t^O$ forever

Old economy’s market value is $M_t^O = E_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} D_s^O ds \right]$, we derive $M_t^O / B_t^O = \Phi(\bar{r}_t, y_t)$

- The profitability parameters are estimated from the data
- The discount rate parameters are calibrated to match the old economy’s average return, volatility, M/B, and the level of interest rate to their empirical counterparts

Table 1

<table>
<thead>
<tr>
<th>Old Economy Profitability</th>
<th>New Economy Profitability</th>
<th>Individ. Firm Profitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_L$</td>
<td>$\bar{\mu}_L$</td>
<td>$\sigma_{LL}$</td>
</tr>
<tr>
<td>0.3574</td>
<td>12.17%</td>
<td>1.47%</td>
</tr>
<tr>
<td>$\sigma_{L,0}$</td>
<td>$\phi^N$</td>
<td>$\sigma_{0,N}$</td>
</tr>
<tr>
<td>$\sigma_{L,0}$</td>
<td>0.3551</td>
<td>2.93%</td>
</tr>
<tr>
<td>$\sigma_{N,N}$</td>
<td>$\phi^i$</td>
<td>$\sigma_{i,0}$</td>
</tr>
<tr>
<td>$\sigma_{i,0}$</td>
<td>0.3891</td>
<td>6.65%</td>
</tr>
<tr>
<td>$\sigma_{i,i}$</td>
<td>8.07%</td>
<td></td>
</tr>
</tbody>
</table>

Stochastic Discount Factor

<table>
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<tr>
<th>$\eta$</th>
<th>$\gamma$</th>
<th>$k_y$</th>
<th>$\bar{y}$</th>
<th>$\sigma_y$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\mu_\varepsilon$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0471</td>
<td>3.9474</td>
<td>0.0367</td>
<td>-0.08%</td>
<td>25.30%</td>
<td>-2.8780</td>
<td>0.3084</td>
<td>-0.0413</td>
<td>2%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Means of Fitted Quantities

<table>
<thead>
<tr>
<th>$E[M/B]$</th>
<th>$E[\mu_{R,t}^{mkt}]$</th>
<th>$E[\sigma_{R,t}^{mkt}]$</th>
<th>$E[r_{f,t}]$</th>
<th>$\sigma[M/B]$</th>
<th>$\sigma[\mu_{R,t}^{mkt}]$</th>
<th>$\sigma[\sigma_{R,t}^{mkt}]$</th>
<th>$\sigma[r_{f,t}]$</th>
<th>$e^O$</th>
<th>$k_\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.77</td>
<td>5.06%</td>
<td>14.47%</td>
<td>6.25%</td>
<td>0.6477</td>
<td>1.72%</td>
<td>2.24%</td>
<td>1.55%</td>
<td>5.67%</td>
<td>0.0139</td>
</tr>
</tbody>
</table>
Many studies argue that the equity premium in 1999/2000 was low:

- Welch (2001): 3% per year
- Fama and French (2002): 2.6% or 4.3%
- Pástor and Stambaugh (2001): 4.8%
- Claus and Thomas (2001): 2.5%
- Gebhardt, Lee, and Swaminathan (2001): 2 to 3%
- Ilmanen (2003): 2%

⇒ values between 1% and 5% seem most plausible to us; we focus on 3%
Plausible Values for Excess Profitability ($\hat{\psi}^N$)

- Expected profitability for Nasdaq seemed higher than for NYSE/Amex ($\hat{\psi}^N > 0$)
  - Average analyst forecasts of long-term earnings growth in March 2000:
    * 28.8% for Nasdaq
    * 15.1% for NYSE/Amex
  - Historical performance
    * Form two portfolios in 1972: Nasdaq and non-Nasdaq
    * Between 1973 and 1999, the average ROE of the Nasdaq portfolio exceeds the ROE of the non-Nasdaq portfolio by 1.35% per year
    * Migration bias
      - Firms that migrate from NYSE/Amex to Nasdaq perform poorly, and firms that migrate from Nasdaq to NYSE/Amex perform well
        - In 1973–99, the excess ROEs are -42.6% and +4.9%, respectively
  - Increasing importance of intangible assets, especially on Nasdaq
Table 2. Nasdaq’s Valuation on March 10, 2000 Assuming Zero Uncertainty

\( \rho_t^N = 9.96\% \) per year, \( c = 1.35\% \) per year, \( E(T) = 20 \) years.

<table>
<thead>
<tr>
<th>( \hat{\psi}^N ) (% per year)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Model-implied M/B with zero uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Actual M/B: 8.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3.33</td>
<td>3.02</td>
<td>2.63</td>
<td>2.23</td>
<td>1.84</td>
<td>1.47</td>
<td>1.12</td>
<td>0.76</td>
</tr>
<tr>
<td>1</td>
<td>4.15</td>
<td>3.70</td>
<td>3.17</td>
<td>2.64</td>
<td>2.14</td>
<td>1.68</td>
<td>1.25</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>5.27</td>
<td>4.62</td>
<td>3.89</td>
<td>3.19</td>
<td>2.53</td>
<td>1.95</td>
<td>1.41</td>
<td>0.90</td>
</tr>
<tr>
<td>3</td>
<td>6.83</td>
<td>5.89</td>
<td>4.87</td>
<td>3.92</td>
<td>3.05</td>
<td>2.29</td>
<td>1.62</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>9.06</td>
<td>7.68</td>
<td>6.23</td>
<td>4.92</td>
<td>3.75</td>
<td>2.74</td>
<td>1.88</td>
<td>1.11</td>
</tr>
<tr>
<td>5</td>
<td>12.28</td>
<td>10.22</td>
<td>8.15</td>
<td>6.31</td>
<td>4.71</td>
<td>3.36</td>
<td>2.23</td>
<td>1.26</td>
</tr>
<tr>
<td>6</td>
<td>17.02</td>
<td>13.92</td>
<td>10.90</td>
<td>8.28</td>
<td>6.04</td>
<td>4.19</td>
<td>2.69</td>
<td>1.45</td>
</tr>
<tr>
<td>7</td>
<td>24.09</td>
<td>19.38</td>
<td>14.91</td>
<td>11.12</td>
<td>7.93</td>
<td>5.36</td>
<td>3.32</td>
<td>1.69</td>
</tr>
<tr>
<td>Panel B: Implied return volatility with zero uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Actual volatility: 41.5% in March 2000, 47% in 2000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>18.09</td>
<td>20.17</td>
<td>21.76</td>
<td>22.93</td>
<td>23.76</td>
<td>24.18</td>
<td>24.10</td>
<td>23.04</td>
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<td>23.92</td>
<td>24.83</td>
<td>25.31</td>
<td>25.22</td>
<td>24.05</td>
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<tr>
<td>3</td>
<td>19.93</td>
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<td>27.81</td>
<td>26.45</td>
</tr>
<tr>
<td>4</td>
<td>20.54</td>
<td>23.30</td>
<td>25.47</td>
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<td>27.85</td>
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<td>6</td>
<td>21.71</td>
<td>24.82</td>
<td>27.34</td>
<td>29.37</td>
<td>31.01</td>
<td>32.15</td>
<td>32.52</td>
<td>31.15</td>
</tr>
<tr>
<td>7</td>
<td>22.25</td>
<td>25.53</td>
<td>28.23</td>
<td>30.44</td>
<td>32.28</td>
<td>33.65</td>
<td>34.27</td>
<td>33.05</td>
</tr>
</tbody>
</table>
Table 3. Nasdaq’s Valuation on March 10, 2000 Assuming Uncertainty of 3% Per Year

\( \rho_t^N = 9.96\% \) per year, \( c = 1.35\% \) per year, \( E(T) = 20 \) years.

<table>
<thead>
<tr>
<th>Excess ROE</th>
<th>Equity Premium (% per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\psi}_N ) (% per year)</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>4.70</td>
</tr>
<tr>
<td>1</td>
<td>6.16</td>
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<tr>
<td>3</td>
<td>11.44</td>
</tr>
<tr>
<td>4</td>
<td>16.17</td>
</tr>
<tr>
<td>5</td>
<td>23.39</td>
</tr>
<tr>
<td>6</td>
<td>34.59</td>
</tr>
<tr>
<td>7</td>
<td>52.23</td>
</tr>
</tbody>
</table>

Panel A: Model-implied M/B with 3% uncertainty
(Actual M/B: 8.55)

Panel B: Implied return volatility with 3% uncertainty
(Actual volatility: 41.5% in March 2000, 47% in 2000)
Table 4. Matching Nasdaq’s Valuation on March 10, 2000

\[ \rho^N_t = 9.96\% \text{ per year, } c = 1.35\% \text{ per year, } E(T) = 20 \text{ years.} \]

<table>
<thead>
<tr>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
<th>Excess ROE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\psi}^N ) (% per year)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>4.39</td>
<td>4.71</td>
<td>5.06</td>
<td>5.43</td>
<td>5.81</td>
<td>6.22</td>
<td>6.67</td>
<td>7.27</td>
</tr>
<tr>
<td>1</td>
<td>3.81</td>
<td>4.17</td>
<td>4.59</td>
<td>5.01</td>
<td>5.44</td>
<td>5.89</td>
<td>6.38</td>
<td>7.03</td>
</tr>
<tr>
<td>2</td>
<td>3.08</td>
<td>3.54</td>
<td>4.04</td>
<td>4.53</td>
<td>5.03</td>
<td>5.54</td>
<td>6.08</td>
<td>6.77</td>
</tr>
<tr>
<td>3</td>
<td>2.08</td>
<td>2.73</td>
<td>3.38</td>
<td>3.98</td>
<td>4.57</td>
<td>5.15</td>
<td>5.75</td>
<td>6.50</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>1.45</td>
<td>2.51</td>
<td>3.32</td>
<td>4.04</td>
<td>4.71</td>
<td>5.39</td>
<td>6.22</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.97</td>
<td>2.43</td>
<td>3.40</td>
<td>4.22</td>
<td>5.00</td>
<td>5.91</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
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<td>3.63</td>
<td>4.56</td>
<td>5.58</td>
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<tr>
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<td>0.00</td>
<td>1.18</td>
<td>2.90</td>
<td>4.06</td>
<td>5.23</td>
</tr>
</tbody>
</table>

Panel A. Uncertainty needed to match the observed M/B

Panel B. Return volatility under implied uncertainty

(Actual volatility: 41.5% in March 2000, 47% in 2000)
Figure 1. Model-predicted distributions of future profitability and average future profitability for Nasdaq.
Model-predicted distribution of the future ratio of Nasdaq book value to NYSE/Amex/Nasdaq book value.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
<th>95</th>
<th>99</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.12</td>
<td>0.16</td>
<td>0.18</td>
<td>0.22</td>
<td>0.27</td>
<td>0.33</td>
<td>0.39</td>
<td>0.43</td>
<td>0.50</td>
</tr>
<tr>
<td>20</td>
<td>0.11</td>
<td>0.17</td>
<td>0.22</td>
<td>0.31</td>
<td>0.43</td>
<td>0.56</td>
<td>0.67</td>
<td>0.73</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Table 6. Matching the Valuations of Selected Technology Firms on March 10, 2000

