**THE UNIVERSITY OF CHICAGO**
**Graduate School of Business**
Business 41912, Spring Quarter 2008, Mr. Ruey S. Tsay

## Solutions to Final Exam

1. (13 pts) Consider the monthly log returns, in percentages, of five U.S. stocks from 1990 to 1999 for 120 observations. The stocks are (1) IBM, (2) Hewlett-Packard (HPQ), (3) Intel (INTC), (4) Merill Lynch (MER), and (5) Morgan Stanley Dean Witter (MWD). Let $\boldsymbol{X}_t = (IBM_t, HPQ_t, INTC_t)'$ and $\boldsymbol{Y}_t = (MER_t, MWD_t)'$ so that $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$ represent returns of high-tech and financial stocks, respectively. Based on the attached output, answer the following questions:

   (a) (2 pts) What are the canonical correlation coefficients between $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$?
   **Answer**: The canonical correlations are 0.404 and 0.052.

   (b) (4 pts) Obtain the first two canonical variates of $\boldsymbol{X}_t$. Show the steps taken to obtain your answer.
   **Answer**: From the output, we obtain the eigenvectors of $\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{21}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}$ as

$$\boldsymbol{G} = \begin{bmatrix} -0.581 & -0.721 \\ -0.814 & 0.693 \end{bmatrix}.$$

   The canonical variates of $\boldsymbol{X}_t$ can be obtained via

$$\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{21}[\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}\boldsymbol{G}] = [\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}\boldsymbol{G}]\boldsymbol{\Lambda},$$

   where $\boldsymbol{\Lambda} = \text{diag}\{0.163, 0.003\}$. In other words, pre-multiplying $\boldsymbol{G}$ by $\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12}$, we can obtain the canonical variates of $\boldsymbol{X}_t$. The results are $(-0.027, -0.448, -0.134)'$ and $(-.007, .020, -.024)'$. [Note that eigenvectors are determined up to a scale factor. For instance, the second canonical variate can be $(-0.213, 0.615, -0.759)'$.]

   (c) (3 points) Let $\rho_1^* > \rho_2^*$ be the two canonical correlations between $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$. Test the null hypothesis $H_o: \rho_2^* = 0$ versus $H_a: \rho_2^* \neq 0$. What is the statistic? Draw your conclusion.
   **Answer**: Using Eq. (10.41), page 565,

$$T = -[120 - 1 - 0.5(2 + 3 + 1)] \ln(1 - 0.0267) = 0.310,$$

   which is distributed as $\chi_2^2$. The p-value is 0.856. Thus, we cannot reject the null hypothesis that $\rho_2^* = 0$.

(d) (4 points) Setup the hypotheses to test the null that $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$ are uncorrelated. Calculate the test statistic? Draw your conclusion.

**Answer**: $H_o : \boldsymbol{\Sigma}_{xy} = \mathbf{0}$ versus $\boldsymbol{\Sigma}_{xy} \neq \mathbf{0}$, where $\boldsymbol{\Sigma}_{xy}$ is the covariance matrix beween $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$. Using Eq. (10.39), we have

$$T = -(120 - 1 - 0.5(2 + 3 + 1))[\ln(1 - 0.00267) + \ln(1 - 0.1631)] = 20.964,$$

which is distributed as $\chi^2_6$. The p-value is 0.002. Thus, we reject the null hypothesis that $\boldsymbol{X}_t$ and $\boldsymbol{Y}_t$ are uncorrelated.

2. (10 pts) Again, consider the five monthly stock returns of Problem 1. The correlation matrix of the stocks is

| | IBM | HPQ | INTC | MER | MWD |
|---|---|---|---|---|---|
| IBM | 1.00 | 0.42 | 0.30 | 0.18 | 0.18 |
| HPQ | 0.42 | 1.00 | 0.45 | 0.36 | 0.38 |
| INTC | 0.30 | 0.45 | 1.00 | 0.25 | 0.24 |
| MER | 0.18 | 0.36 | 0.25 | 1.00 | 0.79 |
| MWD | 0.18 | 0.38 | 0.24 | 0.79 | 1.00 |

Answer the following questions:

(a) (2 points) Use the correlations to construct a distance measure between the five stocks. Write down the distance.

