THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41202, Spring Quarter 2006, Mr. Ruey S. Tsay

Solutions to Homework Assignment #5

1. There are two stock splits in the data file. The first one was 2 for 1 on May 12, 1997 and the second one was 3 for 1 on May 8, 2000. The prior closing price of these two days needs adjustment in computing the volatility.

   Answer: I used R to answer this question. The code used is on the web. The summary statistics of the estimates are

Estimator	Mean	Median	Mini	Maxi
σ_1	1.133	.715	0	16.068
σ_2	.889	.688	.072	8.306
σ_5	.891	.704	.081	7.811
σ_6	1.618	1.288	.146	14.114

2. The estimated volatility series is shown in Figure 1. The mean, median, maximum,and minimum are .287, .264, .131, .607, respectively. Note that the volatility is annualized, i.e. daily volatility times $\sqrt{252}$.

3. S-Plus: If the GJR type of TGARCH models with Gaussian innovations is used, we obtain

$$\begin{aligned} r_t &= 0.070 - 0.037 r_{t-2} - 0.062 r_{t-3} + a_t, \quad a_t = \sigma_t \epsilon_t \\ \sigma_t^2 &= 0.084 \times 10^{-4} + 0.025 a_{t-1}^2 + 0.071 a_{t-1}^2 I(a_{t-1} < 0) + 0.907 \sigma_{t-1}^2 \end{aligned}$$

   where ϵ_t are iid N(0,1) and $I(a_{t-1} < 0)$ is the indicator for $a_{t-1} < 0$. All estimates are significant at the 5% level. This model appears to be adequate with $Q(12) = 12.32(0.42)$ and $Q^*(12) = 9.67(0.64)$ for the standardized residual series and its squared series, respectively.

   If R is used, the fitted model is

$$\begin{aligned} r_t &= 0.0007 + 0.021 r_{t-1} - 0.037 r_{t-2} - 0.062 r_{t-3} + a_t, \quad a_t = \sigma_t \epsilon_t \\ \sigma_t^2 &= 0.086 \times 10^{-4} + 0.024 a_{t-1}^2 + 0.075 a_{t-1}^2 I(a_{t-1} < 0) + 0.906 \sigma_{t-1}^2, \end{aligned}$$

   where all estimates except the AR-1 coefficient are significant at the 5% level. The model is also adequate with Q(10) = 6.31 (0.50) and Q(10) = 9.67(0.29) for the standardized residual series and its squared series, respectively. The leverage effect is significant at the 1% level.

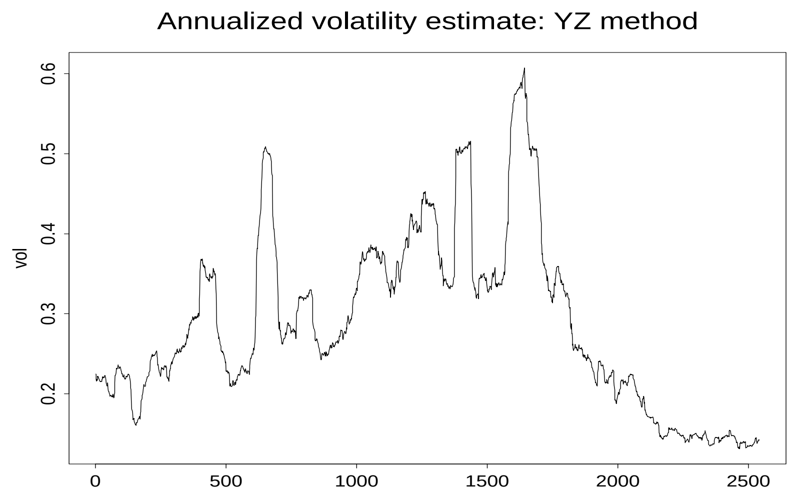

Figure 1: Estimated Volatility for GE stock using Yang and Zhang method

4. Consider the monthly simple returns of GE stock from 1926 to 2003. Use the last three years of data for forecasting evaluation.

