BUS 35132
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FINANCIAL ENGINEERING
Mathematical Models of Option Pricing

Brief Overview

It is a fact that derivatives' markets have been growing fast in the past decade. As of December 2008, the total notional of over-the-counter derivatives was $592 trillion, a 40% increase over its value in December 2006. A similar expansion was registered by the credit derivatives market, which stood at $42 trillion (notional) at the end of 2008. Although both the global derivatives market and the credit derivatives market experienced a decline during the 2008 financial crisis, there is little doubt that they will play a major role in the future. For instance, likely the largely unregulated credit derivatives market will become more regulated, possibly moving to a regulated exchange, but their function as providing insurance on default is too important for it to disappear. The current financial crisis has generated also large trading opportunities, as the dislocation of capital increased the spreads across the board, and numerous apparent "almost arbitrage" opportunities appear available to whoever has capital to invest and the expertise to capture them.

This course covers the analytical and numerical methodologies applied by hedge funds and derivatives trading desks to price complex derivative securities and devise arbitrage strategies. We will apply these methodologies to several case studies, whose topics range from relative value trades in equity options and fixed income instruments, to the pricing of convertible securities using numerical methods. About half of the course is devoted to credit risk and securitization. Numerous profitable opportunities are now available as the government tries to jump start the securitization market again. We will cover case analysis that range from the pricing and hedging of credit derivatives, such as credit default swaps (CDS), to the valuation and risk analysis of cash and synthetic collateralized debt obligations (CDO). We will also discuss and analyze the credit market indices such as TRAC-X.

In a world of increasingly higher sophistication, the valuation of complex derivative securities and the design of arbitrage strategies require the understanding and application of advanced models of option pricing, and their application to real data. This course emphasizes both, and provides students with real world problems to solve.

Required Material

A packet of readings and cases.
Teaching notes:

Available on course web site a few days prior to class

Course web site: *http://faculty.chicagogsb.edu/pietro.veronesi/teaching/BUS439.htm*

Optional Material

a) John C. Hull, *Options, Futures and Other Derivatives*, Sixth Edition, Prentice Hall, 2005, ISBN 0-13-149908-4

b) Darrell Duffie, Kenneth J. Singleton, *Credit Risk*, Princeton University Press, 2003, ISBN 0-691-09046-7

c) Salih N. Neftci, *Principles of Financial Engineering*, Elsevier Inc., 2004, ISBN 0-12-515394-5

d) Robert McDonald, *Derivatives Markets*, 2003, Addison Wesley, ISBN 0-201-75960-1

e) Paul Wilmott, *Paul Wilmott on Quantitative Finance*, John Wiley & Sons, 2000, ISBN 0-471-87438-8

f) Stephen Figlewski, William Silber and Marti Subrahmanyan (Eds.) Financial Options: From Theory to Practice, McGraw-Hill, 1990, ISBN - 1-55623-872-X

g) Martin Baxter and Andrew Rennie, *Financial Calculus: An Introduction to Derivative Pricing*, Cambridge University Press, 1996, ISBN 0-521-55289-3

h) Paul Wilmott, Jeff Dewynne, Sam Howison, *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press, 1993, ISBN 0 -9522082-0-2

Requirements

Strict prerequisites for this course are the following courses:

- 35000: Investments
- 35100: Financial Instruments

Homework assignments (case studies) and final project will require the use of a spreadsheet package, such as Excel, or a programming package (such as C, Fortran, Gauss, Mathematica, Matlab etc.). I will not require the use of one particular package: you may choose. However, I strongly encourage the use of Matlab, as solutions to homeworks and additional files will be in Matlab.

Course Requirements

The course requirements consist of weekly problem sets (case analysis), a midterm exam and a final take-home exam. The solution of most problem sets will be briefly discussed in class, and students are expected to participate to the class discussion.

Problem Sets and class participation

Problem sets based on case studies are handed out every week. Group work is allowed and encouraged, with a limit of 4 students per group. Only one copy of the (joint) homework should be turned in. Make sure that your names, ID and section numbers appears on the cover of the homework. Please try not to change groups during the quarter. If you must change groups, make sure to write a note on the cover of the first problem set after the change.

Midterm Exam

The midterm will take place in class on week 8 and will last 1 1/2 hours. The midterm will be both on theoretical and practical issues. Of course, the use of a computer won't be necessary to solve the exercises, but bring your calculator. You are allowed to bring to the exam a single, two-sided, 8½x11 sheet with formulas.

Regrade policy: If you think that a serious mistake has been committed in grading your exam, you must submit the exam for a complete regrade along with a detailed written explanation of your objection within 10 business days of receiving the graded exam. There is absolutely no guarantee that the grade will not be lowered with the regrade.

Take-Home Final Exam

The take-home final exam will be similar to the homeworks, but more structured, with many questions in increasing order of difficulty. Although the final won't hinge only on programming ability, it will require the use of a computer and the concepts developed in the course. Workgroup is allowed for the final Take-Home Exam. It could well be the pricing of an actual structured derivative product.

Grading

Problem sets, Midterm and Project are graded between 0-100 and then your score is given by the formula:

Score=max (0.3* Problem Sets + 0.3*Midterm + 0.4*Final;
0.3* Problem Sets + 0.4 * Midterm + 0.3 * Final)

If you are a graduating student, you can get a provisional grade (D) if your midterm is above the 25^{th} percentile and you submitted all the homework showing a good work.

