**THE UNIVERSITY OF CHICAGO**
**Graduate School of Business**
Business 41912, Spring Quarter 2006, Mr. Ruey S. Tsay

# Midterm

**Notes**:

1. Open book and notes. The exam time is 90 minutes.
2. Write your answers in a bluebook. Mark the solution clearly.
3. You are required to pledge to uphold the honor code of GSB in the exam.

1. Suppose that $\boldsymbol{X} = (X_1, X_2, X_3)'$ follows a 3-dimensional normal distribution with mean $\boldsymbol{\mu} = (1, 2, 3)'$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

Note that

$$\boldsymbol{S} = 0.308\boldsymbol{e}_1\boldsymbol{e}_1' + 0.6431\boldsymbol{e}_2\boldsymbol{e}_2' + 5.0489\boldsymbol{e}_3\boldsymbol{e}_3',$$

where

$$[\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3] = \begin{bmatrix} 0.591 & 0.737 & 0.??? \\ -0.737 & 0.328 & 0.591 \\ 0.328 & -0.591 & 0.??? \end{bmatrix}.$$

Answer the following equations:

(a) What is the distribution of $\boldsymbol{Z} = (X_1 - X_2, X_2 - X_3)'$?

(b) What is the distribution of $X_2$ given $X_3 = 2$?

(c) Find a linear combination of $\boldsymbol{X}$ of length 1 that has the maximum variance among all possible linear combination.

(d) Find the coefficient $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)'$ of the least squares regression

$$X_3 = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

2. Amitriptyline is prescribed by some physicians as an antidepressant. However, there are also conjectured side effects that seem to be related to the use of the drug: irregular heartbeat, abnormal bllod pressures, and irregular waves on the electrocardiogram, among other things. Data gathered on 17 patients who were admitted to the hospital after an amitriptyline overdose are collected. The two response variables are (a) $Y_1$ = total TCAD plasma level and (b) $Y_2$ = amount of amitriptyline present in TCAD plasma level. The five predictor variables are

- $Z_1$ = Gender; 1 if female, 0 if male.
- $Z_2$ = Amount of antidepressants taken at time of overdose.
- $Z_3$ = PR wave measurement.
- $Z_4$ = Diastolic blood pressure.
- $Z_5$ = QRS wave measurement.

We perform a multivariate multiple linear regression

$$\boldsymbol{Y} = \boldsymbol{Z\beta} + \boldsymbol{\epsilon},$$

to analyze the data, where the first column of $\boldsymbol{Z}$ is a vector of 1. R output is attached. Answer the following questions.

(a) Write down the fitted model.

(b) What are the standard error and $t$-ratio of the least squares estimate of $\beta_{2,1}$, the $(2,1)$th element of $\boldsymbol{\beta}$?

(c) Construct a 95% simultaneous prediction intervals for the two individual responses $Y_{0i}$ at $\boldsymbol{z}_0 = (1, 1, 1200, 140, 70, 85)'$.

(d) Perform a likelihood ratio test for the null hypothesis that $Z_5$ can be dropped from the multivariate linear regression at the 5% level. Draw your conclusion.

3. Consider Problem 6.26 of the textbook that concerning how consumers in Green Bay, Wisconsin would react to an electrical time-of-use pricing scheme. The measurement is $Y = \log(\text{current consumption}) - \log(\text{baseline consumption})$. The summary statistics of the experiment are given below. Test group: $n_1 = 28$, and $\bar{\boldsymbol{X}}_1 = (.153, -.231, -.322, -.339)'$. Control group: $n_2 = 58$, $\bar{\boldsymbol{X}}_2 = (.151, .180, .256, .257)'$. The pooled covariance matrix is

$$\boldsymbol{S}_{\text{pooled}} = \begin{bmatrix} .804 & .355 & .228 & .232 \\ .355 & .722 & .233 & .199 \\ .228 & .233 & .592 & .239 \\ .232 & .199 & .239 & .479 \end{bmatrix}$$

Perform a profile analsyis of the data. Does time-of-use pricing make any difference in electrical consumption at the 5% level? What is the nature of the difference, if any?

