Reporting choices in the shadow of bank runs☆

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ARTICLE INFO

Article history:
Received 14 September 2016
Revised 6 June 2017
Accepted 16 November 2017
Available online 22 November 2017

JEL classification:
E58
G21
G32
M41
M43

Keywords:
Banking
Bank runs
Bank stability
Accounting discretion
Bank transparency

ABSTRACT

This paper investigates banks’ reporting choices in the context of bank runs. A fundamental-based run imposes market discipline on insolvent banks, but a panic-based run closes banks that could have survived with better coordination among creditors. We augment a bank-run model with the bank’s reporting choices. We show that banks with intermediate fundamentals have stronger incentive to misreport than those in the two tails. Moreover, reporting discretion reduces panic-based runs, but excessive discretion also reduces fundamental-based runs. The optimal amount of reporting discretion increases in the bank’s vulnerability to panic-based runs. Finally, a given bank’s opportunistic use of reporting discretion exerts a negative externality on other banks. Our paper answers the call by Armstrong et al. (2016) and Bushman (2016) to understand better the effects of banks’ special features on their reporting choices.

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1. Introduction

Banks are considered as different from non-financial firms and studied in a separate category.1 One special feature of banks is that they are more vulnerable to rollover risk and runs. Banks generally fund their long-term, illiquid assets (e.g., loans, bonds, asset-backed securities, or over-the-counter derivatives) with short-term instruments (e.g., demand deposits, commercial papers, repos, or redeemable equity shares). This maturity mismatch exposes banks to runs by their stakeholders.

☆ We are grateful for comments from Doug Diamond, Phil Dybvig, Zhiguo He, Pierre Liang, Brian Mittendorf, Harish Supra, Ulf Schiller, Liyan Yang, Gaoping Zhang, and participants of workshops at the Accounting Junior Theorist Conference, Duke-UNC Fall Camp, Rice University, SWUFE, University of Minnesota, UT Austin, and University of Toronto. We thank Ziqiong Huang, Guoyu Lin, and Jinzhi Lu for excellent research assistance, and Tim Gray and Roger Meservey for editorial help. All errors are our own. Pingyang Gao gratefully acknowledges financial support from the University of Chicago Booth School of Business, the Centel Foundation/Robert P. Reuss Faculty Research Fellowship, the PCL Faculty Scholarship, and National Natural Science Foundation of China (project 71620107005 “Research of Capital Market Trading System and Stability”). Xu Jiang gratefully acknowledges financial support from the Duke University Fuqua School of Business and the Center for Financial Excellence.

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1 For example, Beatty and Liao (2014) and Acharya and Ryan (2016) provide surveys of accounting research exclusively about banks (see also Bushman (2014) and Bushman (2016)).

https://doi.org/10.1016/j.jacceco.2017.11.005
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How does this special feature of banks affect their reporting choices? Should banks more vulnerable to panic-based runs be allowed more reporting discretion? What determines banks’ preference for reporting discretion? Understanding these questions is important both for interpreting empirical evidence and for policy discussions. On the one hand, a considerable body of empirical evidence shows that banks often have both the ability and the incentive to exploit the reporting discretion built into regulations and financial reporting standards to manage earnings, capital levels, or both, especially in downturns. However, the interpretations and implications of the evidence are still controversial. After reviewing the various arguments, Acharya and Ryan (2016) conclude: “(there are) differing views about whether bank opacity compromises or promotes stability. Some believe that bank transparency is necessary to promote market discipline that is critical for stability. Others believe that bank opacity is necessary to suppress behaviors that compromise stability, such as bank runs.” On the other hand, the literature on firms’ reporting choices has mainly focused on non-financial firms and doesn’t incorporate banks’ special features such as run risk. As a result, the guidance this literature provides for understanding banks’ reporting choices is limited. Armstrong et al. (2016) review the role of financial reporting in corporate governance and discuss the literature’s implications for banks. They emphasize that “a related challenge for researchers has been to understand what is special about banks and other financial institutions.” We respond to this challenge by modeling banks’ reporting choices in the context of bank runs.

We study a stylized model of reporting choices by banks that are vulnerable to runs à la Diamond and Dybvig (1983). Specifically, a bank in our model is characterized by an exogenous maturity mismatch between its assets and financing. This characterization describes some commercial banks, investment banks, and investment funds. These institutions often fund their long-term, illiquid assets with short-term instruments. Such maturity mismatch exposes them to runs or coordination failure among their stakeholders. Following the literature, we use the label “banks” to refer to financial institutions with run risk and “creditors” to their stakeholders with short-term claims.

The bank’s manager observes the fundamentals, issues a potentially biased report to creditors, and prefers less withdrawal. The manager’s cost to bias the report is interpreted as the (inverse) degree of reporting discretion. Upon receiving the potentially manipulated report, creditors decide whether to withdraw from the bank. A benchmark of our model, in which the manager has no reporting discretion, has been studied extensively in the prior literature (e.g., Morris and Shin (2000) and Goldstein and Pauzner (2005)). In their unique equilibrium, runs can be either fundamental-based or panic-based. The former liquidates insolvent banks efficiently, whereas the latter closes banks that are solvent but illiquid. In other words, while runs impose market discipline, they can also be excessive. Moreover, the root cause of panic-based runs is creditors’ strategic uncertainty about other creditors’ decisions (as opposed to their uncertainty about the fundamentals).

Reporting discretion exacerbates the problem of coordination among creditors, as now they have to infer the manager’s reporting behavior in addition to forecasting other creditors’ actions. We use the global games methodology to obtain the model’s unique equilibrium and conduct comparative statics to examine the effects of reporting discretion on the incidence and efficiency of bank runs.

In the unique equilibrium we obtain, creditors withdraw if and only if the signals they receive fall below a common threshold. Moreover, the manager’s reporting strategy is a partial pooling: the strongest banks survive without misreporting, the banks with intermediate fundamentals misreport to pool together to avoid runs, and the weakest ones cannot afford to misreport and thus suffer runs. The equilibrium has three salient features. First, reporting discretion alters the incidence of bank runs. Since in equilibrium creditors play a common threshold strategy, the manager’s misreporting incentive is not monotonic in the bank’s fundamentals. For a bank whose fundamental is just below the run threshold, a small increase in creditors’ beliefs can change a run into a no-run equilibrium, which increases the manager’s payoff by a discrete amount. Thus, the manager’s incentive to influence creditors’ beliefs is the greatest in this scenario. By contrast, when a bank’s fundamental is extremely good or extremely bad, creditors’ withdrawal decisions are less sensitive to changes in their beliefs. So the manager’s incentive to influence those beliefs is weak. This non-monotonic incentive to misreport impedes creditors’ ability to undo the reporting bias. They cannot distinguish between the signals of a weak bank with a larger bias and a stronger one with a smaller bias. Therefore, reporting discretion results in a partial pooling of banks with differing fundamentals.

The second salient feature of the equilibrium is that reporting discretion reduces panic-based runs, but it also impedes fundamental-based runs when it is excessive. By inflating creditors’ beliefs about the fundamentals of weaker banks, reporting discretion induces creditors to be more optimistic about other creditors’ actions, offsetting the excessive pessimism that drives panic-based runs. However, as discretion increases, the probability of runs is reduced still further, to the point that even insolvent banks can survive with inflated reports. Therefore excessive reporting discretion weakens the market discipline on banks. The more severe the coordination failure, the higher the optimal degree of reporting discretion. To the

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2 For example, banks structure commercial paper conduits to inflate capital (e.g., Acharya et al. (2013)), manipulate Basel risk-weights under the internal ratings-based approaches of Basel II (e.g., Marriathasan and Merrouche (2014)), and overstate earnings, assets, and capital through aggressive use of accounting rules over various areas, including loan loss provision (e.g., Bushman and Williams (2012)), impairment (e.g., Vyas (2011)), valuation for level 2 and 3 assets (e.g., Kolev (2009) and Song et al. (2010)), securitization (e.g., Dechow et al. (2010)), deferred tax assets (e.g., Skinner (2008)), or a combination of these areas (e.g., Huizinga and Laeven (2012)). Comprehensive reviews of this empirical evidence are available in a number of recent surveys, including Laux and Leuz (2010), Beatty and Liao (2014), Bushman (2014), Acharya and Ryan (2016), and Bushman (2016). Laux and Leuz (2010) conclude that, during the 2008 financial crisis, “banks used accounting discretion to overstate the value of their assets substantially.”

extent that banks are particularly vulnerable to panic-based runs, banks should be granted more reporting discretion than non-financial firms.

Third, and finally, a given bank’s opportunistic reporting imposes a negative externality on other banks. As reporting discretion increases, the set of banks that inflate reports in equilibrium contains not only those vulnerable to panic-based runs but also those that could have survived in the absence of reporting discretion. In other words, the existence of reporting discretion forces even solvent and liquid banks to inflate their reports in equilibrium. The misreporting by this group generates deadweight loss. Therefore, banks’ preferences for reporting discretion vary, with the weaker banks preferring more reporting discretion than their stronger counterparts.

1.1. The literature review

This paper analyzes banks’ reporting choices in a bank-run setting. It contributes to the literature on costly misreporting (earnings management).4 The prior literature has focused almost exclusively on non-financial firms and is thus not pertinent to our main research question, namely, how reporting discretion affects bank runs. Moreover, the partial-pooling reporting equilibrium in our model offers another explanation for the “kink” phenomenon studied by Burgstahler and Dichev (1997) and Guttman et al. (2006). Our explanation differs from that in Guttman et al. (2006) in a number of important ways. First, they study non-financial firms with no coordination problems among their investors, whereas our primary concern is bank runs induced by investors’ coordination failure. Second, in Guttman et al. (2006) reporting discretion always leads to deadweight loss, but in our model it can reduce panic-based runs. Thus, in terms of policy implications, eliminating reporting discretion is optimal in their model but not necessarily in ours. Third, the forms of the equilibria differ. Even though both feature separation in the two tails, misreporting still occurs in both tails in Guttman et al. (2006) but not in ours. Finally, their model features multiple equilibria, while we use the global games methodology to obtain the unique equilibrium.

Our paper also contributes to the literature on banks’ information environment. This theoretical literature has focused mainly on “symmetric ignorance,” the first type of opacity defined in Acharya and Ryan (2016). It studies the economic consequences of ex ante commitment to transparency in a setting in which all market participants, including banks, have the same information. After a commitment is made, banks cannot misreport the signal’s realizations.5 Our paper complements this literature by studying banks’ ex post reporting choices, the second type of opacity defined in Acharya and Ryan (2016). In our model, the bank decides what to disclose after observing the signal and can distort the reported signal at a cost.

Arya and Mittendorf (2016) construct a model featuring complementarity among firms’ investment decisions. They show that not only does a firm have incentives to make an early investment and publicly disclose it, but also such disclosure benefits non-disclosing firms. By making the private information public, the disclosing firm uses its private information more efficiently, and the non-disclosing firms receive more information for their decision-making. Our paper differs from theirs in that they model neither banks nor firms’ misreporting choices.

Finally, our paper contributes to the global games literature by providing an application to a bank-run setting. Goldstein and Pauzner (2005) and Morris and Shin (2000) have applied the global games methodology to obtain a unique equilibrium in a bank-run setting without reporting choices. We introduce reporting discretion, an important institutional feature of banks.

More broadly, the prior global games literature has studied regime change models in which the regime can take a costly action to influence agents’ beliefs, such as Angeletos et al. (2006) and Edmond (2013). The bank-run model and the regime-change models are often viewed as technically similar in the absence of reporting discretion. However, in the presence of reporting discretion, the two classes of models differ qualitatively. For example, the reporting equilibrium in our model may appear similar to that in Edmond (2013), but they actually differ in a fundamental manner. The pooling equilibrium in Edmond (2013) results from the noise in creditors’ signals. As the noise tends towards 0, the equilibrium converges to a fully separating one. In other words, the pooling in Edmond (2013) is caused by agents’ uncertainty about the fundamental, while in our model it is caused by agents’ uncertainty about other agents’ strategies. This difference in the two models’ driving forces seem to be caused by the different payoff structures.

The rest of the paper is organized as follows. Section 2 describes the model, and Section 3 presents the multiple equilibria with reporting discretion. Section 4 uses the global games methodology to solve for the unique equilibrium with reporting discretion, while Section 5 examines the effects of reporting discretion on the incidence and efficiency of runs. We also discuss their empirical and policy implications. Section 6 concludes. The appendix contains the proofs.

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4 This literature is vast, and we refer readers to some excellent recent surveys, such as Armstrong et al. (2010), Beyer et al. (2010), Ewert and Wagenhofer (2012), and Stocken (2013).

5 A theme in this literature is that ex ante transparency can interact with the coordination friction to generate undesirable consequences. It may induce creditors to put excessive weight on the public information (e.g., Morris and Shin (2002)), distort the private incentive for information acquisition (e.g., He and Manela (2016)), destroy risk sharing (e.g., Goldstein and Leitner (2013)), contaminate market participants’ learning from prices (e.g., Bond et al. (2010), Bond et al. (2012)), and engage in privately optimal but socially wasteful information acquisition (e.g., Dang et al. (2014)). The consequences of ex ante commitment to transparency may also vary across economic cycles (e.g., Bouvard et al. (2015)), Bocken and Schiller (2015) study the consequences of the choice between historical cost and fair value accounting in the context of bank runs. See Goldstein and Sapra (2013) for a recent survey.
2. The model

Our model augments the standard bank-run model à la Diamond and Dybvig (1983) with reporting choices of banks. The bank-run model has been applied not only to banks but also to any financial institutions that fund their long-term illiquid assets with short-term instruments. Following the standard terminology, we call financial institutions with asset-liability mismatch "banks" and their stakeholders with short-term claims "creditors."

Consider a risk-neutral economy with no discounting, one consumption good, three dates (t = 0, 1, 2), one bank, and a continuum [0, 1] of creditors. The bank is characterized by an exogenous maturity mismatch. On the asset side, it has exclusive access to a long-term asset at t = 0 that yields r units at t = 2; r is referred to as the bank’s fundamental and has an improper prior. The asset, however, is illiquid. If proportion l of the investment is withdrawn at t = 1, the return per unit on the remaining asset is

\[ r - \delta l. \]  

(1)

\( \delta > 0 \) captures the cost of premature liquidation. In other words, for 1 unit of investment at t = 0 and early withdrawal l at t = 1, the asset yields l × 1 unit at t = 1 and \((1 - l)(r - \delta l)\) at t = 2.

