**THE UNIVERSITY OF CHICAGO**
**Graduate School of Business**
Business 41202, Spring Quarter 2006, Mr. Ruey S. Tsay

## Solutions to Homework Assignment #4

1. The file "d-bavw9605.txt" contains the daily simple returns of Boeing stock (BA) and CRSP value-weighted index (VW) from 1996 to 2005. The file has three columns containing date, BA return, VW return. The returns include dividends. Convert the simple returns into log returns.

   - Is there any serial correlation in the log returns of BA stock?

     A: No, the Ljung-Box statistic gives Q(10) = 14.33 with p-value 0.16.

   - Is there any ARCH effect in the BA stock log returns?

     A: Yes, the Ljung-Box statistic of $r_t^2$ gives Q(10) = 144.9 with p-value close to zero.

   - Specify a GARCH model for the BA log return using Gaussian distribution for the innovations. Perform model checking and write down the fitted model.

     A: The fitted model is

     $$r_t = .0009 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1),$$

     $$\sigma_t^2 = .05 \times^{-4} + .061 a_{t-1}^2 + .93 \sigma_{t-1}^2.$$

     The Q(10) statistics for the standardized residuals and their squared series are 7.45(.68) and 14.29(.07), respectively, where the number in parantheses denotes p-value. The model is adequate at the 5% level, except for the normality assumption.

   - Find an adequate GARCH model for the log series but using the Student-t distribution for the innovations. Write down the fitted model.

     A: The fitted model is

     $$r_t = 0.0006 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{6.09},$$

     $$\sigma_t^2 = 0.07 \times 10^{-4} + .055 a_{t-1}^2 + .929 \sigma_{t-1}^2.$$

     The model is adequate because the Q(10) statistics of the standardized residuals and their squared series are 7.74(.65) and 14.77(.06), respectively.

2. Consider the daily log returns of VW index in Problem 1.

- Is there any serial correlation in the log returns of VW index?

  A: No, the Ljung-Box statistics give Q(10) = 12.18 with p-value 0.27

- Is there any ARCH effect in the log return series of VW index?

  A: Yes, the Ljung-Box statistics of squared returns show Q(10) = 605.84 with p-value close to zero.

- Build an adequate GARCH model for the log return series of the index using Student-t distribution for the innovations.

  A: The fitted model is

$$r_t = .00077 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{9.88},$$

$$\sigma_t^2 = .0135 \times 10^{-4} + .080 a_{t-1}^2 + .911 \sigma_{t-1}^2.$$

  The model checking statistics show that the model is reasonable. Note that if GARCH(1,2) model is fitted in R, the ARCH(2) estimate is insignificant, confirming that GARCH(1,1) is sufficient.

- Compute 1- to 4-step ahead forecasts for the daily log return and its volatility based on the fitted model.

  A: The forecasts of the return are 0.00077 for all four steps, and the forecasts of volatility are .0060, .0060, .0059, and .0059, respectively, for the 1-step to 4-step ahead.

3. Again, consider the daily log returns of BA stock in Problem 1.

- Find an adequate GARCH-M model for the series with Student-t distribution. Write down the fitted model.

  A: The fitted model is

$$r_t = 0.0009 - 0.713 \sigma_t^2 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{6.08},$$

$$\sigma_t^2 = 0.07 \times 10^{-4} + .056 a_{t-1}^2 + .928 \sigma_{t-1}^2.$$

  The model is adequate because the Q(10) statistics of the standardized residuals and their squared series are 7.68(.66) and 14.62(.07), respectively. The arch.in.mean parameter is insignificant, however.

- Find an adequate EGARCH model for the series. Is the "leverage" effect significant at the 5% level?

  A:The fitted model is

$$r_t = 0.0005 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{6.08},$$

$$\sigma_t^2 = \frac{(1 - 0.31B) g(\epsilon_{t-1})}{1 - 0.859B},$$

$$g(\epsilon_t) = -0.056\epsilon_t + 0.24(|\epsilon_t| - E(|\epsilon_t|)).$$

The model is adequate because the Q(10) statistics of the standardized residuals and their squared series are 10.14(.43) and 5.98(.65), respectively. Yes, the leverage parameter is significant.

4. Consider the monthly simple returns of Alcoa (AA) stock and the S&P 500 index (SP) from 1960 to 2005. The data are in the file "m-aasp6005.txt" with format (date, AA return, SP return) and the sample size is 552. Transform the simple returns into log returns.

- Is there any serial correlation in the monthly log returns of AA stock?

  A: No, the Ljung-Box statistics give Q(10) = 13.61 with p-value 0.19.

- Is there any ARCH effect in the monthly log returns of AA stock?

  A: Yes, the Ljung-Box statistics of the squared return give Q(10) = 35.30 with p-value 0.0001.

- Fit a GARCH(1,1) model to the monthly log returns of AA stock using generalized error distribution for the innovations. Write down the fitted model.

  A: The fitted model is

$$r_t = 0.0057 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{ged}(1.74).$$

$$\sigma_t^2 = 0.0003 + 0.045 a_{t-1}^2 + 0.912 \sigma_{t-1}^2,$$

  where the constant term 0.0003 is insignificant at the 5% level.

- Calculate 1-step to 5-step ahead forecasts based on the fitted GARCH(1,1) model.

  A: The forecasts of return are 0.0057 for all steps, and the 1-step to 5-step ahead forecasts of volatility are 0.08, 0.08, 0.081, 0.081, and 0.081, respectively.

5. Consider the monthly log returns of S&P 500 index from 1960 to 2005 shown in Problem 4.

- Is there any serial correlation in the series?

  A: No, the Ljung-Box statistics show Q(10) = 8.12 with p-value 0.62.

- Is there any ARCH effect in the series?

  A: Yes, the Ljung-Box statistics of squared returns give Q(10) = 21.41 with p-value 0.02.

- Fit a GJR(1,1) model to the SP log return series with generalized error distribution and write down the fitted model.

  A: The fitted model is

$$r_t = .0069 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{ged}(1.43),$$

$$\sigma_t = 2.022 \times 10^{-4} + (0.0085 + 0.1525 N_{t-1}) a_{t-1}^2 + 0.796 \sigma_{t-1}^2,$$

where $N_{t-1}$ is a indicator such that $N_{t-1} = 1$ if $a_{t-1} < 0$ and $= 0$, otherwise. MOdel checking statistics indicate the model is adequate.

- Is the leverage effect significant at the 5% level? If not, is it significant at the 10% level?

  A: Yes, the level effect parameter is 0.152 with p-value 0.061 so that it is significant at the 10% level.