

FIXED INCOME ASSET PRICING

Course Objectives and Overview

The market for fixed income products is skyrocketing, and their complexity is ever increasing. For instance, as of the end of 2009, the U.S. government debt stood at around \$7.2 trillion while the mortgage backed securities market at around \$9 trillion. These numbers represent increases of almost 75% and 30% over their respective numbers at the end of 2005. Derivative securities demonstrated an even larger increase: For instance, as of December 2009, the size (notional) of the interest rate swap market was \$349 trillion, and the OTC forward and options market was at 101 trillion, more than 100% and 130% higher than their sizes four years earlier. In addition, in the aftermath of the 2007 – 2009 financial crisis and the 2008 – 2009 recession, we can expect a substantial increase in the size of the U.S. governments' debt, projected to possibly reach 80% of GDP by 2019, a big increase compared the 40% figure in 2008. Moreover, the recent aggressive expansionary monetary policy of the Federal Reserve may lead to higher future inflation. In such an environment, it is of paramount importance to obtain a deep understanding of the sources of risk of fixed income securities, and the current methodologies used by market participants to price and hedge fixed income products and complex derivative instruments.

This course covers state-of-the art models and techniques required to analyze fixed income instruments, and their derivatives, in modern financial markets. By the end of the course, students will learn (i) the basic concepts of fixed income instruments, such as yield, duration, convexity; (ii) the modern empirical methodologies to describe Treasury and corporate bond data, such as "curve fitting," factor analysis, and default probabilities; (iii) the most recent modeling techniques for fixed income derivative products used in the Street, such as the models of Vasicek, Cox Ingersoll and Ross, Ho and Lee, Hull and White, Black-Derman-Toy, and Heath-Jarrow-Morton; and, importantly, (iv) how to use these models *in practice* to value both traditional derivative instruments, such as Swaps, Bond Options, Caps and Floors, as well as the more recent products, such as Inverse Floaters, Range Notes, Mortgage Backed Securities and Credit Derivatives.

The key feature of *Fixed Income Asset Pricing* is that it strongly emphasizes the *applications* of these models to value real world fixed income products, and their derivatives, by focusing both on the practical difficulties of applying models to the data, as well as on the necessity to use computers to compute prices. The course, which is

mathematical in nature and relies on continuous time methodologies (developed within the course), includes many real world *Case Studies* and *Data Analysis* to allow students to apply these models to a wide range of derivatives and new products, as well as to understand their risk and return characteristics.

Required Material

Book:

Fixed Income Securities: Valuation, Risk, and Risk Management, by Pietro Veronesi, Wiley and Sons, 2010

Teaching notes:

Available on course web site a few days prior to class

Course web site: <http://faculty.chicagobooth.edu/pietro.veronesi/teaching/BUS437.htm>

Recommended Books

- a) John C. Hull, *Options, Futures and Other Derivatives*, 7th Edition, Prentice Hall, 2008, ISBN 0136015867
Classic book on equity and fixed income derivatives. Lots of models, and pricing methodologies. Always a good reference book to have in your library.
- b) Sundaresan S. *Fixed Income Markets and Their Derivatives*, Academic Press, 3rd Ed., 2009, ISBN 978-0123704719
Nice book that explains well the details of the fixed income market, including their institutional characteristics, and it develops a good intuition of the various models by using simple tree-like examples.
- c) Darrell Duffie, Kenneth J. Singleton, *Credit Risk*, Princeton University Press, 2003, ISBN 0-691-09046-7
Great book on credit risk markets and modeling.
- d) Tuckman B. *Fixed Income Securities*, John Wiley and Sons, 2002, ISBN 0-471-06322-3
Nice, simple and comprehensive book on fixed income securities. Highly recommended as an introductory book on the world of fixed income securities and no arbitrage.
- e) Paul Wilmott, *Paul Wilmott on Quantitative Finance*, John Wiley & Sons, 2000, ISBN 0-471-87438-8

A comprehensive book on quantitative finance. It is a good reference book, as it covers both fixed income and option pricing. It is slightly more advanced than other books, but still accessible to many. Students also tend to enjoy the personal writing of Paul Wilmott, full of personal opinions from someone who “has been there.”

