Graduate School of Business, University of Chicago
Business 41202, Spring Quarter 2004, Mr. Ruey S. Tsay

Midterm

GSB Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signiture: **Name**: **ID**:

Notes:

- Open notes and books.
- Write your answer in the blank space provided for each question.
- **Manage** your time carefully and answer as many questions as you can.
- The exam has 6 pages and the computer output has 10 pages. Please **check** that you have all the pages.
- For simplicity, **ALL** tests use the 5% significance level.

Problem A: (30 pts) Answer briefly the following questions.

1. **For problems 1 to 6**, consider the daily log return of the exchange rate between U.S. Dollar and Japanese Yen from January 3, 2000 to March 26, 2004 for 1062 observations. The returns are in percentages. Summary statistics of the log returns from SCA and Splus are attached. See Page 1 of output. What is the sample skewness of the log returns?

2. Is the mean of log returns significantly different from zero? Why?

3. Test the null hypothesis that the excess kurtosis of the log returns is zero. Draw your conclusion.

4. The output also provides the last 3 data points of the percentage log returns. Transform the last data point -0.123 into percentage simple return.

5. Write down the null hypothesis of the Ljung-Box statistic $Q(12)$ for testing that the daily log returns have no serial correlations.

6. Are the log returns of exchange rate serially correlated? Answer this question using the Q(12) statistic.

7. Give two possible applications of asset return volatility.

8. Give two approaches that researchers commonly used to construct or estimate the volatility of an asset return.

9. Define the lag-3 sample partial autocorrelation function (PACF) of the return series $\{r_1, r_2, \cdots, r_T\}$.

10. Describe a method to check the serial correlations of a linear regression model with time series errors.

11. Consider the seasonal MA model $x_t = 0.5 + (1 - 0.4B)(1 - 0.6B^{12})a_t$, where $\{a_t\}$ is a white noise series. Besides the lag-0 serial correlation, how many autocorrelation functions (ACF) of x_t are non-zero? Provide the list of non-zero ACFs.

12. Consider the simple ARCH(1) model

$$r_t = 0.021 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = 0.116 + 0.42 a_{t-1}^2.$$

What is the unconditional variance of a_t? That is, what is $\text{Var}(a_t)$?

13. **For problems 13-15**, consider the following model for the log return r_t of an asset

$$r_t = 0.016 + 0.2 r_{t-1} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = 0.1 + 0.1 a_{t-1}^2 + 0.8 \sigma_{t-1}^2,$$

where $\{\epsilon_t\}$ are *iid* standard Gaussian random variables. What is the expected value of the log return r_t?

14. What is the unconditional variance of a_t?

15. Suppose that $r_{100} = 0.015$, $a_{100} = -0.05$ and $\sigma_{100}^2 = 0.1$. Use $t = 100$ as the forecast origin. What is the 1-step ahead forecast of r_t? What is the 1-step ahead prediction of the volatility?

Problem B. (20 pts) This problem is concerned with the analysis of daily average temperature at the O'Hard International Airport (ORD) from 1979 to 2001. The data for February 29 were deleted resulting in 8395 observations. Because of the strong seasonal pattern, we took the seasonal difference $(1 - B^{365})$ in the analysis. SCA output is attached. See page 2 of output.(Splus took forever to estimate the model. Output is not included.) Due to long seasonality, output is edited to simplify the attachment. Answer the following questions.

1. (2 points) For the temperature data, seasonal differencing is sufficient, i.e. no further first differencing is necessary, why?

2. From the fitted final model, a regular AR(2) factor is $(1 - 0.88B + 0.13B^2)$. Does this factor contain business cycle? Why?

3. (6 points) Write down the final fitted model for the daily temperature at ORD Airport.

4. Is there any serial correlation in the residuals of the fitted model? Use Q(12) and lag-365 ACF of the residuals to answer the question.

5. Is there any evidence of ARCH effects in the residuals of the fitted final model? Why?

Problem C. (20 pts) This problem is a continuation of the analysis of HW#2 concerning monthly simple returns of Decile 1 from 1960 to 2003. It turns out that the seasonal effect shown in ACF is due to the *January effect*. S-plus output is attached. See page 5. Answer the following questions.

1. (4 points) Write down the linear regression model for the decile returns with January indictor as the explanaroty variable. Is the model adequate? Why?

2. (4 points) Consider the ACF of the residuals of the fitted regression model. Is there significant evidence of seasonal effect? Briefly justify your answer.

3. Adding an AR(1) term to the linear regression. Is the AR(1) coefficient estimate significant at the 5% level? Do the residuals of the modified regression show any evidence of ARCH effects? Why?

4. (6 points) Write down the fitted mean equation and volatility equation of the joint model with generalized error distribution.

5. Is the fitted model adequate? Briefly justify your answer.

Problem D. (30 pts) This problem is concerned with daily log returns of Yahoo stock from January 1997 to December 2003 with 1761 observations. Splus output is attached. See page 7 of output. Answer the following questions.

1. (4 points) Is there any ARCH effect in the log return series? Why?

2. Write down the fitted GARCH(1,1) model with conditional Gaussian distribution (both mean and volatility equations).

3. (4 points) Is the fitted GARCH(1,1) model adequate? Why?

4. (6 points) Write down the fitted EGARCH(1,1) model with leverage effect (both mean and volatility equations and the parameter of the conditional distribution used).

5. Is the leverage effect statistically significant? Why?

6. (6 points) To better understand the leverage effect, use the fitted EGARCH(1,1) model to calculate the ratio $\frac{\sigma_t^2(\epsilon=-2)}{\sigma_t^2(\epsilon=2)}$, where ϵ_t denotes the *iid* sequence of the innovations defined in class.