1. Consider five measurements of normal patients and diabetics in the data `Diabetes.txt`. The variables are y1: relative weight; y2: fasting plasma glucose; x1: glucose intolerance; x2: insulin response to oral glucose; x3: insulin resistance. The original data are from Reaven and Miller (1979, Diabetologia). Focus on x variables that are of main interest. Answer the following questions:

(a) Yes, the three variables are normally distributed as the Mardia test fails to reject the normality at the 5% level. The p-values of all three tests are greater than 0.25.

(b) The sample means are (340.82, 171.37, 97.78). The sample covariance matrix is

\[
\hat{\Sigma} = \begin{bmatrix}
1106.4135 & 396.7324 & 108.3836 \\
396.7324 & 2381.8826 & 1142.6377 \\
108.3836 & 1142.6377 & 2136.3961
\end{bmatrix}.
\]

(c) The eigenvalues and their corresponding eigenvectors are given below

```r
> Eigenvalues
[1] 3465.9616 1264.1409 894.5897
```

```r
Eigenvectors
[,1]    [,2]    [,3]
[1,] -0.1548351 0.6374788 0.7547495
[2,] -0.7429870 0.4283895 -0.5142497
[3,] -0.6511500 -0.6403930  0.4073087
```

2. Compute the Kendall’s tau, Spearman’s rho, and the usual Pearson correlation between x variables. (pairwise.)

Answer: The correlations are given below (shown in upper triangular format (x1,x2,x3)

<table>
<thead>
<tr>
<th></th>
<th>Pearson</th>
<th>Kendall</th>
<th>Spearman</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>0.244</td>
<td>0.143</td>
<td>0.219</td>
</tr>
<tr>
<td>x2</td>
<td>0.070</td>
<td>0.054</td>
<td>0.078</td>
</tr>
<tr>
<td>x3</td>
<td>0.507</td>
<td>0.374</td>
<td>0.520</td>
</tr>
</tbody>
</table>

3. Problem 4.6 of the textbook.
(a) $X_1$ and $X_2$ are independent because their covariance is zero.
(b) $X_1$ and $X_3$ are dependent; their covariance is not zero.
(c) $X_2$ and $X_3$ are independent because their covariance is zero.
(d) $(X_1, X_3)$ and $X_2$ are independent because their covariance matrix is zero.
(e) $X_1$ and $X_1 + 3X_2 - 2X_3$ are dependent because their covariance is 6.

4. Parts (b) and (c) of Problem 4.13.

(a) Follow the hint (including those of Problems 4.11 and 4.12)
(b) Simply partition $f(x|\mu, \Sigma) = f(x_1|x_2, \mu_1^*, \Sigma_{11}^*)f(x_2|\mu_2, \Sigma_{22})$, where $\mu_1^* = \mu_1 - \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2)$ and $\Sigma_{11}^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$.

5. Problem 4.21 of the textbook

(a) $N(\mu, \frac{1}{60}\Sigma)$
(b) $\chi_4^2$
(c) $\chi_2^2$
(d) Approximately $\chi_3^2$. 