- f) Jarrow R. *Modeling Fixed Income Securities and Interest Rate Options*, McGraw-Hill, 1996, ISBN0-07-912253-1.

This book explains the world of fixed income pricing and no arbitrage, according to Jarrow. Given his great contributions in the field, it is a nice book to have.

- g) Brigo D. and F. Mercurio *Interest Rate Models: Theory and Practice*, Springer Finance, 2001, ISBN 3-540-41772-9

This book is rather technical, but it contains many applications from two practitioners. In particular, they go through fitting exercises, and discuss pros and cons of various models and methodologies. If you can digest the math at the beginning of the book, it is good to read of the practical problems that arise when one tries to use these models in practice.

- h) Questa G. *Fixed Income Analysis for the Global Financial Market*, John Wiley and Sons, 1999, ISBN 0-471-24653-0.

This book covers the essentials of fixed income markets in a non technical manner. It is full of numerical, real world examples. Not much about pricing, but that is not the purpose of the book. I enjoyed reading it.

- i) John Y. Campbell, Andrew W. Lo, A. Craig MacKinlay, *The Econometrics of Financial Markets*, Princeton University Press, 1997, ISBN 0-691-04301-9

PhD level book on the econometrics of financial markets. It covers also fixed income securities, but more from an “economic” point of view, rather than from a “no-arbitrage” approach.

Administrative Notes

Office hours: Open door policy with the following caveats:

- 1) I will set up an electronic board on my web site (<http://gsbwww.uchicago.edu/fac/pietro.veronesi/teaching>) where I will post frequently asked questions and their answers. Please, check the board before asking me questions, since the answer could already be there.
- 2) Try to ask your questions by e-mail first. In many cases the answer can easily be given by e-mail, and you can expect it within a reasonable time. I will also post the question and the answer on the electronic board. (Of course, I won't disclose who asked the question.)

- 3) If you expect you need a long meeting, please try to arrange a mutually convenient meeting-time with me first.

Teaching Assistant: TBA. He will lead the review sessions, hold office hours and grade the homeworks and the midterm.

Administrative Assistant: Gwen Tillman, HPC525, Phone: 773 702 0053, email: gwen.tillman@chicagobooth.edu

Prerequisites and Requirements

Strict prerequisites for this course are the following courses: Business 35000 and 35100. If you took similar or more advanced courses either at the GSB or at another institution, or you have a good background in math *and* plan to take B35100 concurrently, you must contact me by e-mail to obtain a waiver for the strict requirements.

In addition, you must be familiar with some programming package (such as Gauss, Matlab, Mathematica, Visual Basic, C, Fortran etc.). I will not require the use of one or another packages: you choose.

To get up to speed:

- If you want to refresh your memory/learn about **basic calculus or differential equations** before the class starts, I like the book entitled “Mathematics for Economists” by Carl P. Simon and Lawrence Blume, Norton, 1994, ISBN 0-393-95733-0. Look at Chapters: 2 (all), 4.1, 5 (all), 14.1 – 14.5, 24.1-24.3 (understand the concepts).
- If you want to practice with a little **programming**, I would encourage you to get a programming package (I recommend Matlab if you do not have something else already. You can get the student version from www.mathworks.com. If you do not want to spend money, you can use VBA in Excel, but it is a hard programming language.) Then, set yourself a few simple exercises: You will find a dataset of discount bond yields at the web site (your log-in name and password are the same you use for your e-mail):
- <http://gsbwww.uchicago.edu/fac/pietro.veronesi/teaching/private/BUS437/Data/>
 - You can do the following (1) Import data in the programming package and plot them; (2) Compute simple statistics, such as mean, variances etc.; (3) Compute prices out of yields and viceversa; (4) Compute forward rates etc.
 - We will do all the above, but if you start early, at least you know how to turn on the computer!

Note: Students in the past were able to do all the exercises using only Excel spreadsheets. Although doable, the solutions (prices of complex structured products) would not be accurate. In addition, you will find it hard to do some of the exercises (which would

instead result easy using a programming language, such as Matlab). To get the most out of this course, I strongly recommend that you use a programming language.

