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Accounting Manipulation, Peer Pressure, and Internal Control*

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Abstract

We study firms’ investment in internal controls to reduce accounting manipulation. We first show that peer managers’ manipulation decisions are strategic complements: one manager manipulates more if he believes that reports of peer firms are more likely to be manipulated. As a result, one firm’s investment in internal controls has a positive externality on peer firms. It reduces its own manager’s manipulation, which, in turn, mitigates the manipulation pressure on managers at peer firms. Firms do not internalize this positive externality and thus under-invest in their internal controls over financial reporting. The problem of under-investment provides one justification for regulatory intervention in firms’ internal controls choices.

JEL classification: G18, M41, M48, K22

Key words: accounting manipulation, peer pressure, internal controls, SOX

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1 Introduction

The wave of accounting frauds and restatements in the early 2000s (e.g., Enron, WorldCom) exposed the colossal failure of internal controls over financial reporting in many firms (GAO (2002)). Until the beginning of the 2000s, a firm’s internal controls decisions had long been deemed its private domain and outside the purview of the securities regulations that had traditionally focused on disclosure of such decisions (Ribstein (2002), Coates (2007)). However, the prevalence and magnitude of the failures in firms’ internal controls eroded the support for such practices and eventually led to the passage of the Sarbanes–Oxley Act of 2002 (SOX) in the U.S., and similar legislature in other countries. In addition to enhanced disclosure requirements, SOX also mandated substantive measures to deter and detect accounting fraud. Their mandatory nature has made these measures controversial (e.g., Romano (2005), Hart (2009)). Even for those who felt that something needed to be done about the firms’ internal controls over financial reporting, it may not be clear why it should be done through regulations. Is there a case for regulation that intervenes in firms’ internal controls decisions? Do firms have the right incentives to choose optimal levels of internal controls to assure the veracity of their financial statements? In fact, Romano (2005), in an influential critique of SOX, argues that “The central policy recommendation of this Article is that the corporate governance provisions of SOX should be stripped of their mandatory force and rendered optional for registrants.”

We construct a model to study firms’ investment in internal controls over financial reporting. In the model, firms can invest in costly internal controls to detect and deter managers’ accounting manipulation. We show that such an investment by one firm has a positive externality on its peers. At the core of the channel for this externality is the peer pressure for accounting manipulation among firms: one manager’s incentive to manipulate increases in his expectation that peer firms’ reports are manipulated. As a result, a firm’s investment in internal controls reduces its own manager’s manipulation, which, in turn, mitigates the pressure for manipulation on managers in peer firms. Since the firm does not internalize this

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1These mandates range from independent audit committees, auditor partner rotations, prohibition of non-audit service provided by auditors, and executives’ certification and auditors' attestation to an internal control system.
externality, it under-invests in internal controls over financial reporting. Regulatory intervention can improve the value of all firms by mandating a floor of internal controls investment.

In our model, there are two firms with correlated fundamentals, indexed by $A$ and $B$. Each manager’s payoff is a weighted average of the current stock price and the fundamental value of *his own firm*. Investors rely on accounting reports to set stock prices. Accounting manipulation boosts accounting reports and allows a bad manager who successfully manipulates to be pooled with the good ones. Investors rationally conjecture this pooling result and discount the pool accordingly to break even. In the equilibrium, the bad manager who successfully manipulates obtains an inflated stock price at the expense of the good manager. Accounting manipulation is detrimental to firm value and firms do have private incentives to invest in their internal controls over financial reporting.

In such a setting, peer pressure for accounting manipulation arises. By peer pressure, we mean that one manager manipulates more if he expects that the other firm’s report is more likely to be manipulated in equilibrium. In other words, the two managers’ manipulation decisions are strategic complements in the sense of Bulow, Geanakoplos, and Klemperer (1985). The mechanism could be illustrated in a special case in which two firms’ fundamentals are perfectly correlated and manager $B$ does not manipulate. As a result, manager $A$ will also not manipulate because any successful manipulation will be contradicted by the report from manager $B$. If manager $B$ is expected to slightly manipulate, manager $A$ now anticipates that his fraudulent report is sometimes not contradicted by report $B$ and thus has an incentive to manipulate.

Peer pressure for manipulation creates a positive externality of the firm’s costly investment in internal controls. Firm $B$’s investment in internal controls reduces its own manager’s manipulation. The reduction of manipulation in firm $B$ mitigates the pressure for manipulation on manager $A$, resulting in lower manipulation by manager $A$. However, firm $B$ does not internalize this externality and under-invests in the internal controls. This underinvestment in internal controls by individual firms suggests a rationale for regulatory intervention that imposes some floor of internal controls over financial reporting.

The peer pressure for manipulation is often alleged in practice. One of the best known
and most extreme examples is the telecommunications industry around the turn of the millenium (see Bagnoli and Watts (2010) footnote 1 for detailed references to such allegations). When WorldCom turned to aggressive and, ultimately, illegal reporting practices to boost its performance, peers firms were under enormous pressure to perform. Horowitz (2003) claims: “Once WorldCom started committing accounting fraud to prop up their numbers, all of the other telecoms had to either (a) commit accounting fraud to keep pace with WorldCom’s blistering growth rate, or (b) be viewed as losers with severe consequences.” Qwest and Global Crossing ended up with accounting frauds while AT&T and Sprint took a series of actions that aimed to shore up their short-term performance at the expense of long-term viability. While these companies had plenty of their own problems, the relentless capital market pressure undoubtedly made matters worse (see Sadka (2006)).

Our model generates new empirical predictions. First, one firm’s internal controls quality affects not only its own manipulation but also its peer firms’ manipulation. Second, peer firms’ manipulation decisions are correlated, even after controlling for their fundamentals. For example, one bank’s loan loss provisioning might be increasing in the peer average even after controlling for the bank and its peers’ fundamentals. Finally, our model suggests that SOX, by mandating internal controls measures, could improve firm value. Some recent papers have examined how one firm’s fraudulent accounting affects investment decisions in peer firms (e.g., Gjesdal, Jenkins, and Johnson (2008), Beatty, Liao, and Yu (2013)). Our model suggests an additional effect that one firm’s accounting manipulation and internal controls impose on its peer firms.

1.1 Literature review and contributions

Our paper intends to make two contributions. The first is to provide one rationale for regulating firms’ internal controls over financial reporting. As Leuz and Wysocki (2016) have pointed out, understanding the positive externalities of regulations is crucial for their justification in the first place. Our model suggests that the proposal in Romano (2005)—that the internal controls mandates in SOX should be made optional—is flawed. Competition among firms (or state laws) does not lead to socially optimal investment in internal controls.
The prior literature has studied the externalities of disclosure and of corporate governance. Disclosure alone in our model does not solve the underinvestment problem. In our model, although the firms’ internal controls decisions are perfectly observed by investors and peer managers, the underinvestment problem persists. Internal controls can be viewed as part of the corporate governance. The prior literature has focused on the role of corporate governance in monitoring managerial misbehaviors, whereas we focus on its role in curtailing accounting manipulation.

The other contribution is to offer a rational explanation of the peer pressure for manipulation. Peer pressure for manipulation could arise from product market competition, from agents’ contractual payoff links, or from behavioral assumptions. Our paper complements these explanations from a capital market perspective in that managers intervene in the reporting process to influence capital markets’ inferences regarding their firms. Capital market pressure is often viewed as a major motivation for accounting manipulation (Graham, Harvey, and Rajgopal (2005)). We assume neither contractual links among managers nor complementarity between manipulation costs.

A key driver for the strategic complementarity result is the modeling feature that manipulation degrades the report’s informativeness. This feature does not arise in Stein (1989) because of its dual assumptions: that the report is normally distributed and that manipulation affects only the mean (but not the variance) of the report distribution. We break this “unravelling” result by constraining the message space to be binary, following Chen, Hemmer, and Zhang (2007) and Strobl (2013). The favorable report can come from either the good manager or the bad manager with successful manipulation, preventing investors from perfectly debiasing the report.

Fischer and Verrecchia (2000) break the “unravelling” result in a different way. It departs

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2 For the former, see, e.g., Dye (1990), Admati and Pfleiderer (2000), Lambert, Leuz, and Verrecchia (2007), and Kartik, Lee, and Suen (2017). For the latter, see, e.g., Acharya and Volfpin (2010), Dicks (2012), and Thanassoulis (2012).

Bagnoli and Watts (2001) and Einhorn, Langberg, and Versano (2016) study the product channel. Bagnoli and Watts (2001), Cheng (2011), Glover (2012), Baldegger, Glover, and Xue (2016), and Friedman (2014) exploit the contractual payoff link channel, and Mittendorf (2000), Mittendorf (2008), and Fischer and Huddart (2008) examine the behavioral channel. Nagar and Petacchi (2016) study a model of “enforcement thinning” in which the manipulation decisions among firms are also strategic complements due to the assumption that the regulator’s budget of enforcement against frauds is fixed.

There are other ways to break the “unravelling” result in the literature. For example, Dye and Sridhar
from Stein (1989) by assuming that investors are uncertain about the manager’s preference. This assumption, nonetheless, generates a feature that makes their model less suitable for studying peer pressure. In Fischer and Verrecchia (2000), manipulation sometimes reduces the report’s informativeness, but sometimes does not. As a result, the strategic complementarity between managers’ manipulation decisions might not be robust.

As an extension of Fischer and Verrecchia (2000) to multiple firms, Heinle and Verrecchia (2016) is related to our paper to the extent that both study manipulation decisions in a multi-firm setting. However, the two differ in many important aspects. While we provide a rationale for internal controls regulation and a rational explanation for peer pressure for manipulation, they study neither issue. Instead, they study firms’ ex-ante commitment to disclosure in the presence of other firms and show that “allowing the number of disclosed reports to be endogenous introduces a countervailing force to some of the empirical predictions from the prior literature.” Moreover, they focus on how a manager’s manipulation is affected by the presence of peer firms, whereas we study how a manager’s manipulation is affected by his expectation of peer managers’ equilibrium manipulation. We delineate the relation of these two channels in Section 4. Finally, one may also wonder whether their model could be adapted to study internal controls and peer pressure to reach results similar to ours. Based on our discussion of the feature of the manipulation model in Fischer and Verrecchia (2000), we conjecture that the strategic complementarity between manipulation decisions might not be robust.

More broadly, the manipulation component of our model is also related to the multi-agent career-concern literature (e.g., Meyer and Vickers (1997), Holmstrom (1999), Dewatripont, Jewitt, and Tirole (1999)). The career concern models often focus on endogenous contracting with agents in addition to implicit career concerns and inter-temporal dynamics. Moreover, the actions in these models are productive efforts. Nonetheless, the manipulation component of our model shares a key feature with career concern models in that agents take costly actions

(2004) have also retained the normal structure in Stein (1989) but introduced investors’ uncertainty about the manager’s manipulation cost function. For another example, Guttman, Kadan, and Kandel (2006) show that equilibria with such a feature arise if one adopts an equilibrium selection criterion that favors the manager. We refer readers to some recent surveys for the signal-jamming models, including Ronen and Yaari (2008), Beyer, Cohen, Lys, and Walther (2010), Ewert and Wagenhofer (2012), and Stocken (2013).
in an attempt to influence outsiders’ rational inference. A conceptual difference between our model and multi-agent career concern models such as Meyer and Vickers (1997) is that we focus on how one manager’s manipulation is affected by the equilibrium level of manipulation by other managers, while Meyer and Vickers (1997) exploit how an agent’s effort responds to the mere presence of other agents. We will formally define the former as “strategic complementarity” and the latter “spillover” later in Section 4. Strategic complementarity does not arise in Meyer and Vickers (1997) because efforts do not affect the outcome’s informativeness.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 solves the equilibria and examines the strategic relation among firms’ manipulation and internal controls investment. Section 4 discusses two extensions and Section 5 concludes.

2 The Model

The economy consists of two firms, indexed by A and B. There are four dates, , , and . All parties are risk neutral and the risk-free rate is normalized to 0.

Each firm has a project that pays out gross cash flow , , at . is either high ( ) or low ( ). The prior probability that is 1 is . The firm’s net cash flow at , denoted as , differs from the gross cash flow for two reasons explained below. We refer to the net cash flow as the firm’s long-term value and the gross cash flow as the firm’s fundamental or type.

The payoff function of manager is

\[ U_i = \delta_i P_i + (1 - \delta_i) V_i, \quad i \in \{A, B\}. \] (1)

The manager’s interests are not aligned with the firm’s long-term value . Instead, the manager cares about both the long-term firm value at and the short-term stock price at . measures the manager’s relative focus on the two.

Managers’ concern for short-term stock price performance is empirically descriptive. For

\footnote{We have assumed that both the state and the message spaces are binary. In Section 4 we will study a continuous structure.}
example, Stein (1988) argues that takeovers would force managers to tender their shares at market prices even if they would rather hold the stocks for a longer term. In another example, Narayanan (1985) and Rajan (1994) contend that managers’ reputation concerns could lead them to focus on the short-term stock prices at the expense of the firms’ long-term value. Alternatively, managers’ stock-based compensation or equity funding for new projects could also induce them to focus on firms’ short-term stock price performance.

The stock price $P_i$ at $t = 2$ is influenced by both firms’ accounting reports. Each firm’s financial reporting process is as follows. At $t = 1$, each manager privately observes the fundamental $s_i$. After observing his type $s_i$, each manager issues an accounting report, $r_i \in \{0, 1\}$. The good manager always reports truthfully in equilibrium, i.e., $r_i(s_i = 1) = 1$. The bad manager with $s_i = 0$ may manipulate the report. The probability that the bad manager successfully issues a fraudulent report, i.e., $r_i(s_i = 0) = 1$, is

$$\mu_i \equiv \Pr(r_i = 1|s_i = 0, m_i, q_i) = m_i(1 - q_i).$$

$\mu_i$ is determined jointly by the manager’s manipulation decision, $m_i$, and the firm’s internal controls choice, $q_i$. $m_i \in [0, 1]$ is the bad manager’s efforts to overstate the report. For brevity, we often use $m_i$ to denote the bad manager’s manipulation $m_i(s_i = 0)$ and omit the argument $s_i = 0$ whenever no confusion arises. Manipulation effort $m_i$ is the manager’s choice at $t = 1$ after he has observed $s_i$. $m_i$ reduces the firm’s long-term value by $C_i(m_i)$. $C_i(m_i)$ also has the standard properties of a cost function (similar to the Inada conditions): $C_i(0) = 0$, $C_i'(0) = 0$, $C_i''(m_i) > 0$ for $m_i > 0$, $C_i'(1) = \infty$ and $C_i'' > c$. $c$ is a constant sufficiently large to guarantee that the manager’s equilibrium manipulation choice is unique.