4. Eight men received a certain drug. The changes in blood sugar and blood pressure (both systolic and diastolic) are collected. The sample mean of the changes are (31.25,−0.75,3.125) and the sample covariance matrix and its inverse are given below

$$\boldsymbol{S} = \begin{bmatrix} 1069.64 & 82.5 & 16.96 \\ 82.5 & 17.36 & 6.39 \\ 16.96 & 6.39 & 4.70 \end{bmatrix}, \quad \boldsymbol{S}^{-1} = \begin{bmatrix} .00167 & -.01143 & .00954 \\ -.01143 & .19403 & -.22277 \\ .00954 & -.22277 & .48170 \end{bmatrix}.$$

Answer the following questions:

(a) Test the null hypothesis $H_o : \boldsymbol{\mu} = \mathbf{0}$ versus the alternative hypothesis $H_a : \boldsymbol{\mu} \neq \mathbf{0}$ at the 5% level. Draw your conclusion.

(b) Construct 95% simutaneous confidence intervals for the three changes.

(c) Construct 95% Bonferroni simultaneous confidence intervals for the three changes.

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*** R output for problem 2 of Mid-term Exam, 2006.
*** da stores the data.
> dim(da)
[1] 17  7
> y=cbind(da[,1],da[,2])
> z1=cbind(da[,3],da[,4],da[,5],da[,6],da[,7])
> z=cbind(rep(1,17),z1)

> ztz=t(z)%*%z
> zty=t(z)%*%y
> ztzinv=solve(ztz)
> beta=ztzinv%*%zty
> beta
              [,1]          [,2]
[1,] -2879.4782461 -2728.7085444
[2,]   675.6507805   763.0297617
[3,]     0.2848511     0.3063734
[4,]    10.2721328     8.8961977
[5,]     7.2511714     7.2055597
[6,]     7.5982397     4.9870508
> ztzinv
              [,1]          [,2]          [,3]          [,4]          [,5]
[1,] 10.0884715709 -7.719573e-01  1.420749e-04 -3.992674e-02 -2.055020e-02
[2,] -0.7719573270  3.320454e-01  1.791655e-05  1.896819e-03  2.059691e-03
[3,]  0.0001420749  1.791655e-05  4.690821e-08 -1.386930e-06  1.033346e-06
[4,] -0.0399267408  1.896819e-03 -1.386930e-06  2.289002e-04  3.959865e-05
[5,] -0.0205501985  2.059691e-03  1.033346e-06  3.959865e-05  1.315136e-04
[6,] -0.0178899133  6.205902e-04 -6.848574e-07 -5.414269e-06  3.205873e-05
              [,6]
[1,] -1.788991e-02
[2,]  6.205902e-04
[3,] -6.848574e-07
[4,] -5.414269e-06
[5,]  3.205873e-05
[6,]  1.872992e-04
> eps=y-z%*%beta
> rcov=t(eps)%*%eps
> sigma=rcov/(17-5-1)
> sigma
         [,1]     [,2]
[1,] 79091.66 69606.95
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[2,] 69606.95 85518.99

