

Solutions to Final Exam

Problem A: (34 pts) Answer briefly the following questions.

1. Consider the monthly simple returns of Decile 1 and Decile 10 portfolios from January 1988 to December 2007 for 240 observations. Due to January effects, Decile 1 has some serial correlations at seasonal lags. To remove the effect, an ARIMA model is used. Write down the fitted model for the Decile 1 portfolio returns, including the variance of the innovations.

Answer: The fitted model is $(1-0.999B^{12})(r_{1t}-0.0148) = (1+0.193B)(1-0.976B^{12})a_t$, $\hat{\sigma}_a^2 = 0.00297$.

2. Again, consider the monthly simple returns of Decile 1 of the prior question. Are there any serial correlations in the residuals? Are there any ARCH-effects in the residuals? Why?

Answer: No, there are no residual serial correlations because $Q(24) = 15.13$ with p-value 0.92 for the residuals. No, there are no ARCH effects, either, because $Q(24) = 10.48$ with p-value 0.99 for the squared residuals.

3. The residuals of the prior model for Decile 1 returns are combined with Decile 10 returns to form a bivariate time series. The multivariate Ljung-Box statistics are applied to detect any serial or cross-correlations between the two time series. Write down the null hypothesis for $Q_2(10)$ statistic, where 10 denotes the first 10 lags of serial and cross correlations. What is the asymptotic distribution of $Q_2(10)$? Is the test significant for the two series? Why?

Answer: Let ρ_i be the matrix of lag- i serial and cross-correlations. Then, $H_o : \rho_1 = \rho_2 = \dots = \rho_{10} = 0$. The asymptotic distribution of $Q_2(10)$ is chi-square with 40 degrees of freedom. No, the test is not significant at the 5% level, because it is $Q_2(10) = 32.42$ with p-value 0.797.

4. Describe conditions under which nonsynchronous trading leads to the existence of serial correlations at all lags in the returns of an asset, even though the returns are in fact serially uncorrelated.

Answer: (1) The returns are independent and identically distributed as $N(\mu, \sigma^2)$ with $\mu \neq 0$. (2) The probability of no trade in each time interval is π , which is independent of time.

5. In addition to GARCH-type models, give two alternative approaches to modeling daily stock volatilities.

Answer: Any two of the followings: (1) High-frequency financial data (intradaily log returns), (2) daily high, low, open and closing prices of the stock, (3) implied volatility from the options market.

6. The CBOE VIX index is used as a measure of market volatility. We can use time series models to forecast the VIX index. Write down the fitted model for the $\log(\text{VIX})$ series, including residual variance. Compute 1-step and 5-step ahead forecasts of the VIX index (not $\log(\text{VIX})$).

Answer: The fitted model is $(1 - B) \ln(\text{VIX}_t) = (1 - 0.161B - 0.093B^2)(1 - 0.105B^8)a_t$, with $\hat{\sigma}_a^2 = 0.0037$. The 1-step and 5-step ahead forecasts are $\exp(3.029)$ and $\exp(3.027)$, i.e. 20.68 and 20.64, respectively.

7. Suppose that the price P_t of a stock follows the stochastic diffusion model

$$dP_t = \mu P_t dt + \sigma P_t^{0.5} dw_t,$$

where μ and σ are constant and w_t is the standard Brownian motion. What is the stochastic model for the log price $G(P_t) = \ln(P_t)$?

Answer: $\frac{\partial G(P_t)}{\partial P_t} = \frac{1}{P_t}$, $\frac{\partial G(P_t)}{\partial t} = 0$, and $\frac{\partial^2 G(P_t)}{\partial P_t^2} = -\frac{1}{P_t^2}$. Using the Ito's lemma, we have

$$d \ln(P_t) = \left[\mu - \frac{\sigma^2}{2P_t} \right] dt + \sigma P_t^{-0.5} dw_t.$$

8. Consider a nondividend-paying stock. If the current price of the stock is \$45.50 and the risk-free interest rate is 5% per annum. What is the minimum price of a European call option on the stock with time-to-expiration 3 months and strike price \$43.00?

Answer: $c_t \geq P_t - K \exp[-r(T - t)] = 45.5 - 43 \exp[-0.25 \times 0.05] = \3.03 .