$q_i \in [0, 1]$ denotes the quality of the firm’s internal controls over financial reporting. It is interpreted as the probability that the manager’s overstatement is detected and prevented by the internal controls system. The firm chooses $q_i$ at $t = 0$ at a cost of $K_i(q_i)$. $K_i(q_i)$ has the standard properties of a cost function as well: $K_i(0) = 0$, $K_i'(0) = 0$, $K_i''(q_i) > 0$ for

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6 An alternative approach to model internal controls that has been used in the prior literature is that internal controls directly increase the marginal cost of manipulation. We have checked the robustness of our main result to this representation of internal controls and the detailed analyses are available from the authors upon requests.
$q_i > 0$, $K_i'(1) = \infty$, and $K_i'' > k$. $k$ is a constant sufficiently large to guarantee that the firms’ equilibrium internal controls choice is unique. Unlike $m_i$, the firm’s choice of $q_i$ is publicly observable.

Overall, the bad manager can take actions to inflate the report, but his attempts are checked by the internal controls system.

The firm’s net cash flow (i.e., the long-term firm value) can now be written as

$$V_i = s_i - C_i(m_i(s_i)) - K_i(q_i).$$

$V_i$ is lower than the gross cash flow $s_i$ by two terms, the cost of manipulation $C_i$, and the cost of internal controls $K_i$.7

Finally, the only connection between the two firms is that their gross cash flows, $s_i$, may be correlated. The correlation coefficient $\rho$ can be either positive, negative or zero.8 For ease of exposition, we assume $\rho \geq 0$ in the text but all formal results are proved for both $\rho \geq 0$ and $\rho < 0$.9

In sum, the timeline of the model is summarized as follows:

1. $t = 0$, firm $i$ publicly chooses its internal controls quality $q_i$;
2. $t = 1$, manager $i$ privately chooses manipulation $m_i$ after privately observing $s_i$;
3. $t = 2$, investors set stock price $P_i$ after observing both report $r_A$ and $r_B$;

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7We have modeled the cost of manipulation as a reduction in the firm’s long-term value. Both accrual manipulation and real earnings management are costly to the firm (e.g., Kedia and Philippon (2009)). When the manager engages in accrual manipulation, the cost includes not only the direct cost of searching for opportunities but also the indirect cost of the managers’ distraction and the associated actions to cover up the manipulation. Real earnings management directly distorts the firm’s decisions and decreases the firm’s cash flow. We also check that our results are robust to an alternative model setup in which the cost of manipulation is the manager’s private cost and ex-ante the firm would internalize a part of the manager’s ex-post manipulation cost through an individual participation constraint for the manager. The detailed analyses are available from the authors upon requests.

8For example, if $s_i$ refers to customer preference for American cars, then the gross cash flows for GM and Ford are positively correlated. However, if $s_i$ refers to a firm’s market share, then a higher $s_A$ for GM is likely to indicate a lower $s_B$ for Ford. There is empirical literature documenting the intra-industry information transfer, e.g., Foster (1981), Han, Wild, and Ramesh (1989), and Thomas and Zhang (2008).

9$\rho \in [\rho, 1]$ is bounded from below by $\rho \equiv \max\left( -\frac{\theta_A}{1-\theta_A}, \frac{\theta_B}{1-\theta_B} \right)$, instead of $-1$, $\rho < 0$, and approaches $-1$ (i.e., two Bernoulli variables are perfectly negatively correlated) only if their marginal probabilities satisfy $\theta_A = 1 - \theta_B$.
4. $t = 3$, cash flows are realized and paid out.

The equilibrium solution concept is Perfect Bayesian Equilibrium (PBE). A PBE is characterized by the set of decisions and prices, $\{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_A, r_B)\}$, such that

1. $q_t^* = \arg \max_{q_t} E_0[V_i]$ maximizes the long-term firm value expected at $t = 0$;

2. $m_t^*(s_i) = \arg \max_{m_i(s_i)} E_1[U_i(s_i)]$ maximizes the manager’s payoff expected at $t = 1$ after observing $s_i$;

3. $P_t^*(r_A, r_B) = E_2[V_i(r_A, r_B)]$ is set to be equal to investors’ expectation of the long-term firm value, conditional on both firms’ reports $(r_A, r_B)$;

4. The players have rational expectations at each date. In particular, both the manager’s and investors’ beliefs about the other manager’s manipulation are consistent with Bayes rule, if possible.

3 The Analysis

In this section, we solve the model by backward induction. We first examine how one manager’s manipulation decision is influenced by his expectation of the manipulation of his peer firm’s report and then study the design of internal controls.

3.1 Equilibrium manipulation decisions

3.1.1 Equilibrium manipulation with no correlation

We are interested in how one manager’s manipulation is affected by his expectation of the other manager’s manipulation. We first study a benchmark in which two firms’ fundamentals are not correlated ($\rho = 0$). Investors use only firm A’s report to set price, and two firms’ manipulation decisions are independent. Thus, we also refer to this benchmark as a single-firm benchmark.
Investors at $t = 2$ observe report $r_A$ and use it to update their belief about $s_A$. Denote their posterior belief as $\theta_A(r_A) \equiv \Pr(s_A = 1|r_A)$.

It is straightforward to see $\theta_A(0) = 0$. Since the good firm always issues the favorable report $r_A = 1$ and only the bad firm may issue the unfavorable report $r_A = 0$, investors learn for certain that the firm issuing $r_A = 0$ is a bad type.

Upon observing the favorable report $r_A = 1$, investors are uncertain about the firm type:

\[
\theta_A(1) \equiv \Pr(s_A = 1|r_A = 1) = \frac{\theta_A}{\theta_A + (1 - \theta_A)\mu_A^*}.
\]  
(2)

The denominator is the total population of firms issuing the favorable report $r_A = 1$. It can be issued by either the good firm ($s_A = 1$) or the bad firm with successful manipulation ($s_A = 0$). The population of the former is $\theta_A$ and of the latter is $(1 - \theta_A)\mu_A^*$. $\mu_A^*$ is investors’ conjecture of the probability of the bad type succeeding in manipulation. Since they do not observe the manager’s actual choice of manipulation $m_A$, investors conjecture that the bad manager has chosen $m_A^*$ in equilibrium (and that the good manager does not manipulate). Since $q_A$ is observable to investors at $t = 2$, investors thus conjecture $\mu_A^* = m_A^*(1 - q_A)$.

Equation 2 shows that $\theta_A(1)$ is decreasing in $\mu_A^*$. Upon observing $r_A = 1$ investors are more optimistic when the equilibrium manipulation is lower. If the bad firm cannot manipulate (i.e., $\mu_A^* = 0$), then $\theta_A(1) = 1$ and investors take the favorable report at face value. If the bad firm always succeeds in manipulation (i.e., $\mu_A^* = 1$), then $\theta_A(1) = \theta_A$ and investors completely discard the favorable report. If the probability of manipulation is in between, $\theta_A(1) \in (\theta_A, 1)$. Finally, we measure the information asymmetry between the bad manager $A$ and investors as their expected disagreement about the state. It can be shown to be equal to $\mu_A^*\theta_A(1)$\[10\]. Thus, the equilibrium information asymmetry is increasing in $\mu_A^*$.

Upon observing $r_A$, investors set stock price $P_A^*(r_A)$ to be equal to their expectation of the firm value $V_i$:

\[
P_A^*(r_A) = \theta_A(r_A) + (1 - \theta_A(r_A))(0 - C_A^*) - K_A(q_A).
\]  
(3)

\[10\]The math is as follows: $E_{r_A|s_A=0}[s_A - E[s_A|r_A]] = \Pr(r_A = 1|s_A = 0)\theta_A(1) = \mu_A^*\theta_A(1)$. Since the bad manager’s action is our focus, we refer to the information asymmetry between the bad manager and investors simply as the information asymmetry for expositional ease whenever no confusion may arise.
The good firm generates a gross cash flow of 1. The bad firm generates a gross cash flow of 0 and incurs the manipulation cost of $C_A^* = C_A(m_A^*)$. Both types pay the internal controls cost $K_A(q_A)$.

It is obvious that $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*) > 0$. Investors pay a higher price for the favorable report, despite the manipulation. As a result, the manager who cares about the short-term stock price prefers report $r_A = 1$ to $r_A = 0$.

Anticipating the investors' pricing response to report $P_A^*(r_A)$, the bad manager chooses $m_A$ to maximize his expected utility $E_1[U_A(m_A)|s_A = 0]$ defined in Equation 1. We denote the manager's best response to the investors' conjecture $m_A^*$ as $\tilde{m}_A(m_A^*)$ or simply $\tilde{m}_A$. Its first-order condition is

$$H_A(m_A)|_{m_A = \tilde{m}_A(m_A^*)} = \delta_A \frac{\partial \mu_A}{\partial m_A} \theta_A(1)(1 + C_A^*) - (1 - \delta_A) C_A'(m_A) = 0. \tag{4}$$

The first term in equation 4 is the marginal benefit of manipulation. It increases the firm's chance of issuing the favorable report $r_A = 1$, at a marginal rate of $\frac{\partial \mu_A}{\partial m_A} = 1 - q_A$.

The favorable report, in turn, increases its stock price by $P_A^*(1) - P_A^*(0) = \theta_A(1)(1 + C_A^*)$.

The second term is the marginal cost. Manipulation reduces the firm's future cash flow by $C_A(m_A)$. Since the manager has a stake of $1 - \delta_A$ in the long-term firm value, he bears part of the manipulation cost as well. The manager thus chooses the optimal manipulation level such that the marginal benefit is equal to the marginal cost. We use $H_A(m_A)$ defined in Equation 4 to denote the difference between the marginal benefit and the marginal cost for an arbitrary manipulation $m_A$. Therefore, $\tilde{m}_A(m_A^*)$, defined by $H_A(\tilde{m}_A(m_A^*)) = 0$, characterizes the manager’s best response to investors’ conjecture $m_A^*$. In equilibrium, the manager’s optimal choice is consistent with the investors’ conjecture, that is, $\tilde{m}_A(m_A^*) = m_A^*$. This rational expectations requirement implies that the optimal choice $m_A^*$ is defined by $H_A(m_A^*) = 0$. The regularity conditions that both the manipulation cost and the internal controls cost functions are sufficiently convex guarantees that this fixed point problem has a unique solution, as we prove in the appendix. With the unique equilibrium determined, we conduct comparative

\footnote{We normalize the gross cash flow of the bad type to be 0. As a result, we have negative net cash flow for the bad firm. This can be easily fixed by introducing a positive baseline gross cash flow that is large enough to cover the costs of manipulation and internal controls. Such a setting complicates the notations but would affect none of our formal results.}
statics to understand the determinants of the manager’s optimal manipulation choice.

Lemma 1 When \( \rho = 0 \), there exists a unique equilibrium \( \{ m_A^*(s_A), P_A^*(r_A) \} \) characterized collectively by Equation 4, \( m_A^*(s_A = 1) = 0 \), and Equation 3, \( m_A^*(s_A = 0) \) is increasing in \( \theta_A \) and \( \delta_A \), and decreasing in \( q_A \).

These properties of the equilibrium manipulation decisions are intuitive. First, \( m_A^* \) is increasing in the investors’ prior belief \( \theta_A \). When \( \theta_A \) is higher, the information asymmetry between the bad manager and investors is larger, that is, \( \mu_A^* \theta_A(1) \) is increasing in \( \theta_A \). The bad manager knows \( s_A = 0 \). In contrast, investors receive report \( r_A = 1 \) with probability \( \mu_A^* \). Upon receiving the favorable report, they believe that the firm is good with probability \( \theta_A(1) \). The bad manager takes advantage of this optimism by increasing manipulation. Second, the manager manipulates more if he has a stronger capital market concern. Third, internal controls reduce manipulation. All else being equal, an improvement in internal controls quality detects manipulation more often and reduces the information asymmetry between the bad manager and investors. That is, fixing \( m_A^* \), \( \mu_A^* \theta_A(1) \) is decreasing in \( q_A \). As a result, the bad manager manipulates less.

Our single-firm benchmark is a variant of the signal-jamming model (e.g., Stein [1989]) with an important difference. It has the defining feature that even though the manager attempts to affect investors’ belief through unobservable and costly manipulation, investors with rational expectations are not systematically misled. On average they see through the manipulation and break even. As such, the manipulation eventually hurts the firm value through the distorted decisions.

Our model differs from Stein (1989) in that manipulation reduces the report’s informativeness. As a result, the information asymmetry between investors and the bad manager persists in equilibrium and increases in manipulation. The manager knows his type while investors observe only a report that may have been manipulated. This information asymmetry becomes larger when the manager’s equilibrium manipulation is larger. Facing this information asymmetry, investors rationally discount report \( r_A = 1 \) to price-protect themselves. The same stock price \( P_A^*(1) \) is paid to both the good firm \( (s_A = 1) \) and the bad firm \( (s_A = 0) \).
with successful manipulation. However, such nondiscriminatory discounting implies that the stock price $P_A(1)$ is too low for the good manager, but too high for the bad with successful manipulation. Even though manipulation does not systematically mislead investors, it does reduce the report’s informativeness to investors.

### 3.1.2 Equilibrium manipulation with correlated fundamentals

We now examine the main model with $\rho > 0$ in which investors also use the peer firm’s report in their pricing decisions. We focus on the strategic complementarity between two managers’ manipulation decisions: does manager $A$ manipulate more if he expects that manager $B$ will manipulate more? For instance, suppose that there is a shock that increases manager $B$’s capital market concern, $\delta_B$. Both investors and manager $A$ observe this shock and conjecture that the equilibrium $\mu_B^*$ will be higher (according to Lemma 1). We are interested in the question that how manager $A$ responds to this expected increase in $\mu_B^*$.

We adapt the notation to accommodate the addition of firm $B$. Investors now use both report $r_A$ and $r_B$ to update their beliefs and set the stock price. Denote $\theta_A(r_A, r_B) \equiv \Pr(s_A = 1|r_A, r_B)$ as investors’ posterior about firm $A$ being a good type after observing reports $r_A$ and $r_B$. We also use $\phi$ to denote a null signal. For example, $\theta_A(r_A, \phi)$ is the investors’ posterior after observing $r_A$ but before observing $r_B$. Thus, $\theta_A(r_A, \phi) = \theta_A(r_A)$, which is defined in the single-firm case. Similarly, we denote $P_A(r_A, r_B)$ as the stock price conditional on $r_A$ and $r_B$. In addition, although both investors and manager $A$ observe firm $B$’s internal controls $q_B$ and report $r_B$, neither observes manager $B$’s actual choice of manipulation $m_B$. Thus, both investors and manager $A$ have to conjecture manager $B$’s equilibrium manipulation choices. Rational expectations require that in equilibrium the conjectures by both investors and manager $A$ are the same as manager $B$’s equilibrium choice $m_B^*$. Moreover, since $q_B$ is observable, investors and manager $A$ conjecture that the probability that manager $B$ successfully issues a fraudulent report is $\mu_B^* = m_B^*(1 - q_B)$.