Review Sessions

Weekly review sessions are held throughout the course. In the review sessions, the teaching assistant will give reviews of background material, go through the homework exercises and provide additional clarifications for the material covered in class. Time and location of the review sessions will be announced.

Course Requirements

The course requirements consist of weekly problem sets, a midterm exam and a final take-home exam. The solution of most problem sets will be briefly discussed in class, and students are expected to participate to the class discussion.

Problem sets and class participation

Problem sets will be handed out on a weekly basis. They focus on the actual fitting of term structure models, and the pricing and hedging of fixed income securities in today's challenging financial market. Group work is allowed and encouraged, with the **strict** limit of 4 students per group. Only one copy of the (joint) homework should be turned in. Make sure that your names, ID and section numbers appears on the cover of the homework. Please, try not to change groups during the quarter. If you must change groups, make sure to write a note on the cover of the first problem set after the change.

Most solutions to problem sets will be discussed in class, so students should keep a copy of their solutions handy. Class participation has a weight in the final grade.

Midterm Exam

The midterm will take place in class on week 7 and will last 1 1/2 hours. The midterm will be on both theoretical and practical issues. Of course, the use of a computer won't be necessary to solve the exercises, but bring your calculator. You are allowed to bring to the exam a single, two-sided, 8½x11 sheet with formulas.

Regrade policy: If you think that a serious mistake has been committed in grading your exam, you must submit the exam for a complete regrade along with a detailed written explanation of your objection within 10 business days of receiving the graded exam. There is absolutely no guarantee that the grade will not be lowered with the regrade.

Take-Home Final Exam

The take-home final exam will be similar to the homeworks, but more structured, with many questions in increasing order of difficulty. Although the final won't hinge only on programming ability, it will require the use of a computer and the concepts developed in

the course. Workgroup is allowed for the final Take-Home Exam. It could well be the pricing of an actual structured derivative product.

Grading

Problem sets and class participation, Midterm and Final are graded between 0-100 and then your score is given by the formula:

$$\text{Score} = \max(0.3 * \text{Problem Sets} + 0.3 * \text{Midterm} + 0.4 * \text{Final}; \\ 0.3 * \text{Problem Sets} + 0.4 * \text{Midterm} + 0.3 * \text{Final})$$

Class participation is used for borderline cases, to discriminate between A/B, B/C, C/D and so on.

If you are a graduating student, you can get a provisional grade (D) if your midterm is above the 25th percentile and you submitted all the homework showing a good work.

Honor Code

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. The GSB Honor Code also require students to sign the following GSB Honor Code pledge. "I pledge my honor that I have not violated the Honor Code during this examination," on every examination, as well as on the term project.

Course Outline

Please, note the following class schedule is very preliminary and could be subject to modifications. Reading indicated by "➔" are mandatory.

Class 1 (I) Introduction to Fixed Income Markets

(II) Fixed Income Products and Basic Analysis

- (a) Zeros and Coupon Bonds, Floating Rate Bonds
- (b) FRA, Repos, Strips
- (c) Conventions

➔TN #1: Section 1, Section 2 (up to 2.1)

➔Veronesi, Ch. 1, 5

Additional Readings:

Sundaresan, Ch. 1-5.

Hull, Ch. 4 - 6.

Wilmott, Ch. 38.

(III) Introduction to Continuous Time Models

- (a) Why do we need term structure models?
- (b) Brownian Motions

→TN #2 : Section 0, Section 1 (up to 1.3.2)

→Veronesi, Ch. 14.1-14.3

Additional Readings:

Hull, Ch. 12.1 – 12.2

Wilmott, Ch. 40

Notes: Homework 1 handed out. Due at the beginning of class 2.

Class 2 (I) Fixed Income Products and Basic Analysis (cntd.)

- (d) Duration and Convexity
- (e) Zero coupon curve extraction
- (f) Plain Vanilla Swaps

→TN #1: Sections 2.3, 2.4, 3, 4.1

→Veronesi, Ch. 2-5

Additional Readings

Hull. Ch. 4 – 6.

Wilmott, Ch. 39.

(II) Introduction to Continuous Time Models (cntd.)