<table>
<thead>
<tr>
<th>Excess ROE $\hat{\psi}_i$ (% per year)</th>
<th>Equity Premium (% per year)</th>
<th>Equity Premium (% per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implied Uncertainty (% per year)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MICROSOFT ($516bn): $M/B = 18.79$, $\rho_t = 48.28%$, $c = 0$, Vol = (57.44%, 56.10%)</td>
<td>5.04</td>
<td>5.28</td>
</tr>
<tr>
<td></td>
<td>4.15</td>
<td>4.46</td>
</tr>
<tr>
<td></td>
<td>2.89</td>
<td>3.36</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CISCO ($456bn): $M/B = 39.02$, $\rho_t = 26.58%$, $c = 0$, Vol = (49.81%, 68.88%)</td>
<td>4.59</td>
<td>4.84</td>
</tr>
<tr>
<td></td>
<td>3.67</td>
<td>3.99</td>
</tr>
<tr>
<td></td>
<td>2.34</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>INTEL ($395bn): $M/B = 11.09$, $\rho_t = 28.65%$, $c = 0.0148$, Vol = (45.81%, 68.71%)</td>
<td>5.11</td>
<td>5.38</td>
</tr>
<tr>
<td></td>
<td>4.11</td>
<td>4.47</td>
</tr>
<tr>
<td></td>
<td>2.64</td>
<td>3.22</td>
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</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
### Table 6. Matching the Valuations of Selected Technology Firms on March 10, 2000

<table>
<thead>
<tr>
<th>Excess ROE $\psi^i$ (% per year)</th>
<th>Equity Premium (% per year)</th>
<th>Excess ROE $\psi^i$ (% per year)</th>
<th>Equity Premium (% per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implied Uncertainty (% per year)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBAY ($23.76bn): $M/B = 27.87$, $\rho_t = 7.79%$, $c = 0$, $\text{Vol} = (129.24%, 113.64%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>6.03</td>
<td>6.20</td>
<td>6.38</td>
</tr>
<tr>
<td>-2</td>
<td>5.42</td>
<td>5.62</td>
<td>5.83</td>
</tr>
<tr>
<td>0</td>
<td>4.71</td>
<td>4.94</td>
<td>5.20</td>
</tr>
<tr>
<td>2</td>
<td>3.83</td>
<td>4.13</td>
<td>4.45</td>
</tr>
<tr>
<td>4</td>
<td>2.61</td>
<td>3.06</td>
<td>3.51</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
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<td>2.12</td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>IMMUNEX ($37.56bn): $M/B = 105.70$, $\rho_t = 17.91%$, $c = 0$, $\text{Vol} = (155.94%, 117.71%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>6.54</td>
<td>6.68</td>
<td>6.84</td>
</tr>
<tr>
<td>-2</td>
<td>6.01</td>
<td>6.17</td>
<td>6.35</td>
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<td>5.60</td>
<td>5.81</td>
</tr>
<tr>
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<td>4.73</td>
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<td>3.91</td>
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<td>2.82</td>
<td>3.21</td>
<td>3.60</td>
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<tr>
<td>8</td>
<td>0.56</td>
<td>1.68</td>
<td>2.38</td>
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<tr>
<td>10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>RED HAT ($10.43bn): $M/B = 26.50$, $\rho_t = -10.15%$, $c = 0$, $\text{Vol} = (121.00%, 122.33%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>6.23</td>
<td>6.39</td>
<td>6.57</td>
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<td>5.84</td>
<td>6.04</td>
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<td>5.22</td>
<td>5.45</td>
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<tr>
<td>2</td>
<td>4.22</td>
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<td>4</td>
<td>3.21</td>
<td>3.57</td>
<td>3.94</td>
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<tr>
<td>6</td>
<td>1.57</td>
<td>2.25</td>
<td>2.83</td>
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<tr>
<td>8</td>
<td>0.00</td>
<td>0.00</td>
<td>0.42</td>
</tr>
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</table>
### Table 6. Matching the Valuations of Selected Technology Firms on March 10, 2000

<table>
<thead>
<tr>
<th>Excess ROE $\psi^i$ (% per year)</th>
<th>Equity Premium (% per year)</th>
<th>Implied Uncertainty (% per year)</th>
<th>Implied Return Volatility (% per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>YAHOO</strong> ($98.90bn):** $M/B = 78.41, \rho_t = 10.52% , c = 0, Vol = (75.41%, 90.61%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>5.95</td>
<td>6.12</td>
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<td>5.35</td>
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<td>5.75</td>
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<tr>
<td>4</td>
<td>3.81</td>
<td>4.10</td>
<td>4.40</td>
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<tr>
<td>6</td>
<td>2.67</td>
<td>3.08</td>
<td>3.49</td>
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<td>8</td>
<td>0.00</td>
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<td>10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.63</td>
</tr>
<tr>
<td><strong>AMAZON</strong> ($23.45bn):** $M/B = 88.07, \rho_t = -126.08% , c = 0, Vol = (71.67%, 103.33%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6.94</td>
<td>7.07</td>
<td>7.21</td>
</tr>
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<td>6.12</td>
<td>6.28</td>
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<td>4.77</td>
<td>4.97</td>
<td>5.18</td>
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<td>4.02</td>
<td>4.26</td>
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<td>3.09</td>
<td>3.40</td>
<td>3.72</td>
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<td>16</td>
<td>0.00</td>
<td>0.00</td>
<td>0.67</td>
</tr>
<tr>
<td><strong>PRICELINE</strong> ($15.94bn):** $M/B = 39.58, \rho_t = -264.12% , c = 0, Vol = (128.17%, 133.65%)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>7.82</td>
<td>7.93</td>
<td>8.05</td>
</tr>
<tr>
<td>2</td>
<td>7.42</td>
<td>7.54</td>
<td>7.66</td>
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<tr>
<td>4</td>
<td>7.00</td>
<td>7.12</td>
<td>7.25</td>
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<tr>
<td>6</td>
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<td>8</td>
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<td>6.20</td>
<td>6.36</td>
</tr>
<tr>
<td>10</td>
<td>5.52</td>
<td>5.68</td>
<td>5.85</td>
</tr>
<tr>
<td>12</td>
<td>4.92</td>
<td>5.10</td>
<td>5.29</td>
</tr>
</tbody>
</table>
Figure 2. Model-predicted distributions of future profitability and book value for Amazon.
Panel A. Distribution of future ROE of Yahoo

Panel B. Distribution of future book value of Yahoo

Figure 3. Model-predicted distributions of future profitability and book value for Yahoo.
“Will Yahoo Become the Next Microsoft?”

• Assume that Yahoo is fairly priced according to our model in March 2000
  \( \hat{\psi} = 4\%, 3\% \) equity premium, \( E(T) = 15; \Rightarrow \hat{\sigma^i} = 4.4\% \)

• The distribution of Yahoo’s average ROE over the next 15 years (2000-2014):
  
  1st percentile: -0.97%
  50th percentile: 16.11%
  99th percentile: 33.18%

• Some average ROEs for comparison:
  
  – Microsoft in 1988-1999 (12 years): 44.46%
  – Oracle in 1988-1999 (12 years): 47.19%
  – Dell in 1990-1999 (10 years): 47.95%
  – Cisco in 1992-1999 (8 years): 47.13%

• Probability that Yahoo’s average ROE over the next 12 years will exceed Microsoft’s:
  
  0.66%
The Time Series of Implied Uncertainty

• At each time $t$, we compute Nasdaq’s implied uncertainty (i.e., uncertainty about $\psi^N_t$ that sets Nasdaq’s model-implied M/B equal to the observed M/B)

• Two steps:

  1. Equity premium
     - Given the old economy’s ROE ($\bar{\rho}_t$) and M/B, we invert the pricing formula
       \[(M/B)^O_t = \Phi(y_t; \bar{\rho}_t) \implies y_t = \Phi^{-1} ((M/B)^O_t, \bar{\rho}_t)\]
       at each $t$ to obtain the time series of $y_t$ and the equity premium

  2. Nasdaq’s implied uncertainty
     - Given the equity premium and Nasdaq’s ROE and M/B, we invert the pricing formula
       \[(M/B)^N_t = G (\hat{\sigma}^N; y_t, \rho^N_t) \implies \hat{\sigma}^N = G^{-1} ((M/B)^N_t, y_t, \rho^N_t)\]
       at each $t$ to obtain the time series of implied uncertainty $\hat{\sigma}^N$
Figure 7. Implied equity premium.
Figure 8. Implied uncertainty.
Figure 11. Cross-sectional correlation between implied uncertainty and idiosyncratic return volatility.
Why Did the “Bubble” Burst?

- There is little doubt of what caused tech stock prices to drop in 2000.
  - Nasdaq’s profitability plummeted in 2000.

![Graph showing Nasdaq and NYSE/Amex M/B ratios](image)

![Graph showing Nasdaq and NYSE/Amex profitability](image)
Why Did the “Bubble” Burst?

• Is this large drop consistent with our model?
  
  – Yes
    * A high uncertainty about long term profitability implies large revisions when there are large unexpected events.
    * Our model implies a similar drop in $M/B$ in 2000, and an even larger drop in 2001.

• Return volatility did not move much after March 2000.
  
  – This is also consistent with our model: Uncertainty remained high even after March 2000.
Technological Revolutions and Asset Prices

• The 1990s tech revolution and tech “bubble” was just the last example of a pattern repeated several times in history.

  “Technological revolutions and financial bubbles seem to go hand in hand.”
  “Every previous technological revolution has created a speculative bubble... With each wave of technology, share prices soared and later fell...”
  (The Economist, September 21, 2000)

• Stock prices tend to exhibit bubble-like patterns during technological revolutions
  – Prices rise and then fall, especially for innovative firms
  – Return volatility is high, especially for innovative firms

• Examples:
  – the early 1980s (biotechnology, PC)
  – the early 1960s (electronics)
  – the 1920s (electricity, automobiles)
  – the early 1900s (radio)
Repeated Irrational Exuberance?