9. Describe a consequence in linear regression analysis if the serial correlations of the residuals are overlooked.

Answer: Any one of (1) inflated standard error or (2) biased estimate or (3) erroneous t -ratio statistics.

10. Describe briefly two methods that can be used to model the stock price changes in tick-by-tick transaction data.

Answer: (1) The ADS method and (2) the ordered probit model.

11. Suppose that the monthly log returns, in percentages, of an asset follow the model

$$r_t = 0.1 + 0.2r_{t-1} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1), \quad \sigma_t^2 = 0.95\sigma_{t-1}^2 + 0.05a_{t-1}^2.$$

Suppose that at the forecast origin T , $r_T = 1.0$, $r_{T-1} = -0.5$, and $\sigma_T^2 = 2.05$. What is the 2-step ahead forecast $r_T(2)$ of the return? What is the 2-step ahead volatility forecast $\sigma_T(2)$?

Answer: $r_{T+2} = 0.1 + 0.2r_{T+1} + a_{T+2} = 0.1 + 0.2[0.1 + 0.2r_T + a_{T+1}] + a_{T+2} = 0.12 + 0.04r_T + a_{T+2} + 0.2a_{T+1}$. Consequently, $r_T(2) = 0.12 + 0.04 \times 1 = 0.16$. From the model $a_T = r_t - 0.1 - 0.2r_{T-1} = 1.0 - 0.1 - 0.2(-0.5) = 1.0$. Since it is an IGARCH(1,1) model, we have $\sigma_T^2(2) = \sigma_T^2(1) = 0.95(2.05) + 0.05(1) = 1.998$ so that $\sigma_T(2) = 1.414$.

12. Give one advantage and one disadvantage of using the empirical quantiles to calculate Value at Risk (VaR) of a financial position.

Answer: Advantage: Simplicity or distribution free. Disadvantage: assuming that the future loss cannot exceed the past loss.

13. Consider the daily log returns, in percentages, of Caterpillar Stock from January 1993 to December 2007. The summary statistics are given. Is the distribution of the log returns symmetric against its mean? Perform a proper test to justify your answer.

Answer: The skewness test is $t = -0.1854/\sqrt{6/3778} = -4.65$, which is highly significant. Thus, the distribution is not symmetric with respect to the mean.

14. Again, consider the daily log returns of Caterpillar stock of the prior question. Is the mean of the returns significantly different from zero? Why?

Answer: Yes, because the 95% confidence interval is $[0.0085, 0.1342]$ which does not contain zero.

15. Realized volatility has been used as a measure of daily stock volatility. However, market microstructure may affect the quality of the measure. Give two approaches that can overcome the effects of microstructure noises on the realized volatility.

Answer: (1) subsampling method and (2) use the optimal sampling interval.

16. Consider two stocks. Let their log prices be p_{1t} and p_{2t} , respectively. Describe two conditions needed for implementing pairs trading of the two stocks to obtain a positive profit.

Answer: Any two of (1) both p_{1t} and p_{2t} are unit-root nonstationary, (2) a linear combination $w_t = p_{1t} - \gamma p_{2t}$ is unit-root stationary, (3) the range of w_t is sufficiently large.

17. Consider a univariate AR(3) model, say $x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + a_t$, where $\{a_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance $\sigma^2 > 0$. Write down an explicit error-correction form for the model to perform unit-root test. What is the null hypothesis?

Answer: $\Delta x_t = \gamma x_{t-1} + p_1^* \Delta x_{t-1} + p_2^* x_{t-2} + a_t$, where $\gamma = \phi_3 + \phi_2 + \phi_1 - 1$, $p_2^* = -\phi_3$ and $p_1^* = -\phi_2 - \phi_3$.

Problem B. (30 points) Consider the daily log returns, in percentages, of the Caterpillar and Adobe System Stocks from January 1993 to December 2007. The sample size is 3778 and the tick symbols are CAT and ADBE, respectively. Suppose that you hold a long position of the stocks valued at \$1 million dollars each. (Total is 2 millions.) Answer the following questions based on the attached output.

1. Focus on the CAT stock. A GARCH(1,1) model is fitted to the data. Write down the fitted model, including innovation distributions. Is the fitted model adequate? Why?

