

Graduate School of Business, University of Chicago
Business 41202, Spring Quarter 2006, Mr. Ruey S. Tsay

Midterm

GSB Honor Code:

I pledge my honor that I have not violated the Honor Code during this examination.

Signature:

Name:

ID:

Notes:

- Open notes and books.
- Write your answer in the blank space provided for each question.
- **Manage** your time carefully and answer as many questions as you can.
- The exam has 7 pages and the computer output has 10 pages (5 for R and 5 for S-Plus). Please **check** to make sure that you have all the pages.
- For simplicity, **ALL** tests use the 5% significance level.
- Round your answer to 2 or 3 significant digits.

Circle the output used in your answer: (a) R, (b) S-Plus.

Problem A: (30 pts) Answer briefly the following questions.

1. Suppose that your bank pays interest rate 6% per annum on a quarterly basis, and your goal is to have \$30,000 at the end of 5 years. How much do you have to deposit now in order to achieve the goal?
2. **For questions 2 and 3**, consider the AR(2) model

$$(1 - B + 0.3B^2)r_t = 0.3 + a_t,$$

where $\{a_t\}$ is a sequence of independent standard normal random variables. Does the model imply existence of business cycles? If yes, what is the average period of the cycles?

3. Suppose that the last two data points are $r_{100} = 1.2$ and $r_{99} = -0.6$. What is the 1-step ahead forecast $r_{100}(1)$? What is the 100-step ahead forecast $r_{100}(100)$?
4. **For questions 4 to 8**, consider the monthly log returns, including dividends, of Johnson and Johnson stock from January 1961 to December 2005 for 540 observations. Summary statistics of the log returns from R and S-Plus are attached. Is the mean of log returns significantly different from zero? Why?
5. Suppose one invested \$1 dollar on Johnson and Johnson stock at the very beginning of 1961 and held the investment until the end of 2005. What was the value of the investment at the market closing on December 30, 2005 (the last trading day)?
6. Test the null hypothesis that the log returns is normally distributed. Draw your conclusion. You may use the Jarque-Bera statistic.
7. Given that the observed simple return of Johnson and Johnson stock on December 30, 2005 was -2.672% , what is the corresponding log return?
8. What is the null hypothesis of the Q(10) statistic of the log return series? What is the conclusion based on the Q(10) statistic?
9. Consider a GARCH(1,1) model

$$\sigma_t^2 = 0.15 + 0.07a_{t-1}^2 + 0.78\sigma_{t-1}^2$$

for $a_t = \sigma_t \epsilon_t$, where $\{\epsilon_t\}$ is a sequence of independent standard normal random variables. What is the unconditional variance of a_t ?

10. Give two possible reasons that introduce serial correlations in some daily U.S. stock or index returns.

11. Consider an IGARCH model for $a_t = \sigma_t \epsilon_t$,

$$\sigma_t^2 = 0.145a_{t-1}^2 + .855\sigma_{t-1}^2.$$

Suppose that $a_{100} = 0.02$ and $\sigma_{100} = 0.2$. What is the 1-step ahead volatility forecast $\sigma_{100}(1)$? What is the 10-step ahead volatility forecast $\sigma_{100}(10)$? [Recall that volatility is the conditional standard deviation.]

12. Consider the simple moving average model of order 2, i.e., MA(2),

$$r_t = 0.015 + a_t - 0.3a_{t-2},$$

where $\{a_t\}$ is a sequence of independent and identically distributed normal random variables with mean zero and variance 2. What is the mean of r_t [i.e., $E(r_t)$]? What is the lag-1 ACF of r_t ?

13. Standardized residuals, denoted by $\hat{\epsilon}_t$, are often used to check a fitted GARCH model. What are the purposes of checking the Ljung-Box statistics $Q(10)$ of $\hat{\epsilon}_t$ and $\hat{\epsilon}_t^2$?

