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Graduate School of Business
Business 41202, Spring Quarter 2006, Mr. Ruey S. Tsay

Solutions to Homework Assignment #2

I used R to perform the analysis. You may use any packages of your choice.

1. Answer to Problem 1.

   (a) The first 10 lags of ACF and PACF are

```
> da=read.table("m-dec10.txt")
> d10=da[,2]
> s1=acf(d10,lag.max=10)
> s1
Autocorrelations of series 'd10', by lag
  1      2     3      4     5      6      7      8      9     10
0.020 -0.040 0.013 -0.012 0.084 -0.018 -0.018 -0.027 0.011 0.027
> p1=pacf(d10,lag.max=10)
> p1
Partial autocorrelations of series 'd10', by lag
   1      2     3      4     5      6      7      8     9    10
0.020 -0.040 0.015 -0.014 0.086 -0.023 -0.009 -0.031 0.014 0.016
```

   (b) The Ljung-Box test is used. Q(10) = 6.43 with p-value 0.78. Thus, the null hypothesis is not rejected.

```
> Box.test(d10,10,type='Ljung')
        Box-Ljung test

data:  d10
X-squared = 6.4261, df = 10, p-value = 0.7783
```

2. Answer to Problem 2.

   (a) The ACF is given below. The t-ratio of ρ_{12} is 5.99 with p-value ≈ 0. Thus, ρ_{12} is significantly different from zero.

```
> da=read.table("m-dec1.txt")
> d1=da[,2]
> s1=acf(d1,lag.max=12)
> s1
Autocorrelations of series 'd1', by lag
     0      1      2      3      4      5      6      7      8
 1.000  0.223  0.005 -0.025  0.006  0.011 -0.035 -0.040 -0.085
     9     10     11     12
```

```
 -0.060  0.007  0.095  0.255
> t=.255*sqrt(length(d1))   % Compute t-ratio
> t
[1] 5.991143
> p=2*(1-pnorm(t))  % Compute p-value of the t-ratio
> p
[1] 2.083707e-09
```

(b) The Ljung-Box statistic gives $Q(12) = 77.86$ with p-value close being zero. Reject the null hypothesis.

3. Answer to Problem 3.

   (a) The fitted model is

$$(1 - 0.313B - 0.134B^2 + 0.135B^3)y_t = a_t,$$

   where $y_t = x_t - 0.833$ and x_t is the growth rate of U.S. monthly real GNP and $\text{Var}(a_t) = 0.581$.

   (b) t-ratio of the lag-3 AR coefficient is $t = \frac{-0.1345}{.0646} = -2.08$. The p-value is 0.037 so that the null hypothsis is rejected at the 5% level.

   (c) For the residuals, Q(24) = 23.00 with p-value 0.52 so that the null hypothesis is not rejected.

```
> da=read.table("r-gnp05.txt")
> gnp=da[,4]
> gnp=log(gnp)
> x=diff(gnp)*100   % Compute growth rate series
> m1=arima(x,order=c(3,0,0))
> m1

Call:
arima(x = x, order = c(3, 0, 0))

Coefficients:
         ar1     ar2      ar3  intercept
      0.3132  0.1341  -0.1345     0.8325
s.e.  0.0646  0.0672   0.0646     0.0875

sigma^2 estimated as 0.8513:  log likelihood = -314.63,  aic = 639.26
> tra=-.1345/.0646  % Compute t-ratio of lag 3 coefficient
> tra
[1] -2.082043
> p=2*(1-pnorm(abs(tra))) % Compute p-value
> p
[1] 0.03733851
```

```
> Box.test(m1$residuals,24,type='Ljung')

        Box-Ljung test

data:  m1$residuals
X-squared = 23.0023, df = 24, p-value = 0.5197
```

4. Answer to Problem 4.

   (a) Yes, the fitted AR(3) model contains business cycles with average period 9.31.

   (b) The forecasts and their standard errors are given below.

```
> p1=c(1,-m1$coef[1:3])
> roots=polyroot(p1)
> roots
[1]1.502365+1.202812e+00i -2.007795+1.504327e-16i 1.502365-1.202812e+00i

> Mod(roots)
[1] 1.924541 2.007795 1.924541
> k=2*pi/acos(1.502365/1.924541)
> k
[1] 9.306846

> predict(m1,4)
$pred
Time Series:
Start = 236
End = 239
Frequency = 1
[1] 0.6686921 0.6605904 0.8443961 0.8352130

$se
Time Series:
Start = 236
End = 239
Frequency = 1
[1] 0.9226554 0.9668462 0.9902867 0.9904549

> ar(x)   % select AR(4) mdoel

Call:
ar(x = x)

Coefficients:
      1        2        3        4
 0.2982   0.1467  -0.0997  -0.1057
```

```
Order selected 4  sigma^2 estimated as  0.8614
> m2=arima(x,order=c(4,0,0)) % estimation of AR(4) model
> m2

Call:
arima(x = x, order = c(4, 0, 0))

Coefficients:
         ar1     ar2      ar3      ar4  intercept
      0.2988  0.1491  -0.1009  -0.1072     0.8329
s.e.  0.0648  0.0675   0.0674   0.0648     0.0788

sigma^2 estimated as 0.8413:  log likelihood = -313.27,  aic = 638.53
> tra=-.1072/.0648    % Compute t-ratio and p-value of lag 4.
> pv=2*(1-pnorm(abs(tra)))
> pv
[1] 0.09806231
```

(c) AIC specifies an AR(4) model for the data. The fitted model is

$$(1 - 0.30B - 0.149B^2 + 0.101B^3 + 0.107B^4)y_t = 0.833 + a_t,$$

where $y_t = x_t - 0.833$ and x_t is the growth rate of U.S. real GNP and $\mathrm{Var}(a_t) = 0.8413$. The t-ratios of AR parameters at lags 3 and 4 are less than 2 in modulus. Thus, these two parameters are not significant at the 5% significance level.

5. Answer to Prolem 5.

(a) Fit the model

$$(1 - 0.261B^{12})x_t = (1 + .237B)a_t,$$

where $x_t = R_t - 0.0174$ with R_t being the return series and $\mathrm{var}(a_t) = 0.0054$.

(b) The 1-step to 12-step ahead forecasts and standard errors are

```
Prediction:
Start = 553
End = 564
 [1]  0.00102  0.01447  0.00957 -0.00075  0.02815
 [6]  0.00668  0.02976  0.00988  0.01953  0.00357
[11]  0.01932  0.01503

Standard error:
Start = 553
End = 564
 [1] 0.07342 0.07546 0.07546 0.07546 0.07546 0.07546
 [7] 0.07546 0.07546 0.07546 0.07546 0.07546 0.07546
```