Topics in Dynamic Asset Pricing

Course Presentation.

Pietro Veronesi

Graduate School of Business,
University of Chicago
CEPR, NBER

Course Objectives

- This course has two objectives:

  1. Introduce students to the frontier of research in asset pricing: we will cover a number of models and methodologies have been recently developed in the literature to address intriguing empirical regularities.
  2. Teach students how to write coherent research papers: over the ten weeks I will assign research ideas that students have to developed into research papers (I provide tips). I will "referee" such papers providing then feedback on how papers should be written.
     - By the end of the course, students will learn what it takes to write a good paper, the type of assumptions we must make to "solve the model", when we need to resort on numerical methods, and, importantly, how we confront the model with the data.

- We start by reviewing some (but not all) intruiguing empirical regularities.

A Simple Benchmark Model (Lucas Tree Model)

- Aggregate dividends D_t are i.i.d.

$$\frac{dD_t}{D_t} = \mu_d dt + \sigma_d dB_t$$

- $P_t =$ price of stock that is a claim on these dividends. $r_t =$ risk free rate of return.
- A representative agent has infinite life, power utility over consumption, chooses C_t and asset allocation θ_t to

$$\max_{C_t, \theta_t} E_0 \left[\int_0^\infty e^{-\phi t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$$

- Equilibrium: $C_t = D_t$ and $\theta_t = 1 \Longrightarrow$ SDF $= \lambda_t = e^{-\phi t} C_t^{-\gamma}$

$$P_t = E_t \left[\int_t^\infty \frac{\lambda_\tau}{\lambda_t} D_\tau d\tau \right] = \frac{D_t}{R - \mu_d}$$

- where $R =$ discount rate for risky stock

Implications of Benchmark Model

- A large number of empirical regularities clash with this standard paradigm.

1. **Equity premium puzzle:** Stocks have averaged returns of about 7% over treasuries.

   - This number is high compared to the volatility of consumption, of about 1-2%.
   - The canonical model implies

$$\text{Expected Excess Return} = \gamma \text{Variance of Consumption Growth}$$

   - Even assuming that γ is large, say $\gamma = 10$, we have

$$\text{Expected Excess Return} = 10 \times (.02)^2 = 0.4\%$$

   - We are an order of magnitude off.

Implications of Benchmark Model

2. **Volatility Puzzle 1:** Return volatility (about 16 %) is too high compared to the volatility of dividends (about 7%).

- The same classic canonical model has

$$\frac{P_t}{D_t} = \text{Constant}$$

- This implies

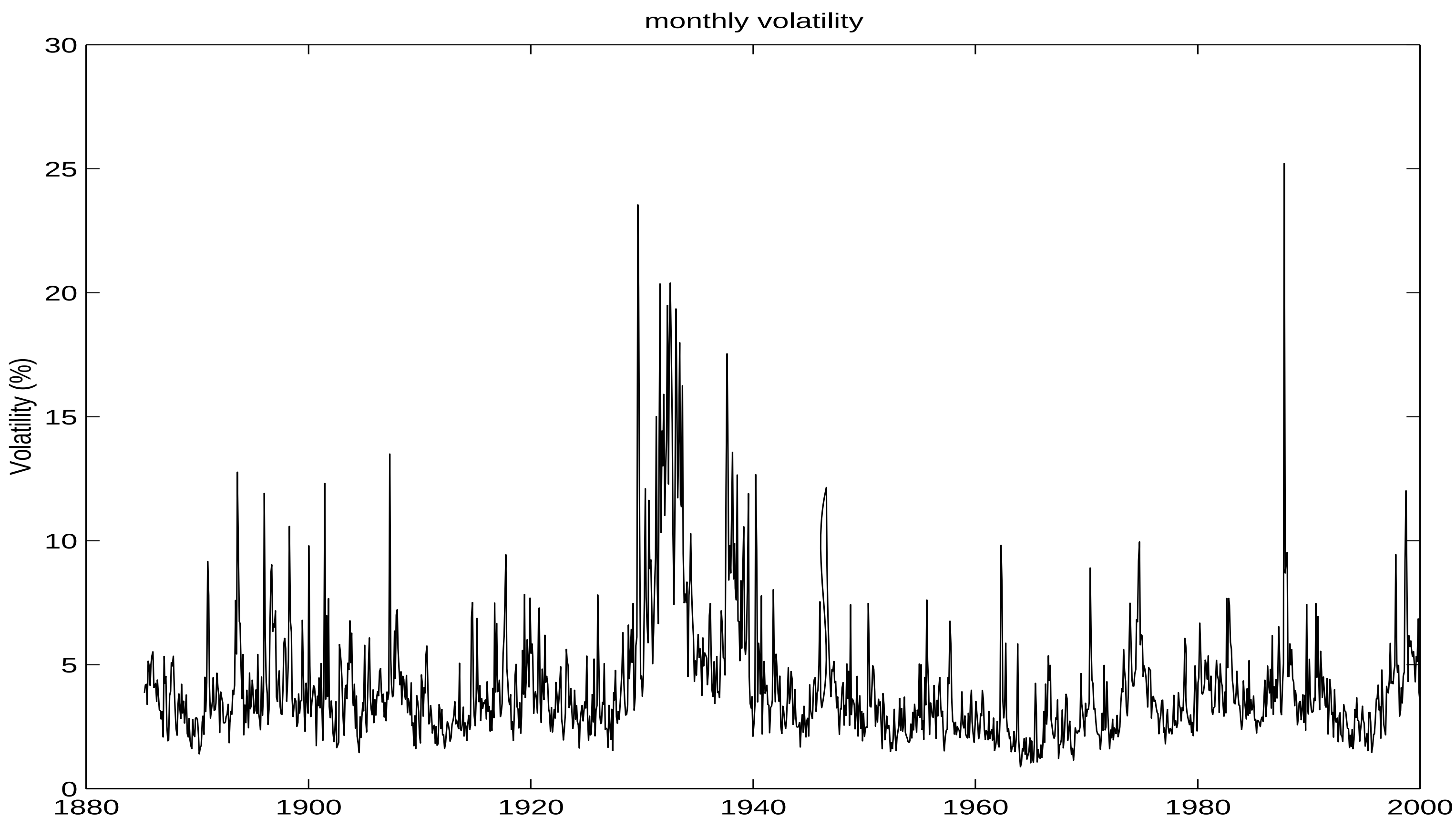
$$\text{Volatility of } \frac{dP_t}{P_t} = \text{Volatility of } \frac{dD_t}{D_t}$$

- Something else must be time varying to make the volatility higher.
- Indeed, the canonical model would imply a constant P/D ratio, which we know it is not.

Implications of Benchmark Model

3. **Volatility Puzzle 2:** Return volatility is not only high, but it is time varying.

- Historically, monthly market return volatility fluctuated between 20 - 25 % in the 30s to less that 2% in the middle of the 1960s.



Implications of Benchmark Model

4. **Risk Free Rate Puzzle:** The usual canonical model implies that the interest rate is given by

$$r = \phi + \gamma\mu_c - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2$$

- If $\gamma = 10$ for instance, using $\mu_c = 2\%$, $\sigma_c = 1\%$ and $\phi = 2\%$ we find $r = 21\%$
- The problem is γ that is too high: If we set $\gamma = 2$ we obtain $r = 6\%$.
- Note the tension between equity premium puzzle (need γ high) and risk free rate puzzle (need γ low).

Implications of Benchmark Model

5. **Predictability 1:** Stock returns are predictable by, say, the dividend price ratio.

- Predictability regression

$$\text{Cumulated Returns } (t \rightarrow t+\tau) = \alpha + \beta \log\left(\frac{D_t}{P_t}\right) + \epsilon_{t,t+\tau}$$

Table: Forecasting Regression

Sample		Horizon (qtrs)			
1948 - 2001		4	8	12	16
	$\log(D/P)$	**.13**	.20	.26	.35
	NW t-stat	(2.13)	(1.65)	(1.34)	(1.29)
	Adj. R^2	.09	.10	.11	.14
Sample		Horizon (qtrs)			
1948 - 1994		4	8	12	16
	$\log(D/P)$	**.28**	**.48**	**.63**	**.78**
	NW t-stat	(4.04)	(4.00)	(4.49)	(5.41)
	Adj R^2	.19	.32	.43	.54

Implications of Benchmark Model

- This result raises a number of issues, such as:

(a) Why are stock return predictable?

(b) Why the regression coefficients (and significance) depend on the time interval used?

(c) What are the implication for an investor who is allocating his wealth between stocks and bonds to maximize his life time utility?

