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Graduate School of Business
Business 41202, Spring Quarter 2007, Mr. Ruey S. Tsay

Solutions to Homework Assignment #4

R output is available, please see hw4s-07-opt.txt

1. The file "d-sbuxsp0106.txt" contains the daily simple returns of Starbucks stock (SBUX) and the S&P 500 composite index from 1996 to 2006. The file consists of date, SBUX return, and S&P returns in three columns. The returns include dividends. Convert the simple returns into **percentage** log returns.

   - Is there any serial correlation in the log returns of Starbucks stock?
     Yes, $Q(10, r_t) = 19.763$ with a p-value of 0.032. The first lag in particular is significant.
   - Is there any ARCH effect in the log returns of Starbucks stock?
     Yes. First we fit a simple AR(1) model to remove the serial correlation found above and save the residuals $\hat{a}_t$. $Q(10, \hat{a}_t^2) = 51.736$, with a p-value near zero.
   - Fit a GARCH(1,1) model for the percentage log return of Starbucks stock using Gaussian distribution for the innovations. Perform model checking and write down the fitted model.

$$\begin{aligned} r_t &= 0.123 - 0.075 r_{t-1} + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 0.015 + 0.019 a_{t-1}^2 + 0.978 \sigma_{t-1}^2 \end{aligned}$$

     From the output we find $Q(10, \hat{\epsilon}_t) = 4.52$, with a p-value of 0.87, and $Q(10, \hat{\epsilon}_t^2) = 3.66$, with a p-value of 0.89. However, the Normality assumption may not be valid since the Jarque-Bera statistics has a p-value near zero.
   - Fit the GARCH(1,1) model again using the Student-t distribution for the innovations. Write down the fitted model.

$$\begin{aligned} r_t &= 0.062 - 0.065 r_{t-1} + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{4.74}(0,1) \\ \sigma_t^2 &= 0.025 + 0.027 a_{t-1}^2 + 0.966 \sigma_{t-1}^2 \end{aligned}$$

2. Consider the daily percentage log returns of S&P 500 index in Problem 1.

   - Is there any serial correlation in the log returns of S&P index?
     No, $Q(10, r_t) = 12.19$ with a p-value of 0.27.
   - Is there any ARCH effect in the log return series of S&P index?
     Yes. $Q(10, r_t^2) = 844$, with a p-value near zero.

- Fit an IGARCH(1,1) model for the log return series of the index using Student-t distribution for the innovations.

$$\begin{aligned} r_t &= 0.039 + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{14.16}(0,1) \\ \sigma_t^2 &= 0.0028 + 0.063 a_{t-1}^2 + 0.937 \sigma_{t-1}^2 \end{aligned}$$

- Compute 1- to 4-step ahead forecasts for the daily percentage log return and its volatility based on the fitted model.
  Forecasts for the daily percentage log return are constant, 0.039, and for the volatility 0.500, 0.502, 0.505, 0.508.

3. Again, consider the daily percentage log returns of Starbucks stock in Problem 1.

- Fit a GARCH(1,1)-M model for the series with Student-t distribution. Write down the fitted model.

$$\begin{aligned} r_t &= 0.126 - 0.065 r_{t-1} - 0.019 \sigma_t^2 + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{4.75}(0,1) \\ \sigma_t^2 &= 0.026 + 0.028 a_{t-1}^2 + 0.966 \sigma_{t-1}^2 \end{aligned}$$

- Is the ARCH-in-mean parameter significant at the 5% level?
  No, its t-stat is -0.76 with a p-value of 0.45.
- Fit a GJR(1,1) model with Gaussian innovations to the log return series. Perform model checking and write down the fitted model.

$$\begin{aligned} r_t &= 0.108 - 0.086 r_{t-1} + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 0.028 + (0 + 0.04 N_{t-1}) a_{t-1}^2 + 0.974 \sigma_{t-1}^2, \end{aligned}$$

  in which $N_{t-1} = I(a_{t-1} < 0)$. From the output we find $Q(10, \hat{\epsilon}_t) = 4.42$, with a p-value of 0.88, and $Q(10, \hat{\epsilon}_t^2) = 3.81$, with a p-value of 0.87. Again, the Normality assumption may not be valid since the Jarque-Bera statistics has a p-value near zero.
- Is the "leverage" parameter significant?
  Yes, its t-stat is 3.76 with a p-value of 0.0002. The Arch parameter, however, is not significant.

4. The data file "m-pg5606.txt" contains the date and monthly simple returns of Procter & Gamble (PG) stock from 1956 to 2006. Transform the simple returns into percentage log returns.

- Is there any serial correlation in the monthly log returns of PG stock?
  No, $Q(10, r_t) = 9.55$ with a p-value of 0.48.

- Fit a GARCH(1,1) model to the monthly percentage log returns of PG stock using generalized error distribution for the innovations. Write down the fitted model.

$$\begin{aligned} r_t &= 1.104 + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim GED_{1.43}(0,1) \\ \sigma_t^2 &= 0.955 + 0.107 a_{t-1}^2 + 0.846 \sigma_{t-1}^2 \end{aligned}$$

- Use the fitted model to calculate 1-step to 5-step ahead forecasts for the percentage log returns series and its volatility.
  Forecasts for the daily percentage log return are constant, 1.104, and for the volatility 2.90, 3.00, 3.08, 3.17, 3.24.

5. The file "d-exuseu.txt" contains the daily exchange rate between U.S. Dollars and Euro from January 1999 to March 20, 2007. Compute the percentage log returns of the exchange rate.

   - Is there any serial correlation in the log return series?
     No, $Q(10, r_t) = 11.88$ with a p-value of 0.29.
   - Is there any ARCH effect in the log return series?
     Yes, $Q(10, r_t^2) = 28.77$ with a p-value of 0.0014.
   - Fit an IGARCH(1,1) model to the log return series using Gaussian innovations. Perform model checking and write down the fitted model. [Note: This is the model used in RiskMetrics to compute VaR.]

$$\begin{aligned} r_t &= 0.011 + a_t \\ a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= 0.016 a_{t-1}^2 + 0.984 \sigma_{t-1}^2 \end{aligned}$$

     From the output we find $Q(10, \hat{\epsilon}_t) = 9.61$, with a p-value of 0.48, and $Q(10, \hat{\epsilon}_t^2) = 11.83$, with a p-value of 0.16. However, $Q(20, \hat{\epsilon}_t^2) = 29.55$, with a p-value of 0.042.

   - Use the model to produce 1-step to 4-step ahead forecasts for the percentage log return series and its volatility.
     Forecasts for the daily percentage log return are constant, 0.011, and constant for the volatility 0.375.