Answer: $r_t = 0.083 + a_t$, $a_t = \sigma_t \epsilon_t$ with $\epsilon_t \sim t_{5.90}^*$, a standardized t-distribution with 5.9 degrees of freedom. $\sigma_t^2 = 0.017 + 0.021a_{t-1}^2 + 0.974\sigma_{t-1}^2$. Yes, the Q-statistics of the standardized residuals and the squared standardized residuals all have high p-values.

2. Use the fitted GARCH(1,1) model, calculate the VaR of the CAT position for the next trading day.

Answer: The 1% quantile value of $t_{5.9}$ is -3.161 . From the fitted model, the 1-step ahead forecast of the return is 0.083 and the 1-step ahead variance forecast is $\sigma_{T+1}^2 = 0.017 + 0.021(-0.906)^2 + 0.974(2.846)^2 = 2.806$. Therefore, $(r_{T+1} - 0.083)/\sqrt{2.806}$ is a standardized t-distribution with 5.9 degrees of freedom. Thus, $\text{VaR} = 0.083 + (-3.161/\sqrt{5.9/3.9}) * \sqrt{2.806} = -4.222$, where the negative sign indicates loss. The VaR of the CAT position is $4.222/100 * 1000000 = \$42220$.

3. A special IGARCH(1,1) model is also fitted to the CAT returns. Use the information to calculate VaR of the CAT position via the RiskMetrics method. What is the corresponding VaR for the next 10 trading days?

Answer: 1-day VaR of the returns is $\text{VaR} = 2.33 \times \sqrt{0.012 \times 0.678^2 + 0.988 \times 2.572^2} = 3.718$. Therefore, $\text{VaR} = \$3.718/100 \times 1000000 = \31780 . The 10-day VaR $= \sqrt{10} \times 31780 = \117574.0 .

4. Apply the traditional extreme value theory to the negative log returns with block size 63. What are the estimates of the three parameters k , α , and β ? Are these estimates statistically significant? Why?

Answer: $k = -0.247$, $\alpha = 1.530$ and $\beta = 3.914$. All three estimates are significantly different from zero because their t -ratios are all greater than 2.0.

5. Based on the EVT estimates of block size 63, what is the VaR of the CAT position for the next trading day?

Answer: Use the evtVaR program, the $\text{VaR} = 4.654/100 \times 1000000 = \46540 .

6. Turn to the approach of peaks over the threshold. Using threshold 2.5, we estimate the parameters of generalized Pareto distribution for the negative CAT returns. Does the log returns have a heavy left tail? Why?

Answer: Yes, the returns have a heavy left tail because $k = -\xi = -0.154$, which is significant and negative.

7. Based on estimated generalized Pareto distribution, what is the VaR of the CAT position for the next trading day? What is the VaR of the position for the next 10 trading days?

Answer: 1-day VaR is $5.121/100 \times 1000000 = \51210 . 10-day VaR is $10^{0.154} \times 51210 = \73005 .

8. Again, based on the fitted generalized Pareto distribution with threshold 2.5, what is the expected shortfall when the 1% VaR is used?

Answer: The expected shortfall is $6.942/100 \times 1000000 = \69420 .

9. Turn next to the ADBE stock. Again, a special IGARCH(1,1) model is fitted. Based on the information available, what is the VaR of the ADBE position for the next trading day.

Answer: $\text{VaR} = 2.33\sqrt{0.023 \times 0.2181^2 + 0.977 \times 2.771} = \38367 .

10. The correlation between the CAT and ADBE stocks is 0.208. What is the VaR of your portfolio of the two stocks for the next trading day?

Answer: $\sqrt{37180^2 + 38367^2 + 2 * 0.208 * 37180 * 38367} = \58718 .

Problem C. (16 pts) Again, consider the daily log returns, in percentages, of the Adobe System stock from January 1993 to December 2007. Now, let us focus on predicting the direction of price movement. Define

$$Y_t = \begin{cases} 1 & \text{if } r_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

where r_t is the log return of ADBE stock at time index t .

1. A 5-2-1 neural network is employed for Y_t . The five input variables are the first five lagged values of the returns, i.e. $x_{1,t} = r_{t-1}, x_{2,t} = r_{t-2}, \dots$. Write down the fitted model for the first node (h_1) of the hidden layer.