<u>Honor Code</u>

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. The GSB Honor Code also require students to sign the following GSB Honor Code pledge. "I pledge my honor that I have not violated the Honor Code during this examination," on every examination, as well as on the term project.

<u>Course Outline and Readings</u>

Please, note the following class schedule is **<u>very preliminary</u>** and could be subject to some modifications. Reading indicated by "➔" are mandatory and must be done before the class meets.

<u>Class 1</u> **Continuous Time Finance**

(a) Stochastic Calculus
(b) The Black and Scholes model
    a. No Arbitrage and Derivatives Pricing
(c) Application 1: Exploiting a No Arbitrage Relation by Dynamic Hedging

➔TN #1
Hull: Ch. 12, 13

Notes:

HW1: Case Study. LTCM: Selling Volatility and Dynamic Replication.
Part 1 due at the beginning of Class 2.
Part 2 due at the beginning of Class 3.

<u>Class 2</u> **Continuous Time Finance (cntd.)**

(d) The Vasicek Model of Interest Rates
    a. No Arbitrage and Bond Pricing
    b. Application 2: Relative Value Trades on the Yield Curve
(e) Multifactor models
    a. Application 3: Relative Performance
    b. Application 4: Stochastic Volatility

➔ TN #1
Hull: Ch. 12, 13, 15
Heston: A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.

Notes:

Part 1 of HW 1 due.

<u>Class 3</u> **(I) Feynman Kac Theorem and Risk Neutral Pricing**

(a) The Feynman Kac solution to the Fundamental Pricing PDE
(b) Risk Neutral Pricing
(c) Examples

➔TN #2

(II) Numerical Methods for Derivative Security Pricing

(a) Approximations to Brownian Motions
(b) Monte Carlo Simulations

➔TN #3

Hull: Ch. 17
➔Courtadon G: An Introduction to Numerical Methods in Option Pricing, in *Financial Options from Theory to Practice* (Ch. 14)
Boyle, P.: Options: A Monte Carlo Approach. *Journal of Financial Economics*. (1976) Pages 323-338
Boyle, P., M. Broadie and P. Glasserman: Monte Carlo Methods for Security Pricing, *Journal of Economic Dynamics and Control* (1997), Pages 1267-1321.

Notes:

Part 2 of HW 1 due.

HW 2: Case Study. Deutsche Bank. Finding Relative Value Trades.
Part I due at the beginning of Class 4. Part II optional.

<u>Class 4</u> **(I) Numerical Methods for Derivative Security Pricing (cntd)**

(c) The Finite Difference Method

➔TN # 3

(II) American Options and Early Exercise: Numerical Methods

(a) Finite Difference Methods

➔TN # 4
Hull. Ch. 11.5 (as a review)

Notes:

Part 1 of HW 2 due (Part 2 is optional)

HW 3: Case Study. DigaMem. Pricing Path Dependent Securities by MC simulations.

Due at the beginning of Class 5.

<u>Class 5</u> **(I) American Options and Early Exercise: Numerical Methods**

(b) Monte Carlo Simulations

➔TN # 4
Hull, Ch 24.7
➔ Longstaff and Schwartz: Valuing American Options by Simulations: A Simple Least Square Approach, (2001), *Review of Financial Studies*, Spring

Notes:

HW 3 due.

HW 4. Case Study: DigaMem. Pricing Convertible Securities by MC Simulations

Due at the beginning of Class 6

<u>Class 6</u> **(I) Exotic Options**

(a) Path Dependent Options
    a. Barrier, Asian, Lookback
(b) Passport Options
(c) Variance Swaps

➔TN # 5

(II) Credit Risk and Defaultable Securities

(a) The Merton Model
    a. Capital Structure Arbitrage
    b. Risk Neutral and Risk Natural Probabilities of Default
    c. Generalizations

(III) Credit Derivatives
    a. Credit Default Swaps

➔TN #6
➔TN #7

➔ Merton R. C., On the Price of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 1974

Notes:

HW 4 due

HW 5: Case Study. First American Bank. Pricing Credit Default Swaps

Part I due at the beginning of Class 7.
Part II due at the beginning of Class 9.

Class 7 **(I) Credit Risk and Defaultable Securities (cntd)**

(b) Intensity Based Models

(II) Credit Derivatives

➔ Duffie and Singleton, Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies*, 1999
➔Lando, D: Modeling Bonds and Derivatives with Default Risk, in *Mathematics of Derivative Securities* (1997)

➔ TN#7
➔ TN#8

Notes:

Part I of HW 5 due.

Class 8 **(I) MIDTERM**

➔TN #6

(II) Credit Derivatives (cntd)

(a) Multi names
(b) Application: Collateralized Debt Obligations

➔ TN #7
Das, S.: Pricing Credit Derivatives. Unpublished Work. (1998) Pages 0-40
Hull and White: Valuing Credit Default Swaps I: No Counterparty Default Risk, *Journal of Derivatives*, 2000

Notes:

Class 9 **Credit Risk: Numerical Methods**

(a) Simulating Default Events
(b) Simulating Correlated Defaults

➔ TN #8

Notes:

Part 2 of HW 5 due.

HW 6: Case Analysis. NexGen: Structuring Collateralized Debt Obligations.

Due at the beginning of Class 10.

Class 10 **(I) Structured Credit Index Products.**

➔ TN #8

(II) Course Wrap up

Notes:

HW 6 due