Implications of Benchmark Model

6. **Predictability 2**: From a basic canonical model, we have

$$\text{Expected Excess Return} = \gamma \text{Variance of Stock Return}$$

- Data show that expected excess returns are time varying (predictability) and variance of stock return is time varying.
- Are they related? Most of the empirical literature shows that there is very little relation between the two.
- For instance, a simple regression

$$\text{Cumulated Returns}\,(t \to t + \tau) = \alpha + \beta\,(\text{Monthly Vol}) + \epsilon_{t,t+\tau}$$

Implications of Benchmark Model

Table: Forecasting Regression

Sample		Horizon (qtrs)			
1925 - 1999		4	8	12	16
	Volatility	−.32	−.30	.82	1.59
	NW t-stat	(−.32)	(−.20)	(.62)	(1.28)
	Adj. R^2	.00	.00	.00	.01
Sample		Horizon (qtrs)			
1948 - 1994		4	8	12	16
	Volatility	**1.05**	1.1	1.00	2.69
	NW t-stat	(1.56)	(0.81)	(0.87)	(1.41)
	Adj R^2	.01	.01	.00	.02

- Using more sophisticated models for volatility, some studies find a significantly positive relation, but some others find a significant negative relation. There is still a considerable debate.

Implications of Benchmark Model

7. **Cross-sectional Predictability Puzzle**: Some type of stocks yield an average return that is not consistent with the canonical model.

- The canonical model implies that expected excess returns of asset i is given by:

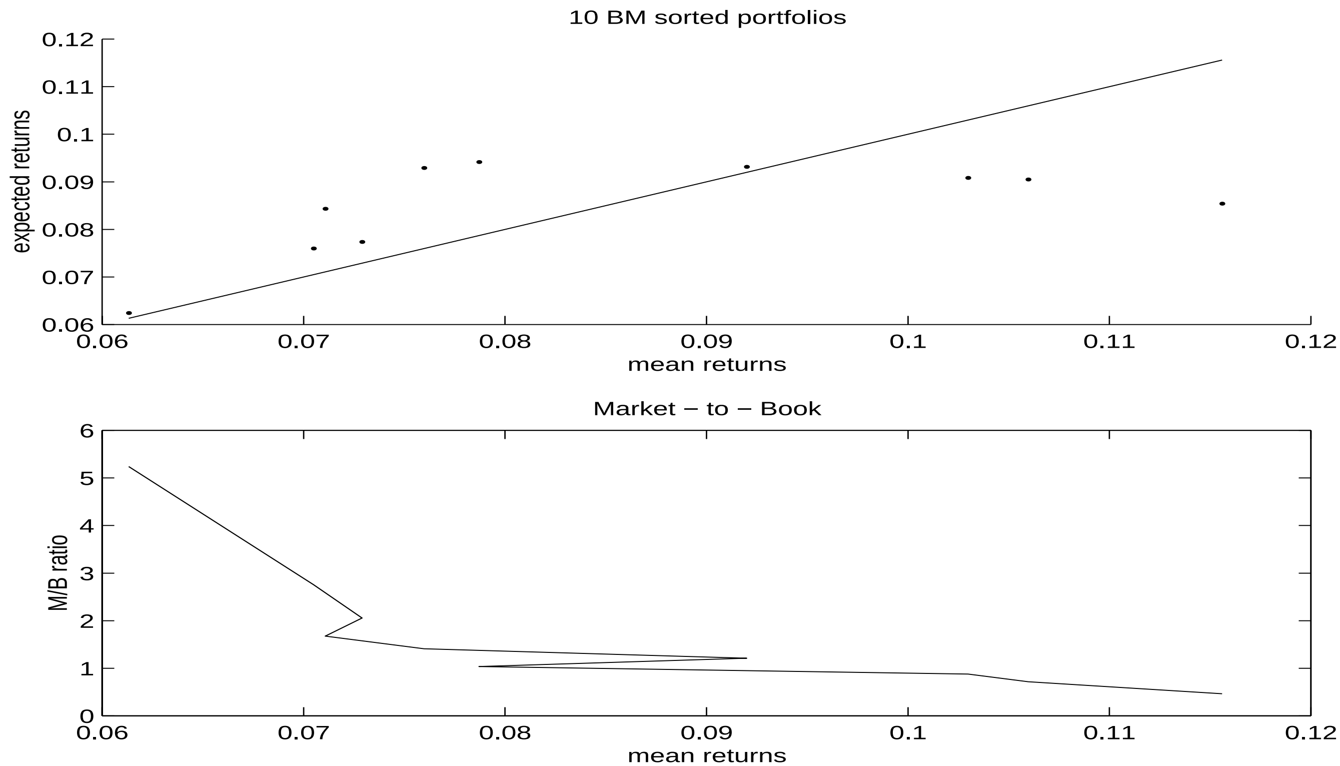
$$\begin{aligned} E\left[\text{Excess Return}_t^i\right] &= \gamma \text{Cov}\ \left(\text{Return}^i, \text{Consumption Growth}\right) \\ &= \beta^i E\left[\text{Excess Return of Mkt Portfolio}\right] \end{aligned}$$

- where

$$\beta^i = \frac{\text{Cov}\ \left(\text{Return}^i, \text{Return Mkt Portfolio}\right)}{\text{Var}\ \left(\text{Return Mkt Portfolio}\right)}$$

- Portfolios of stocks that are sorted by Book-to-Market Ratio or by Size and Book to Market do not satisfy this relation.
- For instance, using Book-to-Market sorted portfolios, we obtain the following

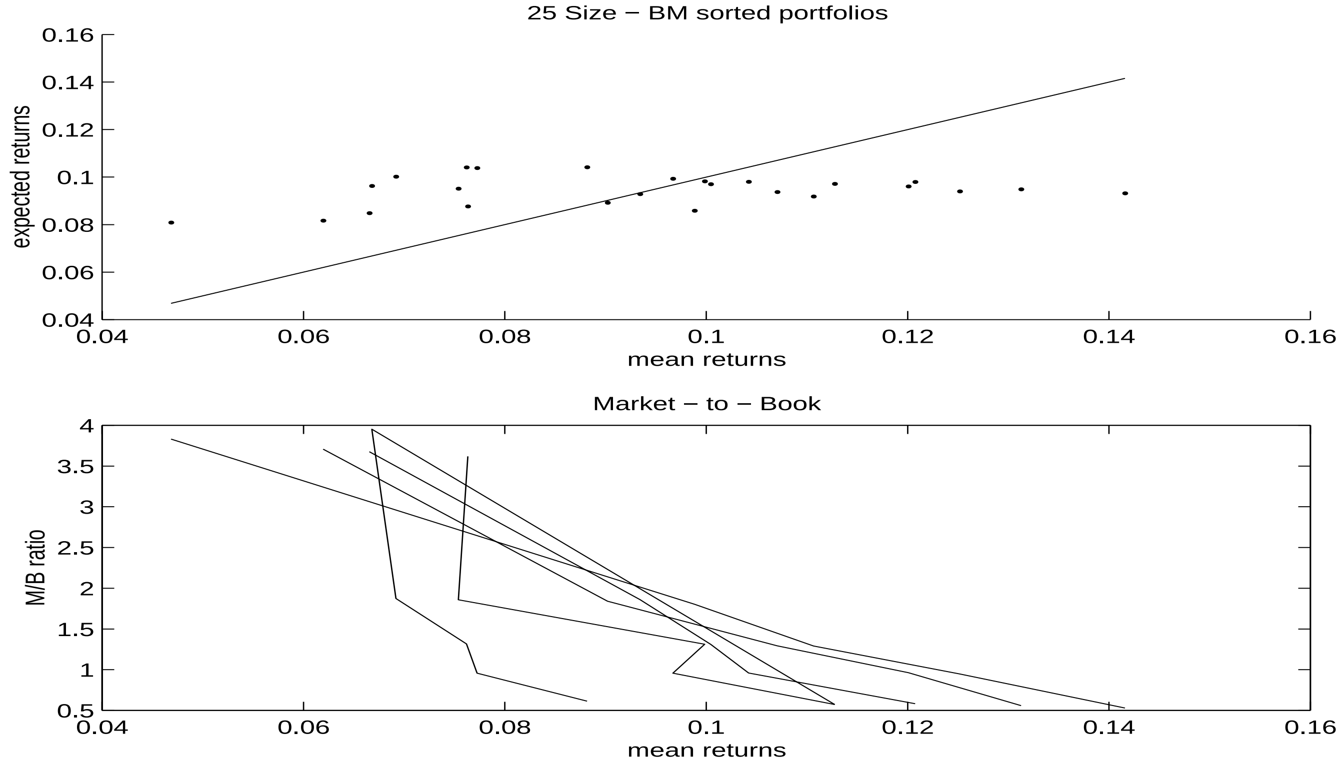
Implications of Benchmark Model



- The top panel shows the the average return on B/M sorted portfolio on the x-axis, and the one implied by the CAPM ($=$ beta $\times$ Average Return of Market Portfolio) on the y-axis
- They should line up, but they don't