- (c) Ito's Lemma and PDEs

→TN #2: Sections 1.4, 2 – 2.4

→Veronesi, Ch 14.4, Ch. 15.1 – 15.2

Additional Readings:

Hull, Ch. 12.5, 25.1

Wilmott, Ch. 40

Notes: (a) Homework 1 due;

(b) Homework 2 handed out. Due at the beginning of class 3.

Class 3 Introduction to Continuous Time Modeling (cntd.)

- (d) Interest Rate Processes
 - a. Vasicek Model
 - i. Examples: Pricing, Hedging and Trading
 - b. CIR Model

- ➔ TN #2: Sections 2 and 3
- ➔ Veronesi, Ch 15.3 – 15.4, Ch. 16.

Additional Readings:
Hull, Ch. 28.2
Wilmott, Ch. 40

- Notes: (a) Homework 2 due;
(b) Homework 3 handed out. Due at the beginning of class 4.

Class 4 Risk Neutral Pricing: Monte Carlo Simulations

- (a) No Arbitrage for one factor models
- (b) Monte Carlo Simulations
- (c) Examples
 - a. Example 1: Pricing an Ammortizing Swap
 - b. Example 2: Convertible Securities

- ➔ TN #3: Sections 1, 2 and 3.
- ➔ Veronesi, Ch. 17.1-17.2, Ch18.1 – Ch 18.2

Additional Readings:
Hull, Ch. 17.6

- Notes: (a) Homework 3 due;
(b) Homework 4 handed out. Due at the beginning of class 5.

Class 5 Risk Neutral Pricing: Trees

- (a) Trees
- (b) Using Risk Neutral Trees
- (c) Examples
 - a. Callable bonds
 - b. Caps and Floors
 - c. Swaps and Swaptions

- ➔ TN #4: Sections 1 – 4.
- ➔ Veronesi, Ch. 9 (skim), Ch. 10 (skim), Ch.11.1 – Ch.11.2

Additional Readings:
Hull, Ch. 11, 28.6
Wilmott, Ch. 4
Tuckman, Ch. 9

- Notes: (a) Homework 4 due;
(b) Homework 5 handed out. Due at the beginning of class 6.

Class 6 No Arbitrage Models

1. Generalized Affine Term Structure Models
 - a. Ho and Lee
 - b. Hull and White
2. Black Derman Toy Model
 - a. Term structure of forward volatilities

➔ TN #5

➔ Veronesi, Ch. 19.1-19.4. Ch. 10.3

Additional Readings:

Hull, Ch. 28.3, 29.

Notes: (a) Homework 5 due;
(b) Homework 6 handed out. Due at the beginning of class 8.

Class 7 I) Midterm

(II) Interest Rate Derivatives: The Market Model

- (a) Black's Model and Forward Risk Neutral Pricing
- (b) Caps, Floors and Collars
 - a. Forward Volatilities and Flat Volatilities
- (c) Swaptions

➔ TN #6

➔ Veronesi, Ch. 20, Ch. 21.1-21.2

Additional Readings

Hull, Ch. 26.

Wilmott, Ch. 42

Class 8 Mortgage Backed Securities

➔ TN #7

➔ Veronesi, Ch. 8, 13

Sundaresan: Ch.9

Notes: (a) Homework 6 due;
(b) Homework 7 handed out. Due at the beginning of class 9.

Class 9 Multi-Factor Interest Rate Models

- (a) Factor Decomposition: Level, Slope and Curvature
- (b) Multifactor Affine Models
 - a. Multifactor Vasicek Model

➔TN #8

➔Veronesi, Ch. 4 (Appendix), Ch. 22

Additional Readings

Hull, Ch. 18.9, 28.3

Wilmott, Ch. 45, 47

Notes: (a) Homework 7 due;
(b) Homework 8 handed out. Due at the beginning of class 10.

Class 10 Multi-Factor Interest Rate Models

- (c) Hull and White Model
- (d) Heath, Jarrow and Morton Model

➔TN #8

➔Veronesi, Ch. 21.6, 22

Additional Readings

Notes: (a) Homework 8 due.
(b) Final handed out.