- The bubble-like stock price behavior is commonly attributed to irrationality (e.g., Shiller, 2000, Perez, 2002, popular press)
  - Investors get too excited about the new technology

- We propose a rational explanation
  - Time-varying nature of uncertainty about the new technology
Our Story

- New technologies have **high uncertainty** about average future productivity
  - This uncertainty makes returns highly volatile

- Initially, this uncertainty is mostly **idiosyncratic**
  - Because the new technology is initially developed on a small scale
  - The idiosyncratic uncertainty increases stock prices (PV 2003, 2006)

- In *technological revolutions*, new technologies are widely adopted
- For those technologies that are eventually adopted by the whole economy, the uncertainty gradually changes from idiosyncratic to **systematic**
  - As a result, discount rates rise and stock prices fall

- The “bubble” in prices is observable **ex post** but unpredictable **ex ante**
  - **Ex post selection bias**: We know ex post that a technological revolution took place, but investors did not know that ex ante
Outline of the Model

• We develop a general equilibrium model with a representative agent

• Two sectors: the “new economy” and the “old economy”
  – Old economy: Large-scale production using old technology
    * Affects the representative agent’s wealth
  – New economy: Small-scale production using new technology
    * Does not affect the representative agent’s wealth

• The representative agent (the social planner)
  1. Sets up the new economy to “experiment” with the new technology
  2. Learns about the average productivity of the new technology
  3. Decides whether/when to adopt the new technology on a large scale

• If the technology is adopted, we call this a technological revolution
Preferences and Technology

- Representative agent has utility from final wealth, \( u(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma} \), with \( \gamma > 1 \)
- The agent is endowed with capital \( B_0 \) at time \( t = 0 \)
- Capital produces output \( Y_t = \rho_t B_t \), and follows \( dB_t = Y_t dt = \rho_t B_t dt \)
- Market clearing: \( W_T = B_T \)

- Productivity \( \rho_t \) follows a mean-reverting process:
  \[
  d\rho_t = \phi (\bar{\rho} - \rho_t) \, dt + \sigma dZ_{0,t},
  \]
  \[
  d\rho_t = \phi (\bar{\rho} + \psi - \rho_t) \, dt + \sigma dZ_{0,t},
  \]
  (under old technology)
  (under new technology)

- The “productivity gain” \( \psi \) is \textit{unobservable}
  - When the new technology arrives at time \( t^* \), \( \psi \) is drawn as normal:
    \[
    \psi \sim N (0, \hat{\sigma}^2_{t^*})
    \]
  - After time \( t^* \), the agent learns about \( \psi \) in a Bayesian fashion
• The agent learns about $\psi$ by observing productivity in the new economy

• Capital used in the new economy, $B_t^N$, is infinitely smaller than $B_t$
  \[ \Rightarrow \text{ the new technology affects } W_T \text{ only if adopted by the old economy} \]

• Capital in the new economy evolves as
  \[
  dB_t^N = \rho_t^N B_t^N dt \\
  d\rho_t^N = \phi \left( \bar{\rho} + \psi - \rho_t^N \right) dt + \sigma_{N,0} dZ_{0,t} + \sigma_{N,1} dZ_{1,t}
  \]

**Learning:** Given the prior $\psi|\mathcal{F}_{t^*} \sim N \left(0, \hat{\sigma}_{t^*}^2\right)$, the posterior of $\psi$ is also normal, $\psi|\mathcal{F}_t \sim N(\hat{\psi}_t, \hat{\sigma}_t^2)$, where

\[
\hat{d}\psi_t = \frac{\hat{\sigma}_t^2 \phi}{\sigma_{N,1}} d\tilde{Z}_{1,t}
\]

\[
\hat{\sigma}_t^2 = \frac{1}{\hat{\sigma}_{t^*}^{-2} + \left( \frac{\phi}{\sigma_{N,1}} \right)^2 \left( t - t^* \right)}
\]
New technology arrives
New economy formed
Learning begins

Agent learns about $\psi$

Agent decides whether to adopt the new technology

New technology adopted by old economy
Mean productivity increases by $\psi$
Technological revolution

Agent consumes $W_T = B_T$

New technology not adopted
Mean productivity unchanged
No technological revolution

$t^{**}$ is chosen to maximize utility, but first we take it as given, for simplicity
Technology Adoption

- The agent chooses if/when to adopt the new technology to maximize utility
- The adoption incurs a proportional conversion cost $\kappa \geq 0$ and it is irreversible

**Proposition 1:** It is never optimal to adopt the new technology at time $t^*$.

- The prior at time $t^*$ is $\psi \sim N \left(0, \hat{\sigma}_{t^*}^2\right)$

**Proposition 2:** The new technology is adopted at time $t^{**} > t^*$ iff

$$\hat{\psi}_{t^{**}} > \psi = -\frac{\log (1 - \kappa)}{A_2 (\tau^{**})} + \frac{1}{2} (\gamma - 1) A_2 (\tau^{**}) \hat{\sigma}^2_{t^{**}},$$

- Adopt if the new technology is perceived as sufficiently productive

**Proposition 3:** It is optimal to begin experimenting with new technology at $t^*$.

- Experimenting provides a valuable option for free
Stock Prices and the Changing Nature of Uncertainty

• Market values of stocks in the old and new economies:

\[ M_t = E_t [PV (B_T)] \quad \text{and} \quad M_t^N = E_t [PV (B_T^N)] \]

• The present value formula \( PV(.) \) is derived endogenously as part of the equilibrium.

• In particular, the \( PV(.) \) formula depends on the probability that the new technology is adopted

  – Adoption probability \( \approx 0 \Rightarrow \) Uncertainty \( \hat{\sigma}_t \) is mostly idiosyncratic
  – Adoption probability \( \uparrow \Rightarrow \) Uncertainty \( \hat{\sigma}_t \) becomes increasingly systematic
  – In a technological revolution, adoption probability increases from \( \approx 0 \) to 1,
    so the nature of Uncertainty \( \hat{\sigma}_t \) changes from idiosyncratic to systematic
  – As uncertainty becomes more and more systematic, the discount rate used in present value
    formula \( PV(.) \) increases.

**Propositions 4 - 6:** Closed-form formulas for \( \pi_t, M_t/B_t \) and \( M_t^N/B_t^N \)
• In a technological revolution, \( \hat{\psi}_t \) increases from \( \hat{\psi}_{t^*} = 0 \) to \( \hat{\psi}_{t^{**}} > \psi > 0 \)

• The increase in \( \hat{\psi}_t \) has two opposing effects on prices:
  – *Cash flow effect*: Expected dividend \( \uparrow \Rightarrow M/B \uparrow \)
  – *Discount rate effect*: Systematic risk \( \uparrow \Rightarrow M/B \downarrow \)

• For the new economy, the cash flow effect tends to prevail initially (close to \( t^* \)), but the discount rate effect prevails in the end (close to \( t^{**} \))
  \( \Rightarrow \) “bubble” in the new economy
Simulations

• We simulate 50,000 samples of shocks in our economy
• Plot average paths of M/B and volatility across simulations
• See how these paths differ depending on whether the new technology was eventually adopted (revolution) or not (no revolution)
  ⇒ Tackle the ex post selection bias

Table 1: Parameters used in Simulations.

<table>
<thead>
<tr>
<th>$\bar{p}_L$</th>
<th>$\psi_{t*}$</th>
<th>$\hat{\sigma}_{t*}$</th>
<th>$\phi$</th>
<th>$\sigma_0$</th>
<th>$\sigma_{N,0}$</th>
<th>$\sigma_{N,1}$</th>
<th>$\kappa$</th>
<th>$t^{**} - t^*$</th>
<th>$T$</th>
<th>$\gamma$</th>
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<td>0.1217</td>
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<td>0.07</td>
<td>0.1</td>
<td>8</td>
<td>30</td>
<td>4</td>
</tr>
</tbody>
</table>
Figure 3. Average M/B and Volatility in Simulations.

(A) Revolution: Market to Book Ratio

(B) No Revolution: Market to Book Ratio

(C) Revolution: Stock Return Volatility

(D) No Revolution: Stock Return Volatility
Figure 4. Beta and Average Stock Return in Simulations.

(A) Revolution: New Economy Beta

(B) No Revolution: New Economy Beta

(C) Revolution: New Economy Return

(D) No Revolution: New Economy Return

(E) Revolution: Old Economy Return

(F) No Revolution: Old Economy Return
Figure 7. Internet Revolution: Theory.

(A) New Economy Beta

(B) Stock Return Volatility

(C) Market Value

(D) Old Economy Productivity
Figure 8. Internet Revolution: Data.

(A) Beta of NASDAQ

(B) Stock Return Volatility

(C) Index Level

(D) Productivity Growth

(A) Beta of NASDAQ

(B) Stock Return Volatility

(C) Index Level

(D) Productivity Growth
American Railroads Before the Civil War

• Early milestones:
  – 1825: First steam locomotive run (John Stevens)
  – 1828: First RR construction begins (Baltimore & Ohio)
  – 1830: First scheduled steam train run (Charleston)

• It was far from obvious in the 1830s-40s that RRs would later come to dominate the transportation industry
  – Competition with other modes of transportation: wagons, steamboats, canals
  – Waterways were cheaper, wagons more flexible

“Far from being viewed as essential to economic development, the first RRs were widely regarded as having only limited commercial application. Extreme skeptics argued that RRs were too crude to insure regular service, that the sparks thrown off by belching engines would set fire to buildings and fields, and that speeds of 20 to 30 miles per hour could be “fatal to wagons, road and loading, as well as to human life.” More sober critics questioned the ability of RRs to provide low cost transportation. [Some] placed “a RR between a good turnpike and a canal” in transportation efficiency.” (Fogel, 1964)
Figure 9.

Rail Consumption in the U.S.

Year

Track-miles

0 500 1000 1500 2000 2500 3000 3500 4000 4500

1830 1835 1840 1845 1850 1855 1860
Railroad Expansion

- Large-scale adoption of RR technology appears to have taken place by 1860
  - 1856: Leap in RR diffusion
    * Two milestone RRs completed
      - Illinois Central, the longest RR in the world (705 miles)
      - Sacramento Valley, the first RR in California
    * First RR bridge across Mississippi, heralding westward expansion

  “By 1860... the RR had emerged not only as the preferred form of transportation but also as the chief weapon of commercial rivalry.” (Klein, 1994)

- Do stock prices agree with this assessment?
Railroad Stock Prices

- We examine RR stock prices in the early days of the RR (1830–1861)
- Nearly all RRs organized as corporations funded by private investors
  - More than half of the $300m+ RR investment in 1850 was stock-financed
- Data compiled by Goetzmann, Ibbotson, and Peng (2001)
  - Monthly individual stock prices for 671 NYSE stocks in 1815 to 1925
  - Annual dividends for a subset of stocks in 1825 to 1870
- We focus on common stocks (exclude 85 preferred stocks and 29 scrips)
- We delete apparent data errors (40 of 15,276 prices; 0.26% of observations)
- We fill in price gaps no more than three months long by linear interpolation
  - Before 1848, uninterrupted price sequences for RR stocks are rare
- We identify RRs by name (284 stocks)
Table 2: Railroads Appearing in our Price Index.