Implications of Benchmark Model

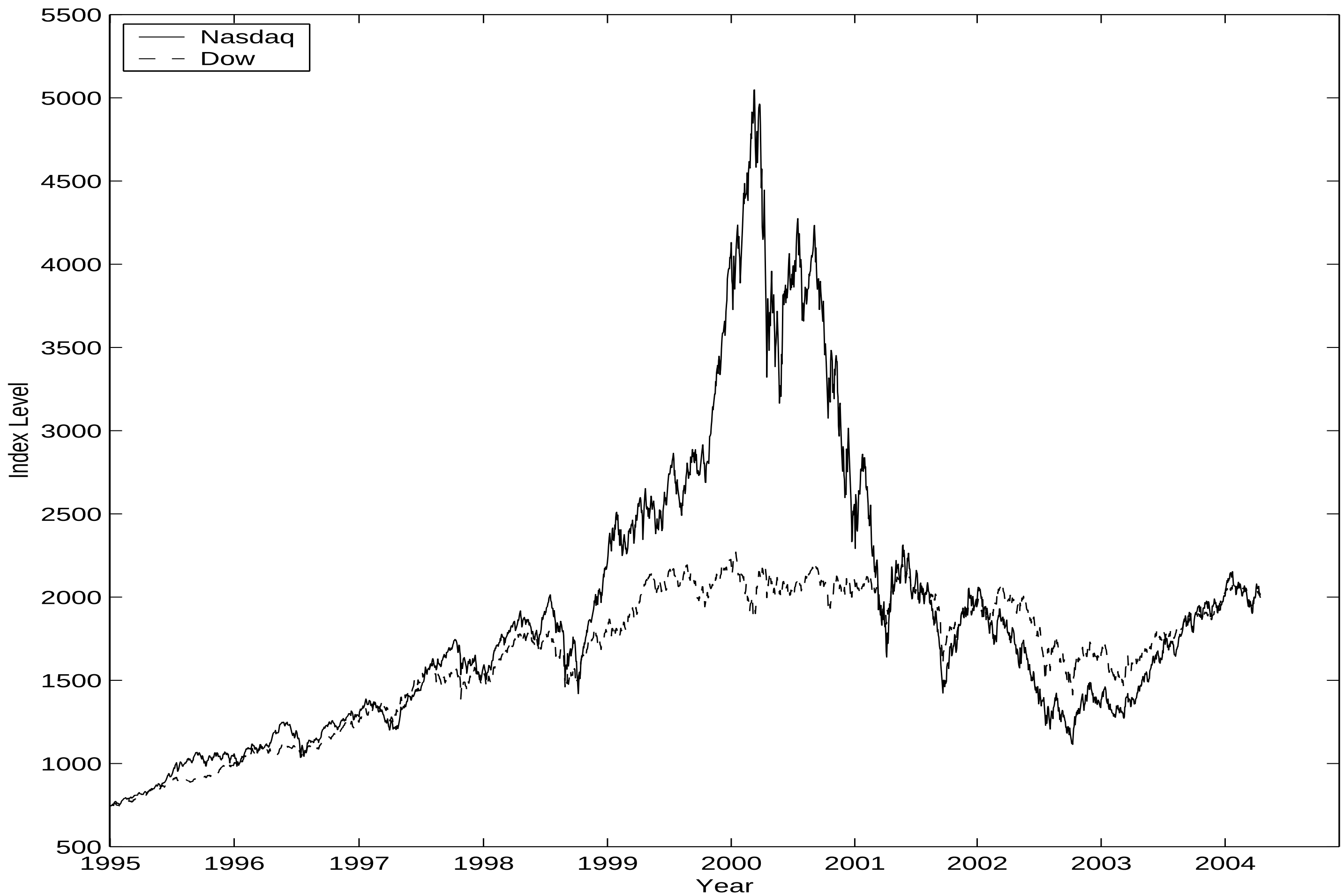
- It is even worse if one uses Size and Book-to-Market portfolios (the so-called FF 25 portfolios)



- Adding to this, momentum portfolios (sorted by past winners and losers) show similar and perhaps more striking pattern.

Implications of Benchmark Model

8. **Tech "Bubble":** Typical to talk about technology bubbles (e.g. late 1990s)



Implications of Benchmark Model

- Was it a bubble?
- Why do stock prices tend to go up and then down around technological revolutions?
- Examples:
  - the early 1980s (biotechnology, PC)
  - the early 1960s (electronics)
  - the 1920s (electricity, automobiles)
  - the early 1900s (radio)

The Link between Time Series and Cross Sectional predictability

- The voluminous "Equity Premium Puzzle" literature developed separately from the equally voluminous "Value Premium" Puzzle literature.
- This is unfortunate:
  - Explanations of one set of facts have equilibrium implications for the second set of facts.
- Consider any theory, rational or behavioral, for the aggregate variation of stock returns
  - What implications does it have for the cross-section of stock returns?
    - In particular, does it imply that the CAPM fails?
  - If so, does a multifactor model, or the conditional CAPM works under the proposed theory?
  - Are the predictions *quantitatively* plausible?
- Similarly, take any successful explanations for the cross-section:
  - What implications does it have for the equity premium, interest rates, Sharpe ratio?
  - What implications does it have for the *conditional* variation of returns?

The Link between Time Series and Cross Sectional predictability

- These questions are important because

  1. They inform on the set of "stories" that are plausible.
     - As a simple example: the Peso Problem explanation of the equity premium has nothing to say about the return differential between value and growth stocks.
     - This does not mean of course that the Peso Problem explanation is not true, but that it cannot be the whole story about the variation in stock returns.

  2. They impose additional constraints that may help in empirical tests.
     - E.g. Conditional CAPM tests show that some factors line up value and growth stocks
     - Are the coefficients in the cross-sectional estimate economically plausible?
       * $\Longrightarrow$ Pitfall in using cross-sectional R^2 and t-stats to declare victory
       * The magnitudes must be in line with the economic model as well.

  3. They yield insights on the variation of premia across asset classes;
     - E.g. The equity premium is time varying, and so is the value spread.
     - Are they related? If so, how? What aggregate factor drives both?
     - Is this index of a time varying market price of risk, or a time varying aggregate risk?

Going one step further: The link to fundamentals

- Some explanations of the cross-section of stock returns declare victory if they find, *empirically*, that value stocks covary more with a given factor than others.
  - Is this satisfactory?
  - Is this an explanation of the value premium puzzle?
    - It is definitely an empirical explanation, but it then raises additional questions.
- For instance:
  - Why do HML and SMB price value and size portfolios?
    - Is this a hardwired result or are these genuine risk factors?
    - Indeed, what is a theory for HML and SMB?
  - Recent research show that value stocks covary with future expected consumption growth.
    - Why should value stocks have higher covariance with future consumption growth?
    - What is the link at the fundamental level between returns on value stocks and future expected consumption growth?
  - Similarly, value stocks are found to have a higher cash flow risk
    - Why should they? Is this endogenous? How is this related to firms' life cycle?

Going one step further: The link to fundamentals

- Only general equilibrium models can provide satisfactory answers to these questions.
  1. We need an empirically plausible model for firms' cash flows:
     - E.g. Multiple trees or production technologies.
     - $\Longrightarrow$ the source of fundamental risk.
  2. We need a model for investor/consumer behavior:
     - $\Longrightarrow$ source of the market price of fundamental risk.
     - $\Longrightarrow$ implications for aggregate market portfolio.
  3. We need to impose market clearing and sum up cash flows to determine aggregate consumption:
     - Market clearing conditions may be powerful sources of variation;
     - They may yield "unexpected" results because of endogenous variation in the stochastic discount factor and endogenous correlation structure.