On the liability side, at t = 0 the bank receives 1 unit of consumption good from each creditor by issuing short-term instruments that entitle creditors to withdraw their investment at either t = 1 or t = 2. At t = 1, each creditor has a decision to make. She either withdraws the 1 unit investment or leave it with the bank. In case of withdrawal, she receives 1 unit of consumption good. Otherwise, if she remains with the bank (and if the aggregate withdrawal by others is l), the creditor receives the asset’s return \(r - \delta l\) at t = 2. Denote creditor i’s withdrawal decision at t = 1 as \(n_i \in [0, 1]\), with \(n_i = 1\) indicating withdrawal. Then the aggregate withdrawal l can be computed as \(l = E[n_i]\). A creditor’s payoff, conditional on fundamental r and aggregate withdrawal l, is summarized in Table 1.

So far we have described the standard bank-run component of the model as in Diamond and Dybvig (1983) and Morris and Shin (2000). Before introducing reporting choices, we pause to discuss some of the model’s assumptions. The key assumption that drives bank runs is the maturity mismatch.\(^6\) Creditors are allowed to withdraw their unit investment at t = 1 on a first-come, first-served basis. To meet the obligation, the bank incurs the cost of liquidating its assets prematurely at t = 1. The remaining creditors ultimately bear the cost of the premature liquidation. The more numerous the creditors who withdraw at t = 1, the less those who remain receive at t = 2. The essence of this mismatch is captured in expression (1) with the assumption \(\delta > 0\). A larger \(\delta\) indicates a more severe mismatch.\(^7\)

The second stark feature of the bank-run framework is that the bank’s capital structure is not modeled: the bank has no equity but is a mutual, distributing returns from its assets to creditors. Note that the feature that creditors receive all the returns serves for convenience only. What drives bank runs is the feature that creditors’ payoffs are more sensitive to the asset performance if they withdraw later. This feature obviously holds even if equity is introduced. As long as the equity is not sufficient to absorb all possible losses, creditors’ claims become risky and are thus more sensitive to the asset return if they choose not to withdraw.

Finally, the global games methodology commonly adopts the improper prior for technical reasons. The improper prior assumption means that the probability density function about the fundamentals is uniform over the entire real line. In other words, the agent has little information a priori and thinks that all outcomes are equally possible. The improper prior is a statistical device to capture the idea that the data (or, later in our model, the report) contains far more information than the agents’ prior knowledge. With the improper prior, the posterior equals the new data since the prior is given no weight. This

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\(^6\) The micro-foundation of the maturity mismatch has been explored in the literature. Goldstein and Pauzner (2005) have proved that demandable deposit contracts with endogenous interest rates can still be efficient, even though they cause panic-based runs. They have also shown that the uniqueness of the run equilibrium is still preserved even in a richer model that faithfully captures the “one-sided complementarity” of a prototypical commercial bank.

\(^7\) \(\delta\) varies across assets. It is probably small for such liquid assets as Treasury notes and large cap stocks. But financial institutions also hold many assets with larger \(\delta\), including loans, small- and mid-cap stocks, corporate bonds, asset-based securities, and over-the-counter derivatives. For example, as of December 31, 2015, the level 1, 2, and 3 financial assets of Goldman Sachs are $153 billion, $432 billion and $24 billion, respectively. In other words, most of the firm’s financial assets are not traded in active markets and thus may have a significant \(\delta\). Moreover, empirical evidence suggests that \(\delta\) can be substantial even for open-end equity mutual funds, which are among the most liquid financial institutions. \(\delta\) is the portfolio rebalancing costs that mutual funds incur to meet redemptions. These costs include the direct costs, such as bid-ask spread, commissions, and price impacts, and the indirect costs when redemptions force fund managers to deviate from their optimal portfolios (e.g., Shleifer and Vishny (1997)). Edeleman (1999) estimates that the average transaction costs to meet redemption are 1.5–2.2% of the average net asset value (NAV). Finally, the empirical evidence on fire sales and runs has been documented in equity markets by Coval and Stafford (2007), in open-end mutual funds by Chen et al. (2010), in repo markets by Gorton and Metrick (2012), and even in money market funds by Kacperczyk and Schnabl (2013). Overall, banks’ maturity mismatch that drives bank runs in our model seems empirically descriptive.
simplifies Bayesian updating.\textsuperscript{8} Morris and Shin (2003) explain the role of the improper prior in the global games paradigm and conclude that “such improper priors are well behaved as long as we are concerned only with the conditional beliefs.”\textsuperscript{9}

Now we can add reporting choices to the standard bank-run model described above. At \( t = 1 \), before creditors decide whether to withdraw, the bank’s manager issues an accounting report that is informative about the fundamental \( r \). For simplicity, we assume that the manager observes \( r \) perfectly and can add a bias to the report at a cost.\textsuperscript{10} In particular, the manager issues a report \( M(r) = r + m(r) \). The bias \( m(r) \) costs the manager privately \( C(k, m) = kc(m) \). The cost function \( C(m) \) has standard properties: \( C(0) = C'(0) = 0, C'(m) > 0 \) for \( m > 0 \), \( C''(m) > 0 \) and \( \log C \) is concave. The log-concavity of \( C(m) \) implies that \( c \) cannot be too convex, that is, \( \frac{c'(m)}{c(m)} \) is decreasing in \( m \). In particular, we assume that \( \lim_{m \to \infty} \frac{c'(m)}{c(m)} = 0 \).\textsuperscript{11} The power functions, i.e., \( c(m) = m^p \) with \( p > 1 \) satisfy all these conditions. \( k > 0 \) measures the cost of misreporting or the (inverse of the) degree of reporting discretion determined by rules and regulations.\textsuperscript{12} As \( k \) tends to \( \infty \), reporting discretion tends to 0, and our model converges to the standard bank-run model of Diamond and Dybvig (1983) and Morris and Shin (2000).

The manager’s net payoff is
\[
w(m; r) = r - l(r, m) - kc(m).
\] (2)

All else equal, the manager benefits from a higher fundamental \( r \) and prefers a smaller fraction of withdrawal \( l \). We take the conflict of interest between creditors and manager as given. While creditors benefit from state-contingent withdrawals, the manager prefers less withdrawal and better fundamentals, as captured succinctly in the manager’s net payoff function in expression (2). The manager’s preference for a higher \( r \) is self-evident. To his preference for less withdrawal, one interpretation might be that the manager simply prefers to run a larger institution. The manager’s compensation, power, job security, reputation, and human capital are all tied to the bank’s size and decrease as withdrawal increases. Thus we assume that, while banks may try to mitigate this conflict of interest, they cannot eliminate it, owing to contracting frictions.

In addition, whether the fundamental \( r \) is or is not included in the manager’s payoff does not affect the main results.

To summarize, the timeline of the model is as follows.

- At \( t = 0 \), the bank receives 1 unit of consumption good and invests it in an illiquid project.
- At \( t = 1 \), the bank manager observes the fundamental \( r \) privately and issues a report \( M(r) = r + m(r) \). Each creditor observes \( M \) and decides whether to withdraw.
- At \( t = 2 \), the remaining investment, if any, pays out.

A Perfect Bayesian equilibrium (PBE) of our model consists of the manager’s reporting strategy \( m(r) \) and each creditor’s withdrawal strategy \( n_l(M) \) and beliefs about the fundamental \( r \) such that (1) both the manager and the creditors maximize their respective objective functions, given their beliefs and the strategies of others; and (2) each creditor uses the Bayes rule, if possible, to update beliefs about \( r \).

3. Preliminary analysis and the multiple equilibria

In this section we characterize the multiple equilibria of our model, resulting from the combination of creditors’ common knowledge about the report and the strategic complementarity of withdrawal decisions. In the following section, we use the global games techniques for refinement.

We set a benchmark with neither coordination failure nor reporting discretion. Suppose there is only one creditor who learns \( r \) perfectly and chooses \( l \) to maximize the expected utility \( l + (1 - l)(r - \delta l) \). It can be shown that \( l^{FB}(r) = 1 \) if \( r < r^{FB} \equiv 1 \) and 0 if \( r \geq r^{FB} \); “FB” stands for “first-best.” A single creditor withdraws from the bank if and only if the bank’s continuation value is lower than liquidation value. We refer to banks with \( r < 1 \) as insolvent, and runs on these banks as

\textsuperscript{8} See DeGroot and Schervish (2002) for more discussions, e.g., page 328.

\textsuperscript{9} We have proved that our main result, Proposition 2, holds with a generic prior distribution as long as the prior is not too precise (and we consider only threshold equilibria). The proof is available upon request.

\textsuperscript{10} The reporting component of our model relates to the signal-jamming literature (as in Stein (1989); see Stocken (2013) for a recent review). As is common in the disclosure literature, the report could be interpreted broadly as any information that the bank provides that is relevant to the asset’s future performance. One such example is the income statement, which measures past performance and is also informative about future performance, given the correlation between past and future fundamentals. To economize on notation, we have assumed that they are perfectly correlated and that the manager knows them perfectly. The manager can influence the income statement by taking some costly actions, and the leeway in this regard depends on accounting rules and regulations.

\textsuperscript{11} This condition is not necessary for our main results, Propositions 1 and 2. It is a sufficient condition to ensure that \( \tau_1(k) < c^0 \) as \( k \) goes to 0 in the limit, a result discussed in Proposition 3. If we assume away this condition, then reporting discretion mitigates but may not eliminate panic-based runs.

\textsuperscript{12} Misreporting should be interpreted broadly. It refers to any influence activities a bank undertakes to improve reported performance without improving its true economic performance (e.g., Milgrom (1988)). At one end of the spectrum, misreporting may be understood literally as aggressive or even illegal application of reporting rules. For example, a bank may set a lower loan loss provision by underestimating borrowers’ default risk. At the other end, misreporting may arise through real activities. For example, a bank can structure a transaction in an economically suboptimal manner so as to satisfy the letter, but not the spirit, of reporting regulations. This type of misreporting is often labeled as “regulatory arbitrage” or “accounting-motivated” transactions, of which the securitizations in Acharya et al. (2013) are one example. Obviously the cost of such activities is influenced by the degree of reporting discretion built into rules and regulations.
fundamental-based runs. Such runs liquidate the insolvent banks, and the liquidation is efficient from the creditors’ standpoint. The manager, by contrast, prefers less liquidation and thus would not liquidate the asset voluntarily even if \( r < 1 \). Thus, there is a conflict of interest between creditors and the manager. To the extent that the fundamental-based runs mitigate this conflict by forcing the efficient liquidation of insolvent banks, they enforce market discipline on banks. We also refer to banks with \( r \in [1, 1 + \delta) \) as solvent but illiquid and banks with \( r \geq 1 + \delta \) as solvent and liquid.

We begin with creditors’ decisions. If a creditor withdraws at \( t = 1 \), she gets 1 unit of consumption good back and guarantees a utility of 1, independent of the report and the choices of other creditors. If she waits, her utility becomes \( r - \delta l \) when a proportion \( l \) of creditors withdraw. The utility from waiting increases in \( r \) and, more importantly, decreases in \( l \). The latter property represents the strategic complementarity of creditors’ withdrawal decisions. The intensity of this complementarity is captured by parameter \( \delta \).

Since \( r \) is not directly observable, a creditor uses the report \( M \), together with rational expectations of the manager’s reporting strategy, to forecast both \( r \) and \( l \). Thus, the expected utility differential between late withdrawal at \( t = 2 \) and early withdrawal at \( t = 1 \) is

\[
\Delta(M) = E[r - \delta l - 1 | M].
\]

(3)

A creditor chooses to withdraw, i.e., \( n_t(M) = 1 \), if and only if \( \Delta(M) < 0 \). As a tie breaker, we assume that a creditor indifferent between early and late elects to stay.

\( \Delta(M) \) suggests that a creditor’s decision depends on her beliefs both about the fundamental \( r \) and about the other creditors’ decisions summarized in \( l \). Moreover, creditors also understand the manager’s misreporting strategy when using report \( M \) to infer both \( r \) and \( l \). Since the manager misreports in order to reduce withdrawals and the maximum possible withdrawal reduction is 1, the manager is willing to add a bias \( m \) only if \( 1 \geq kc(m) \). Denote \( c^{-1}(.) \) as the inverse function of \( c \). The maximum bias the manager is willing to add is \( c^{-1}(\frac{1}{k}) \). Therefore, a creditor’s most pessimistic belief is that the manager has chosen the maximum bias \( (m = c^{-1}(\frac{1}{k})) \) and all the other creditors are withdrawing \( (l = 1) \). Under this belief, a creditor’s expected payoff differential is \( \Delta(M) = E[r|M| - \delta - 1 = M - c^{-1}(\frac{1}{k}) - \delta - 1 \). If \( M \geq \tilde{M} = c^{-1}(\frac{1}{k}) + \delta + 1 \), then \( \Delta(M) \geq 0 \) for any \( l \) and \( M \). Thus waiting is a strictly dominant strategy. We call \( (\tilde{M}, \infty) \) the upper dominance region. When the report is sufficiently favorable, creditors’ withdrawal decisions are independent of others.

Similarly, a creditor’s most optimistic belief is that the manager does not misreport \( (m = 0) \) and that all the other creditors stay \( (l = 0) \). Under this belief, the creditor’s expected payoff differential is \( \Delta(M) = E[r|M| - \delta - 1 = M - \delta - 1 \). If \( M < M = \tilde{M} \), then \( \Delta(M) < 0 \) for any \( l \) and \( M \). Thus, withdrawing is a strictly dominant strategy. We call \( (-\infty, \tilde{M}) \) the lower dominance region. When the report is sufficiently unfavorable, creditors’ withdrawal decisions are again independent of others.

When the report is intermediate, the strategic complementarity of withdrawal decisions becomes important, and multiple equilibria can arise. The multiplicity is complicated by misreporting. To see this, we start with a benchmark with no reporting discretion. This benchmark, in which \( M = r \) and \( \{\tilde{M}, \tilde{M}\} = [1, 1 + \delta) \), has a continuum of equilibria characterized by a threshold \( \bar{r} \in [1, 1 + \delta) \), as has been well-known since Diamond and Dybvig (1983). The least run-prone equilibrium of this continuum is characterized by the lowest possible run threshold \( \bar{r} = 1 \). In this equilibrium, a confident creditor expects all the others to wait, i.e., \( l = 0 \), and thus finds it optimal to wait as well because \( \Delta(\bar{r}) = \bar{r} - \bar{r} - 1 < 0 \). In contrast, the most run-prone equilibrium is characterized by the highest possible run threshold \( \bar{r} \) approaching \( 1 + \delta \) from below. In this equilibrium, a pessimistic creditor expects all the others to withdraw, i.e., \( l = 1 \), and so finds it optimal to withdraw as well because \( \Delta(\bar{r}) = \bar{r} - \bar{r} - 1 < 0 \). In both equilibria, creditors’ initial beliefs about others’ actions are self-fulfilling.