Answer: $h_{1t} = \frac{\exp(2312.3 - 443r_{t-1} - 635r_{t-2} + 990r_{t-3} - 1152r_{t-4} - 272.5r_{t-5})}{1 + \exp(2312.3 - 443r_{t-1} - 635r_{t-2} + 990r_{t-3} - 1152r_{t-4} - 272.5r_{t-5})}$.

2. Write down the fitted model for the output node.

Answer: $z = -0.68 + 0.53h_1 + 0.47h_2 - 0.05r_{t-1} - 0.03r_{t-3} + 0.05r_{t-4} + 0.03r_{t-5}$ and $h(z) = 1$ if $z > 0$ and $h(z) = 0$ otherwise.

3. Alternatively, a logistic linear regression can be used to model Y_t using the five input variables. Write down the fitted logistic linear regression model. Are the estimated coefficients significant? Why?

Answer: $p(Y_t = 1) = \frac{\exp[-0.058 - 0.023r_{t-1} - 0.011r_{t-2} - 0.015r_{t-3} + 0.006r_{t-4} - 0.011r_{t-5}]}{1 + \exp[-0.058 - 0.023r_{t-1} - 0.011r_{t-2} - 0.015r_{t-3} + 0.006r_{t-4} - 0.011r_{t-5}]}$. Except for the coefficient of r_{t-1} , all coefficient estimates are insignificant because their t -ratios are low.

4. To simplify the model, we only keep the lag-1 variable (r_{t-1}) as the independent variable. Write down the fitted logistic linear regression model. If $r_T = -0.47$, what is the probability that the price of the stock would increase at the time index $T + 1$?

Answer: $P(Y_t = 1) = \frac{\exp[-0.061 - 0.023r_{t-1}]}{1 + \exp[-0.061 - 0.023r_{t-1}]}$. With $r_T = -0.47$, $P(Y_{T+1} = 1) = \frac{\exp[-0.061 - 0.023(-0.47)]}{1 + \exp[-0.061 - 0.023(-0.47)]} = 0.487$.

Problem D. (20 points) Consider the adjusted daily closing prices of BHP and RIO stocks from March 21, 2002 to May 30, 2008 as discussed in class. Let the log price be p_{1t} and p_{2t} , respectively. Based on the augmented Dickey-Fuller unitroot test, both series $\{p_{1t}\}$ and $\{p_{2t}\}$ have a unitroot.

1. Use p_{1t} as the dependent variable and p_{2t} as the independent variable. What is the linear combination that gives rise to a mean-reverting process?

Answer: $w_t = p_{1t} - 0.7181p_{2t} - 1.816$. (Note that -1.816 is not critical.)

2. Denote the mean-reverting linear combination by w_t . Unitroot test confirms that it is indeed stationary. The values of w_t for t from 330 to 340 are shown in the attached output. Consider the position: long one share of BHP stock and short γ shares of RIO stock. Let $\delta = 0.03$. Was there an trading opportunity within the time period (assume no problem with order execution)? If yes, what is the resulting profit?

Answer: Yes, there was an opportunity. “Buy 1 share of BHP and short 0.7181 shares of RIO” at 332 and unwind the position at 340. The profit is $0.0406 - (-0.0406) = 0.0812$.

3. Alternatively, use p_{2t} as the dependent variable and p_{1t} as the independent variable. What is the linear combination that gives rise to a mean-reverting process?

Answer: $w_{1t} = p_{2t} - 1.377p_{1t} + 2.478$. (Note, again, the constant 2.478 is not critical.)

4. The values of the linear combination of Question 3 (denoted by w_1 in the output) are given for t from 605 to 630. Based on the values, construct a portfolio and chosen threshold δ that will provide a maximum profit. What is the resulting profit?

Answer: Set $\delta = 0.065$. The portfolio is “Buy 1 share of RIO and short 1.377 shares of BHP.” The profit is $0.0803 - (-0.0695) = 0.1498$.

5. Compare the two linear combinations in Question 1 and 3. The R^2 measure of the two simple linear regressions is the same. Similarly, the two t -ratios for regression coefficients are the same. For the purpose of pairs trading, which linear combination do you prefer? Why?

Answer: The two linear combinations indeed are very close. Note that dividing w_{1t} by -1.377 gives $w_{1t}/(-1.377) = -0.726p_{2t} + p_{1t}$, which is close to w_t . Either one should be fine.