Benchmark Portfolio Allocation Model

- Consider now the model above for stock returns with same preferences, but now we do not impose market clearing ($\theta = 1$).
- In this case, the utility maximization problem of an investor with investment horizon T is

$$J(W_0, 0) = \max_{\{(C_t),(\boldsymbol{\theta}_t)\}} E_0 \left[\int_0^T e^{-\phi t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$$

- subject to the budget constraint

$$dW_t = \{W_t (\theta_t(\mu - r) + r) - C_t\} dt + W_t \theta_t \sigma d\mathbf{B}_t$$

- The solution to this program yields an investment in stocks equal to

$$\text{Fraction of Wealth Invested in Stocks} = \theta_t = \frac{\text{Excess Return on the Stock Market}}{\gamma \text{Variance of Stock Returns}}$$

Implications of Benchmark Portfolio Allocation Model

1. **Portfolio Allocation Puzzle 1:** The typical stockholders holds too little in stocks compared to what a canonical model would require.

   - Using unconditional averages, Excess Stock Return = 7% and Volatility of Returns = .16 %, we obtain

     Table: Portfolio Allocation

	Risk Aversion				
	2	4	6	8	10
Investment	136%	68%	45%	34 %	27 %

   - In contrast, depending on estimates, typical household holds betwee 6 % to 20 % in equity. Conditional on participating to the stock market, these number increase to about 40% of financial assets.

Implications of Benchmark Portfolio Allocation Model

2. **Portfolio Allocation Puzzle 2:** The canonical model with constant investment opportunity set implies that the portfolio allocation should not depend on the age of investor.

   - This is in contrast with the behavior of investors: Investors increase their holdings in equity for the first 1/2 of their life cycle, and decrease it afterwards.

3. **Portfolio Allocation Puzzle 3:** Many investors do not participate in the stock market, while the canonical model would imply always some participation to the market (at worse, short the market).

4. **Portfolio Allocation Puzzle 4:** Many investors invest in own company stocks, especially in their retirement plan. Diversification arguments clearly points at "shorting" the stock, if anything.

Nominal Long Term Bonds in Benchmark Model

- I now introduce an exogenous inflation process, and obtain nominal long term bond prices.
- The log dividend (consumption) $c = log(C)$ and log CPI $q_t = \log Q_t$ grow according to the joint stochastic model

$$\begin{aligned} dc_t &= gdt + \sigma_c dW_{c,t} \\ dq_t &= i_t dt + \sigma_q dW_{q,t} \\ di_t &= (\alpha - \beta i_t)\, dt + \sigma_i dW_{i,t} \end{aligned}$$

  – $i_t =$ is the expected inflation rate $i_t = E_t[dq_t]/dt$.

- The First Order Condition is (recall $\lambda_t = e^{-\phi t} C_t^{-\gamma}$)

$$Z\left(i_t, t; T\right) = E\left[\frac{\lambda_T}{\lambda_t}\frac{Q_t}{Q_T}\right]$$

- yielding

$$Z\left(i_t, t; T\right) = e^{A_0(\tau) - A_\beta(\tau) i_t}$$

- where $A_\beta(\tau)$ and $A_0(\tau)$ are two function of time to maturity $\tau = T - t$

Implications of Benchmark Model

1. The instantaneous nominal rate r_t is given by the constant real rate + inflation risk premium + expected inflation

$$r_t \;=\; \lim_{T-t\to 0} y\left(t;T\right) = -\lim_{\tau\to 0} \frac{A_0\left(\tau\right) - A_1\left(\tau\right) i_t}{\tau} = c + i_t$$

   - where

$$c = \left(\rho + \gamma g - \frac{1}{2}\gamma^2\sigma_c^2\right) - \gamma\sigma_c\sigma_q\rho_{qc} - \frac{1}{2}\sigma_q^2$$

2. The whole yield curve depends on the current expected inflation $i_t = E\left[dq_t\right]/dt$.

$$y\left(t;T\right) = -\frac{\log\left(Z\left(i_t,t;T\right)\right)}{\tau} = -\frac{A_0\left(\tau\right)}{\tau} + \frac{A_\beta\left(\tau\right)}{\tau} i_t$$

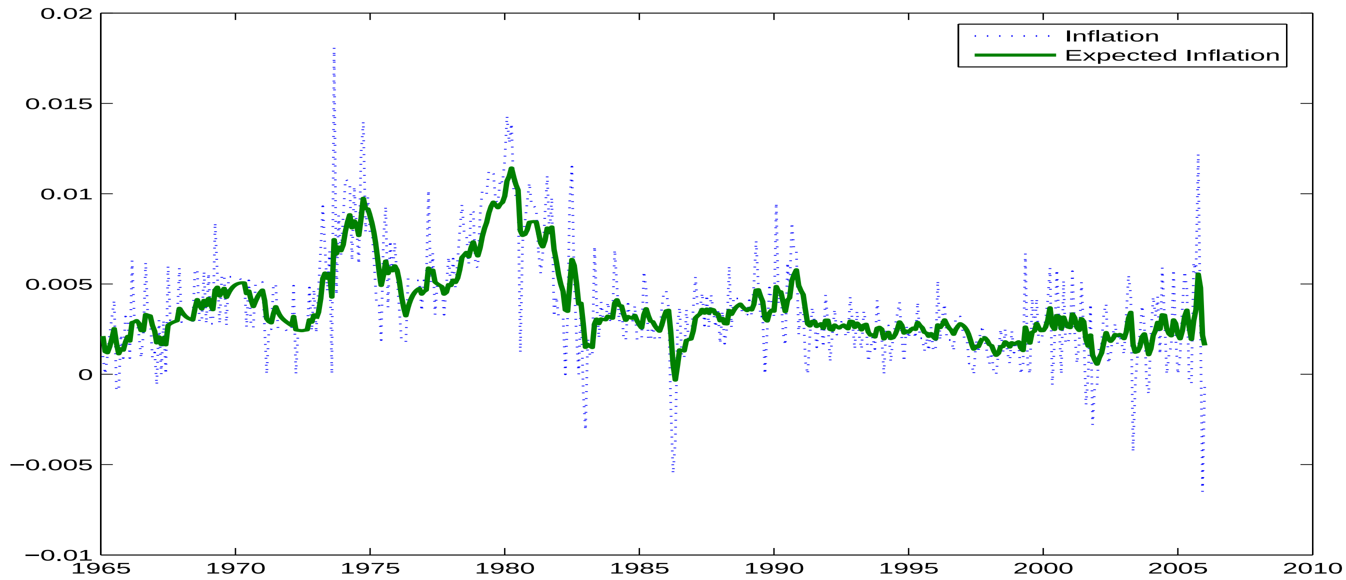
   - In particular, all of the yields are perfectly correlated.

3. The Term Spread (Slope) is

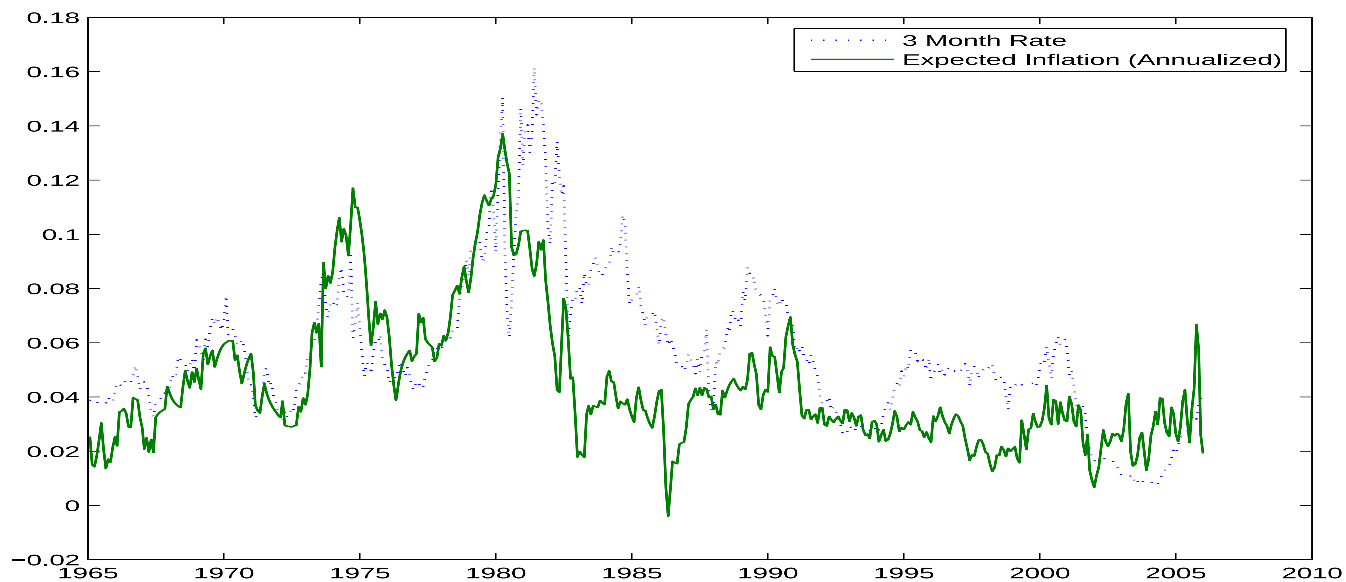
$$y_\infty - r_t = \left(\frac{\alpha}{\beta} - i_t\right) - \frac{1}{\beta}\left(\gamma\sigma_i\sigma_c\rho_{ic} + \sigma_i\sigma_q\rho_{iq}\right) - \frac{\sigma_i^2}{2\beta^2}$$

   - Note that since $\rho_{ic} < 0$ (typically), $\gamma\sigma_i\sigma_c\rho_{ic}/\beta < 0$. Higher risk or risk aversion, the higher the long end of the yield curve.