<table>
<thead>
<tr>
<th>Year</th>
<th>Railroad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1831</td>
<td>Camden &amp; Amboy; Canajoharie &amp; Catskill; Harlem; Ithaca &amp; Oswego</td>
</tr>
<tr>
<td>1832</td>
<td>Boston &amp; Providence</td>
</tr>
<tr>
<td>1833</td>
<td>Boston &amp; Worcester; Brooklyn &amp; Jamaica</td>
</tr>
<tr>
<td>1835</td>
<td>Hudson &amp; Berkshire; Long Island</td>
</tr>
<tr>
<td>1839</td>
<td>Auburn &amp; Syracuse</td>
</tr>
<tr>
<td>1841</td>
<td>Auburn &amp; Rochester</td>
</tr>
<tr>
<td>1844</td>
<td>Housatonic</td>
</tr>
<tr>
<td>1847</td>
<td>Hudson River; Macon &amp; West</td>
</tr>
<tr>
<td>1848</td>
<td>Hartford &amp; New Haven; New York &amp; Erie</td>
</tr>
<tr>
<td>1849</td>
<td>Erie</td>
</tr>
<tr>
<td>1850</td>
<td>Albany &amp; Schenectady; Baltimore &amp; Ohio; Michigan Central; New York &amp; Harlem</td>
</tr>
<tr>
<td>1851</td>
<td>Chemung</td>
</tr>
<tr>
<td>1852</td>
<td>Michigan &amp; Southern</td>
</tr>
<tr>
<td>1853</td>
<td>Cincinnati, Hamilton &amp; Dayton; Cleveland, Columbus &amp; Cincinnati; Cleveland &amp; Pittsburg; Cleveland &amp; Toledo; Galena &amp; Chicago; Illinois Central; Little Miami</td>
</tr>
<tr>
<td>1854</td>
<td>Chicago &amp; Rock Island</td>
</tr>
<tr>
<td>1855</td>
<td>Michigan Southern &amp; Northern Indiana</td>
</tr>
<tr>
<td>1856</td>
<td>Eighth Avenue; Lacrosse &amp; Milwaukee; Macon &amp; Western</td>
</tr>
<tr>
<td>1857</td>
<td>Chicago, Burlington &amp; Quincy; Delaware, Lackawanna &amp; Western; Indianapolis &amp; Cincinnati</td>
</tr>
<tr>
<td>1858</td>
<td>Brooklyn City; Buffalo &amp; State Line; Cleveland, Painesville &amp; Ashtabula</td>
</tr>
</tbody>
</table>
Figure 10. The Railroad Revolution: Data.
Conclusions

• We offer a rational explanation for the bubble-like behavior of stock prices during technological revolutions

• Stock prices of innovative firms
  – initially rise due to good news about the new technology’s productivity
  – ultimately fall as uncertainty changes from idiosyncratic to systematic

• The rise and fall in prices are observable ex post but unpredictable ex ante
  – Ex post, we focus on technologies that led to technological revolutions

• The model makes several empirical predictions about the joint behavior of prices, return volatility and betas.
  – Empirical support for them during both the Internet and Railroad revolutions

• Technological revolutions can be dated based on the behavior of stock prices
  – We find evidence consistent with large-scale adoption of the railroad technology in the United States around year 1857 and Internet by 2002.
  – This theory was recently validated empirically by Barath and Viswanathan (2006).
• Bayesian Bubble (Li and Xue, JF 2009)
  – Uncertainty on whether there is a regime shift in productivity growth leads lofty market valuations
  – Endogenize the increase in uncertainty around technological revolutions

• David and Veronesi (2011)
  – Time variation in uncertainty about growth and inflation regimes brings time variation in covariances between stocks and bonds, as well as time varying relation between price/earnings and volatility, or yields and volatility.
Panel A. Model generated probability of new economy

![Graph showing the probability of a new economy over time.]

Panel B. Model implied market-to-book ratio

![Graph showing the evolution of the model-implied market-to-book ratios.]

**Figure 3.** Model-estimated probability ($Q$) of a new economy and the model-implied market-to-book ratio. In Panel A, the figure traces the evolution of a representative investor’s belief regarding a new economy as captured by the probability $Q$. In Panel B, the figure traces the evolution of the model-implied market-to-book ratios. For the solid line, the initial uncertainty level ($\gamma_0$) is set to be equal to the variance of the noise in the historical TFP growth series. That is, $\gamma_0 = \sigma_{\nu,0}^2$. For the dashed line, $\gamma_0$ is reduced by half. That is, $\gamma_0 = \sigma_{\nu,0}^2/2$. The observed aggregate market-to-book ratio for the IT-producing industry (the dotted line) is presented in the background for comparison.

Source: Li and Xue, Journal of Finance, 2009
Why does IPO volume fluctuate?

How is IPO volume related to stock prices?
Leading Stories Behind Time-Varying IPO Volume

- Market mispricing

- Asymmetric information

- Other (mostly ‘corporate finance’) issues related to the IPO decision
  - Private benefits of control (Zingales, 1995, Benninga, Helmantel and Sarig, 2003)
  - Outside monitoring (Holmström and Tirole, 1993)
  - Diversification (Leland and Pyle, 1977)
Rational IPO Waves

• Pastor and Veronesi (2005, JF) develop a rational symmetric-information model of optimal IPO timing in an environment with time-varying market conditions

• Market conditions vary along three dimensions:
  – Time-varying expected market return
  – Time-varying expected aggregate profitability
  – Time-varying prior uncertainty about average excess profitability

• PV find, theoretically and empirically, that IPO volume is high after
  – Expected market return ↓
  – Expected aggregate profitability ↑
  – Prior uncertainty ↑
Empirical Predictions (1)

- PV’s model is rich in testable predictions:
  1. IPO waves caused by declines in expected market return should be
     - preceded by high market returns
     - followed by low market returns
  2. IPO waves caused by increases in expected aggregate profitability should be
     - preceded by high market returns
     - followed by high aggregate profitability
  3. IPO waves caused by increases in prior uncertainty should be
     - preceded by high disparity between new firms and old firms in terms of their valuations and volatilities

- PV test the model’s implications using data between 1960 and 2002
- Empirical results lend considerable support to all three channels (discount rate, cash flow, and uncertainty).
Empirical Predictions (2)

- The model also predicts that
  1. IPO volume is related more to recent changes in prices than to the level of prices
     - IPOs take place especially after market conditions improve (and prices go up);
       when prices are high, many private firms have already gone public
     - The level of prices matters as well, but much less than changes in prices
  2. IPO valuations are especially high
     - Low discount rate and high expected profitability
     - High prior uncertainty (Pástor and Veronesi, 2003)
  3. After IPO, M/B is predicted to fall, on average
     - Uncertainty declines due to learning
     - Mean reversion in expected return and profitability

- These predictions are also confirmed empirically
Inventors and Investors

• Two types of agents
  1. Inventors: Invent patentable ideas, but lack capital
  2. Investors: Cannot invent ideas, but have capital

• Both types of agents are otherwise identical
  – Same information
  – Same preferences (both maximize expected habit utility from consumption)
  – Same amount of wealth (inventors: human capital; investors: financial capital)

• Timing:

  \[ t_i \quad \tau_i \quad \tau_i + 1 \quad t \quad T_i \]

  - idea is patented
  - decision to go public is taken
  - IPO time
  - patent expires

  \[ h_i = \text{time to expiration} \]
Time-Varying Profitability

• Firm profitability is assumed mean-reverting:

\[ \rho_t^i = \frac{\text{Earnings}_t}{\text{Book Equity}_t} = \frac{Y_t^i}{B_t^i} \]

\[ d\rho_t^i = \phi^i \left( \bar{\rho}_t + \psi^i - \rho_t^i \right) dt + \sigma_{i,0} dW_{0,t} + \sigma_{i,i} dW_{i,t} \]

- \( \bar{\rho}_t \) ... Average aggregate profitability
- \( \psi^i \) ... Average firm-specific excess profitability

• Aggregate \( \bar{\rho}_t \) is also assumed mean-reverting:

\[ d\bar{\rho}_t = k_L (\bar{\rho}_L - \bar{\rho}_t) dt + \sigma_{L,0} dW_{0,t} + \sigma_{L,L} dW_{L,t} \]
Time-Varying Prior Uncertainty

- Average excess profitability $\bar{\psi}^i$ is assumed unobservable.
- After IPO, all agents learn about $\bar{\psi}^i$ by observing realized profitability.
- Before IPO, all agents have a common prior on $\bar{\psi}^i$, with prior uncertainty $\hat{\sigma}_t$.
  - $\hat{\sigma}_t$ is assumed to vary over time as economic conditions change.
  - E.g., technological revolutions could imply high uncertainty.

**Lemma:** If the prior at the IPO time $\tau_i$ is $\bar{\psi}^i \sim N \left( \hat{\psi}^i_{\tau_i}, \hat{\sigma}^2_{\tau_i} \right)$, and investors update their beliefs using the Bayes rule, then the conditional posterior is also normal,

$$\bar{\psi}^i | \mathcal{F}_t \sim N \left( \hat{\psi}^i_t, \hat{\sigma}^2_{i,t} \right)$$

where

$$d\hat{\psi}^i_t = \hat{\sigma}^2_{i,t} \frac{\phi^i}{\sigma_{i,i}} d\tilde{W}_{i,t}, \quad \hat{\sigma}^2_{i,t} = \frac{1}{\frac{1}{\hat{\sigma}^2_{\tau_i}} + \frac{(\phi^i)^2}{\sigma^2_{i,i}} (t - \tau_i)}.$$
Time-Varying Expected Returns

- Assume investors’ risk aversion is time varying, due to the state of the economy
  - When the economy is doing well and economic growth is sustained, risk aversion declines.
    * This is intuitive, as wealth accumulation may induce agents to take larger bets, once the
      some buffer savings are ensured
    * ⇒ discount rate decreases as investors are willing to require a lower compensation for
      risk, given their expectation of future cash flows.
  - When the economy is doing badly and economic growth is slow or even negative, risk aversion
    increases
    * This is also intuitive, as wealth decumulation pushes investors closer to their zero savings,
      and thus make them less willing to take risky bets.
    * ⇒ discount rate increases as investors now require a higher compensation for risk (lower
      price), given their expectation of future cash flows.

- Within the framework of the CAPM, this may imply

\[ E_t[r^i] = r_{f,t} + \beta_t \times (E_t[r^M_t] - r_{f,t}) \]

- where now the risk free rate \( r_{f,t} \), the aggregate expected return on the market \( E_t[r^M_t] \), and
  the market beta \( \beta_t \) may be time varying. (see e.g. PV (2006), Santos and Veronesi (2010)).