Investors use report $r_A$ and $r_B$ to update their belief about $s_A$. Note that $r_A$ and $r_B$ are independent conditional on $s_A$. Thus, investors update their belief in two steps. First, investors use $r_A$ to update their belief from prior $\theta_A$ to posterior $\theta_A(r_A, \phi)$, just as in the
benchmark case of no correlation. Second, treating \( \theta_A(r_A, \phi) \) as a new prior for \( s_A \), investors then use report \( r_B \) to update their belief.\(^{12}\)

As in the benchmark case (Equation 3), the unfavorable report \( r_A = 0 \) reveals \( s_A = 0 \) perfectly. As a result, the additional report \( r_B \) does not affect investors’ belief, i.e., \( \theta_A(0, \phi) = \theta_A(0, 1) = \theta_A(0, 0) \). Thus we focus on the favorable report \( r_A = 1 \).

Conditional on \( r_A = 1 \), report \( r_B \) will also affect investors’ belief about \( s_A \). Because \( s_B \) is positively correlated with \( s_A \), report \( r_B = 1 \) (\( r_B = 0 \)) revises investors’ expectation of \( s_A = 1 \) upward (downward), i.e., \( \theta_A(1, 1) > \theta_A(1, \phi) > \theta_A(1, 0) \). Moreover, as we discussed in the single-firm case, manipulation reduces the informativeness of the favorable report. Thus, upon observing \( r_B = 1 \), investors revise their belief upward by a smaller amount if the equilibrium manipulation \( \mu_B^* \) is higher. Investors attach less credibility to \( r_B = 1 \), as they conjecture that it is more likely to be from a bad manager with successful manipulation. We summarize these discussions in the next lemma.

**Lemma 2** When \( \rho > 0 \), \( \theta_A(1, 1) > \theta_A(1, \phi) > \theta_A(1, 0) \). Moreover, \( \theta_A(1, 1) - \theta_A(1, \phi) \) is decreasing in \( \mu_B^* \).

The equilibrium stock price \( P_A^*(r_A, r_B) \) is now set to be equal to investors’ expectations of the firm value \( V_t \) conditional on \( r_A \) and \( r_B \):

\[
P_A^*(r_A, r_B) = \theta_A(r_A, r_B) + (1 - \theta_A(r_A, r_B))(0 - C_A^*) - K_A(q_A). \tag{5}
\]

With investors’ equilibrium response to the reports, we can verify that, as in the single-firm case, manager \( A \) has incentive to manipulate because \( P_A^*(1, r_B) - P_A^*(0, r_B) = \theta_A(1, r_B)(1 + \ldots

\(^{12}\)The claim is proved as follows. We can rewrite the conditional probability

\[
Pr(s_A|r_A, r_B) = \frac{Pr(s_A|r_B)Pr(r_A|s_A, r_B)}{\sum_{s_A} Pr(s_A|r_B)Pr(r_A|s_A, r_B)} = \frac{Pr(s_A|r_B)Pr(r_A|s_A)}{\sum_{s_A} Pr(s_A|r_B)Pr(r_A|s_A)}.
\]

The first step uses the conditional-probability function definition and the Bayes rule, while the second step utilizes the conditional independence result that \( Pr(r_A|s_A, r_B) = Pr(r_A|s_A) \). The rewriting makes clear that adding report \( r_B \) to investors’ information set is equivalent to replacing investors’ prior of \( Pr(s_A) \) with a new one, \( Pr(s_A|r_B) \). Similarly, we can change the order of \( r_A \) and \( r_B \):

\[
Pr(s_A|r_A, r_B) = \frac{Pr(s_A|r_A)Pr(r_B|s_A, r_A)}{\sum_{s_A} Pr(s_A|r_A)Pr(r_B|s_A, r_A)} = \frac{Pr(s_A|r_A)Pr(r_B|s_A)}{\sum_{s_A} Pr(s_A|r_A)Pr(r_B|s_A)}.
\]
\( C_A^* > 0 \) for any \( \tau_B \).

Anticipating the investors’ pricing response, manager \( A \) chooses manipulation at \( t = 1 \). At the time that manager \( A \) chooses his manipulation, he does not observe report \( r_B \) or manager \( B \)’s actual choice of manipulation \( m_B \). Instead, he conjectures that manager \( B \) will choose manipulation \( m_B^* \) and thus succeed in issuing a fraudulent report with probability \( \mu_B^* = m_B^*(1 - q_B) \). His best response to \( \mu_B^* \), denoted as \( \tilde{m}_A(\mu_B^*) \), is determined by the following first-order condition:

\[
H^A(m_A; m_B^*)|_{m_A = \tilde{m}_A} \equiv \delta_A \frac{\partial \mu_A}{\partial m_A} E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A^*) - (1 - \delta_A) C_A'(m_A) = 0. \tag{6}
\]

Equation (6) is similar to Equation (4) in the single-firm case except that \( \theta_A(1) \) is replaced by \( E_{r_B}[\theta_A(1, r_B)|s_A = 0] \) (after we have imposed investors’ rational expectations about \( m_A^* \)). Manager \( A \) bases his manipulation on his expectation about investors’ belief about his firm averaged over report \( r_B \). Since \( \mu_B^* \) is not affected by manager \( A \)’s actual choice of \( m_A \), we can treat \( \mu_B^* \) as a parameter in Equation (6) and examine the manager’s best (manipulation) response to parameter \( \mu_B^* \). Since \( \tilde{m}_A(\cdot) \) is the manager’s best response to both \( m_B^* \) and \( \mu_B^* \) to save on notations.

**Proposition 1** \( \tilde{m}_A(\mu_B^*) \), defined by Equation (6), characterizes the bad manager’s unique best response to the probability that manager \( B \) issues a fraudulent report. For any interior \( \mu_B^* \), \( \tilde{m}_A(\mu_B^*) \) is increasing in \( \mu_B^* \).

Proposition 1 resembles the peer pressure for manipulation discussed in the introduction: manager \( A \) manipulates more when he expects manager \( B \) to manipulate more.

We first illustrate the intuition behind Proposition 1 with an extreme case in which \( \rho = 1 \). Suppose we start with \( \delta_B = 0 \) (and \( \delta_A > 0 \)) so that \( \mu_B^* = 0 \). Since manager \( B \) never manipulates, investors learn \( s_B \) perfectly upon observing \( r_B \). The perfect correlation implies that investors also learn \( s_A \) perfectly. As a result, the information asymmetry between the bad manager \( A \) and investors vanishes, which in turn deters manipulation by manager \( A \) in equilibrium\(^{13}\) Now suppose \( \delta_B \) has increased from 0 to be positive. Both investors and

\(^{13}\)In presence of firm \( B \), the information asymmetry between the bad manager \( A \) and investors with respect
manager $A$ expect that $\mu_B^* > 0$. We can show that the bad manager $A$ will respond by engaging in a positive amount of manipulation. Upon observing $r_B = 1$, the bad manager $A$ privately knows that $s_B = 1$ with probability 0 but investors believe that there is a positive probability that $s_B = 1$. The manipulation $\mu_B^*$ creates an information asymmetry between the bad manager $A$ and investors about $s_A$. As a result, investors pay a higher price for $r_A = 1$ than $r_A = 0$, resulting in a positive benefit of manipulation. Therefore, as $\mu_B^*$ moves away from 0, manager $A$ starts to manipulate as well.

We now explain the intuition for the general case of $\rho > 0$. We start with investors’ expectation about firm $A$’s type conditional on $r_A = 1$:

$$E_{r_B}[\theta_A(1, r_B)] = \theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)[\theta_A(1, 1) - \theta_A(1, 0)]. \tag{7}$$

Investors’ expectation of $s_A$ is the probability weighted posterior belief conditional on $r_B$. $\mu_B^*$ affects both the distribution of $r_B$ (i.e., $\Pr(r_B = 1|r_A = 1)$) and investors’ posterior belief about $s_A$ upon observing $r_B = 1$ (i.e., $\theta_A(1, 1)$). However, the law of iterated expectations implies that $E_{r_B}[\theta_A(1, r_B)] = \theta_A(1)$, which is independent of $\mu_B^*$. Thus, investors’ rationality implies that the dual effects of $\mu_B^*$ must cancel out each other perfectly so that $\mu_B^*$ does not affect $E_{r_B}[\theta_A(1, r_B)]$.

We now turn to $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$, the bad manager $A$’s expectation of investors’ expectation, which can be written out in a similar way to Equation 7:

$$E_{r_B}[\theta_A(1, r_B)|s_A = 0] = \theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)[\theta_A(1, 1) - \theta_A(1, 0)]. \tag{8}$$

The comparison with Equation 7 reveals that manager $A$’s expectation of investors’ expectation differs from investors’ expectation due to their different information sets. The bad manager $A$ knows $s_A = 0$, but investors observe only $r_A = 1$ and report $r_B$ to update their belief about $s_A$. As $\mu_B^*$ increases, report $r_B$ becomes less informative about firm $A$ (and firm $B$) and the information asymmetry between the bad manager $A$ and investors gets larger.

to $s_A$ is measured in the same way as in the single-firm benchmark except that the expectation is now taken over $r_B$ as well. That is, it is measured as $E_{r_A, r_B|s_A = 0}[s_A - E[s_A|r_A, r_B]] = \mu_A E_{r_B}[\theta_A(1, r_B)|s_A = 0]$. 

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As explained in the single-firm case, the greater information asymmetry, in turn, encourages manager A to manipulate more, making the two managers’ manipulation decisions strategic complements.

Another way to see the intuition is as follows. [Kartik, Lee, and Suen (2017)] have proven a general statistical result that, loosely speaking, when two agents have different priors, either agent (say A) expects a more informative experiment to bring the other’s posterior belief closer to A’s prior. The authors call this the “information-validates-the-prior (IVP) theorem.” The IVP theorem provides another way to view the intuition of Proposition 1.

Before \( r_B \) is provided, bad manager A’s belief about \( s_A \) is lower than investors’. Firm B’s report can be viewed as an informative (albeit endogenous) experiment about \( s_A \). As manager B increases equilibrium manipulation, report B becomes less informative. As a result, manager A expects investors’ belief of \( s_A \) to move upward and further away from his belief and he thus manipulates more accordingly. Therefore, Proposition 1 provides another application of the IVP theorem.

To fully pin down the effect of the internal controls on manipulation, we have to endogenize firm B’s manipulation decision as well. Using the same procedure, we can solve manager B’s best response to manager A’s equilibrium choice, \( \tilde{m}_B^*(m_A^*) \), which is characterized by an equation similar to Equation 6. The optimal manipulation decisions \( (m_A^*, m_B^*) \) are jointly determined by intercepting the two best response functions through \( m_A^* = \tilde{m}_B^*(m_A^*) \) and \( m_B^* = \tilde{m}_A^*(m_B^*) \). Since managers observe the firms’ internal controls choices, \( (m_A^*, m_B^*) \) can be expressed as functions of the firms’ choices of internal controls \( (q_A, q_B) \). Now we can examine the equilibrium effect of internal controls on manipulation.

**Proposition 2** The unique pair of managers’ best responses to firms’ internal controls decisions, \( \{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\} \), are collectively characterized by Equation 6 and its counterpart for manager B. Moreover, \( \frac{dm_A^*(q_A, q_B)}{dq_A} < 0 \) and \( \frac{dm_B^*(q_B, q_A)}{dq_A} < 0 \).

Proposition 2 confirms that a firm’s internal controls have an externality on its peer firm. An improvement in one firm’s internal controls quality reduces not only its own manager’s manipulation (i.e., \( \frac{dm_A^*(q_A, q_B)}{dq_A} < 0 \)), but also the peer manager’s manipulation (i.e.,
\[
\frac{dm^*_{B}(q_B,q_A)}{dq_A} < 0 \]

via mitigation of the peer pressure for manipulation. Lemma 1 shows that a firm’s internal controls directly deter its own manager’s manipulation and reduce \( \mu^*_A \) (the probability that report \( A \) is manipulated). Proposition 1 suggests that, a lower \( \mu^*_A \) results in a lower \( m^*_B \). In other words, firm \( A \)’s internal controls indirectly mitigate the peer pressure for manipulation on manager \( B \), and reduce manager \( B \)’s manipulation. Of course, the reduction of manipulation by manager \( B \) alleviates the pressure on manager \( A \) as well, setting into motion a loop of feedbacks. Through this loop, the effect of a firm’s internal controls on its own manager’s manipulation is amplified.

### 3.2 Equilibrium internal controls decisions

In the previous section, we characterized the managers’ manipulation decisions at \( t = 1 \), taking the firms’ internal controls choices \( q_A \) and \( q_B \) at \( t = 0 \) as observable inputs. In this section, we endogenize the firms’ internal controls decisions. We show that even though firms have a private incentive to invest in internal controls, they under-invest in internal controls due to the externality described in Proposition 2.

After understanding managers’ manipulation decisions, we fold back to \( t = 0 \) and consider firm \( A \)’s private incentive to invest in costly internal controls over financial reporting. Since firm \( A \) does not observe firm \( B \)’s choice of internal controls at the time of choosing \( q_A \), it conjectures that firm \( B \) will choose \( q^*_B \) in equilibrium. Moreover, since managers at \( t = 1 \) will observe firms’ internal controls choices, firm \( A \) anticipates that managers at \( t = 1 \) will respond to its actual choice of \( q_A \) through \( m^*_A(q_A,q_B^*) \) and \( m^*_B(q_B^*,q_A) \). Based on these expectations, the firm value at \( t = 0 \) as a function of its internal controls choice \( q_A \) is

\[
V_{A0}(q_A; q_B^*) = E_0[V_A(q_A, q_B^*)] = \theta_A - \Pr(s_A = 0)C_A(m^*_A(q_A, q_B^*)) - K_A(q_A). \tag{9}
\]

The firm value at \( t = 0 \) has three components. The first is the expected gross cash flow \( E_{C_A}[s_A] = \theta_A \) in the absence of manipulation and internal controls investment. Second, manipulation generates the expected deadweight loss \( \Pr(s_A = 0)C_A(m^*_A(q_A, q_B^*)) \). The existence of this deadweight loss means that the firm has a private incentive to invest in internal
controls to prevent manipulation. Finally, the internal controls investment itself consumes resources and reduces the firm value by \( K_A(q_A) \).

