

DEPARTMENT OF COMMERCE

WORKSHOP TO BE HELD FRIDAY 1ST NOVEMBER, 1974

AT 2PM, ROOM 223 J.D. STORY BUILDING

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Participants please note: the attached note was written in 1972.
I plan to expand the argument, and am keen to hear the Workshop's
comments.

THE VALUE OF A DEBATE

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There can be no debating the merit of "present value" relative to the merit of "market values." This note attempts to show that any such debate is either empty or inconsistent (and therefore empty).¹

Net Present Value

Under some specific assumptions it can be shown that a stream of cash receipts will be valued according to the "present value" rule.² No doubt the rule can be derived from alternative assumptions. But I am aware of only one set of assumptions under which "net present value"

¹Sidney Davidson undoubtedly would have entitled this note: "Old Wine in Empty Bottles."

²The rule states a simple relationship between present and future prices (values). Hence,

$${}_t^p_\tau = {}_t^p_{t+1} \cdot {}_{t+1}^p_{t+2} \cdot {}_{t+2}^p_{t+3} \cdot \dots \cdot {}_{\tau-1}^p_\tau,$$

where ${}_t^p_\tau$ is the price at t of \$1 to be received at τ . The "value" at t of \$ x to be received at τ is ${}_t^p_\tau \cdot x$. The price of \$1 can be expressed in terms of an interest rate, ${}_t^r_\tau$, where:

$$(1 + {}_t^r_\tau) = \frac{{}_\tau^p_\tau}{{}_t^p_\tau} \quad \text{and}$$

$${}_t^p_\tau = \frac{1}{1 + {}_t^r_\tau},$$

since ${}_\tau^p_\tau = 1$. The price (V_t) of a stream of cash receipts $\{x_t\}$ can be shown as the sum of the individual prices:

would describe the valuation process. This particular set of assumptions is described in more detail elsewhere.³ It includes:

1. The assumption that markets are perfect, in the sense that all actors have equal access to the markets regardless of the magnitudes of their transactions and without transactions costs; and
2. the assumption that the future is perfectly certain.

Under the full set of assumptions there exists one and only one rate of interest for all assets for any one period. All investment assets

$$\begin{aligned}
 V_t &= \frac{x_{t+1}}{1 + r_{t+1}} + \frac{x_{t+2}}{1 + r_{t+2}} + \dots + \frac{x_\tau}{1 + r_\tau} \\
 &= \frac{x_{t+1}}{1 + r_{t+1}} + \frac{x_{t+2}}{(1 + r_{t+1})(1 + r_{t+2})} + \dots + \frac{x_\tau}{\prod_{i=t}^{\tau-1} (1 + r_{i+1})}
 \end{aligned}$$

This equation yields the familiar valuation mechanism where both x and r are assumed to be constant over time and $n = \tau - t - 1$:

$$\begin{aligned}
 V_t &= \frac{x}{1 + r} + \frac{x}{(1 + r)^2} + \dots + \frac{x}{(1 + r)^n} \\
 &= x \cdot \frac{(1 + r)^n - 1}{r(1 + r)^n}
 \end{aligned}$$

Numerical solutions for V when $x = \$1$ are reproduced as "present value tables" in many texts.

³See Fama and Miller [2, Chapter 1]. For earlier expositions see Fisher [3, 4] and Hirshleifer [5].

and all security claims to investments earn the same rate. The market values of all investments and claims are identical to the "present values" of their cash flows. And under the assumptions, the market value of each asset or claim is identical to its value to any individual, no matter what his personal time preference for consumption happens to be. The removal of either or both of the above assumptions withdraws the entire basis for considering "present value." Without the assumptions "present value" is an ad hoc procedure, with no conceptual underpinning and no conceptually debatable properties.⁴

An imperfect market might dictate different interest rates for different individuals or different assets. For example, transactions costs create a spread between borrowing and lending rates. Under these conditions, positive and negative cash flows would be "discounted" at different rates. Transactions costs might make the interest rate a function of the size of the cash flow. In both of these cases, valuation would not be conducted according to the "present value" rule.⁵

Market Values

The alleged competitors of net present value are legion. They include market selling prices ("current output prices," "net realizable value,"

⁴"Net present value" has received strange acceptance in the accounting literature as the "ideal" method of valuation. For example, the Committee on Accounting Concepts and Standards of the American Accounting Association wrote in its 1957 Statement [1, p. 4]:

The value of an asset is the money-equivalent of its service potentials. Conceptually, this is the sum of the future market prices of all streams of service to be derived, discounted by probability and interest factors to their present worths.

This strange acceptance belies the process under which ideals are established. "Ideal" valuation procedures can only be implications of particular models, or theories. Since different models can exist for the same purpose, there can be no such thing as the "ideal" valuation concept, except, perhaps, in an empirical sense.

⁵See Hirshleifer [5] for a more detailed illustration. Note that the "present value" rule assumes that r and x are independent: that

"current cash equivalent," "liquidation values") and market buying prices ("current input prices," "current costs," "replacement costs"). These market prices can only differ if there exist market imperfections such as transactions costs. In the absence of market imperfections, buying and selling prices will be identical for every asset and (under perfect certainty and the other assumptions) they will be identical to "net present value."⁶

What Debate?

If markets are imperfect or the future is uncertain, then the "present value" rule has no conceptual basis. It is difficult to imagine how a debate could be conducted under these conditions. "Present value" would have no interpretation as a notion of concern to any decision-maker and could not be evaluated in terms of any decision-maker's needs for information.

If the assumptions are allowed, then "present value" gives identical valuations to those given by the "market value" concepts. Again, it is difficult to imagine how a debate could be conducted under these conditions. "Present" and "market" values would always be identical.

no one asset's cash flows, when invested or disinvested in the market, are large enough to influence market interest rates.

⁶Note that I have defined "capital" markets in a broad fashion, not distinguishing the properties of securities markets from those of other asset markets.

The conclusion, then, is that whether or not one admits to the assumptions of the "present value" rule, there can be no debating its merit relative to the merit of "market" values.⁷

⁷Similar conclusions follow. For example, the American Accounting Association definition (quoted in footnote 4) is internally inconsistent, since it must assume perfect certainty for "present value" to be "conceptually" ideal, yet the definition refers to "probability" factors (and hence assumes uncertainty). It can be shown [6, p. 577] that inconsistent assumptions can lead to any conclusion.

The jointness of cash flows does not alter the analysis. Total or marginal cash flows can be analyzed in the same fashion: if, for either, there is a "present value," then it equals "market" value under the same assumptions that one needs to even consider "present value" in the first place.

REFERENCES

- [1] American Accounting Association, Committee on Concepts and Standards. Accounting and Reporting Standards for Corporate Financial Statements: A.A.A., 1957.
- [2] Fama, Eugene F. and Miller, Merton. The Theory of Finance. New York: Holt, Rinehart and Winston, Inc., 1971.
- [3] Fisher, Irving. The Rate of Interest. New York: The Macmillan Co., 1907.
- [4] _____ . The Theory of Interest. New York: The Macmillan Company, 1930.
- [5] Hirshleifer, J. "On the Theory of the Optimal Investment Decision." The Journal of Political Economy, LXVI (August, 1958), 329-52.
- [6] Thomas, Arthur L. "'Value-itis'--An Impractical Theorist's Reply." The Accounting Review, XXXIX (July, 1964), 574-81.