- Denote \( y_t = \) index of risk tollerance, which is time varying.
Prices and Returns

- Competition \[ \implies M^i_T = B^i_T \text{ at } T = T_i \]
- Assume no dividend payouts and no new equity issues between IPO and patent expiration
  \[ \Rightarrow \text{Clean surplus relation: } dB^i_t = Y^i_t dt = \rho^i_t B^i_t dt \]
- We derive closed-form formulas for stock prices (M/B) and returns
- Stock prices
  - \( \uparrow \) with expected profitability \((\rho^i_t, \bar{\rho}_t, \hat{\psi}^i)\)
  - \( \downarrow \) with the discount rate (through “risk tolerance” \(y_t\))
  - \( \uparrow \) with uncertainty about \(\hat{\psi}^i (\hat{\sigma}^i_{i,t})\)
Optimal IPO Timing

- At time $\tau$, IPO is filed, at $\tau + \ell$, IPO takes place ($\ell = 3$ months, Lowry and Schwert, 2002)
  1. $M_{\tau+\ell}^i$, fair market value of the firm computed earlier, is raised
  2. $f = 7\%$ underwriting fee is paid
  3. Initial investment $B^{t_i}$ is made, production begins
  4. Equity issued in the IPO keeps the markets dynamically complete
- Inventor essentially owns an American option that can be exercised by going public
- Inventor chooses IPO time to maximize the value of his patent:
  \[
  V(\tilde{\rho}_t, y_t, \hat{\sigma}_t, h_i) = \max_{\tau} E_t \left\{ e^{-r\ell} \left( M_{\tau+\ell}^i (1 - f) - B^{t_i} \right) \right\}
  \]
  - The optimal stopping time problem is solved numerically
Panel A. $T = 15, \psi = 0$

Panel B. $T = 15, \sigma = 0$

Panel C. $\psi = 0, \sigma = 0$

Panel D. $\psi = 0, \sigma = 0$

Optimal IPO Timing.
Simulating IPO Waves

- Assume one idea is born every month
  - note that IPO waves arise even when the pace of innovation is constant
- Simulate 10,000 years of data
- Use parameters calibrated to the old economy, as in “bubble”
Panel A. Simulated IPO Volume

Panel B. Simulated Aggregate Market-to-Book Ratio
Table 3
Simulation Evidence: Regressions of IPO Volume on Selected Variables.

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Proxies

- Two proxies for prior uncertainty:

\[
NEWVOL_t = \sigma_{R,t}^{ipo} - \sigma_{R,t}^m,
\]

\[
NEWMB_t = \log \left( \frac{M_t^{ipo}}{B_t^{ipo}} \right) - \log \left( \frac{M_t^m}{B_t^m} \right)
\]

- in the simulation, both proxies are highly correlated with prior uncertainty
  but not with the other two dimensions of market conditions

- Two proxies for expected market return:

\[
MVOL_t = \sigma_{R,t}^m,
\]

Realized returns \( \ldots \) \( R_t^m \)

- in the simulation, both proxies are highly correlated with expected market return
  but not with the other two dimensions of market conditions
Empirical Evidence

• Data
  – IPOs:
    * Jay Ritter’s data: January 1960 through December 2002
    * Number of IPOs is deflated by the number of public firms at previous month-end
  – MKT and MVOL from CRSP
  – Aggregate M/B and aggregate profitability (ROE) from COMPUSTAT
  – Additional proxy for expected profitability:
    * I/B/E/S average forecast of long-term earnings growth
  – Risk free rate = yield on a one-month T-bill

• Tables 5 and 6: Empirical counterparts to Tables 2 and 3
Table 6a, Empirical Evidence: Regressions of IPO Volume on Selected Variables

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### Table 6b, Empirical Evidence: Regressions of IPO Volume on Selected Variables

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<td>0.76</td>
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Figure 7. Monthly time series of proxies for prior uncertainty.
Conclusions

• We develop a model of optimal IPO timing in time-varying market conditions

• The model implies that IPOs take place especially after

  1. expected market return declines
  2. expected aggregate profitability increases
  3. prior uncertainty about average profitability increases

• The model has numerous asset pricing implications for IPO volume. IPO waves should be

  1. preceded by high market returns
  2. followed by low market returns
  3. followed by increases in aggregate profitability
  4. preceded by high disparity between new firms and old firms in their M/Bs and volatilities
  5. related more to recent changes in prices than to the level of prices

• All predictions are met in the data

• We show that prior uncertainty about average profitability

  – can generate IPO waves when it rises
  – helps explain high IPO valuations
  – was high in the late 1990s
In our model, the capital raised in the IPO is immediately invested.

Some firms go public for other reasons (e.g., refinancing), but investment is a key motive:

- 64% of firms state in their prospectus that the reason for their IPO is to finance capital expenditures (Mikkelson, Partch, and Shah, 1997).
- The capital expenditures of IPOs grow significantly more quickly (by more than 100%) than the capital expenditures of industry-matched seasoned firms (Jain and Kini, 1994).
- Private firms’ demands for capital affect IPO volume (Lowry, 2003).
- IPO volume is significantly correlated with aggregate investment growth (our analysis).

Our model can principle address cyclicality in aggregate investment.

Investment waves indeed appear related to recent changes in market conditions:

- Investment growth is related positively to recent market returns, negatively to future market returns, and positively to current and future changes in aggregate ROE.

We believe our model is better suited for studying investment by new firms than old firms:

- Nature of investment projects
- Existing projects
- Learning
- Complete markets
Figure. IPO volume and investment growth. In Panel A, the figure plots the number of IPOs. In Panel B, the figure plots the percentage change in real private nonresidential fixed investment (used e.g. in Barro 1990, Cochrane 1996, and Lamont 2000), extracted from the Bureau of Economic Analysis. Both series are quarterly between 1960 Q1 and 2002 Q4.
### Annual Regressions of Investment Growth on Selected Variables

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| $T$ | 41.00 | 54.00 | 52.00 | 35.00 | 29.00 | 40.00 |
| $R^2$ | 0.13 | 0.25 | 0.02 | 0.33 | 0.01 | 0.00 |
## Semi-annual Regressions of Investment Growth on Selected Variables

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| $T$       | 84.00 | 109.00 | 104.00 | 70.00  | 64.00  | 76.00  |
| $R^2$     | 0.14  | 0.32   | 0.03   | 0.18   | 0.01   | 0.01   |
## Quarterly Regressions of Investment Growth on Selected Variables

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Entrepreneurial Learning and the Post-IPO Performance

• Going public is one of the most important decisions made by private firms
  – Reasons: Diversification, raise capital, exploit good market conditions, etc.

• Problem: Private firm’s future cash flow is highly uncertain.
  – How does this uncertainty affect the IPO decision?
  – How does it affect the firm profitability around the IPO?

• We develop a model of the IPO decision that features
  – Learning about average profitability
  – Tradeoff between benefits of private control and optimal consumption
Results

- The model yields numerous predictions:
  - Profitability drops after IPO, on average
  - Profitability drops more for firms with more volatile profitability
  - Profitability drops less for firms with more uncertain average profitability

- These predictions are supported empirically in a sample of 7,138 U.S. IPOs in 1975-2004.
Panel A. Realized and expected profitability, $\rho_0 = 7\%$
Toy Model: Assumptions

- In periods 0 and 1, entrepreneur decides whether to take her firm public
- Cutoff Rule: IPO occurs if firm’s expected profitability exceeds $\rho$.
  (Threshold rule is optimal in full model, and we derive the endogenous cutoff)
- Firm’s average profitability, $\bar{\rho}$, is unknown
  - Time 0: Prior beliefs about $\bar{\rho}$:
    \[ \bar{\rho} \sim N (\hat{\rho}_0, \hat{\sigma}_0^2) \]
  - Time 1: Entrepreneur observes realized profitability, $\rho$:
    \[ \rho \sim N (\bar{\rho}, \sigma_\rho^2) \]
  - Entrepreneur uses Bayes’ Rule to update beliefs about $\bar{\rho}$
Toy Model Result 1: Profitability is expected to fall after the IPO
Toy Model Result 2: Effects of volatility and uncertainty

- Expected post-IPO drop in profitability is large when
  1. volatility ($\sigma_\rho$) is large
  2. uncertainty ($\sigma_0$) is small

  - Expected (percentage) post-IPO drop in profitability equals $\frac{1/\sigma_0^2}{1/\sigma_0^2 + 1/\sigma_\rho^2}$
  - Realized profitability must rise further above expected profitability when
    1. volatility is higher, because the signal is weaker
    2. uncertainty is lower, because prior beliefs are stronger
The Full Model

- Two types of agents:
  - Investors: endowed with stocks and bonds.
  - Entrepreneur: endowed with patent-protected technology, and wealth $W_0$
- Technology needs initial capital investment $B_0 = W_0$
  - Entrepreneur cannot borrow
- Choice at time $t = 0$
  - (1) Start private firm; (2) Sell patent; (3) Discard patent
- If start private firm, invest $B_0$, start producing.
  - Option to sell the private firm in a IPO at some time $\tau < T$.
    * For most part, IPO time $\tau$ is assumed exogenous.
    * Optimal endogenous $\tau$ yields similar results.
• Firm profitability is assumed mean reverting:

$$\rho_t = \frac{\text{Earnings}}{\text{Book Equity}} = \frac{Y_t}{B_t}$$

$$d\rho_t = \phi(\bar{\rho} - \rho_t) \ dt + \sigma_{\rho,1} dX_{1,t}^{\text{systematic}} + \sigma_{\rho,2} dX_{2,t}^{\text{idiosyncratic}}$$

− Full reinvestment of earnings $$\implies dB_t = \rho_t B_t \ dt$$

• $$\bar{\rho}$$ unobservable to anyone (symmetric information).
  − Prior at time $$t = 0$$
    $$\bar{\rho} \sim N(\hat{\rho}_0, \hat{\sigma}_0^2)$$

− Entrepreneur and investors learn about $$\bar{\rho}$$ using Bayes rule.
Preferences

- Investors: They discount stocks at a given rate $r = \text{risk free rate} + \text{risk premium}$

- Entrepreneur:

  $$\max \ E_t \left[ \int_t^T e^{-\beta(u-t)} \frac{c_{1-\gamma}^{1-\gamma}}{1-\gamma} du + \eta e^{-\beta(T-t)} \frac{W_T^{1-\gamma}}{1-\gamma} \right]$$

  - where $c_t = \text{entrepreneur’s consumption at time } t$; $\gamma = \text{entrepreneur’s risk aversion}$; $W_T = \text{wealth at time } T$

- If firm is private, benefits of private control:

  $$c_t = \alpha B_t$$

- **Learning:** The posterior beliefs is $\bar{\rho} \sim N \left( \hat{\rho}_t, \hat{\sigma}^2_t \right)$

  $$d\hat{\rho}_t = \hat{\sigma}^2_t \frac{\phi}{\sigma_{\rho,2}} d\hat{X}_{2,t}$$

  and

  $$\hat{\sigma}^2_t = \frac{1}{\hat{\sigma}_0^2 + \phi^2 t}$$
The Value of the Firm

• To Investors:
  – Patent expires at $T \implies M_t = E_t \left[ e^{-r(T-t)} B_T \right]$