Firm \( A \) at \( t = 0 \) chooses \( q_A \) to maximize \( V_{A0}(q_A, q_B^*) \) subject to the managers’ subsequent equilibrium manipulation responses \( m_A^*(q_A, q_B^*) \) and \( m_B^*(q_B^*, q_A) \). Differentiating \( V_{A0}(q_A, q_B^*) \) in Equation 9 with respect to \( q_A \), the first-order condition is

\[
\frac{\partial V_{A0}}{\partial m_A^*} \frac{dm_A^*}{dq_A} - K_A'(q_A) = 0.
\]

The first term \( \frac{\partial V_{A0}}{\partial m_A^*} \frac{dm_A^*}{dq_A} \) captures the benefit of internal controls \( q_A \) to firm \( A \) in reducing manipulation, which is balanced by the marginal cost \( K_A'(q_A) \). Note that \( \tilde{q}_A^*(q_B^*) \) implied by the first-order condition is firm \( A \)'s best response to its conjecture of firm \( B \)'s equilibrium internal controls choice \( q_B^* \). We can solve firm \( B \)'s internal controls decision using the same procedure. Taking \( q_A^* \) as given, firm \( B \)'s best response to \( q_A^* \), \( \tilde{q}_B^*(q_A^*) \), is characterized by a similar first-order condition. Intercepting the two best responses, \( q_A^* = \tilde{q}_A^*(q_B^*) \) and \( q_B^* = \tilde{q}_B^*(q_A^*) \), we pin down the firms’ optimal internal controls decisions. Since managers and investors observe the firms’ internal controls decisions, their equilibrium decisions are obtained by substituting \( (q_A^*, q_B^*) \) to their respective best response functions we have characterized earlier. This completes the characterization of the entire equilibrium.

**Proposition 3** The unique equilibrium \( \{q_A^*, q_B^*, m_A^*(s_A), m_B^*(s_B), P_A^*(r_A, r_B), P_B^*(r_B, r_A)\} \) is collectively characterized by Equation 10, Equation 6, \( m_A^*(s_A = 1) = 0 \), Equation 5 and their counterparts for firm \( B \).

After we have solved for the entire equilibrium, we evaluate the efficiency of the privately optimal internal controls investment. Proposition 3 shows that firms do have private incentives to invest in internal controls. But are the privately optimal choices of internal controls efficient from a social perspective? We use the Pareto efficiency criterion. The privately optimal internal controls investment pair \( \{q_A^*, q_B^*\} \) is not efficient if there exists a pair of internal controls investments \( \{q_A', q_B'\} \) such that both firms are weakly and at least one firm is strictly better off under \( \{q_A', q_B'\} \) than under their private choices, i.e., \( V_{A0}(q_A', q_B') \geq V_{A0}(q_A^*, q_B^*) \).
and \( V_{B0}(q'_B, q'_A) \geq V_{B0}(q^*_B, q^*_A) \) with at least one strict inequality. Moreover, the firms under-invest in internal controls if \( q'_A > q^*_A \) and \( q'_B > q^*_B \).

**Proposition 4** The privately optimal choices of internal controls \( \{q^*_A, q^*_B\} \) characterized in Proposition 3 are not Pareto efficient. Moreover, firms under-invest in internal controls.

The key intuition behind Proposition 4 is that the internal controls investment in firm \( A \) exerts a positive externality on firm \( B \). More specifically, Proposition 2 suggests that higher internal controls by firm \( A \) reduces manager \( B \)'s manipulation, \( \frac{dm_B}{dq_A} < 0 \). The reduction in \( m^*_B \) in turn improves the value of firm \( B \). Because of this positive externality, the social planner can implement an upward deviation of \( q_A \) from firm \( A \)'s private optimal choice \( q^*_A \), which benefits firm \( B \). Note that firm \( A \) is not hurt by this deviation because in the neighborhood of \( q_A = q^*_A \), the marginal effect of increasing \( q_A \) on firm \( A \)'s value is zero.

To see the intuition in an alternative way, consider a social planner who maximizes the sum of the two firms’ value. From the social planner’s perspective, the social marginal return of increasing \( q_A \) from the equilibrium level of \( \{q^*_A, q^*_B\} \) is positive, that is,

\[
\frac{\partial V_{A0}}{\partial m^*_A} \frac{dm^*_A}{dq_A} + \frac{\partial V_{B0}}{\partial m^*_B} \frac{dm^*_B}{dq_A} - K'_A(q^*_A) = \frac{\partial V_{B0}}{\partial m^*_B} \frac{dm^*_B}{dq_A} > 0.
\]

The first equality follows because of the first-order condition for \( q^*_A \) in equation 10. The term \( \frac{\partial V_{B0}}{\partial m^*_B} \frac{dm^*_B}{dq_A} \) is positive because of the positive externality firm \( A \)'s internal controls exert on firm \( B \) (Proposition 2) and because of the fact that manipulation reduces the firm value. Therefore, the externality of the firm’s internal controls leads to an underinvestment problem.

Proposition 4 suggests that a coordination failure could exist among firms’ individual choices of internal controls over financial reporting, which provides one rationale for intervention in firms’ internal controls investment. In the presence of peer pressure for manipulation, one firm’s internal controls investment has a positive externality on other firms. A floor of internal controls investment could improve the firm value of all firms. Looking through the lens of Proposition 4 the proposal in Romano (2005) that the internal controls mandates in SOX should be made optional is flawed. Competition among firms (or state laws) does not lead to a socially optimal investment in internal controls.
4 Extensions

4.1 Continuous state and message spaces

We have employed the binary state and message spaces in the baseline model to highlight the economic forces behind our main result: namely there exists a positive externality among firms’ internal controls investments. The positive externality arises from the strategic complementarity between managers’ manipulation decisions. A manager manipulates more when he expects that reports from his peer managers are also more likely to be manipulated. In this section, we extend our analysis to continuous state and message spaces. This extension serves two purposes. First, it helps to identify the broad conditions under which our main result—that internal controls investment has a positive externality—holds. Second, it formalizes the difference between two related notions, strategic complementarity and spillover effect, and better connects our paper to the prior literature.

The continuous extension is the same as the baseline binary model except for the state and message spaces. First, we extend the state space to be continuous. The profitability of each firm, \( s_i, \ i \in \{A, B\} \), is drawn for a normal distribution with mean \( \bar{s} \) and variance \( \sigma_s^2 \). Moreover, \( s_A \) and \( s_B \) are correlated with a correlation coefficient \( \rho \in [-1, 1] \). Second, we also extend the message space to be continuous and modify the manipulation technology accordingly. Specifically, at \( t = 1 \) after observing \( s_i \), manager \( i \) chooses manipulation \( m_i \) at a cost \( C_i(m_i) \) that affects the report \( r_i \) in the following way

\[
r_i = s_i + m_i (1 - q_i) + h(m_i)\tilde{\varepsilon}_i.
\]

\( m_i \geq 0 \) is the manager’s manipulation choice. \( q_i \in [0, 1] \) is the firm’s internal controls investment. \( \tilde{\varepsilon}_i \) is a standard normal random variable that is independent of all other variables in the model. \( h(m_i) \) is an function of \( m_i \) that satisfies \( h(m_i) \geq 0 \). Moreover, we assume either \( h'(m_i) = 0 \) or \( h'(m_i) > 0 \) for any \( m_i \geq 0 \). The former assumes that manipulation does not affect the report’s variance, as used in Stein (1989). An example is \( h(m_i) = h_0 \) with \( h_0 \) being a positive constant. The latter, \( h'(m_i) > 0 \) for any \( m_i \geq 0 \), stipulates that manipulation
strictly reduces the report’s informativeness. An example is \( h(m_i) = m_i \). In Appendix II, we provide one micro-foundation for \( h'(m_i) > 0 \).

**Proposition 5** In this continuous setting,

1. if \( \rho \neq 0 \) and \( h'(m_i) > 0 \) for any \( m_i \geq 0 \), managers’ manipulation decisions are strategic complements, and firms under-invest in internal controls;

2. if \( \rho = 0 \) or \( h'(m_i) = 0 \) for any \( m_i \geq 0 \), managers’ manipulation decisions are independent, and firms’ internal controls investment decisions are Pareto efficient.

Proposition 5 identifies the exact conditions under which both strategic complementarity between managers’ manipulation decisions and inefficiency in firms’ internal controls decisions arise in a continuous setting. As in the baseline model, the inefficiency in firms’ internal controls investment decisions arises from the investment’s positive externality, which in turn results from the strategic complementarity between managers’ manipulation decisions. For two firms with correlated fundamentals, the necessary and sufficient condition for the strategic complementarity between their manipulation decisions is that manipulation reduces the report’s informativeness. That strategic complementarity does not arise in a Stein setting highlights that the driving economic force behind our strategic complementarity result is that manipulation reduces the report’s informativeness, a natural feature in the binary setting.

How robust is the economic force that manipulation reduces the report’s informativeness? In our view, it is at least as robust as, if not more than, the opposite—that is, manipulation does not affect the report’s informativeness. This seems particularly true in our context of studying such an internal controls investment regulation as SOX. The motivation for SOX arose partly from concern about the adverse consequences of manipulation, of which the degradation in information quality was an important part.\(^{14}\)

\(^{14}\)In contrast, Stein (1989) is not concerned about whether manipulation reduces the report’s informativeness. His model was motivated to demonstrate the possibility that myopic behavior (in the form of manipulation) can arise even if investors in the capital market have rational expectations, as an answer to Michael Jensen’s claim that forward-looking capital market should discipline managers from taking short-term activities. The particular feature in Stein—that manipulation does not affect the report’s informativeness—seems unimportant for his argument and makes his model more elegant in making his point. In other words, Stein could have made his point with a setting of our continuous model, even though the extra feature of manipulation reducing informativeness would be distracting to making his point.
The continuous extension also highlights the distinction between two notions: spillover and strategic complementarity. We define spillover as the effect that the presence of report $r_B$ affects manager $A$’s manipulation; that is, spillover effect exists if $m_A^{\ast}|_{\text{SingleFirm}} \neq m_A^{\ast}|_{\text{TwoFirms}}$. In contrast, we define strategic complementarity as the effect that a change in manager $B$’s equilibrium manipulation has on manager $A$’s manipulation; that is, $\frac{\partial m_A^{\ast}}{\partial m_B^{\ast}} > 0$.

We summarize their relation in the following proposition.

**Proposition 6** *In this continuous setting,*

1. spillover exists if and only if $\rho \neq 0$;
2. strategic complementarity exists if and only if $\rho \neq 0$ and $h'(m_i) > 0$ for any $m_i \geq 0$.
3. if $\rho \neq 0$ and $h'(m_i) = 0$ for any $m_i \geq 0$, there exists spillover but no strategic complementarity.

Proposition 6 explains the relation between spillover and strategic complementarity (as we have defined them above). The two notions are related but different. The correlation of two firms’ fundamentals is necessary for both the spillover and strategic complementarity. This correlation is also sufficient for the spillover effect but not sufficient for the strategic complementarity. In other words, a case could exist in which the spillover effect is present, but the strategic complementarity is not. Such a case arises when $\rho \neq 0$ but $h'(m_i) = 0$.

As a result, in a multi-firm variant of Stein setting, there would be spillover but no strategic complementarity.

The distinction between spillover and strategic complementarity is important for our research question. It is the strategic complementarity, not the spillover effect, that drives the positive externality of internal controls investment that our paper focuses on. Internal controls investment has a positive externality if and only if strategic complementarity exists between two managers’ manipulation decisions. In contrast, spillover per se does not lead to the externality of internal controls investment, as shown in Proposition 5. When there is spillover but no strategic complementarity (i.e., $\rho \neq 0$ and $h'(m_i) = 0$), firm $A$’s internal
controls reduce manager A’s manipulation but do not affect manager B’s manipulation. As a result, no externality exists in firms’ internal controls investment.

Since manipulation reduces the report’s informativeness in the binary structure, both the spillover effect and the strategic complementarity arise simultaneously in our baseline model. As a result, it is difficult to see their differences in the binary setting.

4.2 An alternative manipulation technology

In this extension, we consider another alternative manipulation technology, namely that manipulation cost is incurred only when it succeeds. This extension is the same as our main setup with the following differences. First, the probability that the bad manager issues a fraudulent good report depends on two independent events: 1) the manager manipulates the report into a fraudulent good report \( r_i = 1 \) with probability \( m_i \); 2) upon receiving \( r_i \), the internal controls system can detect and prevent the manager’s manipulation with probability \( q_i \). When the internal controls system successfully prevents manipulation, the manager reports \( s_i = 0 \) truthfully; otherwise, the manager issues the fraudulent good report \( r_i = 1 \). For ease of reference, denote the outcome of internal controls as \( IC_i \in \{0,1\} \) with \( IC_i = 1 \) representing the success of internal controls. \( IC_i \) is unobservable to investors. Moreover, we assume that manipulation reduces the firm’s long-term value by \( C_i(m_i) \) only when it succeeds.

We show that all of our main results hold qualitatively. The managers’ manipulation decisions remain strategic complements and the internal controls of one firm still exert a positive externality on the other firm. We summarize these results in the proposition below.

**Proposition 7** In the extension with the alternative manipulation technology,

1. \( \frac{\partial \tilde{m}_A}{\partial m_B} \big|_{m_B = m_B^*} > 0 \), that is, the managers’ manipulation decisions are strategic complements;
2. \( \frac{\partial V_B^*(q_B,q_A)}{\partial q_A} > 0 \), that is, the internal controls of firm A exert a positive externality on firm B.
The proof of Proposition 7 provided in the appendix, mirrors its counterpart for the baseline model except that investors now have to update beliefs about outcomes of the internal controls system above and beyond their beliefs about the fundamentals. While this additional belief update makes the proof more involved, the intuition of the results remains the same. For all purpose and intent, under the alternative manipulation technology, manipulation still reduces and the costly internal controls still improve the report’s informativeness. As the internal controls quality of firm A increases or its manipulation decreases, report $r_A$ becomes more informative. As a result, the information asymmetry between manager $B$ and investors decreases and manager $B$ manipulates less, which increases the value of firm $B$. Hence the strategic complementarity and the externality.

5 Conclusion and Discussion

We have presented a model to show that a firm’s investment in internal controls has a positive externality on peer firms. The core of the mechanism is the strategic complementarity between manipulation decisions. A manager manipulates more if he expects peer firms’ reports are more likely to be manipulated. As a result, a firm’s investment in internal controls benefits not only itself by reducing its own manager’s manipulation, but also peer firms by mitigating the manipulation pressure on peer managers. Without internalizing this positive externality, firms under-invest in internal controls over financial reporting. Regulations that provide a floor of internal controls investment can mitigate the underinvestment problem.