14. Describe two advantages of EGARCH models over GARCH models.

15. Let ρ_k be the lag- k ACF of a monthly time series r_t . Describe a method to test the null hypothesis $H_o : \rho_{12} = 0$ versus the alternative hypothesis $\rho_{12} \neq 0$. Your method should include a decision rule for making inference.

Problem B. (20 pts) Power companies must supply sufficient natural gas to heat all of their customers' homes and businesses, particularly during the coldest days in winter. To forecast the sendouts of natural gas, data on some factors were collected for a major northern city of the U.S. in one winter. The dependent variable Y is the sendout and the explanatory variables are

1. X_1 : degree of heating days (= 65 degree - daily average temperature)
2. X_2 : lag-1 value of X_1 , i.e., $X_{2,i} = X_{1,i-1}$.
3. X_3 : wind speed, a 24 hour average,
4. X_4 : indicator variable for weekends. [$X_4 = 1$ for weekends and = 0, otherwise.]

There are 63 observations, representing 63 consecutive days. R and S-Plus outputs are attached. Use the output to answer the following questions.

1. (4 points) Write down the fitted multiple linear regression model,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + e.$$

2. (4 points) What are the residual variance and R^2 of the fitted multiple linear regression?
3. (4 points) Is the fitted multiple linear regression model adequate? Why? [Hint: use the model checking statistics provided.]
4. (6 points) A seasonal time series model is specified for the residuals with period 7, indicating the weekly cycle of the demand of natural gas. Write down the fitted model.
5. (2 points) The fitted regression model with time series errors is adequate. Why is the coefficient of Weekend indicator (X_4) negative? It suffices to provide a simple explanation.

Problem C. (30 pts) Consider the daily log returns of Qualcomm stock from January 1992 to December 2005 with 3530 observations. R and S-Plus outputs are attached. Because ACFs of the returns are small, the mean equation consists of a constant term only. Answer the following questions. **Note that a model consists of both mean and volatility equations.**

1. (4 points) A GARCH(1,1) model with Student t distribution is fitted to the return series. Write down the fitted model.
2. (8 points) A GJR (or TGARCH) model is also fitted to the return series. Write down the fitted model. Is the model adequate? Why?
3. (4 points) Based on the fitted GJR or TGARCH model, test the null hypothesis that the leverage effect is zero, i.e., $H_o : \gamma = 0$ versus $H_a : \gamma \neq 0$. Draw your conclusion.
4. (5 points) To better understand the leverage effect, use the fitted GJR or TGARCH model to calculate the ratio $\frac{\sigma_t^2(\epsilon_{t-1}=-3)}{\sigma_t^2(\epsilon_{t-1}=3)}$, where $\{\epsilon_t\}$ denotes a sequence of innovations that are independent and identically distributed. For simplicity, you may ignore the constant term of the volatility equation.
5. (4 points) Based on the fitted GJR or TGARCH model, what are the 1-step ahead forecasts of the log return and its volatility at the forecast origin $T = 3530$, the last data point?

Problem D. (20 pts) Consider the quarterly earnings per share of the Carterpillar (CAT) Company from December 1991 to September of 2005 for 56 data points. The data were obtained from First Call. R and S-Plus outputs are attached. Let x_t be the quarterly earnings per share and $w_t = (1 - B)(1 - B^4)x_t$.

1. (4 points) Write down the sample ACFs of w_t at lag 1, 3, 4, and 5.
2. (5 points) Write down the fitted model for x_t , including the variance of the residuals.
3. (4 points) Is there any significant serial correlation in the residuals of the fitted model? Why? [Hint: use the checking statistics provided in the output.]
4. (4 points) Let $T = 56$ be the forecast origin. Based on the fitted model, what are the 1-step and 2-step ahead forecasts of earnings per share for CAT stock?
5. (3 points) Obtain a 95% interval forecast for x_{57} at the forecast origin $T = 56$.