**Answer:** The distance is 1 − correlation. Thus,

| | IBM | HPQ | INTC | MER |
|---|---|---|---|---|
| HPQ | 0.58 | | | |
| INTC | 0.70 | 0.55 | | |
| MER | 0.82 | 0.64 | 0.75 | |
| MWD | 0.82 | 0.62 | 0.76 | 0.21 |

(b) (2 points) Perform the hierarchical cluster analysis using the single linkage method. Show details of the first updating of the distance and draw the dendrogram.

**Answer**: The first cluster is {MER,WMD}. The resulting distance is

| | IBM | HPQ | INTC |
|---|---|---|---|
| HPQ | 0.58 | | |
| INTC | 0.70 | 0.55 | |
| (MER,MWD) | 0.82 | 0.62 | 0.75 |

The dendrogram is shown in Figure 1.

(c) (2 points) Perform the hierarchical cluster analysis using the complete linkage method. Show details of the first updating of the distance and draw the dendrogram.

**Answer**: The first cluster is {MER,MWD}. The resulting distance of the first iteration is

| | IBM | HPQ | INTC |
|---|---|---|---|
| HPQ | 0.58 | | |
| INTC | 0.70 | 0.55 | |
| (MER,MWD) | 0.82 | 0.64 | 0.76 |

The dendrogram is shown in Figure 2.

(d) (2 points) Perform the hierarchical cluster analysis using the average linkage method. Show details of the first updating of the distance and draw the dendrogram.

**Answer**: The first cluster is {MER,MWD}. The resulting distance of the first iteration is

| | IBM | HPQ | INTC |
|---|---|---|---|
| HPQ | 0.58 | | |
| INTC | 0.70 | 0.55 | |
| (MER,MWD) | 0.82 | 0.63 | 0.755 |

The dendrogram is shown in Figure 3.

(e) (2 points) Compare and comment on the three linkage methods. In particular, is there any difference among them in this particular instance?

**Answer**: In this particular instance, the three method give similar results, but the distance measures differ slightly.

3. (15 points) Consider again the monthly log returns of five stocks in Problem 1. The eigenvalues and eigenvectors of the sample correlation matrix are given in the output. Answer the following questions.

(a) (3 points) Assume one common factor only. Obtain an orthogonal factor model using the principal component analysis method. Write down the factor loadings and the specific variances.

**Answer**: The factor loadings and uniquenesses are

| Stock | $f_1$ | $\Psi_i$ |
|---|---|---|
| IBM | 0.54 | 0.71 |
| HPQ | 0.74 | 0.445 |
| INTC | 0.61 | 0.63 |
| MER | 0.79 | 0.38 |
| MWD | 0.79 | 0.37 |