We have presented a stylized model to deliver the intuition for the strategic complementarity between manipulations and the externality of a firm’s internal controls. In particular, the binary structure has dramatically simplified the exposition. However, we believe that the economic forces behind peer pressure for manipulation are more general. The strategic complementarity between the two managers’ manipulations is driven by two features of the model. First, two firms’ fundamentals are correlated. Second, manipulation reduces the report’s informativeness. The first feature is natural. Thus, as long as the second feature is preserved in a richer model in which manipulation leads to information degradation, the
strategic complementarity between the two managers’ decisions is expected to be intact, as we have shown in Section [4].

In addition to the extensions studied in Section [4], there could be other extensions to the baseline model. We have focused on a two-firm economy in the baseline model. It is straightforward to extend the model to \( N > 2 \) firms. A manager manipulates more if he believes that any of his peer firms’ reports is more likely to be manipulated. Conditional on the fundamental \( s_i \), report \( r_i \) and \( r_j \) are independent of each other. As a result, the presence of any additional firms \( k \in N\backslash\{i,j\} \) does not affect the interaction between firm \( i \) and \( j \). In other words, investors can first use all firms’ reports \( r_k \), other than \( r_i \) and \( r_j \), to update their belief about \( s_i \). Treating this posterior as a prior, investors continue to use report \( r_i \) and \( r_j \), as in our baseline model of two firms.\(^{15}\) The peer pressure holds for any pair of firms within the \( N \) firms.

Another possible extension is to relax the assumption that the managers know their firms’ fundamentals perfectly. If managers receive only noisy private information about their firms’ fundamentals, there would be measurement errors in the reports in the absence of manipulation. Specifically, suppose that the fundamental or gross cash flow is \( v_i \in \{0,1\} \). Each manager receives a noisy signal \( s_i \in \{0,1\} \) about \( v_i \) : \( \Pr(s_i = 1|v_i = 1) = \Pr(s_i = 0|v_i = 0) = \tau \in [1/2,1] \). \( \tau \) measures the quality of managers’ signals and our baseline model is a special case of \( \tau = 1 \). With this specification, we can replicate Proposition [1] that manager \( A \) with \( s_i = 0 \) manipulates more if he expects that report \( B \) is more likely to be manipulated. The proof goes through essentially by redefining the fundamental \( s_i \). Even though managers receive noisy signals about the fundamentals, they still know more than the investors in equilibrium and this information asymmetry is still increasing in manipulation.

Finally, we have focused on the capital market pressure as the driver for accounting manipulation, which appears empirically important (e.g., Graham, Harvey, and Rajgopal (2005)). As a result, we have assumed that the two firms are independent, except the correlation of their fundamentals. In practice, peer firms are likely to interact in other areas (such as product markets, labor markets, performance benchmarking, and regulation), which may also

\(^{15}\) See footnote [12] for the discussion of the implications of the conditional independence properties.
lead to interactions of their accounting decisions. As we discussed in the literature review, these other interactions are promising areas for future research.

Appendix I: proofs

Proof. of Lemma 1. For notational ease, we use $C_A = C_A(m_A^\ast)$ whenever no confusion arises. For a given $q_A \in (0,1)$, after we impose the rational expectations requirement, $m_A^\ast$ is determined by the first-order condition:

$$H^A(m_A^\ast) = \delta_A (1-q_A) \theta_A (1) (1 + C_A^\ast) - (1 - \delta_A) C'_A(m_A^\ast) = 0.$$ 

We first verify that the equilibrium is unique, i.e., $H^A(m_A^\ast) = 0$ has a unique solution. First, under our assumption that $C_A$ is $m^{DB}$ convex, we have $\frac{\partial H^A(m_A^\ast)}{\partial m_A^\ast} < 0$. This is because

$$\frac{\partial H^A(m_A^\ast)}{\partial m_A^\ast} = \delta_A (1-q_A) \frac{\partial }{\partial m_A^\ast}[(1 + C_A^\ast) \theta_A (1)] - (1 - \delta_A) C''_A(m_A^\ast) = (\delta_A (1-q_A) \theta_A (1))^2 \frac{1 + C_A^\ast}{1 - \delta_A} + \delta_A (1 - q_A)(1 + C_A^\ast) \frac{\partial \theta_A (1)}{\partial m_A^\ast} - (1 - \delta_A) C''_A(m_A^\ast).$$

The sign of $\frac{\partial H^A(m_A^\ast)}{\partial m_A^\ast}$ is dominated by the sign of $C''_A(m_A^\ast)$. Second, at $m_A^\ast = 0$, $H^A(0) = \delta_A (1-q_A) \theta_A (1; m_A^\ast = 0) > 0$. Finally, at $m_A^\ast = 1$, $H^A(1) = \delta_A (1-q_A) \theta_A (1; m_A^\ast = 1) (1 + C_A (1)) - (1 - \delta_A) C'_A(1) = -\infty < 0$. Therefore, by the intermediate value theorem, the equilibrium $m_A^\ast$ that satisfies $H^A(m_A^\ast) = 0$ is unique.

For any parameter $x \in \{\theta_A, \delta_A, q_A\}$, the application of the implicit function theorem generates

$$\frac{\partial m_A^\ast}{\partial x} = \frac{\frac{\partial H^A(m_A^\ast; x)}{\partial m_A^\ast}}{\frac{\partial H^A(m_A^\ast)}{\partial m_A^\ast}}.$$

The denominator $\frac{\partial H^A(m_A^\ast)}{\partial m_A^\ast} < 0$. As a result, the sign of $\frac{\partial m_A^\ast}{\partial x}$ is the same as that of $\frac{\partial H^A(m_A^\ast; x)}{\partial m_A^\ast}$. In particular, 

$$\frac{\partial H^A(m_A^\ast; \delta_A)}{\partial \delta_A} = (1 - q_A)(1 + C_A^\ast) \theta_A (1) + C'_A(m_A^\ast) > 0,$$

$$\frac{\partial H^A(m_A^\ast; \theta_A)}{\partial \theta_A} = \delta_A (1 - q_A)(1 + C_A^\ast) \frac{\partial \theta_A (1)}{\partial \theta_A}$$

$$= \delta_A (1 - q_A)(1 + C_A^\ast) \frac{\mu_A^\ast}{[\theta_A + (1 - \theta_A) \mu_A^\ast]^2} > 0,$$
\[
\frac{\partial H^A(m^*_A; q_A)}{\partial q_A} = -\delta_A (1 + C^*_A) \theta_A (1) + \delta_A (1 - q_A) (1 + C^*_A) \frac{\partial \theta_A (1)}{\partial q_A} \\
= \delta_A (1 + C^*_A) (-\theta_A (1) + \theta_A (1) (1 - \theta_A (1))) \\
= -\delta_A (1 + C^*_A) \theta_A^2 (1) < 0.
\]

**Proof.** of Lemma 2: We first use the Bayes rule to write out

\[
\theta_A (1, 1) - \theta_A (1, \phi) = \frac{\theta_A (1, \phi)}{\theta_A (1, \phi) + (1 - \theta_A (1, \phi)) \frac{\Pr(r_B | s_A = 0)}{\Pr(r_B | s_A = 1)} - \theta_A (1, \phi) \\
= \frac{\theta_A (1, \phi) (1 - \theta_A (1, \phi)) \left[ 1 - \frac{\Pr(r_B | s_A = 0)}{\Pr(r_B | s_A = 1)} \right]}{\theta_A (1, \phi) + (1 - \theta_A (1, \phi)) \frac{\Pr(r_B | s_A = 0)}{\Pr(r_B | s_A = 1)}}.
\]

\( \theta_A (1, \phi) = \theta_A (1) \) is expressed in Equation 2.

Therefore, \( \theta_A (1, 1) - \theta_A (1, \phi) > 0 \) if and only if \( \frac{\Pr(r_B | s_A = 0)}{\Pr(r_B | s_A = 1)} < 1 \) or equivalently \( \Pr(r_B | s_A = 1) > 1 \). Moreover, since \( \theta_A (1, \phi) \) is independent of \( \mu_B^* \), \( \theta_A (1, 1) - \theta_A (1, \phi) \) is increasing in \( \mu_B^* \) if and only if \( \frac{\Pr(r_B | s_A = 1)}{\Pr(r_B | s_A = 0)} \) is increasing in \( \mu_B^* \).

We then write out

\[
\frac{\Pr(r_B = 1 | s_A = 1)}{\Pr(r_B = 1 | s_A = 0)} - 1 \\
= \frac{\Pr(r_B = 1 | s_B = 1, s_A = 1) \Pr(s_B = 1 | s_A = 1) + \Pr(r_B = 1 | s_B = 0, s_A = 1) \Pr(s_B = 0 | s_A = 1)}{\Pr(r_B = 1 | s_B = 1) \Pr(s_B = 1 | s_A = 1) + \Pr(r_B = 1 | s_B = 0) \Pr(s_B = 0 | s_A = 1)} - 1 \\
= \frac{\Pr(r_B = 1 | s_B = 1) \Pr(s_B = 1 | s_A = 1) + \Pr(r_B = 1 | s_B = 0, s_A = 0) \Pr(s_B = 0 | s_A = 1)}{\Pr(r_B = 1 | s_B = 1) \Pr(s_B = 1 | s_A = 1) + \Pr(r_B = 1 | s_B = 0) \Pr(s_B = 0 | s_A = 1)} - 1 \\
= \frac{\frac{\Pr(r_B = 1 | s_B = 1)}{\Pr(r_B = 1 | s_B = 0)} - 1}{\frac{\Pr(r_B = 1 | s_B = 1)}{\Pr(r_B = 1 | s_B = 0)} - 1} \Pr(s_B = 1 | s_A = 1) + 1 \\
= \frac{\frac{1}{\mu_B} - 1}{\frac{1}{\mu_B} - 1} \Pr(s_B = 1 | s_A = 0) + 1 \\
= \frac{1}{\mu_B} \Pr(s_B = 1 | s_A = 0) + \frac{\sqrt{(1 - \theta_B) \theta_B}}{(1 - \theta_A) \theta_A} \rho \\
\propto \rho
\]
“∝” reads as “has the same sign as.” The second last equality holds because

\[
\frac{\Pr (s_B = 1 | s_A = 1) - \Pr (s_B = 1 | s_A = 0)}{\Pr (s_A = 1)} - \frac{\Pr (s_B = 1, s_A = 1) - \Pr (s_B = 1, s_A = 0)}{\Pr (s_A = 0)}
\]

\[
= \frac{\Pr (s_B = 1, s_A = 1)}{\Pr (s_A = 1)} - \frac{\Pr (s_B = 1) - \Pr (s_B = 1, s_A = 1)}{\Pr (s_A = 0)}
\]

\[
= \theta_A \theta_B + \rho \sqrt{(1 - \theta_A) \theta_A (1 - \theta_B) \theta_B} \frac{\theta_B - \left( \theta_A \theta_B + \rho \sqrt{(1 - \theta_A) \theta_A (1 - \theta_B) \theta_B} \right)}{1 - \theta_A}
\]

\[
= \rho \sqrt{(1 - \theta_B) \theta_B} \frac{(1 - \theta_A)}{(1 - \theta_A) \theta_A}.
\]

The last step holds because \( \mu_B^* \in (0, 1) \) and thus \( \frac{1}{\mu_B^*} > 1 \). Thus, \( \frac{\Pr (r_B = 1 | s_A = 1)}{\Pr (r_B = 1 | s_A = 0)} > 1 \) if and only if \( \rho > 0 \).

Moreover, we can show that \( \frac{\partial}{\partial \mu_B^*} \frac{\Pr (r_B = 1 | s_A = 1) - \Pr (r_B = 1 | s_A = 0)}{\Pr (r_B = 1 | s_A = 0)} \) is increasing in \( \mu_B^* \) if and only if \( \rho < 0 \), because

\[
\frac{\partial}{\partial \mu_B^*} \frac{\Pr (r_B = 1 | s_A = 1) - \Pr (r_B = 1 | s_A = 0)}{\Pr (r_B = 1 | s_A = 0)} \propto - (\Pr (s_B = 1 | s_A = 1) - \Pr (s_B = 1 | s_A = 0))
\]

\[
\propto - \rho.
\]

(11)

The proof for the properties of \( \theta_A (1, 1) - \theta_A (1, 0) \) is similar and hence omitted. Therefore, we have proved Lemma 2.

**Proof.** of Proposition 1: For given interior \( q_A \) and \( \mu_B^* \) and investors’ conjecture \( m_A^* \), manager \( A \)'s best response \( \tilde{m}_A^*(\mu_B^*) \) is determined by the first-order condition:

\[
H^A(m_A^*; \mu_B^*) = \delta_A (1 - q_A) E_{r_B}[\theta_A(1, r_B)|s_A = 0](1 + C_A(m_A^*(\mu_B^*))) - (1 - \delta_A) C_A'(m_A^*) = 0.
\]

The application of the implicit function theorem generates

\[
\frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*} = - \frac{\frac{\partial H^A(\tilde{m}_A^*, \mu_B^*)}{\partial \mu_B^*}}{\frac{\partial H^A(\tilde{m}_A^*, \mu_B^*)}{\partial \tilde{m}_A^*}}.
\]

The denominator \( \frac{\partial H^A(\tilde{m}_A^*, \mu_B^*)}{\partial \tilde{m}_A^*} \) is negative following a similar proof in Lemma 1. Thus, the sign of \( \frac{\partial \tilde{m}_A^*(\mu_B^*)}{\partial \mu_B^*} \) is the same as that of \( \frac{\partial H^A(\tilde{m}_A^*, \mu_B^*)}{\partial \mu_B^*} \). Note that \( \mu_B^* \) shows up in \( H^A(\tilde{m}_A^*; \mu_B^*) \) only through \( E_{r_B}[\theta_A(1, r_B)|s_A = 0] \). Therefore, \( \frac{\partial H^A(\tilde{m}_A^*, \mu_B^*)}{\partial \mu_B^*} \) has the same sign as \( \frac{\partial E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{\partial \mu_B^*} \).

We now prove that \( \frac{\partial E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{\partial \mu_B^*} > 0 \). In words, the bad manager becomes more optimistic about investors’ belief as \( \mu_B^* \) increases.