A key feature of all these runs on solvent but illiquid banks, those with \( r \in [1, 1 + \delta) \), is that they are driven solely by creditors’ pessimistic beliefs about the decisions of other creditors. All creditors agree that a bank with \( r \in [1, 1 + \delta) \) can be saved from runs, because it is solvent. Yet they are uncertain about what the other creditors will do. This strategic uncertainty justifies pessimistic beliefs about aggregate withdrawals and so generates self-fulfilling bank runs. Following the literature, we call runs on solvent but illiquid banks (with \( r \in [1, 1 + \delta) \)) panic-based runs.

Reporting discretion complicates the multiplicity of equilibria, as summarized below.

**Lemma 1.** When the report is common knowledge among creditors, a class of equilibria is characterized by a withdrawal threshold \( M \). In an equilibrium \( M \), the manager misreports by an amount \( \hat{n}(r) = M - r \) for \( r \in [\tilde{M}, c^{-1}(\frac{1}{k})] \), and 0 otherwise. Creditors withdraw if and only if the report is below \( M \). \( \tilde{M} \) can take any value in the interval \([1 + \frac{1}{k}c^{-1}(\frac{1}{k}), 1 + \delta + c^{-1}(\frac{1}{k})]\).

**Lemma 1** shows that the multiplicity that exists in the absence of reporting discretion carries over in its presence as well. This is because the report and hence the fundamental are common knowledge. The additional complexity introduced by reporting discretion is that the lower and upper dominance regions in terms of the report \( M \) have changed. Since the manager’s incentive is biased upward, both the upper bound of the lower dominance region and the lower bound of the upper dominance region (in terms of report \( M \)) have moved up: the former from 1 to \( 1 + \frac{1}{k}c^{-1}(\frac{1}{k}) \) and the latter from \( 1 + \delta \) to \( 1 + \delta + c^{-1}(\frac{1}{k}) \). The two bounds of \( M \) correspond to two extreme equilibria, one generating the smallest incidence of runs and the other the greatest. We illustrate their determination and compare them with the two extreme equilibria in Diamond and Dybvig (1983).

The least run-prone equilibrium in **Lemma 1** is characterized by the lowest possible withdrawal threshold \( \tilde{M} = 1 + \frac{1}{k}c^{-1}(\frac{1}{k}) \). In this equilibrium, the set of banks that suffer runs is \( r \in (-\infty, 1 - \frac{1}{2}c^{-1}(\frac{1}{k})) \). Not only do all solvent but illiquid banks avoid panic-based runs, but some insolvent banks with \( r \in [1 - \frac{1}{2}c^{-1}(\frac{1}{k}), 1] \) also escape fundamental-based
runs. Moreover, the set of banks that misreport is \( r \in [1 - \frac{1}{2} \epsilon^{-1}(\frac{1}{k}), 1 + \frac{1}{2} \epsilon^{-1}(\frac{1}{k})] \). In particular, when \( \delta < \frac{1}{2} \epsilon^{-1}(\frac{1}{k}) \), the solvent banks with \( r \in [1 + \delta, 1 + \frac{1}{2} \epsilon^{-1}(\frac{1}{k})] \) misreport in order to avoid runs, even though they would not be threatened by runs even in the worst equilibrium in Diamond and Dybvig (1983). Therefore, compared with the best equilibrium in Diamond and Dybvig (1983), reporting discretion weakens market discipline on some insolvent banks and imposes a negative externality on solvent ones.

The most run-prone equilibrium in Lemma 1 is characterized by the highest possible withdrawal threshold \( \hat{M} = 1 + \delta + c^{-1}(\frac{1}{k}) \). The set of banks that suffer runs, i.e., \( r \in (-\infty, 1 + \delta) \), is the same as in the worst equilibrium of Diamond and Dybvig (1983). All solvent but illiquid banks, those with \( r \in [1, 1 + \delta] \), suffer panic-based runs. Moreover, the set of banks that engage in misreporting here is \( r \in [1 + \delta, 1 + \delta + c^{-1}(\frac{1}{k})] \), which means that some solvent banks too misreport. Therefore, compared with the worst equilibrium in Diamond and Dybvig (1983), reporting discretion imposes a negative externality on solvent banks without changing the incidence of runs.

Such pair-wise comparison is not very informative, however. Since the economic consequences of reporting discretion differ in these equilibria, it is important to determine which equilibrium is likely to arise.

4. The unique equilibrium with reporting discretion

In this section, we use the global games methodology to refine the multiple equilibria in Lemma 1 and characterize the unique equilibrium that results. The main technique is to relax the assumption that the report is common knowledge among creditors. The implementation takes two steps. In the first step, we introduce noise into creditors’ private signals and prove the existence and uniqueness of the equilibrium. This will be done in the next three subsections. We then take the limit of the unique equilibrium as the noise tends to 0 asymptotically. The resulting equilibrium is unique and captures the situation in which creditors have almost perfect common knowledge about the report.

4.1. The breakdown of common knowledge

Following Morris and Shin (1998), we introduce an arbitrarily small amount of independent noise into each creditor’s beliefs, derive a unique equilibrium, and then take the amount of noise to the limit of 0. Specifically, instead of receiving \( M(r) \), each creditor \( i \) receives a private signal \( x_i \):

\[
x_i = M(r) + \epsilon_i = r + m(r) + \epsilon_i,
\]

where \( \epsilon_i \) is a normal random variable with mean 0 and variance \( \sigma^2 \) and independent across creditors. Even though all creditors receive the common report \( M \), their beliefs about the bank’s fundamental can differ slightly owing to private information or differing interpretations of the report. This heterogeneity of beliefs is captured by the variance \( \sigma^2 \). What matters, though, is the existence of the noise, not its magnitude per se. \( \sigma^2 \) can be arbitrarily small; in Section 4.4 we focus on the equilibrium as \( \sigma^2 \) approaches 0.

Reporting discretion introduces an interaction between creditors’ and the manager’s decisions. In choosing his reporting strategy, the manager has to anticipate the creditors’ use of the report, while the creditors, when they make their withdrawal decisions, have to infer the manager’s reporting strategy. Such interaction is typically tackled by examining each party’s best response to the others’ strategies. However, the direct application of this approach to our model is difficult because there are few restrictions on the possible forms of both the creditors’ withdrawal strategies and the manager’s reporting strategy. Thus, our proof strategy is first to derive some equilibrium restrictions on the strategies of the manager and the creditors, and then proceed with the usual approach of examining best responses.

Specifically, we prove two endogenous equilibrium restrictions in Section 4.2. We first show that the manager’s equilibrium strategy satisfies a monotonicity property (Lemma 2), regardless of creditors’ strategies. With this monotonicity, we then prove that creditors use a common threshold strategy in equilibrium (Lemma 3). These two steps are instrumental in establishing the unique equilibrium but are technical in nature. Readers interested only in the equilibrium characterization could skip Section 4.2 and go directly to Section 4.3.

4.2. Equilibrium reporting monotonicity and common threshold strategies

**Lemma 2.** The manager’s equilibrium report \( M^*(r) = r + m^*(r) \) is non-decreasing in \( r \), regardless of the creditors’ withdrawal strategies.\(^{13}\)

Lemma 2 mirrors a “single-crossing” property: in equilibrium the report of a stronger bank is weakly higher than that of a weaker bank for any possible strategy profile of creditors. We relegate the formal proof to the appendix and illustrate the intuition here. Suppose, for any given profile of creditors’ strategies that determines the aggregate withdrawal \( \hat{M}(r) \), a bank with \( r_L \) finds it optimal to choose report \( M(r_L) = r_L + m^*(r_L) \) with \( m^*(r_L) > 0 \). Now consider the decision of a stronger

\(^{13}\) Since at this stage we are not restricting \( M^*(r) \) to be unique, the non-decreasing of \( M^*(r) \) with respect to \( r \) when \( M^*(r) \) is a correspondence (i.e., not unique) should be understood as follows: \( \forall r_1 < r_2 \) and any \( M_1 \in M^*(r_1) \) and \( M_2 \in M^*(r_2) \), we have \( M_1 \preceq M_2 \).
bank with \( r_H > r_1 \). We argue that it will choose a weakly higher report \( M(r_H) \geq M(r_1) \). On the one hand, since \( r_H > r_1 \), the bank with \( r_H \) needs to add a smaller bias to report \( M(r_2) \) than the bank with \( r_1 \). Since the misreporting cost \( c(m) \) is strictly convex, the marginal cost of an additional bias \( \eta \) at the point of \( M(r_2) \) is strictly smaller for a bank with \( r_H \) than with \( r_1 \). On the other hand, even though we don’t know the form of the creditors’ strategy profile, we do know that the marginal effect of an additional bias \( \eta \) beyond \( M(r_1) \) on creditors’ withdrawal decisions is the same for both banks, because the manager’s marginal payoff from the aggregate withdrawal, i.e., \( \frac{\partial m}{\partial w} \), is independent of \( r \). Therefore, if report \( M(r_1) \) equates \( r_1 \)’s marginal benefit and cost, \( r_H \) must find it profitable to issue a report at least as high as \( M(r_1) \). Hence, in equilibrium, \( M(r_H) \geq M(r_1) \) for any \( r_H > r_1 \) (for any positive \( \sigma^2 \)).

A useful corollary of Lemma 2 is stated below and will be used repeatedly later.

**Corollary 1.** A creditor’s conditional expectation of the fundamental, \( E[r|x_i] \), is non-decreasing in \( x_i \).

The corollary states that, despite the possible misreporting, a creditor’s signal \( x_i \) still satisfies a monotone likelihood property with respect to the fundamental \( r \). A higher \( x_i \) induces creditor \( i \) to be more optimistic about \( r \), which in turn leads to a more optimistic belief about others’ signals and decisions.

With Lemma 2 and Corollary 1, we can prove the following result.

**Lemma 3.** Creditors play a common threshold strategy in equilibrium.

In the absence of reporting discretion, a result similar to Lemma 3 can be proved by the iterated elimination of dominated strategies. With discretion, however, this approach is less convenient. The dominated strategies that can be eliminated are not tight enough to converge, because creditors know nothing more about the manager’s reporting strategy beyond Lemma 2. Instead, we use a proof strategy, borrowed from Goldstein and Pauzner (2005), that relies only on the existence of upper and lower dominance regions discussed above and the monotonicity of \( M(r) \) in Lemma 2. The proof exploits Corollary 1 to show that a creditor becomes more optimistic about both the fundamental and the other creditors’ beliefs about it when she receives a higher signal \( x_i \). Recall that a creditor receiving \( x_i \) calculates her expected utility differential as \( \Delta(x_i) = E[r|x_i] - \delta E[l|x_i] - 1 \) and withdraws if and only if \( \Delta(x_i) < 0 \). The creditor uses the private signal \( x_i \) to forecast both the fundamental \( r \) and other creditors’ decisions summarized by \( l \), and, when doing so she takes into account the effect of the manager’s potential misreporting on her inferences. The discussion following Corollary 1 suggests that \( E[r|x_i] \) is increasing and \( E[l|x_i] \) is decreasing in \( x_i \) for any strategy profile of other creditors. From there, it is a short step to prove Lemma 3.

4.3. Characterizing the unique equilibrium

Lemma 3 simplifies the interaction between the manager and the creditors’ decisions. It indicates that without loss of generality we can consider only the manager’s best response \( m^{BR}(r; \hat{x}) \) to creditors’ common threshold strategy \( \hat{x} \).

**Lemma 4.** When all creditors use the common threshold \( \hat{x} \), the manager’s best response \( m^{BR}(r; \hat{x}) \) is unique (almost everywhere).

We characterize the manager’s reporting decision in the text, leaving the technical details of the proof of uniqueness to the appendix. From the manager’s perspective, expected withdrawal is equal to the probability that a creditor’s signal \( x_i \) is smaller than \( \hat{x} \) (by the law of large numbers). Moreover, \( x_i \) is normally distributed with mean \( r + m(r; \hat{x}) \) and variance \( \sigma^2 \). Thus, the manager with fundamental \( r \) and bias \( m(r; \hat{x}) \) expects an aggregate withdrawal of

\[
I(m; \hat{x}; r) = Pr(x_i < \hat{x}) = \Phi \left( \frac{1}{\sigma} \left( \hat{x} - (r + m) \right) \right).
\]

A bias \( m \) shifts the distribution of creditors’ signals to the right, reducing withdrawals. Substituting Eq. (4) into the manager’s payoff \( w(m; r) \) in Eq. (2), we can write the reporting decision as

\[
\max_{m(r; \hat{x})} \text{w}\left(m(r; \hat{x}); r\right) = r - \Phi \left( \frac{1}{\sigma} \left( \hat{x} - (r + m(r; \hat{x})) \right) \right) - kc\left(m(r; \hat{x})\right),
\]

s.t. \( m(r; \hat{x}) \geq 0 \).

Note that the objective function \( w(m; r) \) is not globally concave in \( m \), because \( \frac{\partial^2 w}{\partial m^2} = -\frac{1}{\sigma^2} \phi \phi' - kc' \) can be positive or negative, depending on the sign and magnitude of \( \phi' \).

Because \( c'(0) = 0 \), we have \( m^{BR}(r; \hat{x}) > 0 \) for any \( \sigma > 0 \). Thus the constraint \( m \geq 0 \) does not bind. A necessary condition for the manager’s interior best response is the first-order condition

\[
\frac{1}{\sigma} \phi \left( \frac{1}{\sigma} \left( \hat{x} - (r + m^{BR}) \right) \right) - kc'(m^{BR}) = 0.
\]

Eq. (FOC) can have multiple critical points, and it is possible that more than one point can be local maxima. Thus we need to compare the value of the objective function \( w(m; r) \) at these points. Accordingly the uniqueness of the optimal \( m^{BR} \) is not guaranteed. The proof of the uniqueness is highly technical and relegated to the appendix.
After characterizing the manager’s best response, we can characterize a creditor’s best response to other creditors’ common threshold \( \tilde{x} \) and the manager’s manipulation \( m_{BR}(r;\tilde{x}) \). A creditor with private signal \( x_i = \tilde{x} \) should be indifferent between withdrawing and staying, i.e., \( \Delta(x_i = \tilde{x}; l(m_{BR}(r;\tilde{x}))) = 0 \).

Finally, rational expectations require that the manager and creditors’ strategies are consistent with each other. Imposing this condition and denoting \( m^*(r) = m_{BR}(r; x^*) \), we obtain an equation that characterizes the creditor’s optimal threshold \( x^* \):

\[
\Delta(x^*; l(m^*(r), x^*)) = 0.
\] (5)

In the appendix, we further prove that there is a unique solution to Eq. (5). Therefore, we have completely characterized the unique equilibrium.

