THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41202, Spring Quarter 2007, Mr. Ruey S. Tsay

Solutions to Homework Assignment #3

The R-output I used is in a separate file. You may look it over to see the analysis I did.

1. Let y_t be the logarithm of the mortgage rate. The fitted model is

$$(1-B)y_t = (1+0.53B-0.05B^2-0.14B^3)a_t, \quad \sigma_a^2 = 0.00069.$$

This ARIMA(0,1,3) model fits the data well. Residual analysis fails to reject the model with Q(12) = 9.65. The associated p-value is 0.38 (after adjusting for degrees of freedom). The 1-step to 4-step ahead forecasts for y_t are 1.805, 1.806, 1.809, and 1.809, respectively.

2. Let x_t be the logarithm of the interest rate. Both series have many high ACFs so that first difference is applied. The final model specified is

$$(1-B)y_t = -0.0006+0.316(1-B)x_t+(1+0.325B-0.072B^2-0.117B^3)a_t, \quad \sigma_a^2 = 0.00039.$$

Model checking shows that the model is adequate with Q(12) = 7.99 for the residuals. The associated p-value is 0.43 (after adjusting for the degrees of freedom). The t-ratio of the $(1-B)x_t$ coefficient is .3161/.0189 = 16.72, which is highly significant.

3. The residuals of the simple linear regression shows a significant lag-1 ACF so that the model used in

$$r_t = 0.0077 + 0.1224\text{jan}_t + (1+0.25B)a_t, \quad \sigma_a^2 = 0.0040,$$

where r_t is the Decile 1 return. The residual checking fails to reject the fitted model with Q(24) = 24.34. The associated p-value is 0.33 after adjusting for the degrees of freedom.

4. The fitted model is

$$(1-B^{12})(r_t - 0.018) = (1+0.24B)(1-0.98B^{12})a_t, \quad \sigma_a^2 = 0.0041,$$

where, again, r_t denotes the Decile 1 return. This model is also adequate with Q(24) = 23.92 for the residuals. The p-value of the Q(24) statistic is 0.30 after adjusting for the degrees of freedom. Note that the seasonal AR and MA parts nearly cancel out. This phenomenon in ARIMA models suggests that the seasonal pattern is deterministic. For the Deceile 1, it indicates the seasonality is due to the January effect as shown in the prior problem.

5. Let x_t be the quarterly earning of AA stock. In the output, I entertained two models. Both are adequate for the series. I selected the final model below because the AIC of the selected model is smaller (or the log-likelihood function of the selected model is higher):

$$(1 - 0.72B)(1 - 0.70B^4)(x_t - 0.32) = a_t, \quad \sigma_a^2 = 0.010.$$

The Q(12) of residuals is 6.82, which is clearly insignificant. The forecasts of the earnings are 0.55, 0.74, 0.84, and 0.61, respectively for the next four quarters.