• To the Entrepreneur:
  – Before IPO $\tau$, entrepreneur’s wealth tied up in the firm;
  – If sell the firm at $\tau$, gets $M_\tau$, which can optimally invest in stocks and bonds
    \[ \implies \text{Expected Utility} = V(M_\tau, \tau) = \frac{M_\tau^{1-\gamma}}{1-\gamma} g(T - \tau) \]
  – If does not sell at $\tau$, let $V^O(B_\tau, \tau)$ be the entrepreneur’s expected utility
    \[ \implies \text{The entrepreneur sells at } \tau \text{ if and only if } V(M_\tau, \tau) \geq V^O(B_\tau, \tau) \]
  – Equivalently, IPO occurs at $\tau$ if and only
    \[ (\text{Market Value of the Firm}) \quad M_\tau \geq P_\tau \quad (\text{Private Value of the Firm}) \]
Corollary 1: An IPO at time \( \tau \) is more likely when

\begin{itemize}
\item[(a)] benefits of private of control, \( \alpha \), are lower
\item[(b)] uncertainty about average profitability, \( \hat{\sigma}_\tau \), is higher
\item[(c)] the idiosyncratic component of the volatility of profitability, \( \sigma_{\rho,2} \), is higher
\item[(d)] current and/or expected profitability, \( \rho_\tau \) and \( \hat{\rho}_\tau \), are higher
\end{itemize}

Corollary 2: An IPO takes place at time \( \tau \) if and only if

\[ \hat{\rho}_\tau > \rho(x_\tau, \hat{\sigma}_\tau, \sigma_{\rho,2}) \]

where \( x_t \) is the “excess” profitability process \( x_t = \rho_t - \hat{\rho}_t \).
The Start of a Private Firm

- The existence of a private firm is endogenous
  - Intuitively, too high uncertainty or volatility of profitability or too low benefits of private control may induce the entrepreneur to discard or sell the patent.
- Let $V_0^O(B_0, 0) = \text{expected utility from starting the company at time 0.}$
- Then, the private company is funded at time 0 if and only if
  $$V_0^O(B_0, 0) \geq \max(V(M_0, 0), V(B_0, 0))$$

- We have close form solution for $V_0^O(B_0, 0)$, but formula is not “self evident”
- Analytical formula is key for feasible simulation exercise, to take into account the endogeneity problem of private firms’ existence.
Profitability Dynamics around an IPO

• Simulate 10,000 paths of shocks from model

• Plot average profitability across paths in which an IPO is optimal
  – Tackle the sample selection bias problem.

• Parameters (Table 1):

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<th>$r$</th>
<th>$\sigma_\pi$</th>
<th>$\sigma_{\rho,1}$</th>
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<td>0.05</td>
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<td>2</td>
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</table>

• Initial profitability, $\rho_0$, equals 7% or 0%. 
Profitability Dynamics around an IPO

Panel A. Realized and expected profitability, $\rho_0 = 7\%$

Panel B. Realized and expected profitability, $\rho_0 = 0$
Profitability Dynamics around an IPO

- Intuition:
  - IPO at \( \tau \) occurs if and only if \( \hat{\rho}_\tau \) above threshold.
  - Since firm did not go public earlier, \( \hat{\rho}_0 \) lower than threshold.
  - **Endogeneity of IPO + Learning:** For firm to go public at \( \tau \), \( \hat{\rho}_t \) must have been pulled up between 0 and \( \tau \):
    * \( \Rightarrow \rho_t \) must have been increasing fast to increase posterior mean \( \hat{\rho}_t \);
    * \( \Rightarrow \) After the IPO, \( \hat{\rho}_t \) is constant (in average), and \( \rho_t \) converges back to its long term mean.

\[ \Rightarrow \text{Drop in Profitability after IPO} \]
Learning and the Average Drop in Profitability after IPO

- Bayesian learning implies that a larger drop in profitability should occur if
  (a) Volatility of profitability is higher;
  (b) Uncertainty on average profitability is lower.

- Why?
  (a) Higher volatility \(\rightarrow\) less precise signal \(\rightarrow\) need larger realization in \(\rho_t\) to pull \(\hat{\rho}_\tau\) above threshold;
  (b) Lower uncertainty \(\rightarrow\) tighter prior \(\rightarrow\) need larger realization in \(\rho_t\) to pull \(\hat{\rho}_\tau\) above threshold;

- Caveat: Threshold level itself depends on volatility and uncertainty.
  - Big simulation exercise to gauge model predictions, and quantify size of drop.
Learning and the Average Drop in Profitability after IPO

Panel A. Realized profitability paths, $\rho_0 = 7\%$

- Base Case
- Lower Uncertainty
- Higher Volatility of Profitability

Panel B. Realized profitability paths, $\rho_0 = 0$

- Base Case
- Lower Uncertainty
- Higher Volatility of Profitability
Endogeneity of Private Firm Existence

• Existence of the private firm is endogenous, and this fact may also impact model’s predictions.
  – E.g. Higher uncertainty makes it easier to pull $\hat{\rho}_\tau$ over threshold.
    * $\implies$ Expected drop is lower.
  – But higher uncertainty implies private firm is less likely to exist.
  – If private firm exits, the private benefits of control may be higher.
    * $\implies$ Threshold goes up and expected drop may increase.

• We perform a large simulation exercise to verify that intuitive relation are still realized once we endogenize existence of firm.
  – Assume $\alpha \in [5\%, 15\%]$ and $\hat{\rho}_0 \in [-20\%, 40\%]$

• $\implies$ Empirical predictions survive.
Table 2
The Average Expected Post-IPO Drop in Profitability ($\tau = 5$)

\[
\sigma_{\rho,1} = \sigma_{\rho,2} \quad (\% \text{ per year})
\]

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Panel A: Average Expected Drop in Profitability (\% per year).
Table 2 (cntd.)

The Average Expected Post-IPO Drop in Profitability ($\tau = 5$)

\[
\sigma_{\rho,1} = \sigma_{\rho,2} \text{ (%) per year) }
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Panel B: Average Stock Return Volatility (% per year).

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Panel C: Average Expected Excess Stock Return (% per year).

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**Endogenous $\tau$ and Average Drop in Profitability after IPO**

- Earlier results obtained for exogenous IPO time $\tau$.
- Endogenous $\tau$ yield the same conclusions.
- Entrepreneur chooses optimal IPO time $\tau^*$

$$
\mathcal{V}(B_t, \rho_t, \hat{\rho}_t, t) = \max_{\tau^*} E_t \left[ \int_t^{\tau^*} e^{-\beta(s-t)} \frac{c_{1-\gamma}}{1-\gamma} ds + e^{-\beta(\tau^*-t)} V(M_{\tau^*}, \tau^*) \right],
$$

- Only numerical solution is available.

- Simulate 100,000 paths of shocks to the economy.
  - To facilitate comparison with previous figure, we average all of the profitability paths that led to an IPO between $\tau = 4.5$ and $\tau = 5.5$. 
Endogenous $\tau$ and Average Drop in Profitability after IPO

A. Realized and expected profitability, $\rho_0 = 7\%$

B. Realized profitability paths, $\rho_0 = 7\%$

C. Realized and expected profitability, $\rho_0 = 0$

D. Realized profitability paths, $\rho_0 = 0$
Empirical Analysis

Data: CRSP, Compustat, SDC, IBES, Jay Ritter’s IPO database

Sample: 7,138 IPOs in the U.S. from 1975-2004

Profitability measure:

- $ROE_{i,s} =$ firm $i$’s return on equity $s$ fiscal quarters after its IPO
- Main dependent variable in tests: $ROE_{i,s} - ROE_{i,0}$
Post-IPO Changes in ROE

Mean $\text{ROE}_{i,s} - \text{ROE}_{i,0}$

$s =$ quarters since IPO

Median
25th percentile
75th percentile

ROE in three periods:
- 1995–2004
- 1985–1994
- 1975–1984
Measuring Volatility and Uncertainty

Profit volatility measure:

• \( VOL(i, s_0) \equiv \text{standard deviation of } ROE_{i,s} \text{ for } s = s_0, \ldots, s_0 + 19 \)
• \( s_0 = 0 \) and (for robustness) \( s_0 = s + 1 \)

Proxy for uncertainty about average profitability:

• Problem: Firms with high uncertainty also tend to have high profit volatility
• However, model predicts volatility and uncertainty have opposite effects
• Guidance from model itself.

• **Corollary 4:** If $\sigma_{\rho,1} = 0$, then

$$dR_t - E_t [dR_t] = M \left( \sigma_{\rho,2}, \hat{\sigma}_0^2, \phi, t \right) (d\rho_t - E_t [d\rho_t]),$$

where

$$M \left( \sigma_{\rho,2}, \hat{\sigma}_0^2; \phi, t \right) = Q_1 (T - t) + Q_2 (T - t) \frac{\hat{\sigma}_t^2}{\sigma_{\rho,2}^2}.$$  

• Implication: The stock price reaction to post-IPO earnings surprises $M$ should be large when uncertainty is high and profit volatility is low.
Separating Uncertainty from Volatility

- $AR_{it} =$ cumulative return of stock $i$ in excess of stock $i$’s industry’s return in a 2-day window around the firm’s $t$-th post-IPO earnings announcement.

- First earnings response coefficient:

$$ERC_1(i) \equiv \frac{1}{13} \sum_{t=0}^{12} \frac{AR_{it}}{(EPS_{it} - E[EPS_{it}]) / BE_{it}}$$

- Second earnings response coefficient:

$$(EPS_{it} - E[EPS_{it}]) / BE_{it} = \gamma_{i0} + \gamma_{i1}AR_{it} + \varepsilon_{it}, \quad t = 0, 1, \ldots, 20$$

$$ERC_2(i) \equiv -\hat{\gamma}_{i1}$$
Post-IPO Changes in ROE: Volatility vs. uncertainty

Panel A

Panel B

Panel C

Panel D
Cross-Sectional Regressions

- Cross-sectional regression model:

\[ ROE_{i,s} - ROE_{i,0} = a + b \, VOL(i, s_0) + c \, ERC(i) + \varepsilon_i \]

- Model predictions:
  1. \( \hat{b} < 0 \): Profitability drops more for firms with higher volatility
  2. \( \hat{c} > 0 \): Profitability drops less for firms with large earnings response [high uncertainty and/or low volatility]
Table 5
Cross-Sectional Regressions

Panel A. One-Year Horizon. (Regressand: $ROE_{i,4} - ROE_{i,0}$)

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</tr>
<tr>
<td>$R^2$</td>
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<td>0.033</td>
<td>0.024</td>
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<td>0.004</td>
<td>0.026</td>
<td>0.014</td>
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</tr>
<tr>
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<td>3,628</td>
<td>2,649</td>
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<td>2,338</td>
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Panel B. Three-Year Horizon. (Regressand: $ROE_{i,12} - ROE_{i,0}$)