**Proposition 1.** For any \( \sigma > 0 \), there is a unique equilibrium. In the equilibrium, the manager adds a bias \( m^*(r) \), and each creditor withdraws if and only if her signal is below threshold \( x^* \). \( m^*(r) \) and \( x^* \) are jointly determined by Eq. (FOC) (evaluated at \( m_{BR}(r; x^*) = m^*(r) \) and \( \tilde{x} = x^* \)) and Eq. (5).

### 4.4. The unique equilibrium in the limit

The last step in the global games methodology is to take the limit of \( \sigma \to 0 \). **Proposition 1** has established the existence and uniqueness of the equilibrium when creditors do not have common knowledge of the report, that is, when \( \sigma > 0 \). The introduction of uncertainty \( \sigma = 0 \) breaks the common knowledge among creditors and is used as a modeling device to refine away the multiple equilibria. The uncertainty itself, however, is not of interest, because our focus is on creditors’ strategic uncertainty (about others’ decisions). Thus, the global games methodology focuses on the equilibrium in the limit of \( \sigma \to 0 \), where runs are driven by creditors’ strategic uncertainty about the other creditors’ actions (not by uncertainty about the report).

**Proposition 2.** As the noise tends to \( 0 \), i.e., \( \sigma \to 0 \), the unique equilibrium \( (m^*(r), x^*) \) converges as follows:

\[
m^*(r) \to \begin{cases} x^* - r & \text{if } r \in [r_1, r_2] \\ 0 & \text{if } r \not\in [r_1, r_2]. \end{cases}
\] (6)

\[
x^* \to r_2.
\] (7)

\( r_1 \) and \( r_2 \) are two constants:

\[
r_1 = 1 + \frac{\delta}{2} - \int_0^{c^{-1}(\frac{1}{r})} kc(t)dt.
\] (8)

\[
r_2 = 1 + \frac{\delta}{2} - \int_0^{c^{-1}(\frac{1}{r})} kc(t)dt + \int_{c^{-1}(\frac{1}{r})}^{\frac{1}{r}} \frac{kr}{k+c(1-R)} dt.
\] (9)

Finally, \( l^*(r) \to 1 \) if \( r < r_1 \) and \( l^*(r) \to 0 \) if \( r \geq r_1 \).

**Fig. 1** illustrates the manager’s and creditors’ equilibrium strategies. The \( x \)-axis is the fundamental \( r \), the dotted line depicts the bias \( m^*(r) \), and the solid line describes the report \( M^*(r) = r + m^*(r) \). The fundamental space along the \( x \)-axis has been decomposed into three parts, each with a particular combination of the manager’s and creditors’ strategies.

The proof of **Proposition 2** is instructive. As \( \sigma \to 0 \), the creditor’s signal \( x_i \) approaches \( M^*(r) = r + m^*(r) \). We start with the manager’s reporting strategy \( m^*(r) \). The proof in the appendix shows that, if \( r > x^* \), the solution to the first-order condition of Eq. (FOC) converges to \( 0 \). Intuitively, a bank with \( r > x^* \) does not misreport, because misreporting is costly but cannot reduce withdrawal (which is already \( 0 \)). Hence, we have

\[
r_2 = x^*.
\] (10)

Similarly, the proof in the appendix shows that, if \( r < x^* \), the solution to the first-order condition of Eq. (FOC) converges to either \( 0 \) or \( x^* - r \). Intuitively, for a bank with \( r < x^* \), the marginal benefit of misreporting is \( 0 \) until the report \( M(r) \) reaches \( x^* \). Thus the bank either reports truthfully, i.e., \( m^* = 0 \), to avoid misreporting cost or misreports by the amount \( m^*(r) = x^* - r \) to avoid runs. As the fundamental deteriorates, the second choice becomes more costly. Thus there exists an \( r_1 \) such that at \( r_1 \) the manager is indifferent between the two choices (and below \( r_1 \) the manager strictly prefers \( m^* = 0 \)). This indifference condition, i.e., \( w(x^* - r_1; r_1) = w(0; r_1) \), gives the equation that determines \( r_1 \) for any given \( x^* \):

\[
1 - kc(x^* - r_1) = 0.
\] (11)

Given the manager’s reporting strategy \( m^*(r) \) characterized by \( r_1 \) and \( r_2 \), creditors understand that the pool of banks generating signal \( x^* \) consists of all banks in the region \( r \in [r_1, r_2] \). Denote the density of the distribution of \( r \) conditional on \( x^* \) by \( f(r|x^*) \) and in the proof we show that \( f(r|x^*) = kc(x^* - r) \). Note that \( f(r|x^*) \) is not uniform because of misreporting. The marginal creditor’s belief about the fundamental thus can be calculated as: \( E[r|x^*] = r_1 + \frac{\int_{0}^{r_1} r c(x^* - r)dr}{c(x^* - r_1)} \). Moreover, we prove in
the appendix that $E[|x^*|] = \Pr[x_1 < x^*] = \frac{1}{2}$. The intuition is that the marginal creditor expects that exactly half of the other creditors have lower signals and hence that exactly half will run. Therefore, the creditors’ indifference condition, Eq. (5), can be written as

$$\Delta(x^*) = r_1 + \int_{r_1}^{x^*} \frac{c(x^* - r)}{c(x^* - r_1)} dr - \frac{1}{2} = 0.$$  

(Eq. 12)

Eqs. (10), (11), and (12) jointly determine $x^*, r_1$, and $r_2$. Solving this system of three equations gives us the closed-form solutions stated in Proposition 2.

Before we study the equilibrium properties, note that Proposition 2 should be properly read as results in the limit of $\sigma \to 0$ (and not at $\sigma = 0$). In the following corollary we prove that all managers misreport as long as $\sigma > 0$.

**Corollary 2.** For any $\sigma > 0$, all managers misreport, that is, $m^*(\sigma) > 0$.

The proof of this corollary is implicit in that of Proposition 1. The underlying intuition is also standard in signal-jamming models. From Eq. (FOC) (evaluated at $m^R(r; x^*) = m^r(r)$ and $x = x^*$), it is clear that the marginal manipulation cost at $m^r = 0$ is 0. However, the marginal benefit of reducing withdrawal is $\frac{1}{2} \phi[\frac{1}{2} (x^* - (r + m^r))] = \frac{1}{2} \phi[\frac{1}{2} (x^* - r)]$ at $m^r = 0$. Since $\phi$ is the density of a standard normal distribution, $\phi > 0$. Thus, the marginal benefit is strictly positive as long as $\sigma > 0$. As a result, when $\sigma > 0$, there can never be an equilibrium in which the manager doesn’t misreport.

However, as $\sigma$ tends to 0, the marginal benefit of misreporting for the manager with $r > x^*$ increases, so the amount of misreporting necessary to avoid runs diminishes. Even though the magnitude of misreporting never becomes 0 as long as $\sigma > 0$, it converges to 0 in the limit of $\sigma \to 0$. This is why we use $\to$ instead of $= $ in Proposition 2. To simplify exposition, we replace $\to$ with $\to$ in the subsequent analysis and state the results accordingly. For example, instead of saying that a manager with $r \not\in [r_1, r_2]$ manipulates an amount that approaches 0 in equilibrium, we simply say that a manager with $r \not\in [r_1, r_2]$ does not manipulate in equilibrium.

**Morris and Shin (2000)** use the global games methodology to refine the multiple equilibria in Diamond and Dybvig (1983). In the resulting unique equilibrium, banks suffer runs if and only if their fundamentals are below threshold $r^{DD} = 1 + \frac{\delta}{2}$. In our model, banks with fundamentals below $r_1$ suffer runs. Note that $r_1(k)$ is a function of $k$. As $k$ increases, misreporting becomes more costly and the misreporting interval $[r_1, r_2]$ narrows. We now confirm that, in the limit of $k \to \infty$, misreporting vanishes and the unique equilibrium in Proposition 2 does converge to the equilibrium in Morris and Shin (2000).

**Corollary 3.** As $k \to \infty$, the manager doesn’t manipulate and creditors run if and only if their signals are below $r^{DD} = 1 + \frac{\delta}{2}$, that is, $\lim_{k \to \infty} r_1 = \lim_{k \to \infty} r_2 = \lim_{k \to \infty} x^* = r^{DD}$. Moreover, $r_2(k) > r^{DD} > r_1(k)$ for any $k > 0$. 

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**Fig. 1. Manager’s equilibrium reporting strategy.**
Fig. 2. Economic consequences of reporting discretion.

5. The analysis

5.1. Comparative statics and the economic consequences of reporting discretion

We conduct comparative statics of the unique equilibrium in Proposition 2 to analyze the effects of reporting discretion on the incidence and efficiency of bank runs. We focus on two questions. First, does reporting discretion reduce the incidence of bank runs in equilibrium, and if so, which types of runs are reduced? Second, who bears the cost of misreporting? The answers to these questions have empirical and policy implications. Fig. 2 provides a graphical summary of the answers.

We review the two types of runs discussed earlier: fundamental-based if \( r < r^{FB} = 1 \) and panic-based if \( r \geq 1 \). From the creditors’ perspective, fundamental-based runs discipline insolvent banks, but panic-based runs close solvent but illiquid banks. In addition, recall that \( k \) is a cost parameter of misreporting and the inverse of the degree of the manager’s reporting discretion. We define \( k \) as a solution to \( r_1(k) = r^{FB} \). That is, \( k \) is the degree of reporting discretion that eliminates panic-based runs without impeding fundamental-based runs. In the proof of, we show that such \( k \) exists and is unique and positive.

**Proposition 3.** Reporting discretion reduces the incidence of runs in equilibrium, i.e., \( \frac{\partial r_1(k)}{\partial k} > 0 \). Moreover, reporting discretion affects panic-based and fundamental-based runs differently.

1. If \( k \geq \bar{k} \), reporting discretion reduces panic-based runs but does not affect fundamental-based runs. That is, \( r_1 \geq r^{FB} \) if \( k \geq \bar{k} \).
2. If \( k < \bar{k} \), reporting discretion eliminates panic-based runs and reduces fundamental-based runs. That is, \( r_1 < r^{FB} \) if \( k < \bar{k} \).
3. The cut-off \( \bar{k} \) is decreasing in \( \delta \).

**Proposition 3** asserts that the manager’s attempt to influence creditors’ withdrawal decisions through costly misreporting does succeed in equilibrium, even as the noise in creditors’ signals approaches 0. The set of banks that suffer runs is \((-\infty, r_1)\), and this set shrinks as reporting discretion increases.

We explain the intuition behind the first result that misreporting influences creditors’ equilibrium withdrawal decisions. Even though in our model creditors can play arbitrary withdrawal strategies, in equilibrium they all choose a common threshold strategy. The binary nature of the threshold strategy means that the manager’s incentive to misreport is asymmetric when the fundamental is near the run threshold. In particular, the marginal benefit of misreporting spikes when the bank’s report \( M(r) \) approaches threshold \( x^* \) from below and decreases as it drifts away from \( x^* \) in either direction. This fundamental-contingent misreporting makes it difficult for creditors to undo the bias in the report and results in a partial pooling of banks with differing fundamentals. Banks with \( r \in [r_1, r_2] \) all send the same report \( M^r(r) = r_2 \). Creditors receiving \( x^* = r_2 \) assign a weighted average belief of \( E[r|x^*] = r^{DD} \) and coordinate on no withdrawal.\(^{14}\)

\(^{14}\) Our model belongs to the signal-jamming literature in which managers misreport in order to influence outsiders’ perception about the fundamental (e.g., Stein (1989); see also reviews in Stocken (2013)). As in Stein (1989), the manager in our model cares both about the fundamental \( r \) and about the creditors’ perceptions of it. The latter is implicit in the manager’s objective function [Eq. (2)] through the aggregate withdrawal \( l \), because \( l \) is determined by creditors’ perceptions about \( r \). The unraveling result common in the literature, namely that the manager biases the report upward and creditors perfectly de-bias the report in equilibrium (see Stein (1989)), doesn’t arise in our model. Instead, we have the partial pooling equilibrium in which the managers
It is useful to highlight the intuition for why reporting discretion reduces runs. Recall that panic-based runs are caused by creditors’ pessimistic beliefs about others’ decisions. Reporting discretion enables banks with \( r < r^{DD} \) to inflate creditors’ beliefs about the fundamental up to \( r^{DD} \). Since creditors use their beliefs about the fundamental to forecast other creditors’ signals and decisions, the inflation of creditors’ beliefs about the fundamental leads to the inflation of their beliefs about others’ decisions as well. Thus misreporting by banks with \( r < r^{DD} \) induces creditors to be more optimistic in both their first-order and higher-order beliefs about the bank’s fundamental. Such optimism offsets the pessimism resulting from the coordination frictions and reduces panic-based runs. Reporting discretion serves as an effective tool to coordinate creditors’ beliefs. As reporting discretion increases, the cost to induce such optimism drops to the point that the induced optimism exceeds the pessimism from the coordination frictions (when \( r_1 < 1 \)). At this point, reporting discretion eliminates panic-based runs and reduces fundamental-based runs.

Parts 1 and 2 of Proposition 3 further specify the types of runs that are reduced by reporting discretion. Because \( \lim_{k \to \infty} r_1(k) = r^{DD} \) and \( \frac{\partial r_1(k)}{\partial k} > 0 \), the set of banks that misreport to avoid runs, i.e., \( [r_1, r^{DD}] \), expands first to those that suffer panic-based runs \( (r \in [1, r^{DD})] \) and then to those that experience fundamental-based runs \( (r \in [r_1, 1]) \). Therefore, some reporting discretion mitigates panic-based runs, but excessive discretion reduces fundamental-based runs as well and so weakens market discipline.

Finally, there exists an intermediate degree of discretion (i.e., \( \hat{k} \)) that eliminates panic-based runs without impeding fundamental-based runs. From the creditors’ perspective this is optimal. Part 3 of Proposition 3 shows that the optimal amount of reporting discretion for creditors increases in \( \delta \). When the coordination friction is greater (a larger \( \delta \)), the creditors prefer more discretion for the manager (a lower \( \hat{k} \)).

Now we examine the second question, i.e., who bears the cost of misreporting. It is obvious that banks with \( r \in [r_1, r^{DD}] \) bear the cost of misreporting in return for the benefit of avoiding runs they would have experienced otherwise. But the set of banks that misreport in equilibrium is \([r_1, r_2]\), larger than \([r_1, r^{DD}]\). Banks with \( r \in [r^{DD}, r_2] \) are compelled to misreport in equilibrium even though they would survive runs in the absence of reporting discretion. That is, discretion by banks with \( r \in [r_1, r^{DD}] \) creates a negative externality for those with \( r \in [r^{DD}, r_2] \).