Figure 1: Dendogram: Simple linkage

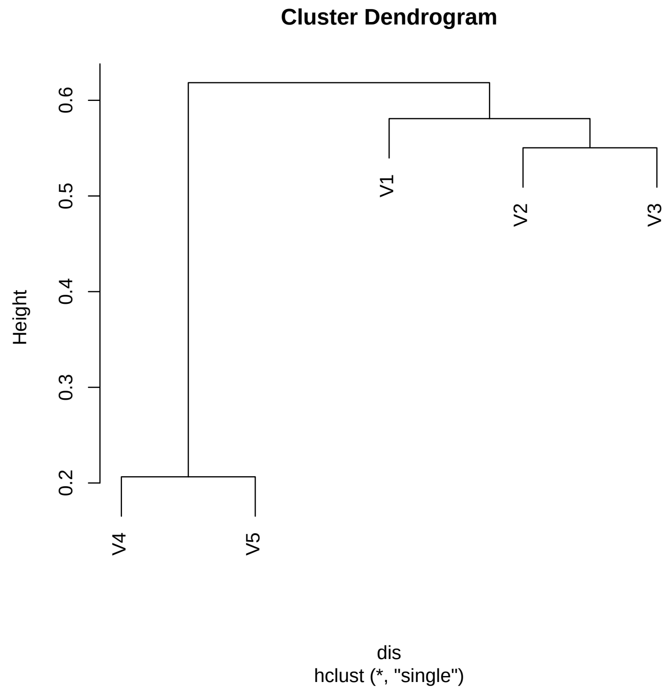



Figure 2: Dendogram: Complete linkage

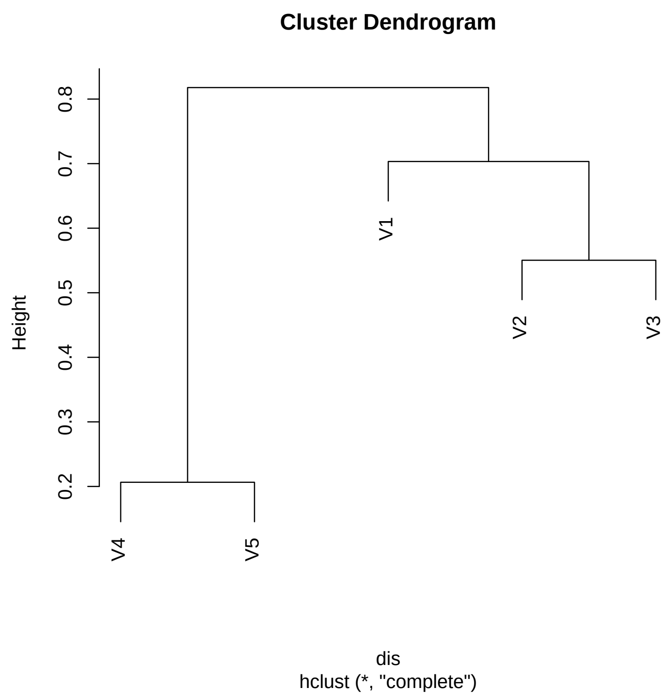
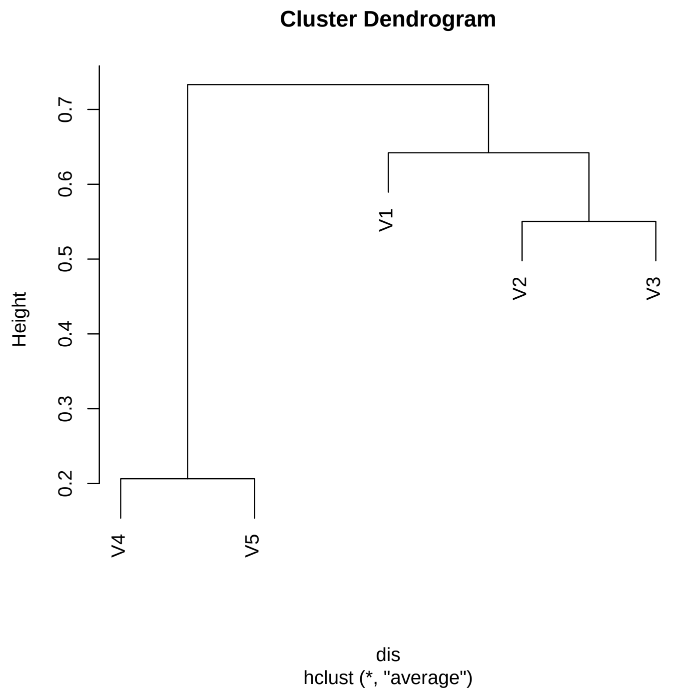


Figure 3: Dendogram: Average linkage




Note that the sign of loading vector can be changed.

(b) (4 points) Assume that there are two common factors. Obtain an orthogonal factor model using the principal component analysis method. Write down the factor loadings and the specific variance.

**Answer**: The factor loadings and uniquenesses are

| Stock | $f_1$ | $f_2$ | $\Psi_i$ |
|---|---|---|---|
| IBM | 0.54 | 0.56 | 0.40 |
| HPQ | 0.74 | 0.34 | 0.33 |
| INTC | 0.61 | 0.43 | 0.44 |
| MER | 0.79 | -0.51 | 0.11 |
| MWD | 0.79 | -0.51 | 0.11 |

(c) (2 points) What is the proportion of total variance explained by the prior two-factor model?

**Answer**: $(2.456+1.145)/5 = 0.72 = 72\%$.

(d) (2 points) The factor analysis using the maximum likelihood method is given in the output. Write down the fitted orthogonal factor model, including factor loadings and the specific variances.