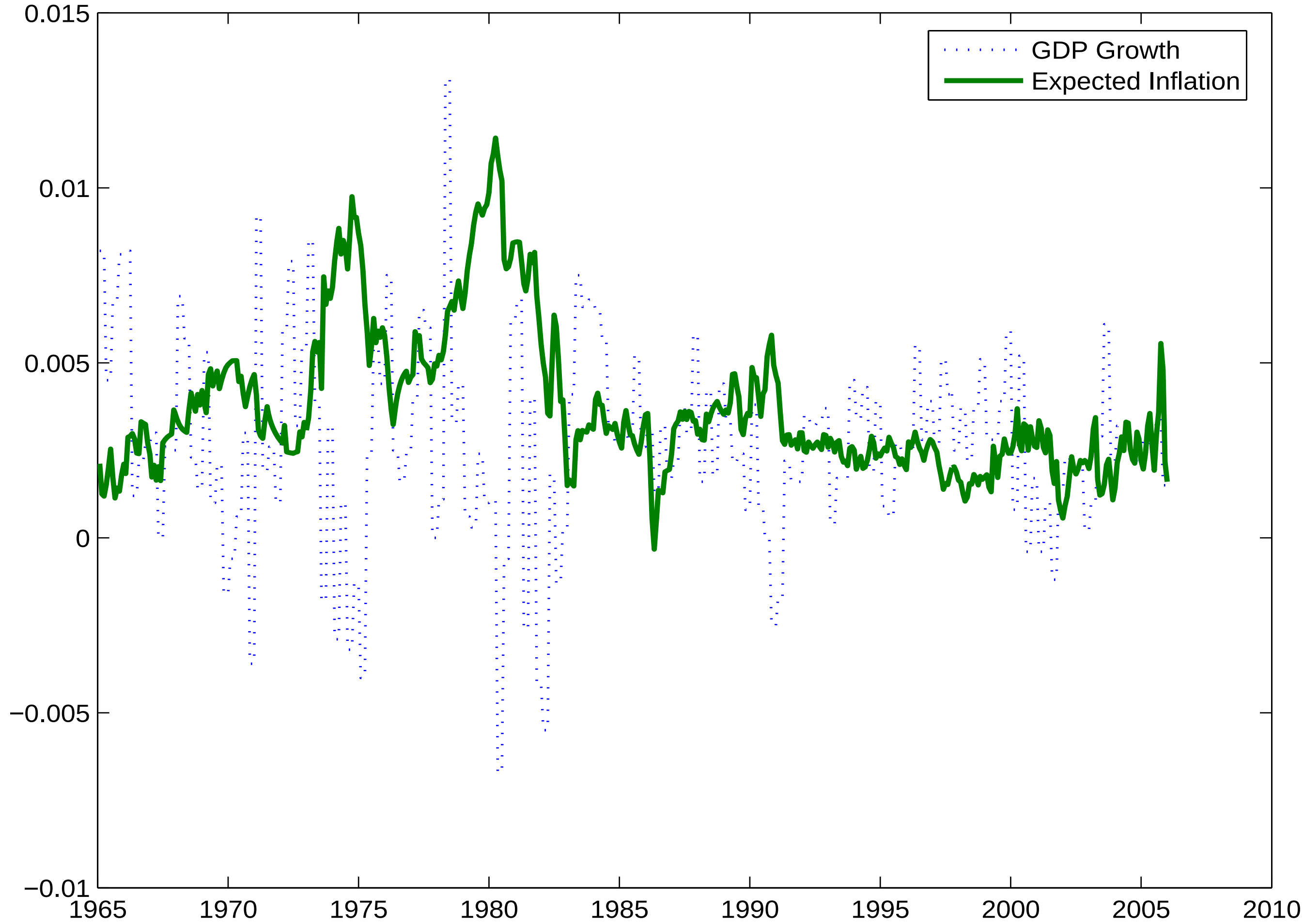
Inflation and Expected Inflation



Inflation and 3-month TBill rate



Expected Inflation and GDP growth



Implications of Benchmark Model

4. The model requires a large risk aversion to produce reasonable yield curves and a reasonable market price of risk λ

   • Using data on inflation and GDP growth ($= C$), we obtain the following parameters for the processes

α	β	g	σ_y	σ_q	σ_i	ρ_{yq}	ρ_{yi}	ρ_{iq}
.0160	0.3805	0.02*	0.02*	0.0106	0.0073	-.1409	-.2894	0.8360

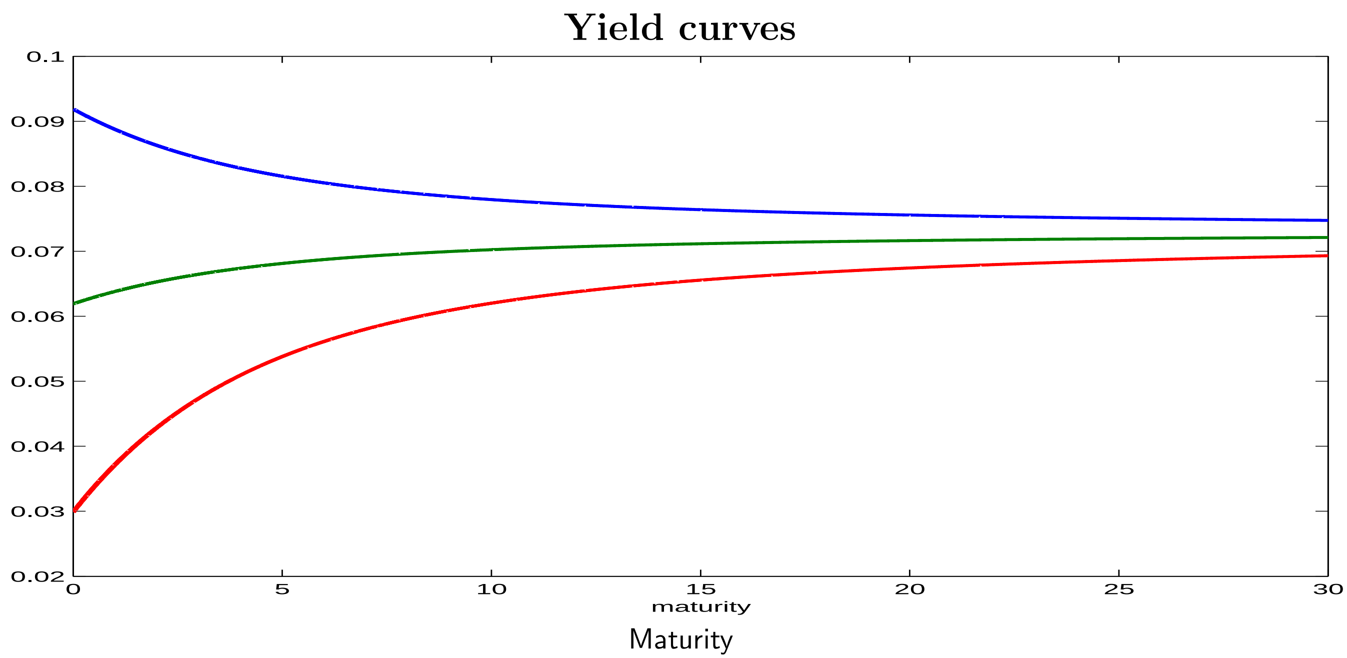
* The estimates of GDP growth were $g = .0321$ and $\sigma_y = 0.0098$, which made it hard to generate sensible yield functions. The parameters assumed are closer to consumption growth