<table>
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<tr>
<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-Value</th>
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<td>VOL(0)</td>
<td>-0.70</td>
<td>-0.84</td>
<td>-22.4</td>
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<td>-18.9</td>
<td>-0.34</td>
<td>-0.66</td>
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<td>(-22.4)</td>
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<td>(-15.3)</td>
<td>(-22.4)</td>
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<td>(-15.3)</td>
<td>(-22.4)</td>
<td>(-18.9)</td>
<td>(-15.3)</td>
</tr>
<tr>
<td>VOL(13)</td>
<td>-0.27</td>
<td>-0.34</td>
<td>-8.68</td>
<td>-0.34</td>
<td>-0.29</td>
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<td>ERC1</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.45</td>
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<td>0.02</td>
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</tr>
<tr>
<td></td>
<td>(3.23)</td>
<td>(0.45)</td>
<td>(0.45)</td>
<td>(3.23)</td>
<td>(0.45)</td>
<td>(0.45)</td>
<td>(3.23)</td>
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<td></td>
<td>(2.13)</td>
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<td>(2.66)</td>
<td>(2.13)</td>
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<td>(2.66)</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.106</td>
<td>0.29</td>
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<td>0.002</td>
<td>0.147</td>
<td>0.093</td>
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<td>4,229</td>
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<td>2,255</td>
<td>2,368</td>
<td>1,365</td>
<td>1,517</td>
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</table>
Table 6  
Cross-Sectional Regressions, Excluding ERCs with Unpredicted Signs

Panel A. One-Year Horizon. (Regressand: \( ROE_{i,4} - ROE_{i,0} \))

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
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<td>-10.3</td>
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<td>1.80</td>
<td>-5.38</td>
<td>-0.06</td>
<td>0.16</td>
<td>-0.74</td>
<td>0.80</td>
<td>1.95</td>
<td>-1.52</td>
<td>0.65</td>
<td>1.62</td>
<td>0.44</td>
</tr>
<tr>
<td>( VOL(0) )</td>
<td>-0.26</td>
<td>0.13</td>
<td>-12.2</td>
<td>-0.18</td>
<td>0.74</td>
<td>-1.52</td>
<td>0.65</td>
<td>1.62</td>
<td>0.44</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( VOL(5) )</td>
<td>-0.19</td>
<td>-0.10</td>
<td>-12.2</td>
<td>-0.09</td>
<td>-0.40</td>
<td>-1.52</td>
<td>0.65</td>
<td>1.62</td>
<td>0.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ERC_1^+ )</td>
<td>0.18</td>
<td>0.11</td>
<td>4.34</td>
<td>0.12</td>
<td>2.62</td>
<td>2.74</td>
<td>0.12</td>
<td>2.74</td>
<td>0.12</td>
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<td></td>
</tr>
<tr>
<td>( ERC_2^- )</td>
<td>11.21</td>
<td>4.78</td>
<td>2.90</td>
<td>12.90</td>
<td>1.16</td>
<td>3.13</td>
<td>12.90</td>
<td>1.16</td>
<td>3.13</td>
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<td></td>
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</tr>
<tr>
<td>( R^2 )</td>
<td>0.000</td>
<td>0.033</td>
<td>0.24</td>
<td>0.011</td>
<td>0.014</td>
<td>0.015</td>
<td>0.014</td>
<td>0.014</td>
<td>0.015</td>
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<td>( N )</td>
<td>5,777</td>
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</table>

Panel B. Three-Year Horizon. (Regressand: \( ROE_{i,12} - ROE_{i,0} \))

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
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<td>-1.52</td>
<td>-0.81</td>
<td>1.76</td>
<td>-1.58</td>
<td>1.55</td>
<td>3.21</td>
<td>-0.75</td>
<td>0.61</td>
<td>1.04</td>
<td>0.61</td>
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<tr>
<td>( VOL(0) )</td>
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<td>1.05</td>
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<td>0.37</td>
<td>-10.8</td>
<td>0.47</td>
<td>0.37</td>
<td>-10.8</td>
<td>0.47</td>
<td>0.37</td>
<td>-10.8</td>
</tr>
<tr>
<td>( VOL(13) )</td>
<td>-0.27</td>
<td>-0.29</td>
<td>-8.68</td>
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<td>-6.55</td>
<td>-0.22</td>
<td>-0.40</td>
<td>-6.55</td>
<td>-0.22</td>
<td>-0.40</td>
<td>-6.55</td>
<td>-0.22</td>
<td>-0.40</td>
<td>-6.55</td>
</tr>
<tr>
<td>( ERC_1^+ )</td>
<td>0.22</td>
<td>0.00</td>
<td>3.32</td>
<td>0.01</td>
<td>0.00</td>
<td>0.19</td>
<td>0.01</td>
<td>0.00</td>
<td>0.19</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( ERC_2^- )</td>
<td>36.59</td>
<td>18.89</td>
<td>6.67</td>
<td>28.85</td>
<td>3.40</td>
<td>5.00</td>
<td>28.85</td>
<td>3.40</td>
<td>5.00</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.000</td>
<td>0.106</td>
<td>0.029</td>
<td>0.007</td>
<td>0.024</td>
<td>0.116</td>
<td>0.081</td>
<td>0.045</td>
<td>0.047</td>
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</tr>
<tr>
<td>( N )</td>
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<td>4,229</td>
<td>2,541</td>
<td>1,532</td>
<td>1,853</td>
<td>1,530</td>
<td>1,853</td>
<td>929</td>
<td>1,206</td>
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</table>
Robustness

• We did an extensive robustness analysis, and found the same results.

1. Definition of variables
   – E.g. Median versus mean $E[\text{EPS}]$; Number of quarters to compute ERC; Horizon of Post-IPO Drop; ROA versus ROE;

2. Controls
   – Mean reversion $\phi$; Firm size; Investment; Leverage; VC Financing.

3. Earnings Management
   – Discretionary Accruals (theory predicts large DCA $\implies$ larger drop)
     * Right sign, but not significant
     * No impact on VOL or ERC coefficient.
We develop a model of the IPO decision that features
- Learning about a private firm’s average profitability
- Tradeoff between benefits of private control and optimal consumption

Model predictions:
- Profitability drops after IPO, on average
- Profitability drops more for firms with more volatile profitability
- Profitability drops less for firms with more uncertain average profitability

Predictions supported empirically in sample of 7,138 U.S. IPOs in 1975-2004
Uncertainty and Expected Returns

• There is large literature documenting that firms with higher earnings dispersion of analysts' forecasts have subsequent lower returns (E.g. Diether, Malloy and Sherbina (2002, JF)).
  
  – Main explanation: short-sales constraints \(\implies\) over-valuation of stocks with large difference of opinion.

• Johnson (2004, JF) argues that
  
  – Dispersion of analysis forecasts is proxy for uncertainty about long-term value of the firm
  – Higher uncertainty increases valuation of stocks that are leveraged
  – Higher valuation leads to lower expected return
Table II
Mean Portfolio Returns by Size and Dispersion in Analysts’ Forecasts

Each month stocks are sorted in five groups based on the level of market capitalization as of the third Thursday of the previous month. Stocks in each size group are then sorted into five additional groups based on dispersion in analyst earnings forecasts for the previous month. Dispersion is defined as the ratio of the standard deviation of analysts’ current-fiscal-year annual earnings per share forecasts to the absolute value of the mean forecast, as reported in the I/B/E/S Summary History file. Stocks with a mean forecast of zero are assigned to the highest dispersion groups, and stocks with a price less than five dollars are excluded from the sample. Stocks are held for one month, and portfolio returns are equal-weighted. The time period considered is February 1983 through December 2000. The table reports average monthly portfolio returns; *t*-statistics in parentheses are adjusted for autocorrelation.

### Mean Returns

<table>
<thead>
<tr>
<th>Dispersion Quintiles</th>
<th>Small</th>
<th>Large</th>
<th>All Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>D1 (low)</td>
<td>1.52</td>
<td>1.45</td>
<td>1.50</td>
</tr>
<tr>
<td>D2</td>
<td>1.12</td>
<td>1.40</td>
<td>1.41</td>
</tr>
<tr>
<td>D3</td>
<td>0.99</td>
<td>1.20</td>
<td>1.32</td>
</tr>
<tr>
<td>D4</td>
<td>0.76</td>
<td>1.07</td>
<td>1.18</td>
</tr>
<tr>
<td>D5 (high)</td>
<td>0.14</td>
<td>0.56</td>
<td>0.83</td>
</tr>
<tr>
<td>D1–D5</td>
<td>1.37*</td>
<td>0.89*</td>
<td>0.67*</td>
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<tr>
<td><em>t</em>-statistic</td>
<td>(5.98)</td>
<td>(3.12)</td>
<td>(2.41)</td>
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</table>

### Mean Dispersion

<table>
<thead>
<tr>
<th>Dispersion Quintiles</th>
<th>Small</th>
<th>Large</th>
<th>All Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>D1 (low)</td>
<td>0.010</td>
<td>0.011</td>
<td>0.012</td>
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<tr>
<td>D2</td>
<td>0.039</td>
<td>0.033</td>
<td>0.030</td>
</tr>
<tr>
<td>D3</td>
<td>0.081</td>
<td>0.062</td>
<td>0.053</td>
</tr>
<tr>
<td>D4</td>
<td>0.172</td>
<td>0.125</td>
<td>0.103</td>
</tr>
<tr>
<td>D5 (high)</td>
<td>1.256</td>
<td>0.963</td>
<td>0.813</td>
</tr>
</tbody>
</table>

*a,b* Statistically significant at the one and five percent levels, respectively.

Source: Diether, Malloy, and Sherbina, Journal of Finance, 2002
Johnson (2004, JF)

- Assume underlying unlevered value of the firm evolves according to
  \[ dV_t = \bar{\rho}V_t dt + \sigma_V V_t dW \]
- In Johnson (2004), investors observe \( \bar{\rho} \) but do not observe \( B_t \), the value of the firm.
- Investors observe signals about the value of the firm, and they believe \( V_t \) it is lognormally distributed, i.e.
  \[ \log(v_t) \sim \mathcal{N}(\hat{m}_t, \hat{\sigma}^2_t) \]
• If the firm has no leverage, then the stock market price is

\[ S_t = e^{-r\tau} e^{\hat{\sigma}_t^2 + \frac{1}{2} e^{(\mu + \text{const.})\tau}} \]

– As in PV(2003,2006), higher uncertainty \( \hat{\sigma}_t^2 \) increases \( S_t \)

• If the firm has leverage in the form of a zero coupon with face value \( K \), then

\[ P_t = S_t \Phi(d_1) - Ke^{-r\tau} \Phi(d_2) \]

• where

\[ d_1 = \frac{\ln \left( \frac{S_t}{Ke^{-r\tau}} \right) + \frac{1}{2}(\hat{\sigma}^2 + \sigma^2 V\tau)}{\sqrt{\hat{\sigma}^2 + \sigma^2 V\tau}}; \quad d_2 = d_1 - (\hat{\sigma}^2 + \sigma^2 V\tau) \]

• This is an extension of Merton’s model of equity for leveraged securities.