   (a) Use lagged returns $r_{t-1}, r_{t-2}, r_{t-3}$ as input, build a 3-2-1 feed-forward network to forecast 1-step ahead returns. Calculate the mean squared error of forecasts.

   We are using 900 observations to estimate the neural network. The final 36 will be used for prediction.

   Let h_j be the jth node in the hidden layer. We have that

$$h_j = f_j\left(\alpha_{0,j} + \sum_{i \to j} w_{i,j} x_i\right),$$

   where $f_j(z)$ is a linear function; and x_i is the value of the ith input node; for $j = 1, 2$, and $i = 1, 2, 3$.

   Then, a 3-2-1 *skip-layer* feed-forward network is

$$o = f_o\left(\alpha_{0,o} + \sum_{i \to o} \alpha_{i,o} x_i + \sum_{j \to o} w_{j,o} h_j\right) \tag{1}$$

   for $j = 1, 2, 3$, and $i = 1, \ldots, 6$; where $f_o(z)$ is also linear.

   The estimates for this model given by Splus are

```
a 3-2-1 network with 14 weights
options were - skip-layer connections  linear output units
 b->h1 i1->h1 i2->h1 i3->h1
```

```
 0.28 -1.02  -2.33  -1.12
b->h2 i1->h2 i2->h2 i3->h2
 0.32 -4.72  -5.46   2.11
 b->o h1->o h2->o i1->o i2->o i3->o
 5.17 -9.93  0.86 -1.52 -4.57 -3.18
```

where $b \to h_j$ represents the constant $\alpha_{0,j}$, for $j = 1, 2$; $i_k \to h_j$ represents the weight $w_{k,j}$, for $j = 1, 2$, and $k = 1, 2, 3$. Similarly, $b \to o$ is $\alpha_{0,o}$; $h_j \to o$ is $w_{j,o}$, for $j = 1, 2$; and $i_k \to o$ corresponds to $\alpha_{k,o}$, for $k = 1, 2, 3$.

Let R_t^e represent the monthly simple return of GE stock. The mean squared error of the forecasts, using the final 36 observations is given by

$$\text{MSE} = \frac{1}{35} \sum_{\ell=1}^{36} (R_{N+\ell}^e - \hat{R}_N^e(\ell))^2 = 0.00635.$$

(b) Again, use lagged returns $r_{t-1}, r_{t-2}, r_{t-3}$ and their signs (directions) to build a 6-5-1 feed-forward network to forecast the 1-step ahead direction of GE stock price movement with 1 denoting up-ward movement. Calculate the mean squared error of forecasts.

Note: Let `rtn` denote a time series in Splus. To create a direction variable for `rtn`, use the command below:

```
drtn = ifelse(rtn > 0, 1, 0)
```

In this case, a neural network without skip-layer seems to give a better fit. Therefore we have that the output for this kind of feed-forward network is

$$o = f_o\left(\alpha_{0,o} + \sum_{j \to o} w_{j,o} h_j\right) \tag{2}$$

for $j = 1, \ldots, 5$, and $i = 1, \ldots, 6$; where $f_o(z) = \exp(z)/(1 + \exp(z))$.

The estimates for this model given by Splus are

```
a 6-5-1 network with 41 weights
options were -
   b->h1  i1->h1  i2->h1  i3->h1  i4->h1  i5->h1  i6->h1
  -50.66  -12.09 -198.81  -48.95   15.39   86.42  -68.71
   b->h2  i1->h2  i2->h2  i3->h2  i4->h2  i5->h2  i6->h2
  -16.24  101.84  445.37  116.86  -14.77 -108.62   25.37
   b->h3  i1->h3  i2->h3  i3->h3  i4->h3  i5->h3  i6->h3
  -64.01  236.62 -147.73  130.79    8.15  -21.25    4.68
   b->h4  i1->h4  i2->h4  i3->h4  i4->h4  i5->h4  i6->h4
   13.59  566.67  195.33  130.60 -117.73  -34.24  -28.24
   b->h5  i1->h5  i2->h5  i3->h5  i4->h5  i5->h5  i6->h5
 -113.79 -368.43   54.84   41.32   37.03  121.34  -75.36
    b->o   h1->o   h2->o   h3->o   h4->o   h5->o
    0.08   91.34    1.00  -66.13   79.03  -91.24
```