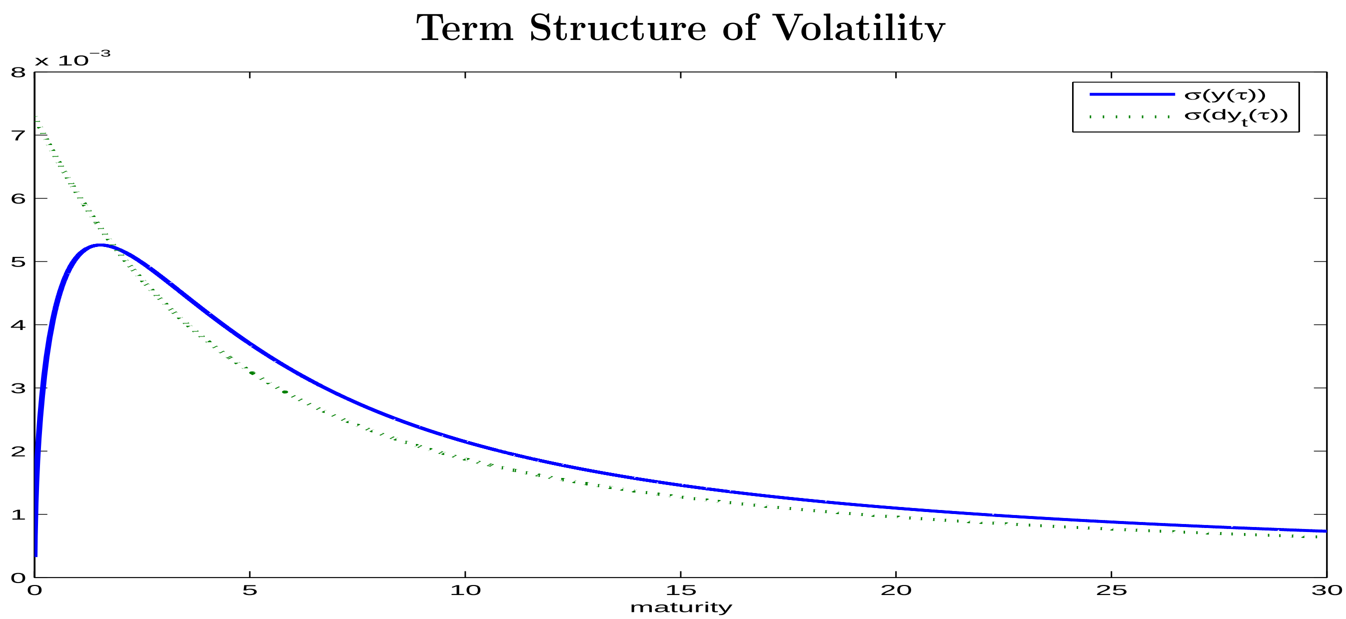
• Using utility parameters $\rho = .1$ and $\gamma = 104$ we get a real rate $c = .02$. $\xi = -0.5931$

• Risk free rate puzzle kicks in:

  – For "reasonable" γ, the interest rate is too high.

  – Lowering γ to $\gamma \approx 0.5$ generates also reasonable yield curves, but they are not upward sloping in average. Moreover, the market price of risk is too low.


Yield curves
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0 5 10 15 20 25 30
maturity
Maturity


Term Structure of Volatility
x 10^-3
8 7 6 5 4 3 2 1 0
σ(y(τ))
σ(dy_t(τ))
0 5 10 15 20 25 30
maturity

Implications of Benchmark Model

5. The volatility of bond yields changes ($\sigma(dy)$) is constant over time but depends on maturity:

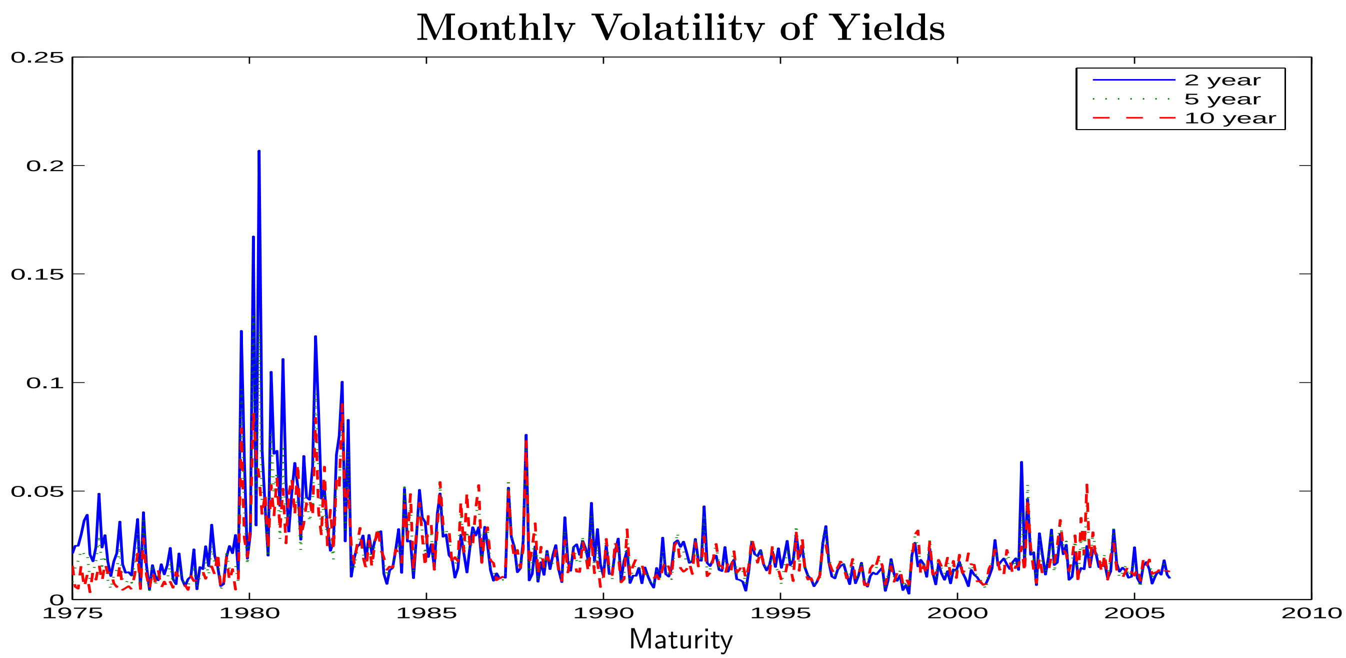
$$\sigma_y(t;T) = \frac{1-e^{-\beta\tau}}{\beta\tau}\sigma_i$$

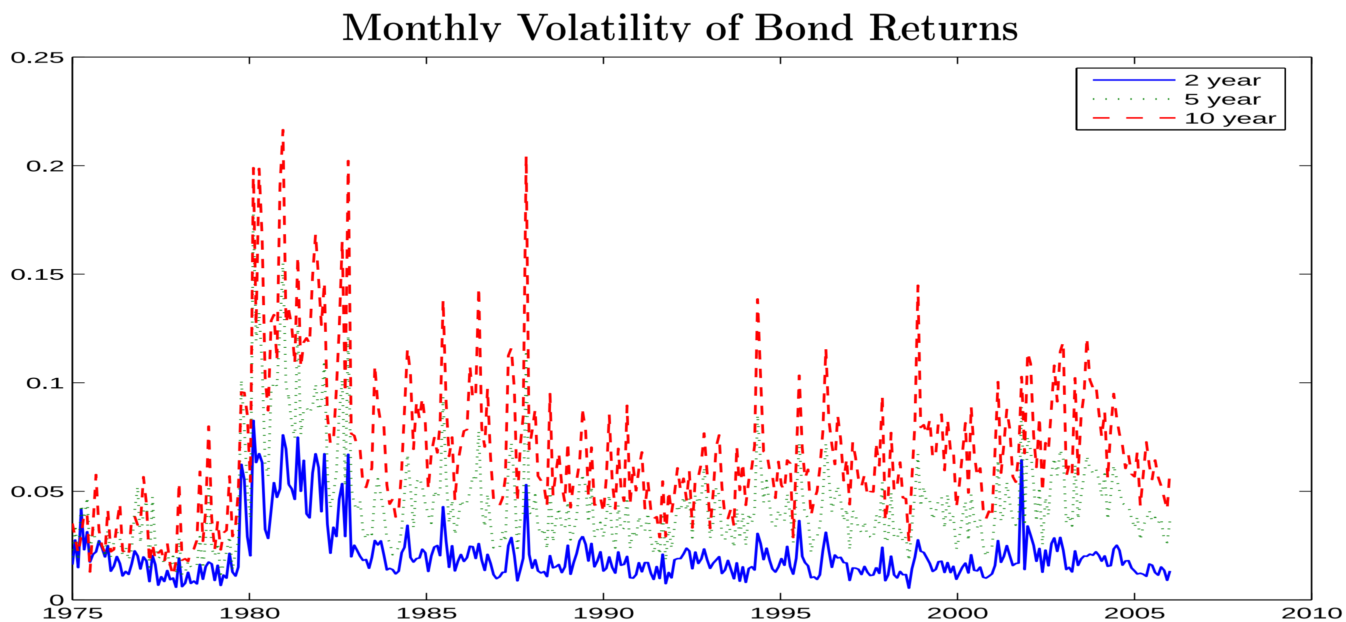
6. The bond risk premium is also constant, and given by

$$E\left[\frac{dZ}{Z}\right]/dt - r_t = \sigma_Z \xi$$

where

- $\sigma_Z =$ vol of$dZ/Z = -A_\beta(\tau)\sigma_i$
- $\xi = \gamma\sigma_c\rho_{ic} + \sigma_q\rho_{iq}$ is **Market Price of (inflation) Risk**
  - No time varying risk premium and no predictability