**Answer**: The factor loadings and uniquenesses are

| Stock | $f_1$ | $f_2$ | $\Psi_i$ |
|---|---|---|---|
| IBM | 0 | 0.52 | 0.72 |
| HPQ | 0.25 | 0.75 | 0.37 |
| INTC | 0.15 | 0.55 | 0.68 |
| MER | 0.77 | 0.23 | 0.36 |
| MWD | 0.98 | 0.18 | 0.01 |

(e) (4 points) For the maximum likelihood method, what is the large sample test statistic for testing $m = 2$ factors when the Bartlett's correction is used? What is the p-value? Draw your conclusion.

**Answer**: From the loadings and uniquenesses, $|\boldsymbol{L}\boldsymbol{L}'\boldsymbol{\Psi}| = 0.20$. From the correlation matrix, $|\boldsymbol{R}| = 0.1996378$. Therefore, the test statistic is T= $(120 - 1 - (2 * 5 + 4 * 2 + 5)/6) \ln(0.2/0.1996378) = 0.21$ with p-value 0.65. We cannot reject the two-fator model.

4. (12 pts) Assume that $n_1 = 11$ and $n_2 = 12$ observations were randomly collected from Populations 1 and 2. Assume also that the observations are bivariate and follow multivariate normal distributions $N_2(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$ for $i = 1$ and 2. Suppose that the summary statistics of the samples are

$$\bar{\boldsymbol{x}}_1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \quad \bar{\boldsymbol{x}}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \boldsymbol{S}_{pooled} = \begin{bmatrix} 4.0 & -1.0 \\ -1.0 & 5.0 \end{bmatrix}.$$

Answer the following questions:

(a) (4 points) Test $H_o : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ versus $H_a : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$ using the Hotelling's two-sample $T^2$-statistic. Draw the conclusion.

**Answer**: Using Result 6.2, $T^2 = 29.90$. The corresponding F-statistic is 14.24 with p-value 0.00014. Thus, the equality in mean vectors is rejected at the 5% level.

(b) (4 points) Construct Fisher's (sample) linear discrimiant function for the two populations.

**Answer**: The Fisher's linear discriminat function is

$$(\bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2)'\boldsymbol{S}_p^{-1}\boldsymbol{x} = -0.95x_1 - 0.79x_2$$

with $\hat{m} = 0.5(\bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2)'\boldsymbol{S}_p^{-1}(\bar{\boldsymbol{x}}_1 + \bar{\boldsymbol{x}}_2) = -0.16$.

(c) (4 points) Assume equal costs and equal prior probabilities. Assign the new observation $\boldsymbol{x}_o = (0, 1)'$ to either population.

**Answer**: For $\boldsymbol{x}_o = (0, 1)'$, $\hat{y} = -0.95 \times 0 - 0.79 \times 1 = -0.79$, which is less tham $\hat{m} = -0.16$. Therefore, $\boldsymbol{x}_o$ is allocated to Population II.

5. (10 pts) Consider the multiple linear regression model

$$\boldsymbol{Y}_{n\times 1} = \boldsymbol{Z}_{n\times(r+1)}\boldsymbol{\beta}_{(r+1)\times 1} + \boldsymbol{\epsilon}_{n\times 1},$$

where $E(\boldsymbol{\epsilon}) = \mathbf{0}$ and $\text{Cov}(\boldsymbol{\epsilon}) = \sigma^2\boldsymbol{I}_n$ with $\boldsymbol{I}_n$ being the $n\times n$ identity matrix, and $\boldsymbol{Z}$ is of full rank $(r+1)$ with the first column consisting of 1. Let $\boldsymbol{H} = \boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'$ be the hat-matrix, $\widehat{\boldsymbol{\beta}}$ be the least squares estimate of $\boldsymbol{\beta}$, and $\widehat{\boldsymbol{\epsilon}} = \boldsymbol{Y} - \boldsymbol{Z}\widehat{\boldsymbol{\beta}}$ be the residual vectors. Prove the following statements:

(a) (2 points) Let $\hat{\epsilon}_i$ be the $i$th element of $\widehat{\boldsymbol{\epsilon}}$. Then, $\sum_{i=1}^n \hat{\epsilon}_i = 0$.
**Proof**: Using $\widehat{\boldsymbol{\epsilon}}'\boldsymbol{Z} = \mathbf{0}$ and the fact that the first column of $\boldsymbol{Z}$ consists of 1's, we have $\sum_{i=1}^n \hat{\epsilon}_i = 0$.