**Proposition 4.** Reporting discretion produces a negative externality for banks with \( r \in [r^{DD}, r_2] \), i.e., \( \frac{\partial r_2(k)}{\partial k} < 0 \). Moreover, if discretion is sufficiently large, even banks that survive runs in the worst equilibrium in Diamond and Dybvig (1983) misreport in order to survive. That is, there exists a constant \( \hat{k} \) such that \( r_2(k) > 1 + \delta \) for \( k < \hat{k} \).

Proposition 4 shows that misreporting by banks with \( r \in [r_1, r^{DD}] \) generates negative externality for banks with \( r \in [r^{DD}, r_2] \). Note that none of these banks are vulnerable to panic-based runs in the unique equilibrium with no reporting discretion (Corollary 3). Moreover, banks with \( r \in [1 + \delta, r_2] \), which is non-empty when \( k < \hat{k} \), never suffer any runs even in the worst equilibrium in Diamond and Dybvig (1983). Nevertheless, given reporting discretion, they all misreport in equilibrium by \( m^*(r) = r_2 - r > 0 \). Intuitively, the equilibrium misreporting by weak banks with \( r \in [r_1, r^{DD}] \) induces rational creditors to discount their reports. Since creditors don’t observe the bank’s fundamental directly, they end up discounting all banks’ reports. The stronger banks with \( r \in [r^{DD}, r_2] \) are “forced” to engage in costly misreporting in order to counter this discounting. Given the equilibrium in Proposition 2, the manager with \( r \in [r^{DD}, r_2] \) who misreports by less than \( r_2 - r \) suffers runs (i.e., \( l = 1 \)); he can avoid this by engaging in a bias of \( r_2 - r \). Since \( k(c(r_2 - r)) < k(c(r_2 - r_1)) = 1 \), the manager finds it optimal to misreport.

In sum, the economic consequences of reporting discretion are mixed. It reduces the incidence of panic-based runs, but can also weaken the market discipline on insolvent banks and compel strong banks to engage in costly misreporting.

### 5.2. The socially optimal degree of reporting discretion

Having examined the economic consequences of reporting discretion for both the manager and creditors, we can now assess the consequences for social welfare. A social planner cares about the utility of both the creditors and the manager. A social planner with all the information and the power to dictate all the decisions would choose no misreporting and liquidate an bank if and only if it is insolvent (i.e., \( r \leq 1 \)). Solvent but illiquid banks should not be liquidated. There would be neither misreporting cost nor liquidation inefficiency.

Against this first-best benchmark, we calculate the social loss associated with the equilibrium in Proposition 2. First, a manager with \( r \in [r_1, r_2] \) manipulates to \( r_2 \) at the cost of \( k(c(r_2 - r)) \). Second, when \( r_1 \geq 1 \), a bank with \( r \in [1, r_1] \) suffers a panic-based run. As a result, the asset yields \( 1 \) at \( t = 1 \), instead of \( r \geq 1 \) at \( t = 2 \), resulting in a net loss of \( r - 1 \). Finally, when \( r_1 < 1 \), a bank with \( r \in [r_1, 1] \) is insolvent but not liquidated. The asset generates \( r < 1 \) at \( t = 2 \), instead of the liquidation value \( 1 \) at \( t = 1 \), leading to a loss of \( 1 - r \). Collecting these terms, we can compute the social loss function as follows:

\[
L(k) = L_{1,1} \int_{r_1}^{r_2} (r_1(k) - r_2) dr + (1 - L_{1,1}) \int_{r_1}^{1} (1 - r_2) dr + \gamma \int_{r_1}^{r_2} k(c(r_2 - r)) dr.
\] (13)

with intermediate fundamental have stronger incentives to misreport than those in the two tails, as we have explained. By contrast, in Stein (1989), the marginal impact of the report’s bias on stock prices is the same for managers with different fundamentals. Thus, managers with different fundamentals choose the same amount of biases, implying that their final reports have to be different and that pooling can’t arise.
\( l_{1,2,3} \) is an indicator function equals to 1 when \( r_1 \geq 1 \) and 0 otherwise. \( \gamma > 0 \) is the weight the social planner places on the manager's misreporting cost relative to the liquidation inefficiency. We have also made it explicit that the social loss \( L(k) \) is a function of reporting discretion \( k \). \( k \) affects the social loss function not only directly through the misreporting cost but also indirectly through its effects on the manipulation threshold \( r_1(k) \) and the run threshold \( r_2(k) \).

We can now solve for the optimal reporting discretion level \( k^* \) that minimizes the social loss function \( L(k) \).

**Proposition 5.** For any finite \( \gamma > 0 \), the optimal reporting discretion \( k^* \) is interior. Moreover, \( k^* > \bar{k} \) and is decreasing in \( \delta \) and increasing in \( \gamma \).

The social planner’s choice of optimal reporting discretion has a trade-off. On the one hand, we know from Part 3 of **Proposition 3** that creditors prefer \( \bar{k} \) that leads to the first-best liquidation. On the other hand, as the degree of reporting discretion increases, the manager misreports more substantially and the aggregate misreporting cost is higher. The social planner chooses an interior \( k^* \) to balance these two effects.

The properties of the socially optimal degree of reporting discretion are also intuitive. Recall that a higher \( k \) indicates less reporting discretion. First, the social planner prefers less reporting discretion than creditors, i.e., \( k^* > \bar{k} \). The creditors are concerned only about liquidation efficiency, whereas the social planner also cares about the misreporting cost. This additional concern for misreporting cost shifts the planner’s preference to a lower degree of reporting discretion. Second, the socially optimal degree of reporting discretion is increasing in the bank’s vulnerability to runs, i.e., \( \frac{\partial k^*}{\partial \delta} > 0 \). When the run risk is greater (a larger \( \delta \)), the benefit of giving the bank more discretion to reduce panic-based runs is also greater, resulting in a lower \( k^* \). Finally, the socially optimal reporting discretion is lower if the social planner is more concerned about misreporting cost than liquidation inefficiency, i.e., \( \frac{\partial k^*}{\partial \gamma} > 0 \). As \( \gamma \) increases, reporting discretion becomes more costly for the social planner, who accordingly chooses a lower degree of reporting discretion.

5.3. The empirical and policy implications

By identifying the specific components of cost and benefit associated with reporting discretion, our model can contribute to empirical studies and policy discussions. It has four testable predictions. First, reporting discretion enables banks to hide unfavorable information at some cost. Second, reporting discretion reduces the incidence of bank runs, both panic-based and fundamental-based. Third, banks’ preferences for reporting discretion differ, with the weaker ones supporting more reporting discretion and the stronger ones preferring less. Finally, the “kink” in earnings should be more prominent for banks given the additional mechanism characterized in this paper.

The results also have policy implications. First, they reconcile the two sides of the debate on the desirability of opacity (e.g., Acharya and Ryan (2016)). On the one hand, too much reporting discretion reduces fundamental-based runs and weakens market discipline (e.g., Calomiris and Kahn (1991)). On the other hand, some reporting discretion does mitigate panic-based runs by better coordination among creditors (e.g., Dang et al. (2014)). The optimal level of opacity depends on the severity of the coordination frictions. To the extent that banks are more prone to panic-based runs than non-financial firms, our results are consistent with the notion that banks should be allowed more reporting discretion than their non-financial counterparts and that those that are more vulnerable to panic-based runs should be accorded more reporting discretion.

To illustrate, consider the new guidance that the FASB issued in the midst of the recent financial crisis to allow banks greater discretion in implementing mark-to-market rules. The guidance could be considered as a reduction in the misreporting cost \( k \). The popular press has alleged that the additional reporting discretion allows managers to “fudge the truth” (see Barr (2009), Bigman and Desmond (2009), Scannell (2009)). Such an allegation is consistent both with our first empirical prediction and with the motivating empirical evidence discussed in the introduction, and one might easily jump to the conclusion that the policy is not desirable. However, our second and third predictions above suggest a different evaluation. Insofar as panic-based runs are prominent in the banking industry and banks are especially prone to them in crises period, the economic consequences of the FASB’s policy change could well be more nuanced than the popular press alleged.

Second, the optimal degree of reporting discretion depends on the regulator’s preferences. This helps shed light on the continuing dispute between the FASB and financial regulators over the primary objective functions of financial reporting. While the FASB’s conceptual framework emphasizes representational faithfulness, which entails a little discretion as possible in our model, financial regulators are concerned about mitigating panic-based runs and thus prefer greater reporting discretion.

6. Conclusion

We study banks’ reporting choices and their economic consequences in a bank-run setting. With no reporting discretion, creditors’ pessimism due to coordination frictions triggers not only fundamental-based runs that close insolvent banks but also panic-based runs that bring down solvent but illiquid banks. We show that, with reporting discretion, the reporting equilibrium features partial pooling. Neither the strongest nor the weakest banks misreport, but those in the middle do in order to be pooled together. Reporting discretion enables these intermediate banks to inflate creditors’ beliefs effectively in equilibrium. Such belief inflation mitigates panic-based runs but can also reduce fundamental-based runs.
The model expands the literature on costly misreporting from non-financial firms to financial institutions, which are characterized by maturity mismatch. We show that this mismatch affects their reporting choices and their economic consequences. As such, the model deepens our understanding of the special features of banks’ information environment.

For tractability we have made a number of assumptions, relaxing which we leave to the future research. First, in the absence of reporting discretion, the report is assumed to reveal the fundamental almost perfectly (as the noise variance $\sigma^2$ approaches 0). In other words, with no reporting discretion there is almost no fundamental uncertainty in the report, which is an important parameter studied in accounting literature. This assumption is carried over from Diamond and Dybvig (1983), whereas the previous conventional wisdom had been that panic-based bank runs originated from fundamental uncertainty (creditors were uncertain about a bank's fundamental). The seminal contribution of Diamond and Dybvig (1983) is to prove that panic-based bank runs are actually driven by strategic uncertainty, that is, creditors’ uncertainty over what other creditors will do. Indeed, panic-based runs are most acute when there is no fundamental uncertainty (or when there is common knowledge about the fundamental). As a result, most bank-run models assume away fundamental uncertainty so as to capture the essence of panic-based runs. While following this assumption, we have shown that managers’ reporting discretion generates endogenous uncertainty through the partial pooling equilibrium and that such endogenous noise mitigates panic-based runs.

Second, we have taken the bank’s maturity mismatch as given. While this assumption enables us to adapt the bank-run model to accommodate reporting choices, it prevents us from modeling the bank’s objective function in a more micro-founded manner. We accordingly assume that the bank’s objective function is increasing in its fundamental and decreasing in withdrawals. While this objective function is justified on empirical grounds, it overlooks the banks’ liquidity transformation role featured in the original model of Diamond and Dybvig (1983). Moreover, we equate fundamental-based runs with market discipline, which could be viewed as a terminological stretch in that we do not micro-found the bank’s objective function.

Third, we have posited a smooth production technology. Creditors’ aggregate withdrawals reduce the return on the remaining investment in a linear way through the parameter $\delta$. This allows us to capture the strategic complementarity of creditors’ decisions cleanly. As such, the model describes investment funds better than commercial banks. For the latter, there is often a notion of a break-point of aggregate withdrawal beyond which the entire project (loan portfolio) must be liquidated. This leads to the problem of one-sided complementarity studied by Goldstein and Pauzner (2005).

Finally, we have focused exclusively on ex post reporting choices. If the banks can commit to an ex ante reporting strategy, however, our model yields the first best (see Cox and Wagenhofer (2009) for an example of such commitment). Arguably, both ex post reporting choices and ex ante reporting commitment (policy) are important in shaping the banks’ information environment. Given the large literature on the ex ante reporting commitment of banks, combining these two strands of the literature is a promising avenue for future research.

7. Appendix

Proof of Lemma 1. We verify the equilibria in the lemma in two steps. First, we prove that given creditors’ threshold strategy $\tilde{M}$, the manager’s reporting strategy $\tilde{m}(r; \tilde{M})$ is the best response. Given run threshold $M$, the manager with $r \geq \tilde{M}$ chooses $m^*(r) = 0$ to minimize misreporting cost without suffering runs. The manager with $r < \tilde{M}$ has two choices. He either misreports by an amount of $\tilde{M} - r$ to just barely avoid a run, resulting in a net payoff $r - kc(\tilde{M} - r)$. Or he doesn’t misreport but suffer runs, leading to a net payoff $r - 1$. Thus, he misreports if and only if $r - kc(\tilde{M} - r) \geq r - 1$, or equivalently, $r \in [\tilde{M} - c^{-1}(\frac{k}{M})], \tilde{M})$, where $c^{-1}(\cdot)$ is the inverse function of $c(\cdot)$.

The second step is to verify the creditor’s best response to the manager’s reporting strategy $\tilde{m}(r; \tilde{M})$ and to other creditors’ threshold strategy $\tilde{M}$. If $M < \tilde{M}$, the creditor expects that $E[l|M] = 1$ and $E[r|M] \leq \tilde{M} - c^{-1}(\frac{k}{M})$, resulting in an expected payoff differential $\Delta(M) \leq \tilde{M} - c^{-1}(\frac{k}{M}) - \delta - 1$. She withdraws if $\tilde{M} < c^{-1}(\frac{k}{M}) + \delta + 1$. If $M = \tilde{M}$, she expects that $E[l|M] = 0$ and $E[r|M] = \frac{\tilde{M} - c^{-1}(\frac{k}{M}) + \tilde{M}}{2}$, resulting in an expected payoff differential $\Delta(M) = \frac{\tilde{M} - c^{-1}(\frac{k}{M}) + \tilde{M}}{2} - 1 \geq 0$. She stays if $\tilde{M} > 1 + \frac{1}{2}c^{-1}(\frac{k}{M})$. Finally, if $M > \tilde{M}$, the creditor expects that $E[l|M] = 0$ and $E[r|M] = M$, resulting in an expected payoff differential $\Delta(M) = M - 1 > \frac{\tilde{M} - c^{-1}(\frac{k}{M}) + \tilde{M}}{2} - 1$. She stays if $M > 1 + \frac{1}{2}c^{-1}(\frac{k}{M})$. Therefore, $\tilde{M}$ is creditors’ best response if $\tilde{M} \in [\frac{1}{2}c^{-1}(\frac{k}{M}) + 1, c^{-1}(\frac{k}{M}) + 1 + \delta]$. □

Proof of Lemma 2. We prove this lemma by contradiction. Suppose that the lemma is not true. In this case there must exist two banks with fundamental $r_H > r_L$, but the bank with $r_H$ reports $M_H < M_L$, where $M_H$ ($M_L$) is the report by the bank with $r_H$ ($r_L$). Note that at this stage we are not restricting the report to be unique. Thus $M_H$ ($M_L$) can be any report that is optimal for the bank with $r_H$ ($r_L$). Since $M_L$ is the optimal report by the bank with $r_L$, we have $w(M_L - r_L; r_L) \geq w(M_H - r_H; r_L)$, which is equivalent to $\frac{1}{k}(l(M_H) - l(M_L)) \geq c(M_L - r_L) - c(M_H - r_L)$. 