Monthly Volatility of Yields
2 year
5 year
10 year
Maturity


Monthly Volatility of Bond Returns
2 year
5 year
10 year

Bond Predictability. Fama Bliss (1987)

- Fama and Bliss classic paper show that bond return are predictable by the forward spread.

$$\text{holding period excess log return} = \alpha + \beta\left(f_t^{(n)} - y(t,1)\right) + \epsilon_t$$

- where $n =$ horizon (in years)

TABLE 2—FAMA-BLISS EXCESS RETURN REGRESSIONS

Maturity n	β	Small T	R^2	$\chi^2(1)$	p-val	EH p-val
2	0.99	(0.33)	0.16	18.4	⟨0.00⟩	⟨0.01⟩
3	1.35	(0.41)	0.17	19.2	⟨0.00⟩	⟨0.01⟩
4	1.61	(0.48)	0.18	16.4	⟨0.00⟩	⟨0.01⟩
5	1.27	(0.64)	0.09	5.7	⟨0.02⟩	⟨0.13⟩

Notes: The regressions are $rx_{t+1}^{(n)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \varepsilon_{t+1}^{(n)}$. Standard errors are in parentheses "( )", probability values in angled brackets "⟨ ⟩". The 5-percent and 1-percent critical values for a $\chi^2(1)$ are 3.8 and 6.6.

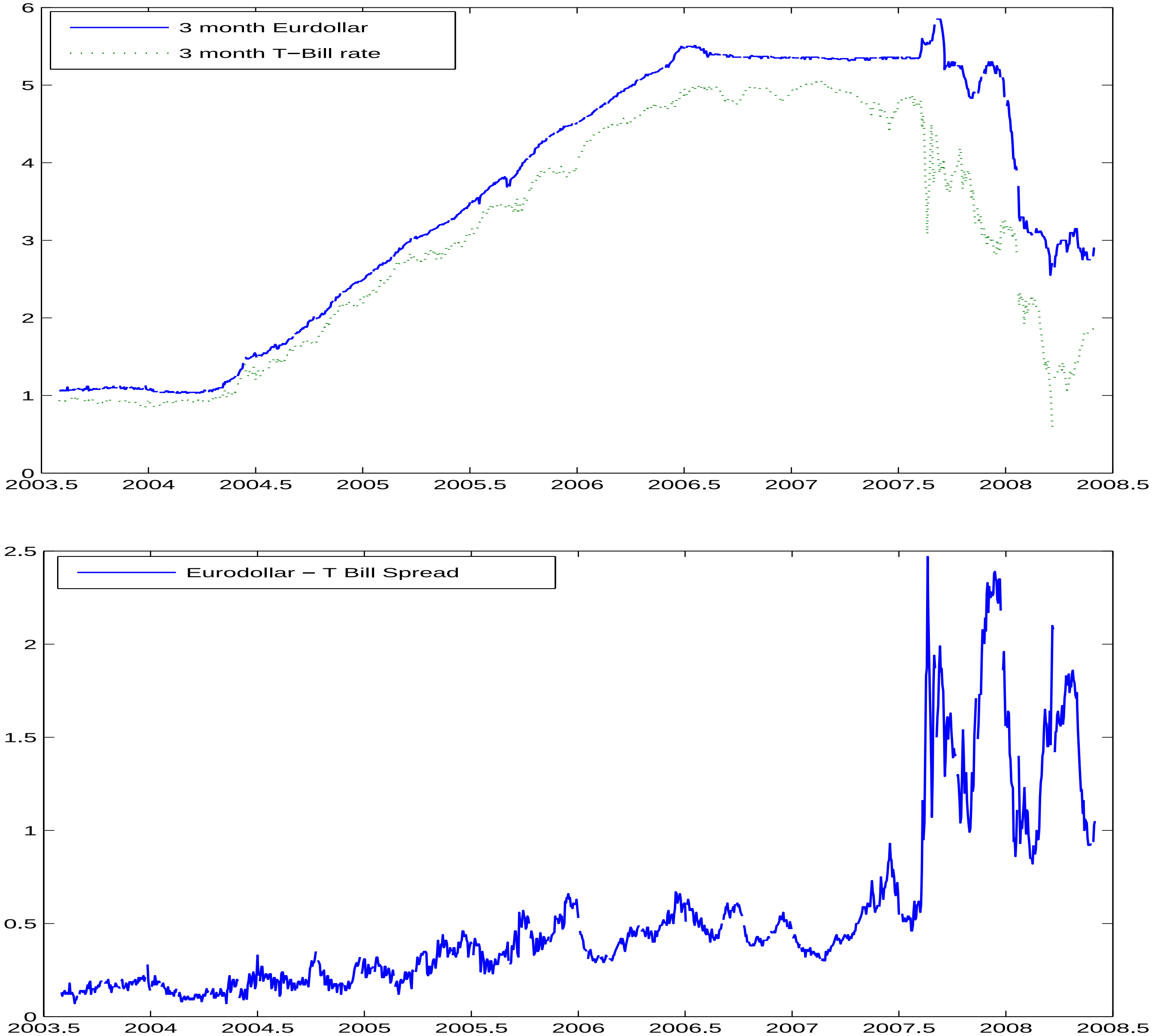
Source: Cochrane and Piazzesi (2005, AER)

- However, evidence from Euro, UK, Japan is much less clearcut. What's different there?

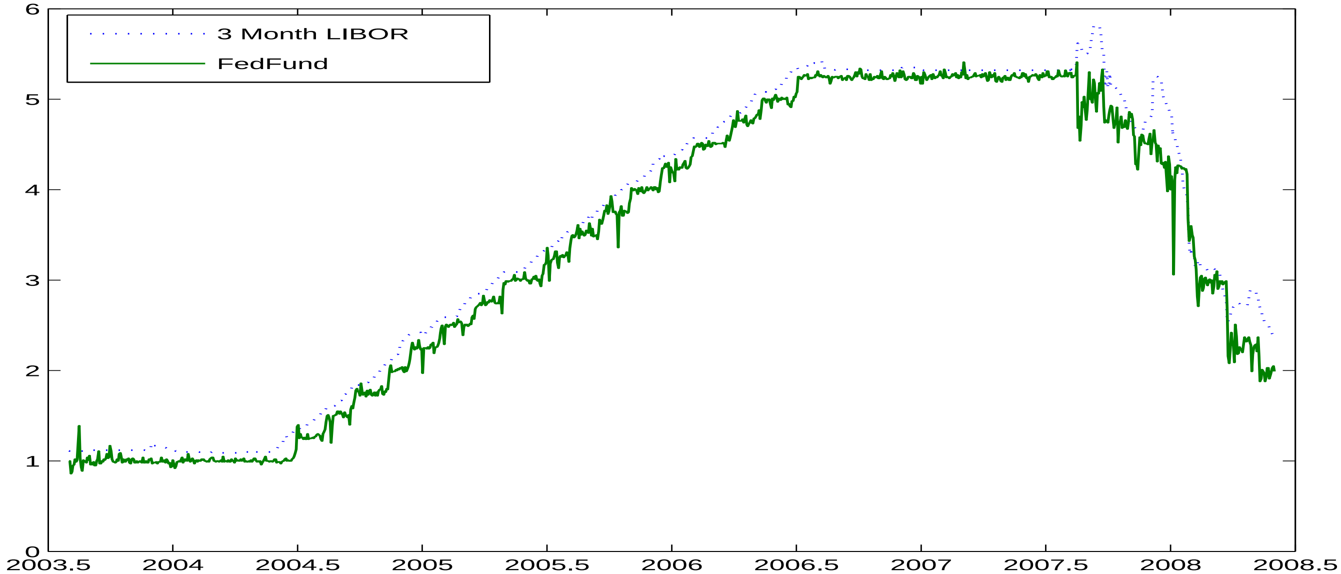
Bond Predictability. Cochrane and Piazzesi (2003)