(b) (3 points) $E(\widehat{\boldsymbol{\epsilon}}\widehat{\boldsymbol{\epsilon}}') = (\boldsymbol{I} - \boldsymbol{H})\sigma^2$.
**Proof**: Since $\widehat{\boldsymbol{\epsilon}} = (\boldsymbol{I} - \boldsymbol{H})\boldsymbol{Y}$ and $\boldsymbol{I} - \boldsymbol{H}$ is an idempotent matrix, we have

$$\begin{aligned} E(\widehat{\boldsymbol{\epsilon}}\widehat{\boldsymbol{\epsilon}}') &= (\boldsymbol{I} - \boldsymbol{H})E(\boldsymbol{Y}\boldsymbol{Y}')(\boldsymbol{I} - \boldsymbol{H}) \\ &= \sigma^2(\boldsymbol{I} - \boldsymbol{H})\boldsymbol{I}(\boldsymbol{I} - \boldsymbol{H}) \\ &= \sigma^2(\boldsymbol{I} - \boldsymbol{H}). \end{aligned}$$

(c) (5 points) If $r = 1$, i.e. simple linear regression, then

$$h_{jj} = \frac{1}{n} + \frac{(z_j - \bar{z})^2}{\sum_{i=1}^n (z_i - \bar{z})^2},$$

where $h_{jj}$ is the $(j, j)$th element of $\boldsymbol{H}$ and $z_j$ is the $j$th element of the 2nd column of $\boldsymbol{Z}$.
**Proof**: For the simple linear regression, we have

$$\boldsymbol{Z}'\boldsymbol{Z} = \begin{bmatrix} n & \sum_{i=1}^n z_i \\ \sum_{i=1}^n z_i & \sum_{i=1}^n z_i^2 \end{bmatrix}, \quad (\boldsymbol{Z}'\boldsymbol{Z})^{-1} = \frac{1}{d}\begin{bmatrix} \sum_{i=1}^n z_i^2 & -\sum_{i=1}^n z_i \\ -\sum_{i=1}^n z_i & n \end{bmatrix},$$

where $d$ is the determinant of $\boldsymbol{Z}'\boldsymbol{Z}$. Note that

$$d = n\sum_{i=1}^n z_i^2 - (\sum_{i=1}^n z_i)^2 = n\left[\sum_{i=1}^n z_i^2 - n\bar{z}^2\right].$$

On the other hand,

$$\sum_{i=1}^n (z_i - \bar{z})^2 = \sum_{i=1}^n (z_i^2 - 2z_i\bar{z} + \bar{z}^2) = \sum_{i=1}^n z_i^2 - n\bar{z}^2.$$

Consequently, $d = n\sum_{i=1}^n (z_i - \bar{z})^2$. Finally,

$$h_{jj} = [1, z_j](\boldsymbol{Z}'\boldsymbol{Z})^{-1}[1, z_j]'$$

$$
\begin{aligned}
&= \frac{1}{d}\left[\sum_{i=1}^{n} z_i^2 - 2n\bar{z}z_j + nz_j^2\right] \\
&= \frac{1}{d}\left[\sum_{i=1}^{n} z_i^2 - n\bar{z}^2 + n\bar{z}^2 - 2n\bar{z}z_j + nz_j^2\right] \\
&= \frac{1}{d}\left[\left(\sum_{i=1}^{n} z_i^2 - n\bar{z}^2\right) + n(\bar{z}^2 - 2\bar{z}z_j + z_j^2)\right] \\
&= \frac{1}{d}\left[d/n + n(\bar{z} - z_j^2)^2\right] \\
&= \frac{1}{n} + \frac{(z_j - \bar{z})^2}{\sum_{i=1}^{n}(z_j - \bar{z})^2}.
\end{aligned}
$$

6. (10 pts) Assume that $\boldsymbol{X}$ is a $p$-dimensional normal random vector with mean zero and covariance matrix $\boldsymbol{\Sigma}$. Suppose that a radnom sample $\{\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n\}$ of $n$ observations is available. Let $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\boldsymbol{X}'\boldsymbol{X}$, where $\boldsymbol{X}' = (\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n)$. Prove the following statements:

   (a) (4 points) $\widehat{\boldsymbol{\Sigma}}$ is the maximum likelihood estimate of $\boldsymbol{\Sigma}$.