Similarly, since \( M_H \) is the optimal report by the bank with \( r_H \), we have \( w(M_H - r_H; r_H) \geq w(M_L - r_H; r_H) \), which is equivalent to

\[
\frac{1}{k} \left( I(M_H) - I(M_L) \right) \leq c(M_L - r_H) - c(M_H - r_H).
\]

Combining the two we get

\[
c(M_L - r_L) - c(M_H - r_L) \leq c(M_L - r_H) - c(M_H - r_H).
\] (14)

Since \( c''(m) > 0 \), \( c(m + a) - c(m) \) is strictly increasing in \( m \) for any constant \( a > 0 \) and \( M_L - M_H > 0 \). Eq. (14) implies that \( (M_H - r_L) - (M_H - r_H) = r_H - r_L < 0 \), which leads to contradiction. \( \square \)

**Proof of Corollary 1.** To show that \( E[r|x_i] \) is increasing with respect to \( x_i \), consider the likelihood ratio of the conditional distribution of \( x_i \) on \( r_1 \) and \( r_2 \) with \( r_1 > r_2 \),

\[
f_{x_i|r_1}(r_1) = \frac{1}{2\pi \sigma^2} e^{-\frac{(r_1 - M(r_1))^2}{2\sigma^2}},
\]

\[
f_{x_i|r_2}(r_2) = \frac{1}{2\pi \sigma^2} e^{-\frac{(r_2 - M(r_2))^2}{2\sigma^2}}.
\]

By Lemma 2, we know that \( r_1 > r_2 \) implies \( M(r_1) > M(r_2) \). Therefore, \( f_{x_i|r_1}(r_1) \) increases with respect to \( x_i \). This implies that the conditional distribution of \( r \) on \( x_i \), \( g(r|x_i) \), satisfies the monotone likelihood ratio property (MLRP). MLRP implies first-order stochastic dominance, which in turn implies that \( E[r|x_i] \) is increasing with respect to \( x_i \). \( \square \)

**Proof of Lemma 3.** We prove the lemma in two steps. First we show that any equilibrium must be a (single) threshold strategy equilibrium, and then prove that any (single) threshold strategy equilibrium must be symmetric, i.e., all creditors must have the same threshold. Both steps are proved by contradiction.

To prove the first step, we begin by proving the existence of upper and lower dominance regions, following a procedure similar to what we have used in Section 3 to establish the dominance regions when the report is common knowledge. Specifically, we can prove that it is a dominant strategy for a creditor with signal \( x_i \geq \bar{x} \equiv 1 + c^{-1}(\frac{1}{2}) + \delta \) to stay and with signal \( x_i < \bar{x} \equiv 1 \) to withdraw. Now we prove that any equilibrium must be a (single) threshold strategy equilibrium.

A creditor's strategy is a function \( n: R \rightarrow [0, 1] \), where \( n(x) = 1(0) \) means that a creditor with signal \( x \) withdraws (stays). We are interested in the strategy profiles \( \{n_i\}_{i \in [0,1]} \). Let \( l = \int_0^1 f(x) n_i \, dx \) be the equilibrium withdrawal population where \( f(x) \) is the density function of signal \( x_i \). Denote \( x_B = \text{sup} \{x_i < x_B : \Delta(x_i; l(.)) < 0\} \). Intuitively, \( x_B \) represents the highest signal below which a creditor prefers to withdraw. Because of the existence of the upper dominance region, we have \( x_B < \bar{x} \).

Now suppose there exists an equilibrium strategy that is not characterized by a single threshold. This means that there exists a signal \( x_i < x_B \) such that a creditor observing a signal below \( x_i \) would stay. Denote \( x_A \) to be the largest of them, i.e., \( x_A = \text{sup} \{x_i < x_B : \Delta(x_i; l(.)) \geq 0\} \). Because of the existence of the lower dominance region, we have \( x_A > \bar{x} \). One example of such a non-single-threshold strategy equilibrium is illustrated in Fig. A.3.

In such an equilibrium, since \( M(r) \) is monotone (Lemma 2), \( \Delta(x_i; l(.)) \) is continuous in signal \( x_i \). Therefore, creditors with threshold signal \( x_A \) and \( x_B \) are indifferent between withdrawing and staying:

\[
\Delta(x_B; l(.)) = \Delta(x_A; l(.)) = 0.
\] (15)

Now we prove that

\[
\Delta(x_A; l(.)) - \Delta(x_B; l(.)) = (E[r|x_A] - E[r|x_B]) - \delta(E[l|x_A] - E[l|x_B]) < 0,
\]

which contradicts the indifference condition above.

The first term \( E[r|x_A] - E[r|x_B] \) is non-positive because of Corollary 1 and \( x_A < x_B \). We now show that \( \delta(E[l|x_A] - E[l|x_B]) \) is positive.

The creditors who withdraw consist of three categories: those with \( x_i < \bar{x} \), those with \( x_i \in (x_A, x_B) \), and some interior fraction of those with signal \( x_i \in (x_A, x_B) \). Moreover, conditional on creditor \( i \)'s own signal \( x_i \), creditor \( j \)'s signal \( x_j \) is normally distributed with mean \( x_j \) and variance \( 2\sigma^2 \). In other words, \( f(x_j|x_i) \sim \mathcal{N}(x_j, 2\sigma^2) \). Thus, by the law of large numbers, the
aggregate withdrawal is

\[ E[I|x_i] = \int_{-\infty}^{x_i} f(x_j|x_i)dx_j + \int_{x_i}^{x_A} f(x_j|x_i)n_jdx_j + \int_{x_A}^{x_B} f(x_j|x_i)dj \]

\[ = \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_A) \right] + \int_{x_i}^{x_A} \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_j - x_A) \right] n_jdx_j + \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_B - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_A - x_j) \right] \]

for \( i = A, B \).

Taking the difference, we have

\[ E[I|x_A] - E[I|x_B] \]

\[ = \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_B) \right] + \int_{x_i}^{x_A} \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_j - x_A) \right] n_jdx_j - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_A - x_B) \right] \]

\[ \geq \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_B) \right] + \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_B - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_A - x_B) \right] - 1 \]

\[ = \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_B) \right] + \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_B - x_A) \right] - \Phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_A - x_B) \right] \]

\[ > 0. \]

The inequality in the second step results from \( n_j(x_j) \geq 0 \) and \( \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_j - x_A) \right] - \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x_j - x_B) \right] > 0 \) for any \( x_j < x_A < x_B \). The third step utilizes the property of the CDF of a normal distribution: \( \Phi(X) = 1 - \Phi(-X) \). The fourth step follows from the fact that \( \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_A) \right] - \phi \left[ \sqrt{\frac{1}{2\sigma^2}} (x - x_B) \right] > 0 \) because \( x_A < x_B \).

The second step of the proof is to show that all creditors must use the same threshold. The proof is by contradiction as well. Suppose there exists an equilibrium in which creditors do not have the same threshold. Then denote \( S \) as the set of all thresholds used by any positive mass of creditors in equilibrium. Denote \( x_C = \inf S \) and \( x_D = \sup S \). We have \( x_C < x_D \). Intuitively, \( x_C \) is the lowest threshold adopted by creditors and \( x_D \) the highest. One example of such an equilibrium is illustrated in Fig. A.4.

In such an equilibrium, creditors with threshold signal \( x_C \) and \( x_D \) are indifferent between withdrawing and staying. Thus we have

\[ \Delta(x_C; I(.)) = \Delta(x_D; I(.)) = 0. \quad (16) \]

We now prove that

\[ \Delta(x_C; I(.)) - \Delta(x_D; I(.)) = \left( E[r|x_C] - E[r|x_D] \right) - \delta \left( E[I|x_C] - E[I|x_D] \right) < 0, \]

which contradicts the indifference condition.

The first term \( E[r|x_C] - E[r|x_D] \) is non-positive because of Corollary 1 and \( x_C < x_D \). We now prove that the second term \( \delta \left( E[I|x_C] - E[I|x_D] \right) \) is positive.
In this equilibrium, the creditors who withdraw consist of two categories: those with \(x_j < x_C\) and an interior fraction of those with \(x_j \in [x_C, x_D]\). Conditional on creditor \(i\)'s own signal \(x_i\), creditor \(j\)'s signal \(x_j\) is normally distributed with mean \(x_j\) and variance \(2\sigma^2\). In other words, \(f(x_j|x_i) \sim N(x_i, 2\sigma^2)\). Therefore, by the law of large numbers, the aggregate withdrawal is thus

\[
E[I|x_i] = \int_{x_i}^{x_C} f(x_j|x_i) \, dx_j + \int_{x_C}^{x_D} f(x_j|x_i) \, n_j(x_j) \, dx_j
\]

\[
= \Phi\left( \frac{1}{2\sigma^2}(x_C - x_i) \right) + \int_{x_C}^{x_D} \Phi\left( \frac{1}{2\sigma^2}(x_j - x_i) \right) \, n_j(x_j) \, dx_j,
\]

Following a procedure similar to what we have used to prove \(E[I|x_A] - E[I|x_B] < 0\) in the first step, we prove that \(E[I|x_C] - E[I|x_D] < 0\) (the exact derivation is available from the authors upon request).

\[\square\]

**Proof of Lemma 4.** In the text, we characterized the manager's best response to a common withdrawal threshold \(\hat{x}\) in terms of first- and second-order conditions. They are reproduced here for ease of reference, and for simplicity we have also suppressed the dependence of \(m^{BR}\) on \(r\) and \(\hat{x}\).

\[
\frac{1}{\sigma} \phi\left( \frac{1}{\sigma} \left( \hat{x} - (r + m^{BR}) \right) \right) = k' \left( m^{BR} \right),
\]

\[\text{(FOC)}\]

\[
-\frac{1}{\sigma^2} \phi\left( \frac{1}{\sigma} \left( \hat{x} - (r + m^{BR}) \right) \right) \cdot k'' \left( m^{BR} \right) < 0.
\]

\[\text{(SOC)}\]

There are two possible cases.

**Case 1:** \(r \geq \hat{x}\). \(\phi\left( \frac{1}{\sigma} \left( \hat{x} - (r + m^{BR}) \right) \right) > 0\) and thus the second-order condition is satisfied. The first-order condition is therefore both necessary and sufficient for optimality. Since the first-order condition has a unique solution, \(m^{BR}(r; \hat{x})\) is unique.

**Case 2:** \(r < \hat{x}\). The second-order condition is not satisfied globally. There may be multiple solutions that satisfy both the first- and second-order conditions above. The optimal \(m^{BR}\) is then achieved by comparing \(w(m; r)\) evaluated at those solutions. However, we prove that the optimal \(m^{BR}(r; \hat{x})\) is unique for almost every \(r\) for any given \(\hat{x}\) by contradiction. For simplicity we suppress the dependence of \(m^{BR}(r)\) on \(\hat{x}\).

Suppose \(m^{BR}(r)\) is not unique for all \(r\) on a set of non-zero measure. Since every set of a positive measure in a Lebesgue space contains a closed subset of a positive measure, there must exist at least one closed, non-zero measure set on which \(m^{BR}(r)\) is not unique. Denote this set as \(T\). For any \(r \in T\), there are at least two optimal solutions, denoted as \(m_1^{BR}(r)\) and \(m_2^{BR}(r)\), respectively. Without loss of generality, we assume that they are ordered as \(m_1^{BR}(r) - m_2^{BR}(r) \geq \kappa(r) > 0\). Set \(\tau = \min\kappa(r) > 0\). Since both \(m_1^{BR}\) and \(m_2^{BR}\) have to satisfy the first-order condition, they are continuous functions of \(r\). This implies that \(\kappa(r)\) is continuous in \(r\), which further implies that \(\tau\) is well-defined as \(\kappa(r)\) is continuous and \(T\) is closed. Moreover, \(\tau\) will be achieved by some \(r^*\) on \(T\). Therefore, \(\exists \varepsilon > 0\) s.t. \(r \geq \varepsilon > 0\). To see why this is true, suppose otherwise. Then \(\tau = 0\), implying that \(\kappa(r) = 0\), which is in contradiction. We thus have \(\tau \geq \varepsilon > 0\).

We first prove that \(M(r)\) is almost everywhere strictly increasing in \(T\). We know that \(M(r) = r + m^{BR}(r)\) is non-decreasing in \(r\) from Lemma 2. Now suppose there exists a non-zero measure, closed subset of \(S \subseteq T\) such that \(M(r)\) is a constant for any \(r \in S\). That is, for any two different points in \(S\), \(r_0\) and \(r_0\), we have \(M(r_0) = M(r_0)\). Since \(M(r) = r + m^{BR}(r)\), we then have \(m^{BR}(r_0) = m^{BR}(r_0)\). Thus, \(c'(m) > 0\) on \(S\), we have \(c'(m^{BR}(r_0)) \neq c'(m^{BR}(r_0))\). The combination of \(M(r_0) = M(r_0)\) with \(c'(m^{BR}(r_0)) \neq c'(m^{BR}(r_0))\) implies that it is impossible for both \(m^{BR}(r_0)\) and \(m^{BR}(r_0)\) to satisfy the first-order condition at the same time, contradicting the claim that \(m^{BR}(r_0)\) and \(m^{BR}(r_0)\) are the optimal solutions. Therefore, \(M(r)\) is almost everywhere strictly increasing in \(T\). Since \(M(r)\) is a correspondence on \(T\), “strictly increasing” here means \(\forall r_1, r_2 \in V\), \(r_1 < r_2\) and any \(s_1 \in M(r_1)\) and \(s_2 \in M(r_2)\), we have \(s_1 < s_2\).