> kronecker(sigma,ztzinv)
               [,1]          [,2]         [,3]          [,4]          [,5]
 [1,] 797914.009706 -61055.389981 11.236944164 -3.157872e+03 -1.625349e+03
 [2,] -61055.389981  26262.026594  1.417050110  1.500225e+02  1.629044e+02
 [3,]     11.236944      1.417050  0.003710048 -1.096946e-01  8.172903e-02
 [4,]  -3157.872390    150.022536 -0.109694600  1.810410e+01  3.131923e+00
 [5,]  -1625.349406    162.904357  0.081729026  3.131923e+00  1.040163e+01
 [6,]  -1414.943019     49.083508 -0.054166512 -4.282235e-01  2.535578e+00
 [7,] 702227.760065 -53733.596885  9.889404164 -2.779179e+03 -1.430437e+03
 [8,] -53733.596885  23112.671147  1.247116747  1.320318e+02  1.433688e+02
 [9,]      9.889404      1.247117  0.003265137 -9.653997e-02  7.192804e-02
[10,]  -2779.178744    132.031758 -0.096539968  1.593304e+01  2.756341e+00
[11,]  -1430.436687    143.368784  0.071928040  2.756341e+00  9.154259e+00
[12,]  -1245.262340     43.197389 -0.047670836 -3.768708e-01  2.231510e+00
               [,6]          [,7]          [,8]         [,9]         [,10]
 [1,] -1.414943e+03 702227.760065 -53733.596885  9.889404164 -2.779179e+03
 [2,]  4.908351e+01 -53733.596885  23112.671147  1.247116747  1.320318e+02
 [3,] -5.416651e-02      9.889404      1.247117  0.003265137 -9.653997e-02
 [4,] -4.282235e-01  -2779.178744    132.031758 -0.096539968  1.593304e+01
 [5,]  2.535578e+00  -1430.436687    143.368784  0.071928040  2.756341e+00
 [6,]  1.481381e+01  -1245.262340     43.197389 -0.047670836 -3.768708e-01
 [7,] -1.245262e+03 862755.907989 -66017.011582 12.150106211 -3.414495e+03
 [8,]  4.319739e+01 -66017.011582  28396.190973  1.532205650  1.622140e+02
 [9,] -4.767084e-02     12.150106      1.532206  0.004011543 -1.186089e-01
[10,] -3.768708e-01  -3414.494580    162.214008 -0.118608851  1.957531e+01
[11,]  2.231510e+00  -1757.432236    176.142660  0.088370675  3.386437e+00
[12,]  1.303733e+01  -1529.927329     53.072244 -0.058568314 -4.630228e-01
              [,11]         [,12]
 [1,] -1.430437e+03 -1.245262e+03
 [2,]  1.433688e+02  4.319739e+01
 [3,]  7.192804e-02 -4.767084e-02
 [4,]  2.756341e+00 -3.768708e-01
 [5,]  9.154259e+00  2.231510e+00
 [6,]  2.231510e+00  1.303733e+01
 [7,] -1.757432e+03 -1.529927e+03
 [8,]  1.761427e+02  5.307224e+01
 [9,]  8.837068e-02 -5.856831e-02
[10,]  3.386437e+00 -4.630228e-01
[11,]  1.124691e+01  2.741630e+00
[12,]  2.741630e+00  1.601764e+01
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```
> z0=as.vector(c(1,1,1200,140,70,85))
> z0=as.matrix(z0)
>
> t(beta)%*%z0
         [,1]
[1,] 729.5248
[2,] 575.7255

> 1+t(z0)%*%ztzinv%*%z0
         [,1]
[1,] 1.236048

> qf(.95,2,10)
[1] 4.102821
>
> sigmaf=rcov/17
> det(sigmaf)
[1] 803335966
>  %%% Reduced model %%%
> x1=cbind(da[,3],da[,4],da[,5],da[,6])
> x=cbind(rep(1,17),x1)
> xtx=t(x)%*%x
> xty=t(x)%*%y
> xtxinv=solve(xtx)
> betar=xtxinv%*%xty
> betar
              [,1]          [,2]
[1,] -2153.7312674 -2252.3696576
[2,]   650.4750615   746.5058569
[3,]     0.3126339     0.3246085
[4,]    10.4917756     9.0403586
[5,]     5.9506326     6.3519604
> epsr=y-x%*%betar
> rcovr=t(epsr)%*%epsr
> sigmar=rcovr/17
> det(sigmar)
[1] 1134405317
```