We first write out the investors’ belief about \( s_A = 1 \) before they observe \( r_B \) but after they
observe $r_A = 1$:

$$
\theta_A(1, \phi) \equiv \Pr(s_A = 1|r_A = 1) = \Pr(r_B = 0|r_A = 1)\theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)\theta_A(1, 1) = \theta_A(1, 0) + \Pr(r_B = 1|r_A = 1)(\theta_A(1, 1) - \theta_A(1, 0)).
$$

The second step writes out the expectation and the third regroups the terms. This gives us Equation 7 in the text which we reproduce here:

$$
\theta_A(1, 1) - \theta_A(1, 0) = \frac{\theta_A(1, \phi) - \theta_A(1, 0)}{\Pr(r_B = 1|r_A = 1)}. \tag{12}
$$

We can similarly write out the bad manager’s expectation about the investors’ expectation of $s_A = 1$ as

$$
E_{r_B}[\theta_A(1, r_B)|s_A = 0] = \Pr(r_B = 0|s_A = 0)\theta_A(1, 0) + \Pr(r_B = 1|s_A = 0)\theta_A(1, 1) = \theta_A(1, 0) + \frac{\Pr(r_B = 1|s_A = 0)(\theta_A(1, \phi) - \theta_A(1, 0))}{\Pr(r_B = 1|s_A = 0)}.
$$

Again the first step writes out the expectation and the second regroups the terms. The third step plugs in Equation 12. The last step writes out the total probability of $\Pr(r_B = 1|r_A = 1)$ and reorganizes the terms. Note that $\mu_B^*$ only affects the likelihood ratio $\frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)}$ in the last equality. Thus, we have

$$
\frac{\partial E_{r_B}[\theta_A(1, r_B)|s_A = 0]}{\partial \mu_B^*} \propto - (\theta_A(1, \phi) - \theta_A(1, 0)) \frac{\partial}{\partial \mu_B^*} \frac{\Pr(r_B = 1|s_A = 1)}{\Pr(r_B = 1|s_A = 0)} \rho > 0.
$$

The second step relies on expression 11, the result from the proof in Lemma 1. Therefore, regardless of $\rho$, $E_{r_B}[\theta_A(1, r_B)|s_A = 0]$ is increasing in $\mu_B^*$. Lastly, since $\mu_B^* = m_B^*(1 - q_B)$ is increasing in $q_B$ and decreasing in $q_B$, $m_A^*(\mu_B^*)$ is increasing in $m_B^*$ and decreasing in $q_B$. ■

**Proof.** of Proposition 2 We first prove that the equilibrium $(m_A^*(q_A, q_B), m_B^*(q_B, q_A))$ is unique in two steps. First, we solve for manager A’s unique best response $m_A^*(m_B^*)$. This part is similar to the proof in Lemma 1 because manager A’s best response problem (after imposing the investors’ rational expectations) is essentially a single firm problem with given $m_B^*$ and $q_B$.

Second, we plug manager A’s best response into manager B’s first-order condition and show that manager B’s optimization has a unique solution as well. By substituting the best
response \( \tilde{m}_A^* (m_B^*) \) into \( H^B(\tilde{m}_B^*; m_A^*) = 0 \) and obtain

\[
\frac{dH^B(\tilde{m}_B^*; m_A^*)}{dm_B^*} = \frac{\partial H^B(\tilde{m}_B^*; m_A^*)}{\partial \tilde{m}_B^*} + \frac{\partial H^B(\tilde{m}_B^*; m_A^*)}{\partial m_A^*} \frac{\partial \tilde{m}_B^*}{\partial \tilde{m}_A^*}
\]

where the second step is from \( \frac{\partial \tilde{m}_B^*}{\partial \tilde{m}_A^*} = -\frac{\partial \tilde{m}_B^*}{\partial m_A^*} \). When \( C_A \) and \( C_B \) are \( m^{BR} \) convex, it is easy (but tedious) to verify that the numerator is positive (the Hessian matrix of the objective function is negative definitive). The denominator is negative from the first step. Thus, \( \frac{dH^B(\tilde{m}_B^*; m_A^*)}{dm_B^*} < 0 \). Therefore, \( H^B(\tilde{m}_B^*; m_A^*(\tilde{m}_B^*)) \) is decreasing in \( \tilde{m}_B^* \), and by the intermediate value theorem, \( H^B(\tilde{m}_B^*; m_A^*(\tilde{m}_B^*)) = 0 \) has a unique solution \( m_B^* (q_B, q_A) \). In addition, \( m_A^* (q_A, q_B) = m_A^* (m_B^* (q_B, q_A)) \) is also unique. Now we can write the first-order condition of manager \( A \) as \( H^A(\tilde{m}_A^* (q_A, q_B); m_B^* (q_B, q_A)) \).

To derive \( \frac{dm_A^*}{dq_A} \) and \( \frac{dm_B^*}{dq_A} \), the application of the multivariate implicit function theorem generates

\[
\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{dm_A^*}{dq_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_B^*} \frac{dm_B^*}{dq_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{d\mu_B^*}{dq_A} = 0,
\]

which can be simplified into

\[
\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} \frac{dm_A^*}{dq_A} + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_B^*} \frac{dm_B^*}{dq_A} + (1 - q_B) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{d\mu_B^*}{dq_A} = 0,
\]

\[
\frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} \frac{dm_B^*}{dq_A} + \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \frac{d\mu_A^*}{dq_A} + (1 - q_A) \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \frac{d\mu_A^*}{dq_A} = 0.
\]
Solving the two equations gives

\[
\frac{dm_A^*}{dq_A} = - \left[ \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} \frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1 - q_B) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \right]
\]

\[
\frac{dm_B^*}{dq_A} = \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \left( \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} (1 - q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} m_A^* \right)
\]

We have shown that in the unique equilibrium, the denominator is positive. Hence the signs of \(\frac{dm_A^*}{dq_A}\) and \(\frac{dm_B^*}{dq_A}\) are determined by their numerators, respectively. First,

\[
\frac{dm_A^*}{dq_A} < - \left[ \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} \frac{\partial H^B(m_B^*; m_A^*)}{\partial m_B^*} + (1 - q_B) \frac{\partial H^A(m_A^*; m_B^*)}{\partial \mu_B^*} \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \right]
\]

From a proof similar to that of Lemma 1, we have \(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0\) and \(\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} < 0\). Proposition 1 shows \(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} > 0\) and \(\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0\). As a result, \(\frac{dm_A^*}{dq_A} < 0\).

Similarly,

\[
\frac{dm_B^*}{dq_A} = \frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} \left( \frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} (1 - q_A) + \frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} m_A^* \right)
\]

where \(\frac{\partial H^B(m_B^*; m_A^*)}{\partial \mu_A^*} > 0\), \(\frac{\partial H^A(m_A^*; m_B^*)}{\partial q_A} < 0\), \(\frac{\partial H^A(m_A^*; m_B^*)}{\partial m_A^*} < 0\). As a result, \(\frac{dm_B^*}{dq_A} < 0\).

**Proof.** of Proposition 3 The proof of the uniqueness of the internal controls equilibrium is similar to that of the manipulation choice in Proposition 2. In short, when \(K_A(q_A)\) is \(m^{BR}\) convex, the LHS of the first-order condition of \(q_A\), \(\frac{\partial V_{A0} dm_A^*}{\partial m_A^* dq_A} - K_A'(q_A)\), is decreasing in \(q_A\), making the best response \(\hat{q}_A^*(q_B^*)\) unique. Substituting \(\hat{q}_A^*(q_B^*)\) into manager B’s best response gives

\[
\frac{\partial V_{B0} dm_B^*}{\partial m_B^* dq_B} |_{q_A = \hat{q}_A^*(q_B^*)} - K_B'(\hat{q}_B) = 0.
\]

When \(K_B(q_B)\) is \(m^{BR}\) convex, the LHS \(\frac{\partial V_{B0} dm_B^*}{\partial m_B^* dq_B} |_{q_A = \hat{q}_A^*(q_B^*)} - K_B'(\hat{q}_B)\) is decreasing in \(\hat{q}_B\), making the solution of \(q_B^* = \hat{q}_B^*(q_B^*)\) unique. As a result, the equilibrium decisions \(q_A^* = \hat{q}_A^*(q_B^*)\), \(m_A^* = q_A^* q_B^*)\), and \(m_B^* = q_B^* q_A^*)\) are also unique.

**Proof.** of Proposition 4 We show that there exists a pair of internal controls levels \((q_A', q_B')\) with \(q_A' < q_A^*\) and \(q_B' > q_B^*\) such that \(V_{A0}(q_A', q_B') > V_{A0}(q_A^*, q_B^*)\) and \(V_{B0}(q_B', q_A^*) > V_{B0}(q_B^*, q_A^*)\). To see this, set \(q_A' = q_A^* + \epsilon\) and \(q_B' = q_B^* + \epsilon\), where \(\epsilon > 0\) is an arbitrarily small positive number. By a first-order Taylor expansion,

\[
V_{A0}(q_A', q_B') = V_{A0}(q_A^*, q_B^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_A}(q_A' - q_A^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B}(q_B' - q_B^*)
\]

\[
= V_{A0}(q_A^*, q_B^*) + \frac{\partial V_{A0}(q_A^*, q_B^*)}{\partial q_B} \epsilon.
\]
The second step is by the first-order condition of \( q_A^* \), i.e., \( \frac{\partial V_A( q_A^*, q_B^*)}{\partial q_B} = 0 \). Recall that

\[
\frac{\partial V_A( q_A^*, q_B^*)}{\partial q_B} = \frac{\partial V_A( q_A^*, q_B^*)}{\partial m^*_A} \frac{\partial m^*_A( q_A^*, q_B^*)}{\partial q_B} > 0.
\]

The inequality is because \( \frac{\partial V_A( q_A^*, q_B^*)}{\partial m^*_A} = -\text{Pr}(s_A = 0)C'_A(m^*_A) < 0 \) and \( \frac{\partial m^*_A( q_A^*, q_B^*)}{\partial q_B} < 0 \) from Proposition 5. As a result, \( \frac{\partial V_A( q_A^*, q_B^*)}{\partial q_B} \epsilon > 0 \) and \( V_A( q_A^*, q_B^*) > V_A( q_A^*, q_B^*) \). The proof for \( V_B( q_B^*, q_A^*) > V_B( q_B^*, q_A^*) \) is similar. □

**Proof.** of Proposition 5 We first characterize the managers’ equilibrium manipulation decisions \( \{m_A^*( q_A, q_B), m_B^*( q_B, q_A)\} \) given the two firms’ internal controls \( (q_A, q_B) \). We focus on manager A’s manipulation decisions first. Upon observing both reports and given investors’ conjecture of manipulation \( (\hat{m}_A, \hat{m}_B) \), investors set a price for firm A,

\[
P_A^*( r_A, r_B) = E[ V_A( r_A, r_B, \hat{m}_A, \hat{m}_B)] = E_{s_A}[ s_A| r_A, r_B, \hat{m}_A, \hat{m}_B] - C_A(\hat{m}_A) - K_A(q_A).
\]

We conjecture an equilibrium in which investors’ conjecture \( (\hat{m}_A, \hat{m}_B) \) is independent of the profitability \( (s_A, s_B) \) and verify this conjecture when we derive the equilibrium.

To characterize the price, we first derive \( E_{s_A}[ s_A| r_A, r_B, \hat{m}_A, \hat{m}_B] \). As in our baseline model, we decompose the derivation of \( E_{s_A}[ s_A| r_A, \hat{m}_A, r_B, \hat{m}_B] \) into two steps. First, we use \( (r_B, \hat{m}_B) \) to obtain investors’ posterior belief about \( s_A \). Second, we treat this posterior as the prior about \( s_A \) and then use report \( r_A \) to update. Specifically, upon observing \( r_B \) and \( \hat{m}_B \), the triplets \( (r_B, s_A, \hat{m}_B) \) are jointly normally distributed. Using properties of multivariate normal distributions, the distribution of \( s_A \) conditional on report \( r_B \) and \( \hat{m}_B \) is normal with mean and variance:

\[
E_{s_A}[ s_A| r_B, \hat{m}_B] = \bar{s} + \beta_A(\hat{m}_B)(r_B - \hat{m}_B(1 - q_B) - \bar{s}), \quad \text{with} \quad \beta_A(\hat{m}_B) = \frac{\rho \sigma_s^2}{\sigma_s^2 + [h(\hat{m}_B)]^2};
\]

\[
\text{Var}[ s_A| r_B, \hat{m}_B] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(\hat{m}_B)]^2}.
\]

Treating this conditional distribution as the prior of \( s_A \), investors use \( (r_A, \hat{m}_A) \) to update their beliefs about \( s_A \). We have

\[
E_{s_A}[ s_A| r_A, \hat{m}_A, r_B, \hat{m}_B] = \beta_A(\hat{m}_A, \hat{m}_B)(r_A - \hat{m}_A(1 - q_A)) + (1 - \beta_A(\hat{m}_A, \hat{m}_B))E_{s_A}[ s_A| r_B, \hat{m}_B],
\]

with

\[
\beta_A(\hat{m}_A, \hat{m}_B) = \frac{\text{Var}[ s_A| r_B, \hat{m}_B]}{\text{Var}[ s_A| r_B, \hat{m}_B] + [h(\hat{m}_A)]^2}.
\]

\( \beta_A(\hat{m}_A, \hat{m}_B) \) is the response coefficient of firm A’s price to report \( r_A \) conditional on investors’ conjecture of both managers’ manipulation choices \( \hat{m}_A \) and \( \hat{m}_B \).

Given the price \( P_A^*( r_A, r_B) \), the payoff function of manager A is

\[
U_A = \delta A P_A^*( r_A, r_B) + (1 - \delta A)V_A,
\]
where the firm value
\[ V_A = s_A - C_A(m_A) - K_A(q_A). \]

The first-order condition on the manager’s choice \( m_A \) is given by
\[
\frac{\partial U_A}{\partial m_A} = \delta_A \beta_A(m_A, \tilde{m}_B)(1 - q_A) - (1 - \delta_A) C_A''(m_A) = 0.
\]

\( m_A \) affects the price \( P_A^*(r_A, r_B) \) only through affecting the report \( r_A \) and affects \( V_A \) only through affecting the manipulation cost \( C_A(m_A) \). Also notice that from the first-order condition, the optimal \( m_A \) depends on neither \( s_A \) or \( s_B \), thus confirming our conjecture on the equilibrium in the beginning. Imposing the rational expectations conditions that \( m_A^* = \tilde{m}_A \) and \( m_B^* = \tilde{m}_B \) and defining \( H^A(m_A, m_B) = \frac{\partial U_A}{\partial m_A} \) manager \( A \)’s optimal response \( \tilde{m}_A^*(m_B^*) \) is a solution to a fixed point problem:
\[
H^A(m_A, m_B^*) = \delta_A \beta_A(m_A^*, m_B^*)(1 - q_A) - (1 - \delta_A) C_A''(m_A) = 0. \tag{15}
\]

Equation (15) shows that the larger the impact of \( r_A \) on the price \( P_A^* \) (a larger \( \beta_A(m_A^*, m_B^*) \)), the larger the benefit of manipulation for manager \( A \) and the more he manipulates. In addition, since \( \frac{\partial H^A}{\partial q_A} < 0 \) and \( \frac{\partial m_A^*}{\partial q_A} < 0 \).