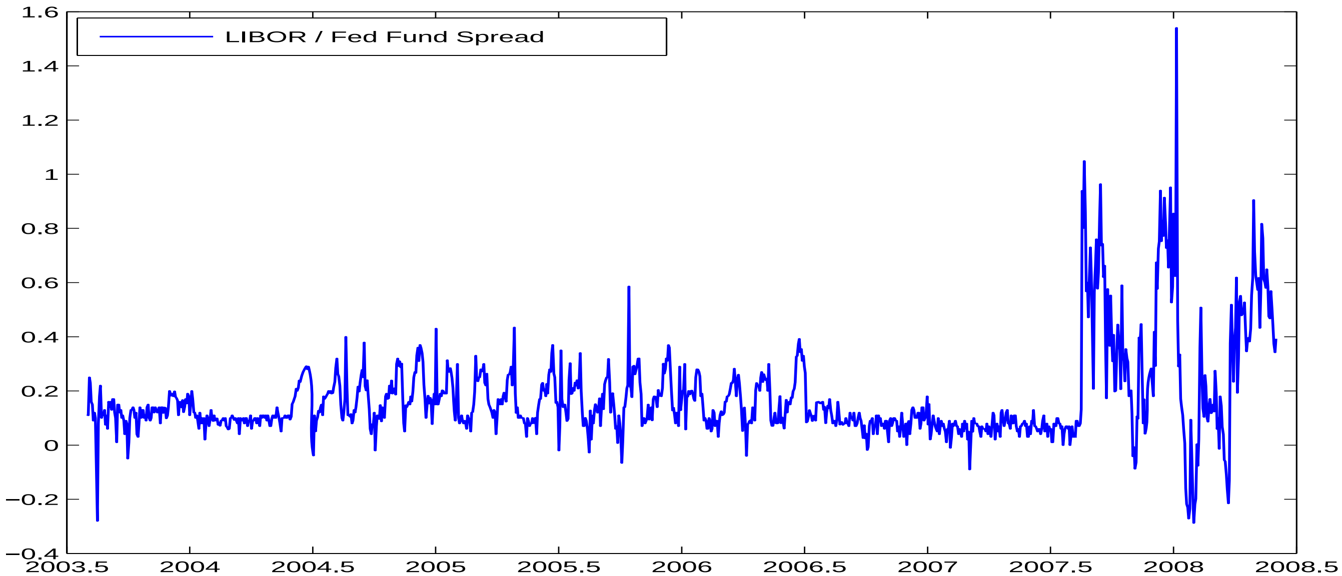
- Cochrane and Piazzesi (2003) show that there is a single combination of forwards that explain bond excess returns.
  - What is an economic model that generates that effect?
  - Intriguinly, Cochrane Piazzesi factor works also outside US, while Fama Bliss regressions do not. What is the factor capturing?

Credit Risk (and Credit Crisis)

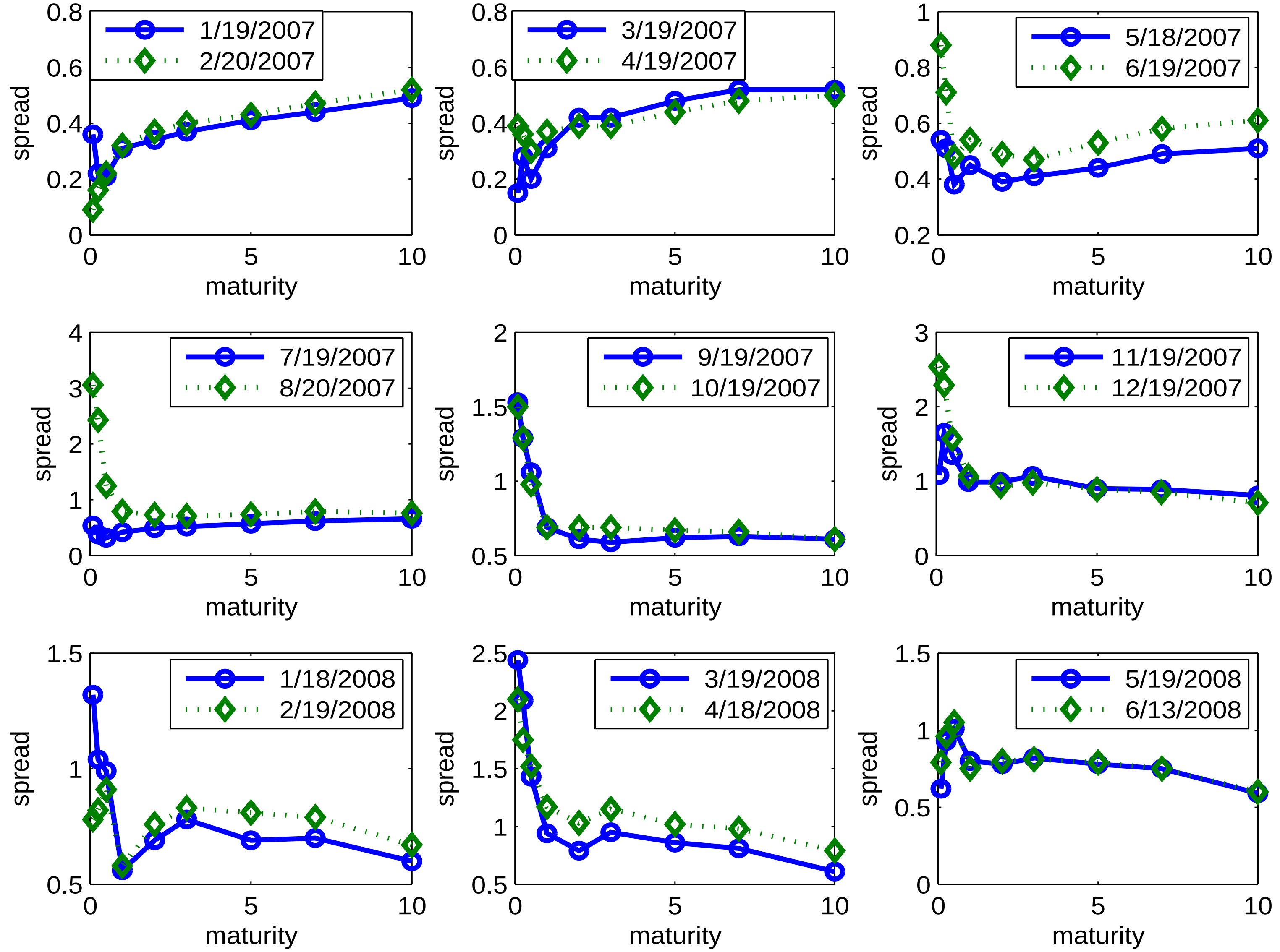


Credit Risk (and Credit Crisis)





The Term Structure of Credit Spreads between January 2007 and June 2008



This Course Covers

- Benchmark model of portfolio selection and stock returns
- Portfolio allocation models with
  - Time varying investment opportunities
  - Other preferences (e.g. Epstein Zin, Ambiguity Aversion etc.)
  - Incomplete information (learning)
- Incomplete information, learning and stock and bond returns
  - Valuation with uncertainty in long term growth.
- Different preference specifications and look at time series versus cross-sectional predictability
  - Habit formation, long run risk, and the cross-section
- Modern Term Structure Models
  - Reduce form, no arbitrage models (affine, quadratic etc) and empirical implications
  - Macro-based model and monetary policy
- Macro-economic based models of credit risk
- Wish list
  - Models of liquidity and volume
  - Rare Events

Requirements

- Homework:
  - I will assign three research ideas during these weeks.
  - These are ideas that you won't find in any paper
  - Your assignments will be to develop such research ideas into coherent papers. This will involve (a) solving a model; (b) obtain predictions; (c) check the predictions in the data, through testing or calibration.
  - The paper must have the form of a paper, with an introduction, body of the paper, data analysis, conclusion, appendix.
  - I will be the referee: this way you will get a feedback on what I did not like of the paper and how it should be written.
  - You can work in groups, but with a limit of 3 per group.
- Midterm
  - There will be a midterm around week 7 or 8. Essentially 1 1/2 hour on the material covered in class.
- Final Paper
  - There is a final paper you can develop. This is a paper of your choice.
- Grading assigns 30%, 30%, 5%, 35% to homeworks, midterm, class participation, and final project.