Denote \(V\) as the largest closed subset of \(T\) in which \(M(r)\) is strictly increasing in \(r\). Denote \(r = \min_{r \in V} r\) and \(r_0 = \min_{r \in V} r\). By definition \(r_0 \geq \Gamma\). Thus, \(M(r_0) \geq \Gamma + m^{BR}(r_0) \geq \Gamma + r + m^{BR}(r) > \Gamma + \frac{r}{2} + m^{BR}(r) = M(r) + \frac{r}{2}\). The first inequality utilizes \(r_0 \geq \Gamma\) and the fact that \(M(r)\) is non-decreasing in \(r\), and the second inequality uses the defined ordering of \(m^{BR}(r)\) and \(m^{BR}(r)\). Since \(V\) is closed, we can find points in \(V\) that are arbitrarily close to each other. Denote \(r_0 \in V\) as a point that is infinitesimally greater than \(r_0\). Then we have \(M(r_0) > r_0 + m^{BR}(r_0) > r_0 + m^{BR}(r_0) = \Gamma + \frac{r}{2} + m^{BR}(r) = M(r) + \frac{r}{2}\). The first inequality utilizes the result that \(M(r)\) is strictly increasing in \(r\) for any \(r \in V\). So we have established that \(M(r_0) > M(r) + \frac{r}{2}\) and \(M(r_0) > M(r) + \frac{r}{2}\). Since there are infinitely many points on \(V\) such that \(M(r)\) is strictly increasing in \(r\), we can establish a similar inequality as above infinitely many times, resulting in \(M(T) \rightarrow +\infty\) where \(r = \max_{r \in V} r\). This, however, cannot be true as this implies that \(m^{BR}(r) \rightarrow +\infty\) and \(c'(m^{BR}(r)) \rightarrow +\infty\). This cannot be optimal as the maximum benefit from reducing \(l\) is 1, which is finite. Therefore, \(m^{BR}(r)\) is unique for almost every \(r\) and the lemma is proved. \(\square\)
Proof of Proposition 1. We need to prove that there is a unique solution, which we denote as \(x^*\), to Eq. (5), i.e.,
\[
\Delta \left( x^*, l(x^*, r^*) \right) = 0,
\]
or, equivalently, \(E[r - \delta l(x^*)] = 1\). We analyze \(E[r|x^*]\) and \(E[l|x^*]\) separately. Below we show that \(E[r|x^*]\) is strictly increasing in \(x^*\) and that \(E[l|x^*]\) is a constant, establishing that \(E[r - \delta l|x^*]\) is strictly increasing in \(x^*\). This strict monotonicity together with the existence of dominance regions proves the proposition. To see this, note that when \(x^* > x\), we are in the upper dominance region and \(E[r - \delta l|x^*] > 1\). Similarly, when \(x^* < x\), we are in the lower dominance region and \(E[r - \delta l|x^*] < 1\). Thus there is a unique solution to equation \(E[r - \delta l|x^*] = 1\) if \(E[r - \delta l|x^*]\) is strictly increasing in \(x^*\).

First, we show that \(E[r|x^*]\) is strictly increasing in \(x^*\), which follows through by showing that \(g(r|x)\) satisfies strict MLRP. From the proof of Corollary 1, we know that \(E[r|x^*]\) is weakly increasing in \(x^*\). To prove that \(E[r|x^*]\) is strictly increasing in \(x^*\), we need to show that \(g(r|x^*_2)\) strictly first-order stochastically dominates \(g(r|x^*_1)\) on at least a set of non-zero measure for any \(x^*_2 > x^*_1\). Again from the proof of Corollary 1, we know this is implied if
\[
\frac{1}{2\pi \sigma^2} e^{-\frac{|x^*_1-M(r_1)|^2}{2\sigma^2}} = \frac{1}{2\pi \sigma^2} e^{-\frac{|x^*_2-M(r_2)|^2}{2\sigma^2}}
\]
is strictly increasing in \(x_1\) for any \(r_1 > r_2 \in S\) on a set \(S\) of non-zero measure. Suppose this is not true. Then this implies that \(M(r_1) = M(r_2) = M\) for some constant \(M\) for all \(r\) except possibly on a set of zero measure. The first-order condition implies that for all \(r\),
\[
kc'\left( m^*(x^*, r) \right) = \frac{1}{\sigma} \phi \left( \frac{1}{\sigma} (x^* - M) \right)
\]
Since the right hand side of the first-order condition is a constant of \(r\), this implies that \(m(x^*, r)\) has to be a constant of \(r\) for those \(r \geq r^\ast\), except possibly on a set of zero measure. But if this is the case, then \(r + m(x^*, r)\) cannot be a constant of \(r\), resulting in contradiction.

The second step is to prove that \(E[l|x^*]\) is non-decreasing in \(x^*\). As is shown below, conditional on one creditor having private signal \(x^*\), the other creditors’ private signal distribution is symmetric around \(x^*\), implying that \(E[l|x^*] = Pr(x_j < x^*|x_j = x^*) = \frac{1}{2}\).

More formally,
\[
E[l|x^*] = E[Pr(x_j < x^*)|x^*]
\]
\[
e = E \left[ Pr \left( r + m^*(r) + \epsilon_j < x^* \right) \right]
\]
\[
e = E \left[ Pr \left( x^* - \epsilon_i + \epsilon_j < x^* \right) \right] = E \left[ Pr \left( \epsilon_j < x^* \right) \right]
\]
\[
= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_j^2}{2\sigma^2}} d\epsilon_j \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_i^2}{2\sigma^2}} d\epsilon_i
\]
\[
= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_i^2}{2\sigma^2}} d\epsilon_i \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_j^2}{2\sigma^2}} d\epsilon_j
\]
\[
= \int_{-\infty}^{+\infty} \Phi \left[ \frac{\epsilon_j}{\sigma} \right] \phi \left[ \frac{\epsilon_i}{\sigma} \right] d\left( \frac{\epsilon_i}{\sigma} \right) \int_{-\infty}^{+\infty} \Phi[t] \phi[t] dt
\]
\[
= \int_{-\infty}^{+\infty} \Phi[t] dt \Phi[t] dt
\]
where the third equality follows from changing variables \(\epsilon_i = x^* - \left( r + m^*(r) \right) \) and the last step uses the properties that \(\epsilon_i\) and \(\epsilon_j\) are i.i.d. with mean zero. Denoting this integral as \(I\) and using integration by parts, we have
\[
I = \Phi[t] \Phi[t] \bigg|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \phi[t] \Phi[t] dt = 1 - I.
\]
Therefore, \(E[l|x^*] = I = \frac{1}{2}.\)

Proof of Proposition 2. The proposition is proved by taking the proper limits of \(m^*(x^*) = m^BR(r; x^*)\) from Eq. (FOC) and \(x^*\) from Eq. (5). We proceed in four steps.

Step 1: we prove the following claim: as \(\sigma \to 0\), \(m^*\) converges to either 0 or \(x^* - r\). For ease of notation in this step, we suppress the dependence of \(m^*\) on \(r\) and \(x^*\) and instead denote \(m^*\) as \(m^*(\sigma)\). For ease of reference, we also reproduce the first- and second-order conditions for the manager’s misreporting choice evaluated at \(m^BR(r; x^*) = m^*\) and \(\tilde{x} = x^*\).

\[
\frac{1}{\sigma} \phi \left( \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right) - kc'\left( m^* \right) = 0
\]

(FOC)
\[-\frac{1}{\sigma^2} \phi' \left( \frac{1}{\sigma} (x^* - (r + m^*)) \right) - kc''(m^*) < 0 \tag{SOC}\]

We consider two cases.

Case 1: \( r \geq x^* \). As \( \sigma \to 0 \), \( kc'(m^*) = \frac{1}{\sigma} \phi \left( \frac{1}{\sigma} (x^* - (r + m^*)) \right) \to 0 \). Therefore, \( m^*(\sigma) \to 0 \).

Case 2: \( r < x^* \). The proof in this case is by contradiction. Since \( m^*(\sigma) \leq c^{-1}(\frac{1}{k}) < +\infty \) for any \( \sigma > 0 \), we know that \( m^*(\sigma) \) is continuous and bounded. This implies that \( m^*(\sigma) \) must converge to some number as \( \sigma \to 0 \). If the limit is neither 0 nor \( x^* - r \), it must be sufficiently away from both 0 and \( x^* - r \). But this implies \( x^* - (r + m^*(\sigma)) \neq 0 \) and \( \frac{1}{\sigma} \phi \left( \frac{1}{\sigma} (x^* - (r + m^*(\sigma))) \right) \to 0 \). The first-order condition (Eq. (FOC)) then results in \( kc'(m^*(\sigma)) \to 0 \) and \( m^*(\sigma) \to 0 \), resulting in a contradiction.

Step 2: after establishing the claim that \( m^*(\sigma) \) must converge to either 0 or \( x^* - r \), we pin down \( m^* \) for given \( x^* \). We again consider two cases.

Case 1: \( r \geq x^* \). In this case, \( m^*(\sigma) \) converges to 0. The manager’s payoff is

\[
\lim_{\sigma \to 0, m^*(\sigma) \to 0} W(m^*(\sigma); r) = \lim_{\sigma \to 0, m^*(\sigma) \to 0} r - \Phi \left( \frac{1}{\sigma} (x^* - (r + m^*(\sigma))) \right) - kc(m^*(\sigma)) = r.
\]

In words, the manager with \( r \geq x^* \) does not suffer any withdrawal even without misreporting and thus does not misreport.\(^{15}\)

Case 2: \( r < x^* \). In this case, \( m^*(\sigma) \) converges to either 0 or \( x^* - r \). The manager now compares the payoffs from two reporting choices, \( m^*(\sigma) \to 0 \) or \( m^*(\sigma) \to x^* - r \).

If \( m^*(\sigma) \to 0 \), the manager’s payoff is

\[
W_0 = \lim_{\sigma \to 0, m^*(\sigma) \to 0} W(m^*(\sigma); r) = \lim_{\sigma \to 0, m^*(\sigma) \to 0} r - \Phi \left( \frac{1}{\sigma} (x^* - (r + m^*(\sigma))) \right) - kc(m^*(\sigma)) = r - 1.
\]

If \( m^*(\sigma) \to x^* - r \), the manager’s payoff is

\[
W_1 = \lim_{\sigma \to 0, m^*(\sigma) \to x^* - r} W(m^*(\sigma); r) = \lim_{\sigma \to 0, m^*(\sigma) \to x^* - r} r - \Phi \left( \frac{1}{\sigma} (x^* - (r + m^*(\sigma))) \right) - kc(m^*(\sigma)) = r - kc(x^* - r).
\]

This expression, Eq. (17), will be derived in Step 4.

The manager chooses \( m^*(\sigma) \to 0 \) if \( W_0 \geq W_1 \) and \( m^*(\sigma) \to x^* - r \) if \( W_0 < W_1 \). We have

\[
W_0 - W_1 = kc(x^* - r) - 1.
\]

Note that \( W_0 - W_1 \) is decreasing in \( r \). Thus we can define \( r_1 \) with

\[
kc(x^* - r_1) - 1 = 0 \tag{18}
\]

such that the manager misreports \( m^*(\sigma) \to x^* - r \) if and only if \( r \in [r_1, x^*] \). Combined with Case 1, we can also define \( r_2 \) such that

\[
r_2 = x^* \tag{19}.
\]

Therefore, combining Case 1 and Case 2, we conclude that, for any given \( x^* \), as \( \sigma \) tends to 0, the manager misreports \( m^*(\sigma) \to x^* - r \) if \( r \in [r_1, r_2] \) and \( m^*(\sigma) \to 0 \) otherwise.

Step 3: We pin down \( x^* \) by taking the creditors’ indifference condition (Eq. (5)) to the limit of \( \sigma \to 0 \), which can be reproduced and written out here:

\[
\lim_{\sigma \to 0} E[r|x^*] = \lim_{\sigma \to 0} \delta E[l|x^*] = 1. \tag{20}
\]

Note first, from the proof of Proposition 1, that

\[
\lim_{\sigma \to 0} E[l|x^*] = \lim_{\sigma \to 0} \Pr(x_j < X^*|x^*) = \lim_{\sigma \to 0} \frac{1}{2} = \frac{1}{2}. \tag{21}
\]

\(^{15}\) When \( r = x^* \), since \( m^*(\sigma) > 0 \) for any \( \sigma > 0 \), we have \( x^* - (r + m^*(\sigma)) < 0 \), and thus \( \frac{1}{\sigma} \left( x^* - (r + m^*(\sigma)) \right) \to -\infty \) and \( \Phi \left( \frac{1}{\sigma} (x^* - (r + m^*(\sigma))) \right) \to 0 \). Thus the manager’s payoff is \( r - kc(x^* - r) = r \).
Thus we only need to calculate \( E[r|x^*] \) as \( \sigma \to 0 \). We again explicitly write out the dependence of \( m^* \) on \( r \). Note that

\[
E[r|x^*] = E[r + m^*(r, \sigma) - m^*(r, \sigma)|x^*] = E[r + m^*(r, \sigma)|x^*] - E[m^*(r, \sigma)|x^*] = x^* - E[m^*(r, \sigma)|x^*].
\]

(22)

The third equality is true because for the creditor \( i \) with threshold signal \( x_i = x^* \), \( x_i = r + m^*(r, \sigma) + \epsilon_i \).

Therefore, we only need to derive \( E[m^*(r, \sigma)|x^*] \) as \( \sigma \to 0 \). We can write

\[
E[m^*(r, \sigma)|x^*] = \int_{-\infty}^{+\infty} m^*(r, \sigma)f(r|x^*)dr = \left( \int_{-\infty}^{r_1} + \int_{r_1}^{x^*} + \int_{x^*}^{+\infty} \right) m^*(r, \sigma)f(r|x^*)dr
\]

We have shown in Step 1 that \( m^*(r, \sigma) \to x^* - r \) for \( r \in [r_1, r_2] \) and \( m^*(r, \sigma) \to 0 \) otherwise. It can be proved that this convergence is (almost everywhere) uniform (the proof is available upon request) and that the converged conditional density is \( f(r|x^*) = \frac{k\epsilon^c(x^* - r)}{\int_{r_1}^{r_2} k\epsilon^c(x^* - r)dr} = k\epsilon^c(x^* - r) \) for \( r \in [r_1, r_2] \) and 0 otherwise. Uniform convergence implies that we can interchange the sequence of the limit and integral. Thus,

\[
\lim_{\sigma \to 0} E[m^*(r, \sigma)|x^*] = \lim_{\sigma \to 0} \left( \int_{-\infty}^{r_1} + \int_{r_1}^{x^*} + \int_{x^*}^{+\infty} \right) m^*(r, \sigma)f(r|x^*)dr
\]

\[
= \int_{-\infty}^{r_1} \lim_{\sigma \to 0} m^*(r, \sigma)f(r|x^*)dr + \int_{r_1}^{x^*} \lim_{\sigma \to 0} m^*(r, \sigma)f(r|x^*)dr + \int_{x^*}^{+\infty} \lim_{\sigma \to 0} m^*(r, \sigma)f(r|x^*)dr
\]

\[
= \int_{r_1}^{x^*} \lim_{\sigma \to 0} m^*(r, \sigma)f(r|x^*)dr
\]

\[
= \int_{r_1}^{x^*} \lim_{\sigma \to 0} m^*(r, \sigma)\epsilon^c(x^* - r)\epsilon^c(x^* - r)dr
\]

\[
= x^* - \frac{\epsilon^c(x^* - r)}{\epsilon^c(x^* - r)}
\]

\[
= x^* - r_1 - \int_{r_1}^{x^*} c(x^* - r)dr
\]

\[
= x^* - r_1 - \frac{\epsilon^c(x^* - r)}{\epsilon^c(x^* - r)}
\]

(23)

Inserting \( \lim_{\sigma \to 0} E[m^*(r, \sigma)|x^*] \) from Eq. (23) into Eq. (22) results in

\[
\lim_{\sigma \to 0} E[r|x^*] = r_1 + \frac{\int_{r_1}^{x^*} c(x^* - r)dr}{c(x^* - r_1)}.
\]

(24)

Substituting \( \lim_{\sigma \to 0} E[r|x^*] \) from Eq. (24) and \( \lim_{\sigma \to 0} E[l|x^*] \) from Eq. (21) into the creditors’ indifference condition (Eq. (20)), we have

\[
\frac{1}{\sigma} \Phi \left( \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right) = k\epsilon^c(x^* - r).
\]

(25)

In sum, as \( \sigma \to 0 \), \( m^*(r) \to x^* - r \) for \( r \in [r_1, r_2] \) and 0 otherwise. Moreover, \( x^*, r_1 \) and \( r_2 \) are jointly determined by three equations, Eqs. (18), (19) and (25). Solving these three equations, we obtain the closed-form solutions for \( x^*, r_1 \) and \( r_2 \) as expressed in Eqs. (6), (8), and (9). Thus we have completed the characterization of the equilibrium as \( \sigma \to 0 \).

