THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41202, Spring Quarter 2007, Mr. Ruey S. Tsay

Solutions to Homework Assignment #6

1. Problem 6.2 of the textbook.

   Answer: Take partial derivatives, we have $\partial F_{t,T}/\partial P_t = e^{r(T-t)}$, $\partial F_{t,T}\partial t = -rP_t e^{r(T-t)}$, $\partial^2 F_{t,T}\partial P_t^2 = 0$. Using Ito's Lemma, we obtain

$$\begin{aligned} dF_{t,T} &= (e^{r(T-t)}\mu P_t - rP_t e^{r(T-t)})dt + e^{r(T-t)}\sigma P_t dw_t, \\ &= (\mu - r)F_{t,T}dt + \sigma F_{t,T}dw_t. \end{aligned}$$

2. Problem 6.10 of the textbook.

   Answer: The lower bound of the put price is

$$\$[47\exp(-0.06 * 0.5) - 44] = \$1.61,$$

   which is greater than the put price of \$1.0. An arbitrageur can borrow \$45 for six months to buy both the put option and the stock. At the end of six months, the arbitrageur repays the loan at \$45exp(0.06∗0.5) = \$46.37. If $P_T < \$47$, the arbitrageur exercises the option to sell the stock for \$47, repays the loan, and makes a profit of \$(47-46.37) = \$0.63. If $P_T > \$47$, the arbitrageur discards the option, sells the stock to repay the loan and makes an even greater profit.

3. Problem 3.

   - Calculate the VaR of your position for the next trading day using the RiskMetrics method, using $\alpha = 0.98$, $r_{2770} = 4.792$ and $\sigma_{2770} = 1.869$, where 2770 is the sample size. $\sigma^2_{2771} = .98(1.869)^2 + 0.02(4.792)^2 = 3.883$. The VaR for the log return $2.326 \times \sqrt{3.883} = 4.5832$. The VaR for the position is $\$1000000 \times 4.5832/100 = \$45832$.
   - Based the 1-step ahead forecast of the fitted GARCH(1,1) model, we have mean = 0.1615 and variance = 9.488. Therefore, 1% quantile of the predicted log return is $0.1615 - 2.326 \times \sqrt{9.488} = -7.0032$. The VaR for the position is $\$7.0032 \times 1000000/100 = \$70032$.
   - Based on the forecast of the fitted GARCH(1,1) model with Student-t innovations, we have mean = 0.0668 and variance 7.852. Also, the degrees of freedom is 4.75. The 1% quantile is $0.0668 - (3.44/\sqrt{4.75/2.75})\sqrt{7.852} = -7.268$. The VaR of the position is $\$1000000 \times 7.268/100 = \$72680$.

4. Problem 4. For the case of block size $n = 21$, the estimates are $k = -0.352$, $\sigma = 1.775$ and $\beta = 4.282$. The standard errors are 0.081, 0.154, and 0.178, respectively. The 1% VaR is $4.282 + \frac{1.775}{-0.352}[1 - (-21\ln(0.99))^{-0.352}] = 7.9584$. The VaR for the position is $\$1000000 \times 7.9584/100 = \$79584$. For the POT method with threshold 4%, the VaR is $\$1000000 \times 8.544/100 = \$85440$. The expected shortfall is $124,600. The estimated parameters are xi $= 0.2742$ and beta $= 1.5966$. Both estimates are significantly different from zero at the 5% level.

5. Problem 5. For the AIG stock the prediction of the volatility is $\sigma^2_{2771} = 0.95(0.636)^2 + 0.05(-0.306)^2 = 0.3890$. The 1% quantile is then $2.326 \times \sqrt{0.3890} = 1.451$. The VaR is then $\$500000 \times 1.451/100 = \$7255$. The VaR for the combined portfolio is

$$\sqrt{45832^2 + 7255^2 + 2 \times (0.152)(45832)(7255)} = \$47479.$$