Similarly, we can solve for the manager \( B \)'s best response \( \tilde{m}_B^*(m_A^*) \). In particular, \( \tilde{m}_B^*(m_A^*) \) is a solution to the following fixed point problem:
\[
H^B(m_B, m_A^*) = \delta_B \beta_B(m_B^*, m_A^*)(1 - q_B) - (1 - \delta_B) C_B''(m_B) = 0. \tag{16}
\]

The equilibrium manipulation choices \( (m_A^*, m_B^*) \) are the joint solutions to Equation (15) and (16).

We now prove the existence and uniqueness of the equilibrium. To prove the existence of the best response \( \tilde{m}_A^*(m_B^*) \) from \( H^A(\tilde{m}_A^*, m_B^*) = 0 \), consider an \( m_A \in [0, +\infty) \).

\[ \lim_{m_A \to 0} H^A(m_A, m_B^*) = \delta_A \beta_A(0, m_B^*)(1 - q_A) > 0 \text{ and } \lim_{m_A \to \infty} H^A(m_A) = \delta_A \beta_A(\infty, m_B^*)(1 - q_A) - \infty < 0 \]

since \( \beta_A(\infty, m_B^*) < 1 \) is finite. By the intermediate value theorem, \( m_A^*(m_B^*) \) always exists. In addition, \( m_A^*(m_B^*) \) is also unique if \( C_A \) is sufficiently convex and \( H_A^* < 0 \). For the other best response \( \tilde{m}_B^*(m_A^*) \), following similar steps, we can also verify its existence and uniqueness given that \( C_B \) is sufficiently convex and \( H_B^* < 0 \).

Now we prove the existence of the equilibrium \( \{m_A^*(q_A, q_B), m_B^*(q_B, q_A)\} \). Substituting the best response \( \tilde{m}_B^*(m_A^*) \) into \( H_A^* = 0 \) gives
\[
H_A^*(m_A^*, \tilde{m}_B^*(m_A^*)) = 0.
\]

At \( m_A = 0, \tilde{m}_B^*(0) > 0 \) and is finite, and \( \lim_{m_A \to 0} H_A(m_A, \tilde{m}_B^*(m_A)) = 0 \); at \( m_A = \infty, \tilde{m}_B^*(\infty) > 0 \) and is finite, \( \delta_A \beta_A(\infty, \tilde{m}_B^*(\infty))(1 - q_A) \) is finite and smaller than \( \infty \), i.e., \( \lim_{m_A \to \infty} H_A(m_A, \tilde{m}_B^*(m_A)) < 0 \). By the intermediate value theorem, an equilibrium always exists. In addition, \( m_A^* \) is also unique if \( \frac{\partial H_A}{\partial m_A} = H_A^* + H_B^* \frac{\partial \tilde{m}_B}{\partial m_A} < 0 \). This satisfies if \( C_A \) is sufficiently convex.

With the unique equilibrium characterized, we turn to the strategic relation between two managers’ manipulation decisions. We focus on analyzing the effect of \( m_B^* \) on \( \tilde{m}_A^*(m_B^*) \) and the other case is similar.

From \( H_A^*(\tilde{m}_A^*(m_B^*), m_B^*) = 0 \) and by the implicit function theorem, we have \( \frac{\partial \tilde{m}_A^*}{\partial m_B^*} + H_A^* \frac{\partial \tilde{m}_A^*}{\partial m_B^*} = 0 \), which gives \( \frac{\partial \tilde{m}_A^*}{\partial m_B^*} = -\frac{H_A^*}{H_B^*} \). Recall that for \( C_A \) sufficiently convex,
$H_A$ is decreasing in $\tilde{m}_A^*$ in the unique equilibrium, i.e., $H_A^A \leq 0$, the sign of $\frac{\partial \tilde{m}_A^*}{\partial m_B^*}$ is the same as $H_B^A$, i.e.,
\[
\frac{\partial \tilde{m}_A^*}{\partial m_B^*} \propto H_B^A = \delta_A (1 - q_A) \frac{\partial \beta_A (m_A^*, m_B^*)}{\partial m_B^*}.
\]
That is, the sign of $\frac{\partial \tilde{m}_A^*}{\partial m_B^*}$ is determined by how $m_B^*$ affects $\beta_A (m_A^*, m_B^*)$.

Consider first the case of $\rho = 0$. Since the two firms’ profitability are uncorrelated, $\text{Var}[s_A | r_B, m_B^*] = \sigma_s^2$, which makes $\beta_A (m_A^*, m_B^*) = \frac{\text{Var}[s_A | r_B, m_B^*]}{\text{Var}[s_A | r_B, m_B^*] + h(m_A^*)} = \frac{\sigma_s^2}{\sigma_s^2 + h(m_A^*)^2}$. Therefore, $\beta_A (m_A^*, m_B^*)$ is independent of $m_B^*$ and $H_B^A = 0$. That is, the two firms’ manipulation choices are strategically independent.

Second, consider $\rho \neq 0$ and $h'(m_i) = 0$, i.e., manipulation does not affect the informativeness of the report. We have the conditional variance $\text{Var}[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + h(m_B^*)^2}$ independent of $m_B^*$ and thus so is $\beta_A (m_A^*, m_B^*) = \frac{\text{Var}[s_A | r_B, m_B^*]}{\text{Var}[s_A | r_B, m_B^*] + h(m_A^*)}$. As a result, $\tilde{m}_A^*$ is independent of $m_B^*$. In sum, for $\rho = 0$ or $h'(m_i) = 0$, internal controls produces no externality with $\frac{dm_A^*}{dq_A} = 0$.

Lastly, consider $\rho \neq 0$ and $h'(m_i) > 0$, i.e., manipulation reduces the informativeness of the report. We have the conditional variance $\text{Var}[s_A | r_B, m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + h(m_B^*)^2}$ increases in $m_B^*$ since $h'(m_i) > 0$ and thus so is $\beta_A (m_A^*, m_B^*) = \frac{\text{Var}[s_A | r_B, m_B^*]}{\text{Var}[s_A | r_B, m_B^*] + h(m_A^*)}$. As a result, $\tilde{m}_A^*$ is increasing in $m_B^*$. In addition, since $\frac{dm_A^*}{dq_A} < 0$, the strategic complementarity between the manipulation decisions implies that $\frac{dm_B^*}{dq_A} < 0$.

We now derive firms’ internal controls decisions. As in the baseline model, the firm’s ex ante payoff $V_{i0}$ can be reduced into
\[
V_{i0} = \tilde{s} - C_i (m_i^*) - K_i (q_i) .
\]
Firm $A$ at $t = 0$ chooses $q_A$ to maximizes $V_{A0}$. Differentiating $V_{A0}$ with respect to $q_A$, the first-order condition is
\[
\frac{\partial V_{A0}}{\partial m_A^*} \frac{dm_A^*}{dq_A} - K'_A (q_A) = -C'_A (m_A^*) \frac{dm_A^*}{dq_A} - K'_A (q_A) = 0 ,
\]
which is similar to the one in the baseline model. Thus following similar proofs in Proposition 3, we show that there exists a unique pair of optimal private choices of internal controls $(q_A^*, q_B^*)$ by the firms that solves the first-order conditions.

If $h'(m_i) > 0$ and $\rho \neq 0$, then given the positive externality result $\frac{dm_B^* (q_A, q_B)}{dq_A} < 0$, the proof for the existence of Pareto improvements is identical to the one in Proposition 4.

If $h'(m_i) = 0$ or $\rho = 0$, $\frac{dm_B^* (q_A, q_B)}{dq_A} = 0$. That is, improving $q_A$ produces no benefit to firm $B$ as it affects neither $m_B^*$ nor $C_B (m_B^*)$. As a result, the privately optimal choice $q_A^*$ by firm $A$ is also socially optimal and thus Pareto efficient.

\textbf{Proof.} of Proposition 6 We have already derived the conditions for the strategic complementarity in Proposition 5. Therefore, we focus on examining the sufficient and necessary condition for the spillover effect. We find that the spillover effect exists if and only if the two
firms’ profitability are correlated, \( \rho \neq 0 \). To see this, we first compute a single manipulation \( A \)'s manipulation decision. Following similar analyses in the two-firm case, one can verify that the manager \( A \)'s equilibrium decision \( m_A^* \) is a solution to a fixed point problem:

\[
H^A(m_A) = \delta_A \beta_A(m_A^*)(1 - q_A) - (1 - \delta_A) C'_A(m_A) = 0.
\]

\( \beta_A(m_A^*) \) is the response coefficient of firm \( A \)'s price to report \( r_A \) given only firm \( A \)'s report. It is straightforward to show that

\[
\beta_A(m_A^*) = \frac{\sigma_s^2}{\sigma_s^2 + [h(m_A^*)]^2}.
\]

To analyze the spillover effect, consider first the case \( \rho = 0 \), i.e., the two firms’ profitability are independent. At \( \rho = 0 \), it is straightforward to verify that

\[
Var[s_A|r_B,m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(m_B^*)]^2} < \sigma_s^2,
\]

that is, the additional report of firm \( B, r_B \), provides no information about firm \( A \)'s profitability \( s_A \) and thus does not affect investors’ conditional variance about \( s_A \). In addition,

\[
\beta_A(m_A^*,m_B^*) = \frac{Var[s_A|r_B,m_B^*]}{Var[s_A|r_B,m_B^*] + [h(m_A^*)]^2} = \frac{\sigma_s^2}{\sigma_s^2 + [h(m_A^*)]^2},
\]

that is, \( \beta_A(m_A^*,m_B^*) = \beta_A(m_A^*) \). As a result, the first-order condition in the two-firm case, Equation [15], reduces into the one in the single-firm case. The presence of firm \( B \) does not affect the equilibrium \( m_A^* \) and the spillover effect does not exist.

With \( \rho \neq 0 \), however, we have

\[
Var[s_A|r_B,m_B^*] = \sigma_s^2 - \frac{\rho^2 \sigma_s^4}{\sigma_s^2 + [h(m_B^*)]^2} < \sigma_s^2,
\]

that is, the additional report of firm \( B, r_B \), provides information about firm \( A \)'s profitability \( s_A \) and thus reduces investors’ conditional variance about \( s_A \) compared with investors’ prior variance \( \sigma_s^2 \). In addition,

\[
\beta_A(m_A^*,m_B^*) = \frac{Var[s_A|r_B,m_B^*]}{Var[s_A|r_B,m_B^*] + [h(m_A^*)]^2} < \frac{\sigma_s^2}{\sigma_s^2 + [h(m_A^*)]^2},
\]

that is, with the presence of firm \( B \), firm \( A \)'s report has a smaller impact on the price \( P_A^* (\beta_A(m_A^*,m_B^*) < \beta_A(m_A^*)) \) as firm \( A \) contains less incremental information. As a result, manager \( A \)'s incentive to inflate \( r_A \) is weakened and \( m_A^* \) decreases with the presence of firm \( B \).
Overall, the sufficient and necessary condition for the spillover effect is \( \rho \neq 0 \).  

**Proof.** of Proposition[17] We first prove the strategic complementarity result. As in our main analysis, we solve the model by backward induction. For a manager with \( s_A = 0 \), the firm value is

\[
V_A = -1_{IC_A = 0} C_A (m_A) - K_A (q_A).
\]

\( 1_{IC_A = 0} \) is an indicator function and takes a value of 1 if \( IC_A = 0 \). In other words, manipulation reduces the firm value by \( C_A (m_A) \) only when the internal controls system fails (\( IC_A = 0 \)).

We now derive the price \( P_A \). Notice that \( r_B \) is independent of \( r_A \) and \( IC_A \), conditional on \( s_A \). Therefore, investors update their belief in two steps. First, investors use \( r_B \) to update their belief from prior \( \theta_A \) to posterior \( \theta_A (\phi, r_B) \equiv \Pr (s_A = 1 | r_B) \). Second, treating \( \theta_A (\phi, r_B) \) as a new prior for \( s_A \), investors then use report \( r_A \) to update their belief about \( s_A \) and \( IC_A \).

More specifically, in the first step,

\[
\begin{align*}
\theta_A (\phi, 0) &= \Pr (s_A = 1 | r_B = 0) - \Pr (s_A = 1 | s_B = 0), \\
\theta_A (\phi, 1) &= \frac{\theta_B}{\theta_B + \mu^*_B (1 - \theta_B)} \left[ \Pr (s_A = 1 | s_B = 1) - \Pr (s_A = 1 | s_B = 0) \right] + \Pr (s_A = 1 | s_B = 0).
\end{align*}
\]

\( \mu^*_B \equiv \mu^*_B (1 - q_B) \) is the effective level of manipulation that affects the informativeness of \( r_B \). Notice that \( \theta_A (\phi, 0) \) is independent of \( \mu^*_B \) whereas \( \theta_A (\phi, 1) \) moves towards 0 when \( \mu^*_B \) increases. As a result, an increase in \( \mu^*_B \) (a higher \( m^*_B \) or a lower \( q_B \)) makes \( \theta_A (\phi, r_B) \) less volatile.