   **Proof**: Since the mean is zero, the likelihood function is

$$
L(\boldsymbol{\Sigma} = \frac{1}{(2\pi)^{np/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left[-tr\left(\boldsymbol{\Sigma}^{-1}(\sum_{j=1}^{n} \boldsymbol{X}_j\boldsymbol{X}_j')\right)\right].
$$

   By Result 4.10 and $\boldsymbol{B} = \sum_{j=1}^{n} \boldsymbol{X}_j\boldsymbol{X}_j'$, the MLE of $\boldsymbol{\Sigma}$ is $\frac{1}{n}\sum_{j=1}^{n} \boldsymbol{X}_j\boldsymbol{X}_j' = \frac{1}{n}\boldsymbol{X}'\boldsymbol{X}$.

   (b) (2 points) $E(\widehat{\boldsymbol{\Sigma}}) = \boldsymbol{\Sigma}$.

   **Proof**: $E(\widehat{\boldsymbol{\Sigma}}) = \frac{1}{n}\sum_{j=1}^{n} E(\boldsymbol{X}_j\boldsymbol{X}_j') = \boldsymbol{\Sigma}$.

   (c) (4 points) When $n < p$, $\widehat{\boldsymbol{\Sigma}}$ is singular. Show that the non-zero eigenvalues of $\boldsymbol{X}'\boldsymbol{X}$ are the same as those of $\boldsymbol{X}\boldsymbol{X}'$, which is a $n \times n$ matrix. *Hint: use the identity*

$$
|\boldsymbol{I}_p - \boldsymbol{X}'\boldsymbol{X}| = |\boldsymbol{I}_n - \boldsymbol{X}\boldsymbol{X}'|.
$$

   **Proof**: For$\lambda \neq 0$, from the identity,

$$
\begin{aligned}
|\lambda\boldsymbol{I}_p - \boldsymbol{X}'\boldsymbol{X}| &= \lambda^p|\boldsymbol{I}_p - \boldsymbol{X}'(\lambda^{-1}\boldsymbol{X})| \\
&= \lambda^p|\boldsymbol{I}_n - (\lambda^{-1}\boldsymbol{X})\boldsymbol{X}'| \\
&= \lambda^{p-n}|\lambda\boldsymbol{I} - \boldsymbol{X}\boldsymbol{X}'|.
\end{aligned}
$$

   Hence the non-zero eigenvalues of $\boldsymbol{X}'\boldsymbol{X}$ are the same as the non-zero eigenvalues of $\boldsymbol{X}\boldsymbol{X}'$.

7. (15 pts) Anacondas are some of the largest snakes in the world. Jesus Ravis and his associates capture a snake and measure its (i) snout vent length (cm) or the length from

the snout of the snake to its vent where it evaculates waste and (ii) weights (kilograms). A sample of these measurements is shown in Table 6.19 of the textbook (p. 357). Some summary statistics of the data are given in the output. Use the information to asnwer the following questions:

(a) Test the equality of the two covariance matrices between male and female snakes.

**Answer**: Apply the Box-M test. The test statistic is 100.33 with p-value close to zero. Thus, reject the equality in the covariance matrix.

(b) Test the equality of the means between male and female snakes based on the result of part 1 (i.e. to pool or not to pool the covariances).

**Answer**; Since the covariance matrices are not the same, we do not pool the covariance matrices. The Hotelling $T^2$ statistic is 76.92 and the corresponding F-statistic is 37.75 with p-value close to zero. Thus, the equality of means is also rejected.

(c) Construct the 95% Bonferroni confidence intervals for the mean differences between males and females on both length and weight.