The incidence of runs can be calculated as follows: \( l^*(r) = \Phi \left( \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right) \) converges to 0 if \( r \geq r_1 \) and to 1 if \( r < r_1 \). This completes the proof, except for some technical details to be supplied in Step 4.

Step 4: We prove equation (17). When \( r < x^* \), from the first-order condition, we have \( \lim_{\sigma \to 0} \frac{1}{\sigma} \Phi \left( \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right) = k\epsilon^c(x^* - r) \). Since \( c(x^* - r) \) is finite, it must be that \( \Phi \left( \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right) \to 0 \), which implies that \( \frac{1}{\sigma} \left( x^* - (r + m^* \sigma) \right) \to +\infty \) or \( -\infty \). Next we prove that \( \frac{1}{\sigma} \left( x^* - (r + m^* \sigma) \right) \to -\infty \) by contradiction. Suppose the opposite is true, that is,
\[
\frac{1}{\sigma} \left[ x^* - \left( r + m^*(\sigma) \right) \right] \to +\infty. \text{ We use the property } \phi'(X) = -X\phi(X) \text{ to rewrite the second-order condition as }
\]
\[
SOC = -\frac{1}{\sigma^2} \phi \left[ \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right] - kc''(m^*)
\]
\[
= \frac{\left( x^* - (r + m^*) \right)}{\sigma^3} \phi \left[ \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right] - kc''(m^*)
\]
\[
= \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \phi \left[ \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right] - kc''(m^*).
\]

If \( \frac{1}{\sigma} \left[ x^* - \left( r + m^*(\sigma) \right) \right] \to +\infty \), then \( \lim_{\sigma \to 0} SOC > 0 \), which is a contradiction. Therefore, \( \frac{1}{\sigma} \left[ x^* - \left( r + m^*(\sigma) \right) \right] \to -\infty \), which generates the manager’s payoff in equation (17). \( \square \)

**Proof of Corollary 2.** For any \( \sigma > 0 \),
\[
\frac{\partial}{\partial m^*} w(m^*; r)|_{m^*=0} = \left( \frac{1}{\sigma} \phi \left[ \frac{1}{\sigma} \left( x^* - (r + m^*) \right) \right] - kc'(m^*) \right)|_{m^*=0}
\]
\[
= \frac{1}{\sigma} \phi \left[ \frac{1}{\sigma} \left( x^* - r \right) \right] - kc'(0)
\]
\[
= \frac{1}{\sigma} \phi \left[ \frac{1}{\sigma} \left( x^* - r \right) \right]
\]
\[
> 0.
\]

Thus, at \( m^*(\sigma) = 0 \), the net benefit of manipulation is strictly positive. Hence, \( m^* > 0 \) as long as \( \sigma > 0 \). \( \square \)

**Proof of Corollary 3, Proposition 3 and Proposition 4.** We group the proofs of these three results together because they all involve the comparative statics of \( r_1(k) \) and \( r_2(k) \), whose closed-form solutions are given in Proposition 2. They are reproduced below for convenience with the replacement of \( r^{DD} = 1 + \frac{\delta}{k} \) and \( y(k) \equiv c^{-1}\left( \frac{1}{k} \right) \):
\[
r_1(k) = r^{DD} - \int_0^{y(k)} kc(t)dt,
\]
\[
r_2(k) = r^{DD} - \int_0^{y(k)} kc(t)dt + y(k).
\]

Later we will use the following intuitive properties of \( y(k) \):
\[
c(y) = \frac{1}{k},
\]
\[
\frac{\partial y}{\partial k} = -\frac{1}{k^2} \frac{1}{c'(y)} = \frac{c^2(y)}{c'(y)},
\]
\[
\lim_{k \to +\infty} y(k) = 0.
\]

We first show that \( r_2(k) > r^{DD} > r_1(k) \) for any finite \( k \). This is because
\[
r^{DD} - r_1 = \int_0^y kc(t)dt > 0
\]
\[
r_2 - r^{DD} = y - \int_0^y kc(t)dt > y - \int_0^y kc(y)dt = 0.
\]

The inequality utilizes the property \( c'(t) > 0 \) for \( t > 0 \) and the last equality employs the property \( c(y) = \frac{1}{k} \).

We now prove that \( \lim_{k \to +\infty} r_2(k) = \lim_{k \to +\infty} r_1(k) = r^{DD} \).

\[
\lim_{k \to +\infty} \left( r_2 - r_1 \right) = \lim_{k \to +\infty} \int_0^{y(k)} kc(t)dt = \lim_{k \to +\infty} \int_0^{y(k)} c(t)dt \frac{c(t)dt}{c(y)} \leq \lim_{k \to +\infty} \int_0^{y(k)} \frac{c'(y)dt}{c'(y)} = \lim_{k \to +\infty} y = 0.
\]

Thus, we have proved Corollary 3.
To prove $\frac{\partial r_1}{\partial k} > 0$, we differentiate $r_1(k)$ with respect to $k$:

\[
\frac{\partial r_1}{\partial k} = - \frac{\partial}{\partial k} \int_0^y kc(t) \, dt = - \frac{\partial y}{\partial k} kc(y) - \int_0^y c(t) \, dt = \frac{c^2(y)}{c'(y)} - \int_0^y c(t) \, dt = \frac{c(y)}{c'(y)} \left( c(y) - c'(y) \int_0^y c(t) \, dt \right) = \frac{c(y)}{c'(y)} \left( \int_0^y c'(t) c(t) \, dt - c'(y) \int_0^y c(t) \, dt \right) > \frac{c(y)}{c'(y)} \left( \int_0^y c'(t) c(t) \, dt - c'(y) \int_0^y c(t) \, dt \right) = 0.
\]

The inequality holds because the log-concavity of $c(t)$ implies that $\frac{c'(t)}{c(y)}$ is strictly decreasing in $t$ (Bagnoli and Bergstrom (2005)).

Now we use the intermediate value theorem to prove the existence of unique $\hat{k}$. We have already proved $\frac{\partial r_1}{\partial k} > 0$. Second, we have proved above that $\lim_{k \to 0} r_1(k) = r^{DD} > r^{FB} = 1$. Finally, we have $\lim_{k \to 0} r_1(k) < 1$ because

\[
\lim_{k \to 0} r_1 = r^{PD} - \lim_{k \to 0} \int_0^y kc(t) \, dt = r^{PD} - \lim_{k \to 0} \frac{\int_0^y c(t) \, dt}{c(y)} = r^{PD} - \lim_{y \to +\infty} \frac{\int_0^y c(t) \, dt}{c(y)} = r^{PD} - \lim_{y \to +\infty} \frac{c(y)}{c'(y)} < 1.
\]

The fourth equality utilizes L'Hospital's rule. Therefore, the intermediate value theorem implies that there exists a unique $\hat{k} > 0$ such that

\[
r_1(\hat{k}, \delta) = r^{FB}.
\]

Differentiating it with respect to $\delta$ and reorganizing the terms, we have

\[
\frac{\partial \hat{k}}{\partial \delta} = \frac{\partial r_1(\hat{k})}{\partial k} \bigg|_{k=k} = - \frac{1}{\partial r_1} \bigg|_{k=k} < 0.
\]

Thus we have proved Proposition 3.

Finally we prove Proposition 4. We have $\frac{\partial r_2(k)}{\partial k} < 0$ because

\[
\frac{\partial r_2(k)}{\partial k} = \frac{\partial}{\partial k} \left( y(k) - \int_0^y kc(t) \, dt + r^{PD} \right) = \frac{\partial y}{\partial k} - \left( \int_0^y c(t) \, dt + kc(y) \frac{\partial y}{\partial k} \right) = - \int_0^y c(t) \, dt < 0.
\]

Now we use the intermediate value theorem to prove the existence of unique $\hat{k}$ such that $r_2 > 1 + \delta$ for $k < \hat{k}$. We have already proved $\frac{\partial r_2(k)}{\partial k} < 0$. Second, we have proved above that $\lim_{k \to +\infty} r_2(k) = r^{DD} = 1 + \frac{\delta}{2} < 1 + \delta$. Finally, we have $\lim_{k \to 0} r_2(k) >$
1 + δ because

\[
\lim_{k \to 0} r_2 = r^{DD} + \lim_{k \to 0} \left( y - \int_0^{y(k)} kc(t)dt \right)
\]

\[
= r^{DD} + \lim_{k \to 0} \left( y - \int_0^{\gamma} c(t)dt \right)
\]

\[
= r^{DD} + \lim_{y \to +\infty} \left( \frac{yc(y) - \int_0^{\gamma} c(t)dt}{c(y)} \right)
\]

\[
= r^{DD} + \lim_{y \to +\infty} \left( \frac{c(y) + yc'(y) - c(y)}{c'(y)} \right)
\]

\[
= r^{DD} + \lim_{y \to +\infty} y = \infty
\]

The fourth equality utilizes L'Hospital's rule. Therefore, the intermediate value theorem implies that there exists a unique \( k \) such that \( r_2(k) = 1 + \delta \). \( r_2 > 1 + \delta \) if and only if \( k < \bar{k} \). Thus, we have proved Proposition 4.

\[\Box\]

**Proof of Proposition 5.** Recall that the social loss function is given by Eq. (13) and reproduced here for convenience.

\[L(k) = I_{r_{1} \geq 1} \int_1^{r_{1}(k)} (r - 1)dr + (1 - I_{r_{1} \geq 1}) \int_{r_{1}(k)}^{1} (1 - r)dr + \gamma \int_{r_{1}(k)}^{r_{2}(k)} kc(r - r)dr.\]

We first show that the total misreporting cost \( \int_{r_{1}(k)}^{r_{2}(k)} kc(r - r)dr \) is decreasing in \( k \). Using change of variables \( t = r_{2} - r \) and denoting \( y = c^{-1}(\frac{1}{k}) \), we have

\[\int_{r_{1}}^{r_{2}} kc(r_{2} - r)dr = \int_{0}^{r_{2} - r_{1}} kc(t)dt = \int_{0}^{c^{-1}(\frac{1}{k})} kc(t)dt = \int_{0}^{y} kc(t)dt = r^{DD} - r_{1}.\]

Thus,

\[\frac{\partial}{\partial k} \int_{r_{1}}^{r_{2}} kc(r_{2} - r)dr = -\frac{\partial r_{1}}{\partial k} < 0.\]

Now we consider the effect of \( k \) on the liquidation inefficiency, the first two terms in \( L \).

\[I_{r_{1} \geq 1} \int_1^{r_{1}(k)} (r - 1)dr + (1 - I_{r_{1} \geq 1}) \int_{r_{1}(k)}^{1} (1 - r)dr\]

\[= \left\{ \begin{array}{ll}
\int_{1}^{r_{1}(k)} (r - 1)dr & \text{if } r_{1}(k) \geq 1 \\
\int_{r_{1}(k)}^{1} (1 - r)dr & \text{if } r_{1}(k) < 1.
\end{array} \right.\]

The derivative with respect to \( k \) is

\[\left\{ \begin{array}{ll}
(r_{1} - 1) \frac{\partial r_{1}}{\partial k} & \text{if } r_{1}(k) \geq 1 \\
-(1 - r_{1}) \frac{\partial r_{1}}{\partial k} & \text{if } r_{1}(k) < 1.
\end{array} \right.\]

\[= (r_{1} - 1) \frac{\partial r_{1}}{\partial k}.
\]

This implies that the effect of \( k \) on \( L \) can be summarized as

\[\frac{\partial L(k)}{\partial k} = (r_{1} - 1) \frac{\partial r_{1}}{\partial k} - \gamma \frac{\partial r_{1}}{\partial k}
\]

\[= (r_{1} - 1 - \gamma) \frac{\partial r_{1}}{\partial k}.
\]

Since \( \frac{\partial r_{1}}{\partial k} > 0 \), \( \frac{\partial L(k)}{\partial k} \) is negative if \( r_{1} < 1 + \gamma \) and positive if \( r_{1} > 1 + \gamma \). Thus, \( L(k) \) reaches its minimum at \( r_{1}(k) = 1 + \gamma \). In other words, the socially optimal \( k^{*} \) is such that

\[r_{1}(k^{*} ; \delta) = 1 + \gamma.\]  

(26)

Now we examine the properties of \( k^{*} \). First, \( k^{*} > \bar{k} \), because \( \frac{\partial r_{1}}{\partial k} > 0 \) and \( 1 + \gamma > r^{PB} \). Second, differentiating Eq. (26) with respect to \( \delta \) and reorganizing the terms, we have

\[\frac{\partial k^{*}}{\partial \delta} = -\frac{\partial r_{1}}{\partial k} \bigg|_{k=k^{*}} = -\frac{1}{\frac{\partial r_{1}}{\partial k} \bigg|_{k=k^{*}} < 0.}\]
Finally, differentiating Eq. (26) with respect to $\gamma$ and reorganizing the terms, we have
\[
\frac{\partial k^*}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left| k^* \right| > 0.
\]

\[\Box\]

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jacceco.2017.11.005

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