In the second step, given the new prior \( \theta_A (\phi, r_B) \), the price is given by

\[
P_A = E_{IC_A} \left[ \theta_A^{IC_A} (r_A, r_B) + \left( 1 - \theta_A^{IC_A} (r_A, r_B) \right) (0 - 1_{IC_A = 0} C_A^* | r_A) \right] - K_A (q_A),
\]

\( \theta_A^{IC_A} (r_A, r_B) \equiv \Pr (s_A = 1 | r_A, r_B, IC_A) \) is investors' posterior about firm \( A \) being a good type conditional on the reports \{\( r_A, r_B \)\} and the internal control outcome \( IC_A \). Note that investors don’t observe \( IC_A \) and thus take expectation with respect to it. Collecting \( V_A \) and \( P_A \), the manager’s expected payoff is given by

\[
E_1 [U_A | s_A = 0] = \delta_A E_{r_A, r_B} \left[ E_{IC_A} \left[ \theta_A^{IC_A} (r_A, r_B) + \left( 1 - \theta_A^{IC_A} (r_A, r_B) \right) (0 - 1_{IC_A = 0} C_A^* | r_A, r_B) \right] | s_A = 0 \right]
- (1 - \delta_A) E_{IC_A} \left[ I_{IC_A = 0} C_A (m_A) - K_A (q_A) \right].
\]
Writing out expectations with respect to \( r_A \) and \( IC_A \),

\[
E_1[U_A|s_A = 0] = -\delta_A [q_A + (1 - m_A)(1 - q_A)] C_A^* E_{rb} \Pr (IC_A = 0|r_A = 0, r_B) |s_A = 0] + \delta_A m_A (1 - q_A) E_{rb} \Pr (IC_A = 1|r_A = 1, r_B) |s_A = 0] + \delta_A m_A (1 - q_A)(1 + C_A^*) E_{rb} \Pr (IC_A = 0|r_A = 1, r_B) \theta_A^0(1, r_B) |s_A = 0] - \delta_A m_A (1 - q_A) C_A^* E_{rb} \Pr (IC_A = 0|r_A = 1, r_B) |s_A = 0] - (1 - \delta_A)(1 - q_A) C_A (m_A) - K_A(q_A).
\]

The equality uses \( \theta_A^0(0, r_B) = 0 \) and \( \theta_A^0(1, r_B) = 1 \). Taking the first-order condition with respect to \( m_A \) gives manager \( A \)'s best response \( \tilde{m}_A^* \) to \( m_B^* \):

\[
H_A(\tilde{m}_A^*; m_B^*) = \delta_A E_{rb}[g(\theta_A(\phi, r_B)) |s_A = 0] - (1 - \delta_A) C_A' (\tilde{m}_A^*) = 0.
\]

The function \( g(\theta_A(\phi, r_B)) \equiv (1 + C_A^*) \Pr (IC_A = 0|r_A = 1, r_B) \theta_A(1, r_B) + \Pr (IC_A = 1|r_A = 1, r_B) + C_A^* \Pr (IC_A = 0|r_A = 0, r_B) - \Pr (IC_A = 0|r_A = 1, r_B) \). To prove that \( \frac{\partial g(\theta_A(\phi, r_B))}{\partial m_B} |m_B = m_B^* > 0 \), we need to show that \( E_{rb}[g(\theta_A(\phi, r_B)) |s_A = 0] \) is strictly increasing in \( \theta_A(\phi, r_B) \). Writing out \( g(\theta_A(\phi, r_B)) \) gives that

\[
g(\theta_A(\phi, r_B)) = (1 + C_A^*) \theta_A(\phi, r_B) + (1 - \theta_A(\phi, r_B)) m_A^* (1 - q_A) q_A \theta_A(\phi, r_B) + \theta_A(\phi, r_B) + (1 - \theta_A(\phi, r_B)) m_A^* (1 - q_A)\]

\[
+ C_A^* \frac{1 - q_A - m_A^* (1 - q_A)}{1 - m_A^* (1 - q_A)} (1 - q_A) \left[ \theta_A(\phi, r_B) + m_A^* (1 - q_A) (1 - \theta_A(\phi, r_B)) \right] = \theta_A(\phi, r_B) + (1 - \theta_A(\phi, r_B)) m_A^* (1 - q_A)\]

\[
+ C_A^* \frac{1 - q_A - m_A^* (1 - q_A)}{1 - m_A^* (1 - q_A)} \left[ \theta_A(\phi, r_B) + m_A^* (1 - q_A) (1 - \theta_A(\phi, r_B)) \right].
\]

The first step uses \( \theta_A^0(1, r_B) = \frac{\theta_A(\phi, r_B)}{\theta_A(\phi, r_B) + (1 - m_A^*) (1 - \theta_A(\phi, r_B))} \).

\[
\Pr (IC_A = 1|r_A = 1, r_B) = \frac{\Pr (r_A = 1|IC_A = 1) \Pr (IC_A = 1)}{\Pr (r_A = 1|IC_A = 1) \Pr (IC_A = 1) + \Pr (r_A = 1|IC_A = 0) \Pr (IC_A = 0)}\]

\[
= \frac{q_A \theta_A(\phi, r_B)}{q_A \theta_A(\phi, r_B) + (\theta_A(\phi, r_B) + m_A^* (1 - \theta_A(\phi, r_B)) (1 - q_A))}\]

\[
= \frac{q_A \theta_A(\phi, r_B)}{\theta_A(\phi, r_B) + m_A^* (1 - q_A) (1 - \theta_A(\phi, r_B))}.
\]
and

\[
Pr(IA = 1 | r_A = 0, r_B) = \frac{Pr(r_A = 0 | IC_A = 1) Pr(IA = 1)}{Pr(r_A = 0 | IC_A = 1) Pr(IA = 1) + Pr(r_A = 0 | IC_A = 0) Pr(IA = 0)}
\]

\[
= \frac{(1 - \theta_A(\phi, r_B)) q_A}{(1 - \theta_A(\phi, r_B)) q_A + (1 - \theta_A(\phi, r_B))(1 - m^*_A)(1 - q_A)}
\]

\[
= \frac{1 - m^*_A (1 - q_A)}{q_A}.
\]

As a result,

\[
E_{r_B} \left[ g(\theta_A(\phi, r_B)) | s_A = 0 \right] = \sum_{r_B} g(\theta_A(\phi, r_B)) \frac{Pr(r_B | s_A = 0)}{Pr(s_A = 0)}
\]

\[
= \sum_{r_B} g(\theta_A(\phi, r_B)) \left(1 - \theta_A(\phi, r_B)\right) \frac{Pr(r_B)}{Pr(s_A = 0)}
\]

\[
= E_{r_B} \left[ g(\theta_A(\phi, r_B)) \left(1 - \theta_A(\phi, r_B)\right) \right] \frac{Pr(s_A = 0)}{Pr(s_A = 0)}.
\]

The second equality uses \(Pr(r_B | s_A = 0) = \frac{(1 - \theta_A(\phi, r_B)) Pr(r_B)}{Pr(s_A = 0)}\). We verify that \(\frac{g(\theta_A(\phi, r_B))(1 - \theta_A(\phi, r_B))}{Pr(s_A = 0)}\) is strictly concave in \(\theta_A(\phi, r_B)\). In addition, recall that an increase in \(\mu_B^*\) (a higher \(m_B^*\) or a lower \(q_B\)) makes \(\theta_A(\phi, r_B)\) less volatile. Therefore, by the second-order stochastic dominance, \(E_{r_B} \left[ g(\theta_A(\phi, r_B)) \left(1 - \theta_A(\phi, r_B)\right) \right] \frac{Pr(s_A = 0)}{Pr(s_A = 0)}\) increases in \(\mu_B^*\) and \(m_B^*\), which in turn makes the two managers’ manipulation decisions strategic complements, i.e., \(\frac{\partial \mu_B^*}{\partial m_B} | m_B = m_B^* > 0\). In addition, notice that in \(E_{r_B} \left[ g(\theta_A(\phi, r_B)) | s_A = 0 \right]\), either \(q_B\) or \(m_B^*\) affects \(E_{r_B} \left[ g(\theta_A(\phi, r_B)) | s_A = 0 \right]\) and \(m_A^*\) only through changing the effective level of manipulation \(\mu_B^*\).

We now turn to the positive externality result. The ex-ante value of firm \(B\) is given by

\[
V_B(q_B, q_A) = \theta_B - E[1_{IC_B = 0}] Pr(s_B = 0) C_B(m_B^*) - K_B(q_B)
\]

\[
= \theta_B - (1 - q_B)(1 - \theta_B) C_B(m_B^*) - K_B(q_B).
\]

Therefore,

\[
\frac{\partial V_B(q_B, q_A)}{\partial q_A} = -(1 - q_B)(1 - \theta_B) C_B(m_B^*) \frac{\partial m_B^*}{\partial q_A}
\]

\[
= -(1 - q_B)(1 - \theta_B) C_B(m_B^*) \frac{\partial m_B^*}{\partial \mu_A} \frac{\partial \mu_A}{\partial q_A}.
\]

The second equality is because as with \(q_B, q_A\) can affect \(m_B^*\) only through changing \(\mu_A^* = m_A^*(1 - q_A)\). We have already proven that \(\frac{\partial m_B^*}{\partial \mu_A} > 0\). Therefore, to show that \(\frac{\partial V_B(q_B, q_A)}{\partial q_A} > 0\), we only need to show that \(\frac{\partial q_A}{\partial q_A} < 0\).

To derive the sign of \(\frac{\partial q_A}{\partial q_A}\), we rewrite \(A^B(\tilde{m}_B^*; m_B^*) = 0\) and its counterpart for manager \(B, H^B(\tilde{m}_B^*; m_B^*) = 0\), as \(H^A(\mu_A^*; \mu_B^*) = 0\) and \(H^B(\mu_B^*; \mu_A^*) = 0\) by replacing \(m_A^* = \frac{\mu_A^*}{1 - q_A}\) and
\(m_B^* = \frac{\mu_B^*}{1-q_B}\). By the implicit function theorem,

\[
\frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial q_A} + \frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial \mu_A^*} \frac{\partial \mu_A^*}{\partial q_A} + \frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial \mu_B^*} \frac{\partial \mu_B^*}{\partial q_A} = 0,
\]

\[
\frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial q_A} + \frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial \mu_A^*} \frac{\partial \mu_A^*}{\partial q_A} + \frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial \mu_B^*} \frac{\partial \mu_B^*}{\partial q_A} = 0.
\]

Recall that \(\frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial q_A} = 0\) (\(q_A\) affects \(H^B(\mu_B^*; \mu_A^*)\) only through changing \(\mu_A^*\)). Thus we can derive \(\frac{\partial \mu_A^*}{\partial q_A}\) as:

\[
\frac{\partial \mu_A^*}{\partial q_A} = -\frac{\frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial \mu_B^*} \frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial q_A}}{\frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial \mu_B^*} \frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial q_A} - \frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial \mu_A^*} \frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial q_A}}.
\]

As proved in the baseline model, the denominator is positive in the unique equilibrium. In the numerator of \(\frac{\partial \mu_A^*}{\partial q_A}\), \(\frac{\partial H^B(\mu_B^*; \mu_A^*)}{\partial q_A} < 0\) in the unique equilibrium. Therefore, \(\frac{\partial \mu_A^*}{\partial q_A}\) has the same sign with \(\frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial q_A}\).

To see the sign of \(\frac{\partial H^A(\mu_A^*; \mu_B^*)}{\partial q_A}\), notice that

\[
H^A(\mu_A^*; \mu_B^*) = \frac{\delta_A}{Pr(s_A = 0) E_{r_B}} \theta_A(\phi, r_B)(1 - \theta_A(\phi, r_B))
\]

\[
+ C_A \left( \frac{\mu_A^*}{1 - q_A} \right) (1 - q_A) (1 - \mu_A^*)
\]

\[
- C_A \left( \frac{\mu_A^*}{1 - q_A} \right) \left( \frac{1 - \mu_A^*}{1 - \mu_A^*} + \frac{(1 - \theta_A(\phi, r_B)) \mu_A^*}{\theta_A(\phi, r_B) + (1 - \theta_A(\phi, r_B)) \mu_A^*} \right) (1 - \theta_A(\phi, r_B))
\]

\[
- (1 - \delta_A) C'_A \left( \frac{\mu_A^*}{1 - q_A} \right)^2 < 0.
\]

The last inequality is because in the first term, \(C_A \left( \frac{\mu_A^*}{1 - q_A} \right)\) increases in \(q_A\) since \(C'_A > 0\) and \(C_A \left( \frac{\mu_A^*}{1 - q_A} \right)\) decreases in \(q_A\) as

\[
\frac{\partial}{\partial q_A} \left( C_A \left( \frac{\mu_A^*}{1 - q_A} \right) (1 - q_A) \right)
\]

\[
= C'_A \left( \frac{\mu_A^*}{1 - q_A} \right) \frac{\mu_A^*}{1 - q_A} - C_A \left( \frac{\mu_A^*}{1 - q_A} \right)
\]

\[
= C'_A \left( \frac{\mu_A^*}{1 - q_A} \right) \frac{\mu_A^*}{1 - q_A} - C_A(0) - C'_A \left( \frac{\mu_A^*}{1 - q_A} \right) \frac{\mu_A^*}{1 - q_A} - C_A(\xi) \left( \frac{\mu_A^*}{1 - q_A} \right)^2
\]

\[
= -C_A(0) - C'_A(\xi) \left( \frac{\mu_A^*}{1 - q_A} \right)^2 < 0.
\]
The third equality uses the second-order Taylor expansion and \( \xi \in \left(0, \frac{\mu^*_A}{1-q_A}\right) \) is some constant.

The last inequality is due to \( C_A > 0 \) and \( C_A'' > 0 \). In the second term, \( C'_A \left( \frac{\mu^*_A}{1-q_A}\right) \) increases in \( q_A \) since \( C_A'' > 0 \). Overall, every term in \( H^A(\mu^*_A; \mu^*_B) \) decreases in \( q_A \) and thus \( \frac{\partial H^A(\mu^*_A; \mu^*_B)}{\partial q_A} < 0 \).

As a result, \( \frac{\partial \mu^*_A}{\partial q_A} < 0 \), which in turn leads to \( \frac{\partial V_q^B(q_B; q_A)}{\partial q_A} > 0 \).

Appendix II: micro-foundation for the continuous extension

In this section, we provide a micro-foundation for the information structure used in the continuous state and message extension. Suppose that after observing \( s_i \), manager \( i \) chooses manipulation \( m_i \) at a cost \( C_i(m_i) \) that adds \( m_i \) Bernoulli errors \( \{\eta_l\}_{l=1}^{m_i} \) into \( s_i \). Each error generates either \(-1\) or \( 1 \) with \( \Pr(\eta_l = 1) = p \in (0,1] \). The manager knows the realization of the errors and will only add the errors with a realization of \( 1 \).

Define a new variable \( \tilde{\eta}_l \) as a Bernoulli experiment with success rate \( p \), that is, \( \Pr(\tilde{\eta}_l = 1) = p \) and \( \Pr(\tilde{\eta}_l = 0) = 1 - p \).

The report is then given by

\[
\begin{align*}
    r_i &= s_i + \sum_{l=1}^{m_i} \tilde{\eta}_l.
\end{align*}
\]

Using the central limit theorem, we can approximate the distribution of \( \sum_{l=1}^{m_i} \tilde{\eta}_l \) by \( \sum_{l=1}^{m_i} \eta_l \sim N(m_i p, m_i p (1 - p)) \), when \( m_i \) is sufficiently large. Therefore, the function \( h(m_i) \) in the continuous extension is given by

\[
    h(m_i) = \sqrt{m_i p (1 - p)}.
\]

For \( p \in (0,1) \), \( h'(m_i) = \frac{1}{2} \sqrt{\frac{p(1-p)}{m_i}} > 0 \) whereas at \( p = 1 \), \( h'(m_i) = 0 \). Note that this micro-foundation has its limitation in that \( h(m_i) \) is not an exact expression due to approximation under the central limit theorem.

References


\(^{16}\)It is worth noting that in this micro-foundation, \( m_i \) is restricted to be a positive integer by construction, whereas in the continuous extension, \( m_i \) is a positive real number.


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