where $b \to h_j$ represents the constant $\alpha_{0,j}$, for $j = 1, \ldots, 5$; $i_k \to h_j$ represents the weight $w_{k,j}$, for $j = 1, \ldots, 5$, and $k = 1, \ldots, 6$. Similarly, $b \to o$ is $\alpha_{0,o}$; $h_j \to o$ is $w_{j,o}$, for $j = 1, \ldots, 3$; and $i_k \to o$ corresponds to $\alpha_{k,o}$, for $k = 1, \ldots, 6$.

Note that the output o represents a probability of the direction of the GE return going up. Hence, even when the "true" response variable is either 0 or 1 (go up or down), the fitted values give the likelihood of such a movement to happen. Therefore, we must set a criterion to decide whether the answer means "it goes up" or "it goes down".

Let $\hat{d}$ be the forecast direction of the return. A naïve criterion is simply to set $\hat{d} = 1$ if $o \geq 0.5$, and zero otherwise.

The mean squared error of the forecasts is

$$\text{MSE} = \frac{1}{35} \sum_{\ell=1}^{N} (d_{N+\ell} - \hat{d}_N(\ell))^2 = 11.514$$

where d is the true direction.

5. There are many ways to construct intraday 5-minute returns. This problem uses the average transaction price within each 5-minute bin to compute the returns. Alternatively, one can use the last transaction price within each 5-minute bin to compute the returns. The properties of the returns will depend on how the returns are constructed.

   Use the average transaction price of 5-minute bin: (a) Yes, the series is serially correlated with $\rho_1 = 0.30$ and $Q(10) = 172$, which is highly significant. (b) Based on the result of Chapter 3, we compute the daily volatility in two ways. First, we simply use the sum of squares of intraday 5-minute log returns. This is equivalent to assuming the returns are independent (an assumption which is clearly violated here). Second, we assume the 5-minute log returns have a significant lag-1 serial correlation (an assumption which appears to be more reasonable here). Figure 2 shows the two volatility series. The series in part (b) of the figure is preferred.

   Use the last transaction price within each 5-minute bin. In this case, the 5-minute intraday log returns do not have significant serial correlation. Specifically, we have $Q(10)$ = 12.4 with p-value 0.26.

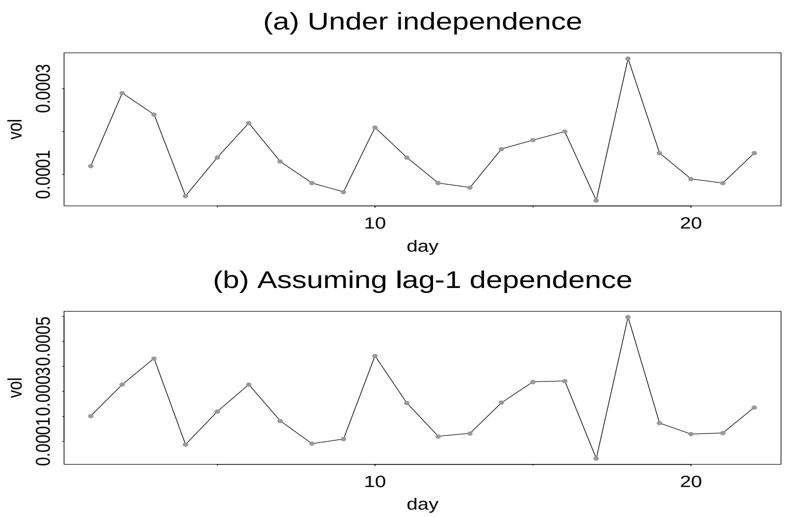

Figure 2: Daily volatility series of 3M stock in December 1999 based on intraday 5-minute log returns. (a) Assume the 5-minute returns are independent. (b) Assume the 5-minute returns have lag-1 serial correlation.