**Answer**: The 95% C.I.s for the differences in means are [84.8,151] and [21.2, 38.8], respectively.

8. (15 pts) Consider the monthly 30-year fixed mortgage rates ($M_t$) and the 2-year Treasury Constant Maturity interest rates ($I_t$) of the U.S. from June 1976 to May 2008 for 384 observations. Let $\boldsymbol{Y}_t = (M_t, I_t)'$ be the 2-dimensional dependent variable and $\boldsymbol{X}_t = (1, \boldsymbol{Y}'_{t-1}, \boldsymbol{Y}'_{t-2}, \boldsymbol{Y}'_{t-3}, \boldsymbol{Y}'_{t-4})'$ be an 9-dimensional independent variable, including the first element being 1. This is equivalent to employing a VAR(4) model for $\boldsymbol{Y}_t$. Let $\boldsymbol{Z}_t = (1, \boldsymbol{Y}'_{t-1}, \boldsymbol{Y}'_{t-2}, \boldsymbol{Y}'_{t-3})'$ be a 7-dimensional independent variable. Thus, using $\boldsymbol{Z}_t$ is equivalent to fitting a VAR(3) model to $\boldsymbol{Y}_t$. More presicely, we write the VAR(3) model as

$$\boldsymbol{Y}'_t = \boldsymbol{Z}'_t \boldsymbol{\beta}_3 + \boldsymbol{\epsilon}'_t$$

and the VAR(4) model as

$$\boldsymbol{Y}'_t = \boldsymbol{X}'_t \boldsymbol{\beta}_4 + \boldsymbol{e}'_t,$$

where

$$\boldsymbol{\beta}_4 = \begin{bmatrix} \boldsymbol{\beta}_3 \\ \boldsymbol{\beta}_* \end{bmatrix} \quad \text{with} \quad \boldsymbol{\beta}_* \quad \text{being a } 2 \times 2 \text{ matrix.}$$

Some R output is attached. Use the information to answer the following questions.

(a) (4 points) Test the hypothesis $H_o : \boldsymbol{\beta}_* = \mathbf{0}$ versus $H_a : \boldsymbol{\beta}_* \neq \mathbf{0}$. Under the normality assumption, what is the maximum likelihood ratio test statistic? Draw your conclusion.

**Answer**: Apply Result 7.11, the test statistic is

$$-(380 - 9 - 0.5(2 - -8 + 6 + 1)) \ln(0.005237/0.0052484) = 0.78$$

which, compared with chi-square with 4 degrees of freedom, is insignifcant. Thus, we cannot reject $H_o$.

(b) (2 points) Write down the fitted VAR(3) model, including covariance matrix of the residuals.

**Answer**:The fitted model is

$$\boldsymbol{Y}_t' = \boldsymbol{Z}_t' \begin{bmatrix} 0.156 & 0.099 \\ 1.011 & -0.575 \\ 0.483 & 1.632 \\ -0.161 & 0.621 \\ -0.484 & -0.714 \\ 0.094 & -0.065 \\ 0.054 & 0.093 \end{bmatrix} + \boldsymbol{\epsilon}_t', \quad \text{Cov}(\boldsymbol{\epsilon}_t) = \begin{bmatrix} 0.051 & 0.054 \\ 0.054 & 0.166 \end{bmatrix}.$$

(c) (3 points) Focus on the VAR(3) fit. Let $\beta_{ij}$ be the $(i,j)$th element of $\boldsymbol{\beta}_3$. Test the hypothesis $H_o : \beta_{11} = 0$ versus $H_a : \beta_{11} \neq 0$. What is the $t$-ratio? What is the associated p-value? Draw your conclusion.

**Answer**: The $t$-ratio is 2.88 with p-value $4.27 \times 10^{-3}$. THus, the estimate is significantly different from zero.

(d) (3 points) Again, focus on the VAR(3) fit. Test the hypothesis $H_o : \beta_{21} = 0$ versus $H_a : \beta_{21} \neq 0$. What is the $t$-ratio? What is the associated p-value? Draw your conclusion.

**Answer**: The $t$-ratio is 16.02 with p-value close to zero. Thus, the estimate is significantly different from zero.

(e) (3 points) Again, focus on the VAR(3) fit. What is the estimated covariance between $\beta_{21}$ and $\beta_{22}$?

**Answer**: $0.0544 \times 0.0786 = 0.0043$.