• Important, Johnson (2004) shows formally that

higher uncertainty \( \hat{\sigma}_t^2 \) \( \implies \) lower expected return
Figure 1. Risk premium for the levered firm. The figures show the expected excess return for a levered firm under the model of Section II. The different panels correspond to different choices of the cash-flow horizon $T$. The firm has a value of $S = 100$ and an unlevered expected excess return of 0.05. Face value of debt $K$ varies along the left axis and the amount of parameter uncertainty $\omega^{1/2}$ varies along the right axis.

Source: Johnson, Journal of Finance, 2004
### Table I

**Return Regressions**

The table shows results from monthly Fama–MacBeth regressions of returns on measures of analyst forecast dispersion and leverage, and their product. The variable DISP1 is the standard deviation of current-fiscal-year forecasts divided by the mean of the forecasts. The variable DISP2 is the standard deviation divided by the firm’s most recently reported asset value. Both measures are transformed into percentile rank form. The leverage measure $L$ is the most recently reported book value of debt divided by the sum of that debt and the month-end market value of equity. The data are monthly observations from January 1983 through December 2001. The rightmost column is the arithmetic average of the $R^2$’s of the individual regressions. The $t$-statistics are in parentheses.

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Dispersion</th>
<th>Leverage</th>
<th>Interaction</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
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<td>DISP1, $L$</td>
<td>$-0.0059$</td>
<td>$0.0049$</td>
<td>$-0.0044$</td>
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<tr>
<td></td>
<td>(2.46)</td>
<td>(1.24)</td>
<td>(1.19)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-0.0058$</td>
<td>$0.0071$</td>
<td>$-0.0113$</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>(2.68)</td>
<td>(2.23)</td>
<td>(3.28)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-0.0041$</td>
<td>$0.0103$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.44)</td>
<td>(2.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DISP2, $L$</td>
<td>$-0.0035$</td>
<td>$0.0035$</td>
<td>$-0.0124$</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(0.79)</td>
<td>(2.27)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-0.0027$</td>
<td>$0.0089$</td>
<td>$-0.0097$</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(2.32)</td>
<td>(2.02)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.0017$</td>
<td>$0.0074$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(1.45)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Johnson, Journal of Finance, 2004
A recent paper by Amstrong, Banerjee, Corona (2011) argues that higher uncertainty in the firms’ beta also decreases the discount rate.

To make the argument in a simple setting, consider again the Gordon growth model.

Consider a given firm and you know the growth rate is 3% and the CAPM beta is $\beta = 1$.

Let $r_f = 3\%$ risk free rate, and $r^M = 10\%$ return on the market, the discount rate is

$$r = r_f + \beta(r^M - r_f) = 10\%$$

- The P/D ratio is $P/D = (1 + g)/(r - g) = 14.71$.
- Let $D(0) = 1$, so that $D(1) = 1.03$, and thus the total ex-post return is

$$1 + R = \frac{P(1) + D(1)}{P(0)} = \frac{D(1)P/D + D(1)}{P(0)} = \frac{1.03 \times 14.71 + 1.03}{14.71} = 1.1$$

- We can expect a realized return $R = 10\%$, which equals the discount rate $r = 10\%$, and thus ex-post and ex-ante expected returns are equal.
Beta uncertainty and expected returns

- Consider now the case in you are uncertain about the proper beta to apply.
  - The market beta could be $\beta = 0.4$ or $\beta = 1.6$, with equal probabilities.
  - The discount rate could be $r = 5.8\%$ or $r = 14.2\%$ with equal probability.
    * If $r = 5.8\% \implies P/D = 36.78$ and if $r = 14.2\% \implies P/D = 9.20$.
  - Given that $D(0) = 1$, we have that the price today is
    $$P(0) = 0.5 \times 36.78 + 0.5 \times 9.20 = 22.99$$
  - The price is much higher than when $\beta$ is known, even if $E[\beta] = 1$, and thus $E[r] = 10\%$.
- What is the ex-ante expected return?
  - Assume we do not learn anything about the discount between today and next year.
  - Then we can compute
    $$1 + R = \frac{P(1) + D(1)}{P(0)} = \frac{D(1)P/D + D(1)}{P(0)} = \frac{1.03 \times 22.99 + 1.03}{22.99} = 1.0748$$
  - The proper expected return is $E[R] = 7.48\% < 10\%$.
  - $\implies$ higher uncertainty about market beta leads to a lower expected return.
Beta uncertainty and expected returns

- What if you learn something about the proper discount rate between this year and next year?
- Let’s consider the extreme that by next year the true $\beta$ will be revealed.
- We can compute the realized return between this year and next year as follows
  - If we find out that $\beta = 0.4$, then next year $P/D = 36.78$ and thus the return will be
    
    \[ 1 + R_u(t, t+1) = \frac{P_u(1) + D(1)}{P(0)} = \frac{D(1)[P/D]_u + D(1)}{P(0)} = \frac{1.03 \times 36.78 + 1.03}{19.62} = 1.6928 \]
  - If we find out that $\beta = 1.6$, then next year $P/D = 9.20$ and thus the return will be
    
    \[ 1 + R_d(t, t+1) = \frac{P_d(1) + D(1)}{P(0)} = \frac{D(1)[P/D]_d + D(1)}{P(0)} = \frac{1.03 \times 9.20 + 1.03}{19.62} = 0.4568 \]
  - The expected return as of time 0 is then
    
    \[ E[R(t, t+1)] = 0.5 \times 0.6928 + 0.5 \times (-0.5432) = .0748 = 7.48\% \]
  - The same result as before.

- The bottom line is that for some securities it is harder to estimate the proper discount rate.
- This very fact, implies that in equilibrium, expected returns will be lower for such firms than in the case in which there is less uncertainty.
• Amstrong, Banerjee, Corona (2011) consider a much more elaborate model in which investors must learn about “beta”

• They assume $\beta \sim N(b, V_\beta)$, and obtain expected returns from a formal pricing model.

• The main result is that higher $V_\beta$ implies lower equilibrium expected return

• They test their model implication, and find that indeed, higher $V_\beta$ is associated with lower future returns.
Table 2  
The effect of factor loading uncertainty on the cross-section of expected returns  

This table presents the results from a Fama-MacBeth estimation of the following cross-sectional regression:

\[
r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \lambda_t b_{i,t} + \gamma_t V_{\beta,i,t} + \kappa_t b_{i,t}^2 + \text{controls}_{i,t} + \varepsilon_{i,t+1},
\]

where \(b_{i,t}\) and \(V_{\beta,i}\) are estimates from firm-specific first-stage, 60-month rolling window regressions of excess log returns \(r_{i,t+1} - r_{f,t}\) on market excess (log) returns \(r_{m,t+1} - r_{f,t}\). The controls include factor loadings on the SMB, HML, UMD portfolios and aggregate volatility risk (denoted by \(b_{i,SMB}, b_{i,HML}, b_{i,UMD}\) and \(b_{i,\Delta VIX}\), respectively). The t-statistics (reported below the coefficient estimates) are computed using Newey-West standard errors using 60 lags. The sample is 564 monthly observations from January 1964 through December 2010. The average number of firms in the cross-section is 1830, and the smallest cross-section has 561 firms. The estimates for \(b_{i,\Delta VIX}\) are available for a shorter sample since the data on the VXO index used to estimate them is only available from January 1986. Finally, the average and median adjusted \(R^2\)'s from the cross-sectional regressions are reported.

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<td>-4.52</td>
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<tr>
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<td>(b_{i,SMB})</td>
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<td>Mean Adj(R^2)</td>
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<td>23%</td>
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<td>28%</td>
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<tr>
<td>Median Adj(R^2)</td>
<td>16%</td>
<td>16%</td>
<td>18%</td>
<td>22%</td>
<td>23%</td>
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Concluding remarks

1. Stock valuation and learning about profitability
   - New valuation model that ensures that prices are positive, even if earnings are negative, and firms pays no dividends
     - Look at growth rate in book value, and concentrate on market-to-book ratios
   - Higher uncertainty about long-term profitability implies higher prices, ceteris paribus
   - Higher uncertainty also increases volatility, which should be then declining over time
   - Evidence consistent with prediction when uncertainty is proxied by lower age

2. Explaining technological ”bubbles”
   - Uncertainty effect is especially strong when aggregate discount rate is small
   - The late 1990s witnessed a technological revolution, which increased uncertainty about long-term growth and volatility
   - Moreover, unprecedented economic boom (longest in US history) justify low discount rates
     - Extreme prices in the late 1990s can be rationalized within a standard calibration
     - Model also explains why volatility was so high, why the price dropped (but volatility didn’t) in 2000
3. Stock price ”bubbles” during technological revolutions

- More generally, “bubbles” in the stock price of innovative firms tend to accompany technological revolution

- Due to ex-post endogenous time varying risk in technological revolutions
  - If ex post a technology becomes ”revolutionary” it must be adopted for mass production
  - \( \implies \) Its risk must have gone from being idiosyncratic to systematic
  - Should observe increase in “betas” and volatilities of innovative firms before the new technology is adopted

- Evidence from the 1990s tech bubble and the railroad revolution are consistent with the model’s predictions
Concluding remarks

4. Long-term uncertainty and IPO-waves
   - IPO’s come in waves
     - Endogenous market timing of entrepreneurs.
     - Entrepreneurs take their private firm’s public when their valuation increase
       * If economic growth increases
       * If aggregate equity premium decreases
       * If uncertainty increases
     - Numerous predictions are supported in the data.

5. Entrepreneurial learning and the post-IPO underperformance
   - Widespread observation that firm’s profitability declines after IPO
   - Explanation: Learning about long-term profitability and a cutoff rule on when the entrepreneur should take the firm public
     - To increase expected profitability above threshold, we need a large realization of current profitability
     - *ex-post selection bias*: Because current realized profitability is larger than expected profitability, on average, post-IPO profitability must drop compared to before IPO profitability.
   - Empirical evidence is consistent with learning story.
6. Uncertainty, leverage, and firms’ cost of capital

- Large controversy on why higher dispersion of analysts forecasts is related to lower future returns
  - Short-sale constraints and behavioral explanations
- Uncertainty about firm value implies that expected return decrease as leverage increase
  - Higher *idiosyncratic* uncertainty about firm value increases the value of equity as an option
  - The required premium declines, as higher leverage move the risk of the firm from equity holders to debt holders
  
  * Evidence consistent with the claim. Controlling for leverage there is less of an effect of dispersion of forecasts on future returns.

- In addition, higher uncertainty about CAPM beta also decrease the risk premium
  - Convexity effect on the discount rate: The average required risk premium decreases as uncertainty on the discount rate increases, as the price increases.
  - Evidence indeed shows that the variance of estimated beta partly explains future returns.